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# Modern computer simulation for the design of concrete catenary shell structures

Joo Hong Lee[1a], Hyerin Lee[*2] and Thomas H.-K. Kang[1b]

[1]*Department of Architecture & Architectural Engineering, Seoul National University, Seoul, Republic of Korea*
[2]*Department of Disaster Prevention Engineering, Chungbuk National University, Cheongju, Republic of Korea*

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**Abstract.** The purpose of this study was to model and design a concrete catenary shell using a modern computer program without performing experiments. The modeling idea stems from the study by Pendergrast, but he listed supplementary items that should be improved in his paper. This study aims to resolve those issues and overcome the drawbacks of the study by Pendergrast. The process of experiment for the design of a catenary shell was reproduced by Grasshopper script. In order to ensure credibility, two models designed from the Grasshopper script were analyzed using a finite element program, SAP2000; one is a square-based catenary shell and the other is a special catenary shell called as the Naturtheater Grötzingen shell, which was completed in 1977. First, the developed modeling approach was proved to be reasonable from the analysis of the square-based shell. The reliability was further confirmed by a comparison between the current and previous analysis results for the Naturtheater Grötzingen shell.

**Keywords:** computer programming; spatial structure; concrete catenary shell; algorithm; grasshopper; rhinoceros

## 1. Introduction

In order to create large space, spatial structures such as funicular structures with cables, membrane structures, and shell structures are often adopted (Bhattacharjya *et al.* 2015, Gu and Huang 2015, Xu *et al.* 2015, Gil Pérez *et al.* 2016, Gil Pérez *et al.* 2017, Cao *et al.* 2017, Labbafi *et al.* 2017a, Labbafi *et al.* 2017b). Among them, concrete shell structures have more gravity load than other structures, and mostly rely on compressive resistance within the shell due to the characteristics of concrete (Yang *et al.* 2014). Therefore, it is important to shape and design the concrete shell properly such that compressive forces are dominant within the entire concrete shell.

There are several types of concrete shell structures: catenary shell, hemisphere shell and cylinder shaped shell. Unlike hemisphere and cylinder shaped shells, the catenary shell has irregular shape and thus its design is more complicated. The shape of a catenary curve can be found by hanging a heavy uniform flexible cord (e.g., chain) or membrane freely. This is one of the most resistible shapes for gravity load, and the shell with this shape is called as a catenary shell.

The concrete catenary shell design has been carried out through the experiment of hanging fabric and the measurement of the curved shape during experiment. This study aims to simplify such a process of catenary shell design, which is expected to contribute to the digitalization of concrete shell design.

Due to the rapid progress of computational power, digitalization allows the application of concrete shell structures to a broader range of building design. In particular, it has a strong advantage in designing roofs, given that a shell structure can enlarge space utilization by reducing the number of columns. The digitalization of concrete shell design can accelerate a concrete catenary shell structure to be an attractive alternative for a less constrained and more economical roof design.

## 2. Previous studies

In the past shell design, the form-finding method developed by Isler (1961) has been popular. This form-finding method is based on measuring the shape of real hanging fabric mixed with mortar, etc. Isler (1961) designed many shell structures using the form-finding method, such as Deitingen Service Station in Switzerland (see Fig. 1(a)) and the Naturtheater Grötzingen in Germany (see Fig. 1(b)).

Isler’s method is one of the basic experimental and empirical methods, and is not compatible with mathematical or computer-based design. As such, it is not easy to digitalize this form-finding method and the experimental tools shown in Fig. 1 are necessary. In Fig. 2, the mechanical form-finding rigs are used to simulate uniformly distributed loads. Once the form is determined, the measurement jig is used for accurate measurement of each rig’s length. The Isler’s method had been widely used before the 21st century, though it is known as a

∗Corresponding author, Research Professor
E-mail: hyerinlee@chungbuk.ac.kr
[a]Master’s Student
E-mail: dkdkman@snu.ac.kr
[b]Associate Professor
E-mail: tkang@snu.ac.kr


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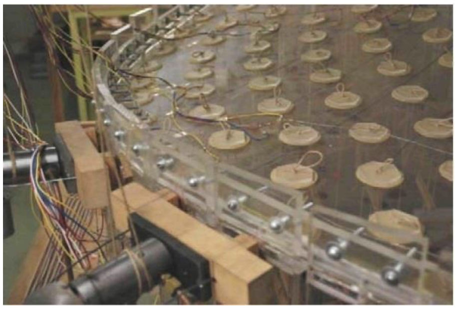
(a) Deitingen Service Station (Wikipedia 2017)

(b) Naturtheater Grötzingen (Maurer *et al.* 2013)

Fig. 1 Deitingen service station and Naturtheater Grötzingen

(a) Mechanical form-finding rig



(b) Loading and measurement rig

Fig. 2 Previous experimental form-finding method by Isler (Chilton 2011)

combination of complicated and inconvenient procedures.

To address the aforementioned issue and digitalize the design process, Pendergrast (2010) developed a design tool replacing Isler's form-finding method by computer simulation. The tool, however, is the stand-alone program that has less compatibility with other computer-aided design programs or finite element packages. The design tool of Pendergrast (2010) is difficult to be applied to practical design processes despite of its significance. As well as Pendergrast (2010) acknowledged the limitation of applicability, he suggested alternatives and other possibilities. One of them is the use of Grasshopper for simulation. The Grasshopper is a graphical algorithm editor

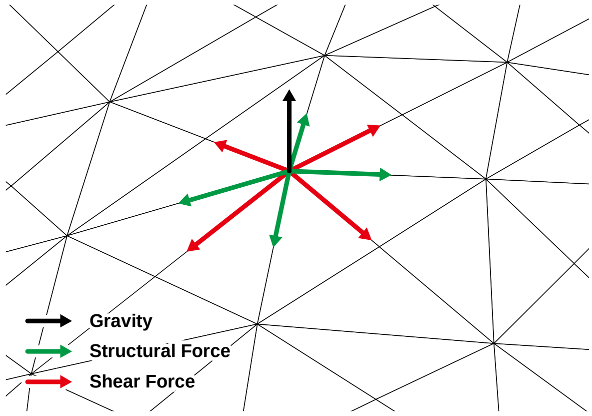



Fig. 3 Annotation on fraction of wireframe

integrated with 3D modeling packages in Rhino program (Davidson 2015).

In this study, an innovative shell design program is developed using the Grasshopper and concrete catenary shells are designed correspondingly. In addition, the validity is checked by performing finite element analysis for the designed catenary shells and by comparing the analysis results with the previous information.

## 3. Computer simulation using grasshopper

Pendergrast (2010) used braced grid wireframes for simulating the behavior of fabric on computer. According to the algorithm of Pendergrast (2010), the gravity force and secondary reaction forces are applied to nodes as shown in Fig. 3.

Each element of wireframes should work as a spring to reflect the behavior of fabric, but the Grasshopper has no spring element. Hence, an additional plugin, the Kangaroo plugin, is used for modeling spring elements in this study. It is an interactive physics and constraint solver plugin (Piker 2010), and gravity force and nodal interactions are applicable to the elements with this plugin.

The Pendergrast algorithm is reproduced in the Grasshopper script. The fact that the wireframe shown in Fig. 3 can simulate the behavior of fabric is also mentioned in the Kangaroo manual written by the developer of the plugin (Piker 2012).

Fig. 4 shows the full script of Grasshopper, with more detailed step-by-step procedures indicated as follows:

A. Set the parameters with slider bars and toggle switch by users. The parameters can be changed for form-finding while the simulation is executed.

B. Make a surface element from 4 points.

C. Divide the surface element into small surface elements, using Domain function and Isotrim function. All small surface elements have the same size.

D. Find 4 points of each small surface element, and draw grids and braces using the points.

E. Convert the grids and braces into Kangaroo Spring elements.

F. Apply gravity loads to the points.

G. Set Anchor Points and make them able to move using

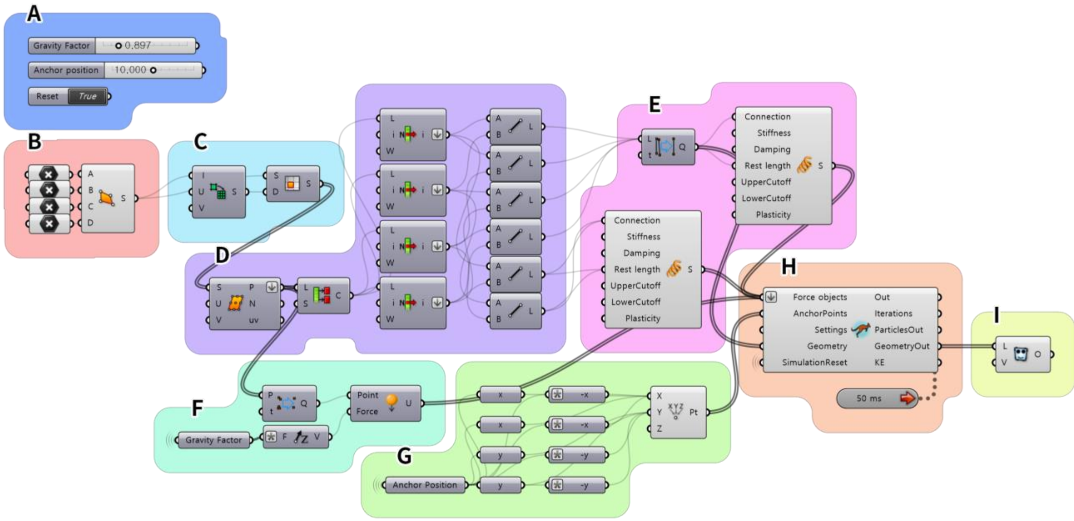



Fig. 4 Grasshopper script for form-finding of concrete catenary shell using flat square element

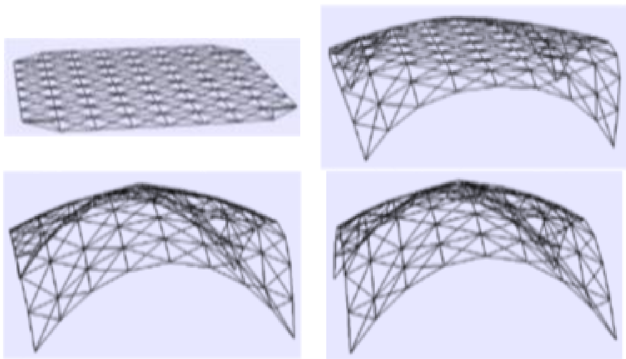
Number Slider.

H. Input the tentative parameters resulting from Step E, F and G into Kangaroo Physics Engine, and execute the simulation of hanging experiment.

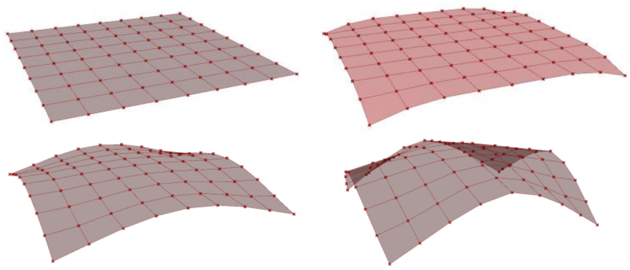
I. After stabilization of the shell model shape, make grids and braces to a Mesh object.

In order to establish the wireframe, a flat square surface is divided into smaller elements, and each smaller surface element is converted into spring elements. The conversion is necessary to allow for the elongation of each wireframe member. In this way, the concrete catenary shell can be generated.

The Mesh object from Step I should be exported as a DXF file, edited using AutoCAD, and imported into SAP2000, a finite element analysis program (Abell 2012). For Step I, the plugin named WeaverBird is used in addition to Kangaroo. The WeaverBird is a topological modeler consisting of existing subdivision and transformation operators (Piacentino 2009), but it is used only for Step I in this study.



(a) Pendergrast



(b) Grasshopper

Fig. 5 Form-finding and final shell shapes

## 4. Verification in comparison with Pendergrast's study

Pendergrast (2010) developed a stand-alone program written in C++ and executed a simulation. Pendergrast's program makes it possible to create various user interfaces for imposing loads on points, moving anchor points (only on the *x*-*y* plane), etc. On the other hand, the program has some limitations. First, the anchor points must exist on the *x*-*y* plane. This feature makes it difficult for the users to generate various shell shapes. In addition, the output file is readable only within Pendergrast's program and cannot be exported into other CAD programs.

On the other hand, the program developed by the authors allows for the movement of each anchor point in the *z*-direction as well. Also, it is compatible with commercially available CAD programs. Furthermore, the developed program utilizes only the built-in and plug-in functions of Grasshopper so that any structural engineers can use it with little computational cost, though the user interface is designed with built-in basic visual components of Grasshopper such as Slider and Toggle Button.

### *4.1 Comparison of final shell shape generated through Form-Finding analysis*

Fig. 4 shows models from the simulations by the design tool of Pendergrast and Grasshopper script, respectively. Both are in the state of force equilibrium. Pendergrast (2010) did not show the stiffness of spring elements, the size of each grid element, and the magnitude of applied loads. Those parameters were assumed arbitrary in making a flat-square-based shell with the Grasshopper script.

Although the differences might exist, the final shapes are similar as shown in Fig. 5.

### *4.2 Stress-Deformation analysis*

Pendergrast did not perform the actual stress-deformation analysis using the final shell shape, but performed the form-finding analysis. Therefore, it is not adequate to compare the stress-deformation analysis results from the methods by Pendergrast and Grasshopper script on equal terms. In this section, the stress-deformation analysis is carried out using SAP2000 only for the developed Grasshopper model.

The span ($L$) of the shell structure is about 8000 mm, and the shell thickness ($d$) is set to 250 mm. To determine whether it is a thick or thin shell, Eq. (1) is used.

$$\frac{d}{L} = \frac{250\,\text{mm}}{8000\,\text{mm}} = \frac{1}{32} < \frac{1}{20} \tag{1}$$

where $L$: span of the shell structure, $d$: shell thickness

A thick shell behaves like a plate whose ratio of thickness and span is between 1/20 and 1/10, approximately. Thus, the ratio of a thin shell should be under 1/20. In this case, the ratio is 1/32, so it can be considered as a thin shell. The anchor points have the fixed boundary conditions. Based on the analysis results, the average stress is calculated by Eq. (2).

$$f_n = \frac{f}{d} \tag{2}$$

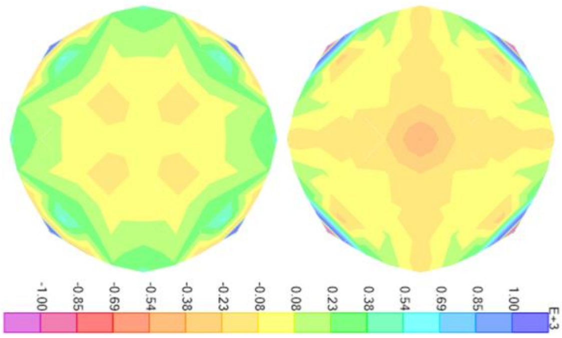
where $f_n$: value of normal force diagram (kN/m), $f$: average in-plane force, $d$: thickness of shell element

The maximum and minimum normal forces per unit length are summarized in Table 1. In addition, the distribution of maximum normal force per unit length at the mid-surface is shown in Fig. 6. Fig. 6(a) and Fig. 6(b) show those in the local $x$-axis and $y$-axis, respectively. There is no tensile force along the local $y$-axis. There is moderate tensile force along the local $x$-axis, but it is less than 5% of the compressive force. Moreover, the area under tensile force is about 50% of that under compressive force. It implies that compressive force is dominant in the model, and one can consider the structure as a shell.

Fig. 7 shows the distribution of maximum normal stresses at the top (see Fig. 7(a)) and bottom (see Fig. 7(b)) of the model. The maximum normal stress is 4.54 MPa and the minimum is -3.54 MPa, and they appear near the support anchor points. The absolute values of the maximum and minimum are relatively small; thus, the satisfactory performance of the shell will be achieved by placing a small amount of reinforcing bars near the anchor points. It should be noted that the normal stress is between -0.2 MPa and 0.2 MPa except for the anchor point regions. It confirms that the model behaves as a shell.

Table 1 Maximum and minimum values of local normal forces (unit: kN/m)

| Axis | Max. | Min. |
|---|---|---|
| $x$-axis | 152 | -208.9 |
| $y$-axis | -19.2 | -541.2 |




(a) Top (b) Bottom

Fig. 7 Distribution of maximum normal stresses (MPa)

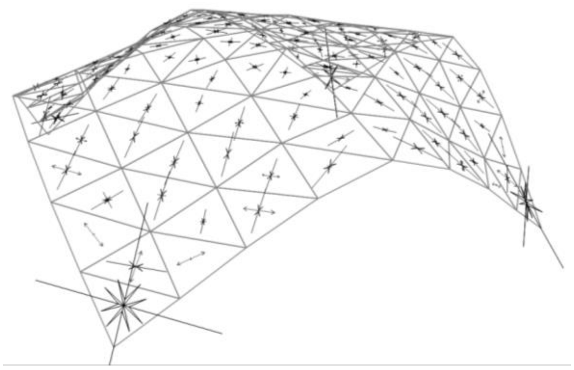


Fig. 8 Flow and magnitude of maximum and minimum normal stresses

Fig. 8 shows the flow and magnitude of maximum and minimum normal stresses. The flow is represented by the arrow direction, and the magnitude is represented by its length. Even though some tensile stresses exist near the anchor points, the model is compression-dominated as previously confirmed in Table 1, and Figs. 6 and 7. Overall, the model generated by Grasshopper script is considered as a concrete catenary shell.

## 5. Verification in comparison with Maurer *et al*.'s study

Maurer *et al*. (2013) did perform both form-finding analysis and stress-deformation analysis of the Naturtheater Grötzingen, a concrete shell structure built in Germany (see Fig. 1(b)). This structure was designed by the form-finding method of Isler (1961), Maurer *et al*. (2013) reproduced the concrete shell structure through the same method. In this study, the Naturtheater Grötzingen is reproduced using the Grasshopper script and compared to the model of Maurer *et al*. (2013).

### *5.1 Comparison of final shell shape generated through Form-Finding analysis*

Applying the height and span of the shell model analyzed by Maurer *et al*. (2013), a new model is generated

(a) Maurer *et al*.

(b) Grasshopper

Fig. 9 Comparison of Naturtheater Grötzingen model by two methods

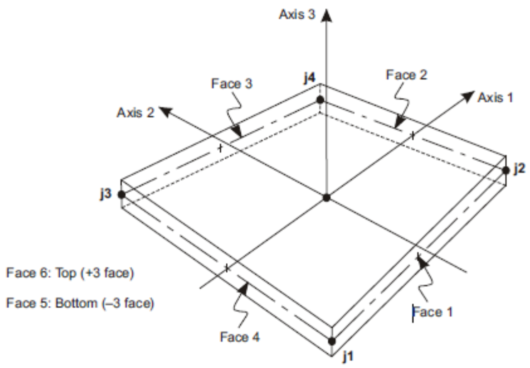



Fig. 10 Four-node quadrilateral shell element (kN/m) (SAP2000 2017)

by using Grasshopper. For some unclear properties, the assumed values are used in the analysis. The scale is the same as Maurer *et al*.'s model whose width, depth, and height are 42 m, 28 m, and 10 m, respectively. Fig. 9 shows some differences in the final shape of two models. Even though the overall shape is similar, the Grasshopper model has the following distinct features: relatively straight legs and the peak point location, which are resulted from the less smooth surface.

### *5.2 Comparison of force and stress generated through Stress-Deformation analysis*

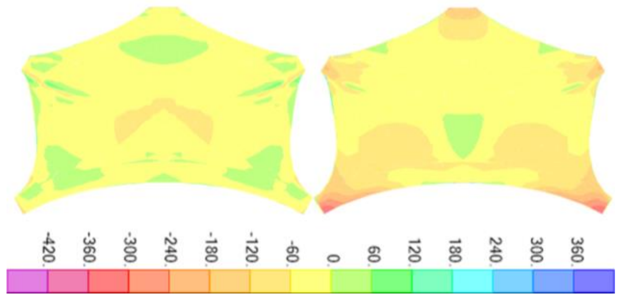
As was done in Maurer *et al*.'s analysis, four-node quadrilateral shell elements with a thickness of 105 mm are used (see Fig. 10), the density of concrete is 2,400 kg/m$^3$, and the compressive strength of concrete is 28 MPa. Referring to Eq. (1), the ratio between the thickness and span is about 1/420; thus, this concrete shell is considered as a thin shell. The behavior of the shell under concrete self-weight only is examined.

The plan of shell is much more complicated than the square model shown in Fig. 5, but both models in Fig. 9 do not have an "ideal" catenary shape. For example, some elements in the middle of the shell are under a small degree of moment and tensile force.

The analysis results are summarized in Table 2. Except for the maximum and minimum tensile stresses, the model by Grasshopper provides smaller compressive and tensile forces/stresses, which are less than 50% of the corresponding forces/stresses by Maurer *et al*. (2013).

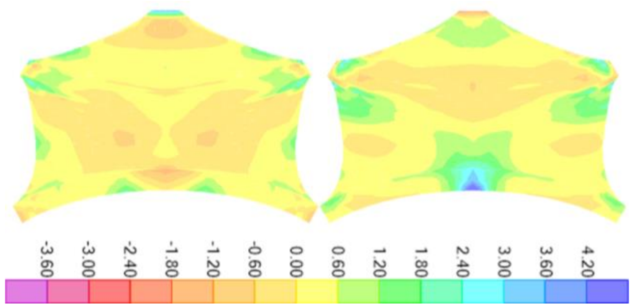
Table 2 Comparison of normal forces and stresses of two models

| Model Type | Force Type | Max. Normal Force (kN/m) | Max. Normal Stress (MPa) | Min. Normal Stress (MPa) |
|---|---|---|---|---|
| Grasshopper | Tension | 393.3 | 8.1 | 2.7 |
| | Compression | 478.1 | 3.9 | 10.6 |
| Maurer *et al*. | Tension | 1152 | 15 | 3 |
| | Compression | 2785 | 14 | 33 |




(a) Local *x*-axis (b) Local *y*-axis

Fig. 11 Distribution of maximum normal force per unit length at the mid-surface (kN/m)




(a) Top (b) Bottom

Fig. 12 Distribution of maximum normal stress (MPa)

In Fig. 11, tensile force appears near the support anchor points in the direction of local x-axis. Additionally, it occurs near the peak point and along the arch lines between two support anchor points in the direction of local y-axis. Despite of the existence of tensile force, the average tensile force is just about 5 to 20 kN/m, while the average of compressive force is about 50 kN/m. The area under compression is larger than that under tension. The maximum compressive and tensile forces are 478.1 kN/m and 393.3 kN/m, respectively. The analysis results indicate that the shell is in the compression-dominant state. On the other hand, both the maximum compressive and tensile forces in the analysis of Maurer *et al*.'s model are over twice those in the developed Grasshopper model (5.8 times and 2.9 times for compressive and tensile forces).

According to Fig. 12, the largest tensile and compressive stresses appear around the anchor points at the upper-left and upper-right corners, and along the arch between the lower-left and lower-right anchor points. The maximum tensile stress is 8.1 MPa near the upper-left and upper-right anchor points, and the tensile stress along the arch between


Fig. 13 Comparison of maximum and minimum stresses between Maurer *et al*. and Grasshopper models (MPa)

the lower-left and lower-right anchor points reaches up to approximately 5.6 MPa. Given that the tensile strength of concrete is about 2 MPa, steel reinforcement is essential in this region.

As shown in Fig. 13, there are substantial differences of maximum and minimum stresses, except for the minimum tensile stress. The values from the Grasshopper model are less than those from Maurer *et al*.'s model in all cases. The largest difference between Maurer *et al*.'s model and the developed Grasshopper model occurs in the minimum compressive stress (3.5 times difference).

The reproduced model of the Naturtheater Grötzingen by using Grasshopper algorithm and Pendergrast's form-finding approach is similar to the previous Maurer *et al*.'s model that utilized Isler's form-finding method in terms of the shell shape and stress distribution pattern. Some differences found in analysis include the location of peak point, the existence of double curvature, and the magnitude of absolute stress values. Maurer *et al*. (2013) mentioned that his model did not reflect the as-built shell perfectly. For example, the Naturtheater Grötzingen has double curvature at the arch parts between anchor points. Relatively large shell elements were used in Maurer *et al*.'s model, and the height and width were both 3 m. On the contrary, the height and width of the shell elements used in the Grasshopper model were about 1 m such that the double curvature is reproduced in the Grasshopper model. Considering that the double curvature is simulated and that the maximum and minimum stresses of the Grasshopper model are smaller, the developed model could be more efficient in generating concrete catenary shell.

## 6. Conclusions

The form-finding method by Isler (1961) has been applied to the shell design, even though it is a complex and inconvenient method. To overcome its inadequacy for computer-aided design, Pendergrast developed a program which generates concrete catenary shells using a computer, but it still has features to be improved. In this study, a program is developed to generate concrete catenary shell structures using Grasshopper. The followings are discovered in the process of the examination:

1. There are no significant differences between the catenary shell models developed with Pendergrast's program and Grasshopper program.
2. The catenary shell model using Grasshopper worked as a structural shell.
3. Reproducing the Naturtheater Grötzingen, a shell structure designed by the form-finding method of Isler (1961), Maurer *et al*.'s method and Grasshopper produced somewhat different analysis models, but both produced the models working as a shell structure in terms of the behavior obtained from analysis.
4. The models by Maurer *et al*.'s method and Grasshopper are not identical to the as-built structure, and the assumed values were adopted for some unclear properties. Through the stress-deformation analysis of the computational models, Maurer *et al*.'s model had greater stresses than the developed model using Grasshopper. In particular, the compressive stress of the Maurer *et al*.'s model was 3.5 times greater than that of the model using Grasshopper.

The developed model can use the widely available Grasshopper program with ease, whereas the Maurer *et al*.'s approach requires the manual experimental measurement. The process of hand measurement often results in the model's asymmetry, but the developed model created perfectly symmetric shell. Furthermore, the developed algorithm is original and has its significant contribution to the state-of-the-art in form-finding methods.

In this study, the stress-deformation analysis was conducted under the assumption that the shell structure made with normal strength concrete was under self-weight only. A further study should include nonlinear structural analyses or dynamic analyses to consider the cases under additional loads such as dead loads due to permanent attachments, snow loads, seismic loads, etc. In particular, the applicability of ultra-high performance concrete (UHPC) to thin shell structures need consideration.

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