Computers and Concrete, *Vol. 21, No. 1 (2018) 95-102*
DOI: https://doi.org/10.12989/cac.2018.21.1.095

Analysis of reinforced concrete corbel beams using Strut and Tie models

Jafarali Parol*, Jamal Al-Qazweeni[a] and Safaa Abdul Salam[b]

Sustainability and Reliability of Infrastructures Program, Energy and Building Research Center,
Kuwait Institute for Scientific Research, Kuwait

(Received May 23, 2016, Revised July 16, 2017, Accepted November 17, 2017)

Abstract. Reinforced concrete corbel beams (span to depth ratio of a corbel is less than one) are designed with primary reinforcement bars to account for bending moment and with the secondary reinforcement placed parallel to the primary reinforcement (shear stirrups) to resist shear force. It is interesting to note that most of the available analytical procedures employ empirical formulas for the analysis of reinforced concrete corbels. In the present work, a generalized and a simple strut and tie models were employed for the analysis of reinforced corbel beams. The models were benchmarked against experimental results available in the literature. It was shown here that increase of shear stirrups increases the load carrying capacity of reinforced concrete corbel beams. The effect of horizontal load on the load carrying capacity of the corbel beams has also been examined in the present paper. It is observed from the strut and tie models that the resistance of the corbel beam subjected to combined horizontal and vertical load did not change with increase in shear stirrups if the failure of the corbel is limited by concrete crushing. In other words, the load carrying capacity was independent of the horizontal load when failure of the beam occurred due to concrete crushing.

Keywords: reinforced concrete; strut and tie; corbel beam

1. Introduction

Corbels are short cantilever reinforced concrete beams, whose length to depth ratio is less than one. Corbels are used to support precast beams, or to connect beams of structural frames at an expansion joint. The load transfer in corbels is predominantly through shear. The classical Euler Bernoulli (EB) beam theory, where the shear strain is neglected, is not quite valid in the analysis and design of corbels as depth to length ratio is less than one. Creep and shrinkage is also an important issue in corbels. Creep and shrinkage effects are taken into account in the corbel design by considering a minimum value of horizontal load while designing the corbel (ACI 318M-11 2011). In general, the behavior of corbel is not easy to describe analytically, and most of the design codes suggest a hybrid approach based on the simplified EB theory and empirical equations for the design of corbel (ACI 318M-11 2011, PCI 2004).

Kriz and Rath (1965) have conducted extensive experiments on reinforced corbel beams. They examined the effect of various parameters (type of loading, span to depth ratio, reinforcement details, etc.), and they proposed empirical formula for estimating the load carrying capacity of reinforced corbel beams. Later, Mast (1968) introduced a semi empirical approach based on shear-friction theory. Using this theory, Mast compared a number of experimental results presented by Kriz and Rath (1965). He showed that shear friction theory predicts the load carrying capacity of corbel beams quite accurately. Later in 1974, Hermansen and Cowan introduced a cohesion effect on Mast's design formulas. This model offered better results compared with experimental results. Somerville (1974) employed hybrid approach for the corbel beam analysis. In his method he used EB theory for estimating tension and compression. This approach is questionable as EB theory is not quite appropriate to model short beams, as shear stress significantly influences the behavior of short beams. According to Somerville, the approach proposed by the Hermansen and Cowan is most suited for the analysis of short corbels.

It has been shown by Mattock *et al*. (1976) that the use of shear friction theory for the corbel beams with length to depth ratio less than 0.5 is unwarranted. Mattock *et al*. (1976) have conducted further study on the corbel beams intended to extend the shear friction theory to corbel beams subjected to combined shear and horizontal loads. In their experiments, 26 out of 28 samples were with shear stirrups. They showed that a minimum amount of horizontal stirrups must be provided in corbels to eliminate the premature diagonal tension failure. They also recommended that the yield strength of stirrups should not be less than half of the yield strength of the main reinforcement.

Fattuhi and Hughes (1989a, 1989b), Fatuhi (1990, 1994) examined the behavior of concrete corbels using laboratory experiments. It has been shown that shear strength of concrete corbels with main bars and certain amount of steel fibers could be comparable to those corbels with main reinforcement and horizontal stirrups. They observed that

∗Corresponding author, Associate Research Scientist
E-mail: jparol@kisr.edu.kw
[a]Program Manager
E-mail: jqazweni@kisr.edu.kw
[b]Senior Research Associate
E-mail: ssalam@kisr.edu.kw

http://www.techno-press.org/?journal=cac&subpage=8
ISSN: 1598-8198 (Print), 1598-818X (Online)

the failure mode changed from being shear to flexural either when steel fibers were used as a replacement for horizontal stirrups or by using low volume of main reinforcement. They observed that the efficiency of the fibers was inversely proportional to the shear span-to-depth ratio. Their experiments were on corbels subjected to vertical loads only. The effect of column load on strength of corbels was investigated by Fattuhi (1990). According this paper, the column load did not influence the strength of corbels. It has been shown by several investigators that the amount of secondary reinforcement affects the ultimate load carrying capacity of corbels (Mattock *et al*. 1976, Yong and Balaguru 1994, Foster *et al*. 1996, Compione *et al*. 2005).

Niedhoff (1961) was the first to propose a truss analogy for the analysis of reinforced corbel beams. He modelled the main reinforcement as the horizontal tie and concrete as an inclined strut. Since then this method was quite well accepted among researchers and engineers for modelling D-Region in reinforced concrete beams. Mehmet and Freitag (1967) employed modified strut and tie model with two tension members (horizontal main reinforcements and inclined stirrups). The proposed model was a statically indeterminate system. Later Hagberg (1966) examined the aforementioned two models using the principle of minimum energy theory. He observed that statically determinate truss model had the smallest deformation energy and thus it would satisfy the compatibility conditions better. Hagberg (1983) later proposed a two layer strut and tie model that included one layer of horizontal stirrups in the tension ties. Recently, he investigated capability of EN 1992 (2008) and the European Concrete Platform (2008) to reproduce experimental results of corbel beams behaving as strut-and-tie models. In his study he employed models he presented earlier in 1983. Zeilinski and Ragotti (1995) used the strut and tie model to analyze their experimental work. They proposed design formulas to obtain the maximum amount of reinforcements for beams reinforced with horizontal and inclined bars. Huwang *et al*. (2000), Russo *et al*. (2006) used the strut and tie model to predict the shear strength of reinforced corbel beams. They compared their results with experimentally observed values from the literature.

Campione (2009a, 2009b) employed strut and tie macro model to predict the flexural behavior of plain and fibre reinforced concrete. The effect of traditional steel bars and transverse stirrups were compared with the effect produced by the fiber reinforced concrete. The strut and tie model was able to predict the experimental results very accurately. In his strut and tie model steel yielding and concrete crushing were accounted.

It is worth mentioning that the strut and tie model in general, is a linearized method and allows the design engineer to see only the flow of internal force transfer in the corbel due to any external load combinations. The method cannot explicitly track the crack propagation, unlike nonlinear finite element method. However, in the STM, implicitly accounts the damage progression either through tie or strut.

Finite element method was employed by several investigators for the examination of reinforced concrete corbel beams (Will *et al*. 1972, Renuka Prasad *et al*. 1993, Strauss *et al*. 2006, Manzoli *et al*. 2008, Abdul Razak and Muhammd Ali 2011a, 2011b, Syroka *et al*. 2011, Rezai *et al*. 2013, Rejane Martins *et al*. 2014). Most of the investigators used smeared crack approach in their constitutive models. Manzoli *et al*. (2008) used three dimensional finite element models. The results of their studies were compared against experimental results by Mehmet and Frietag (1967). Syroka *et al*. (2001) examined the influence of characteristic length of finite elements and tensile fracture energy in the predicted finite element results. They also showed using the finite element analysis that presence of horizontal stirrups increased the ductility and ultimate load carrying capacity.

Abdul-Razzak and Mohammd Ali (2011a, 2011b) developed material models using regression analysis from the experimental data. Their study showed that the finite element results and the experimental results were in good agreement for the load deflection and crack patterns. Recently Rezai *et al*. (2013) examined the influence of the ratio of shear stirrups and main reinforcement on the failure mechanism and ultimate load carrying capacity. According to their study, load carrying capability of reinforced concrete corbels was enhanced considerably until the shear stirrups reached 0.3%. Rejane Martins *et al*. (2014) examined the influence of mesh size on the predictive capability of finite element analysis. According to them, mesh size did not influence the predicted load carrying capacity of corbel beams. This conclusion can be attributed to the fact that, all the finite element size that they had used in their study should have already been in the converged region.

Kumar (2010) used artificial neural network to model the steel fiber reinforced concrete without shear reinforcement. His study results were comparable with the experimental results. However, according to authors opinion, artificial network is not quite common among civil engineers and practically not very useful in the investigation of reinforced concrete corbels, in particular, and reinforced concrete beams, in general.

Though several analytical methods are available in the literature, the PCI design handbook (2004) and ACI 318-11 (2011) are the most commonly employed guidelines by practicing engineers for the design of reinforced concrete corbel beams. These design guidelines recommend the use of a minimum percentage of shear stirrups. As per the American Concrete Institute (ACI) design guidelines, the secondary reinforcement should not be less than approximately 50% of the primary reinforcement (ACI 318-11 2011). Additionally, it is recommended to uniformly distribute the secondary reinforcement to over a two-third of the depth of the corbels. It is interesting to note that, despite the use of minimum percentage of shear stirrups, none of these design guidelines account shear stirrups while estimating the load carrying capacity of the corbels. This is against all common practices in predictions of strength of reinforced concrete practice and contradictory to test results. On the other hand, it was shown by several investigators that horizontal reinforcement will increase the ultimate load carrying capacity of corbel beams. Yassin et al (2015) presented an excellent review on reinforced concrete corbels. They reviewed experimental and analytical models for the analysis of corbels and strengthening of damaged corbels using normal, high strength and self-compacting

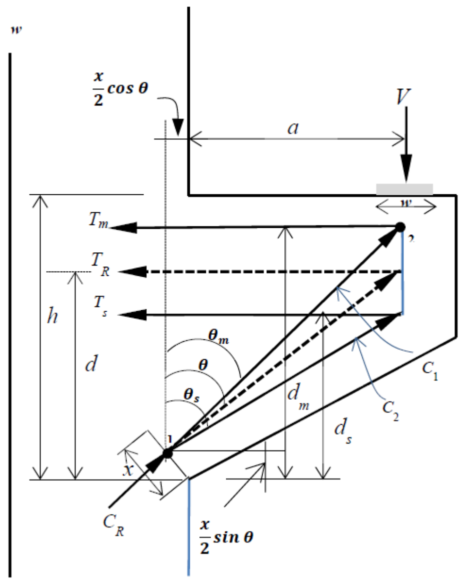

Fig. 1 Generalized strut and tie model

concrete. In particular, they presented various strut and tie models. They did not discuss about the effect of distribution of shear stirrups capacity of corbel beams.

In the present study, two strut and tie models are presented. These models are used to analyze behavior of reinforced concrete corbel beams. The effect of secondary reinforcement on the load carrying capacity of corbel beams is examined. It is shown in the present study that the secondary reinforcement and its distribution influence the ultimate load carrying capacity of the corbel beams. Furthermore, effect of horizontal load on the vertical load carrying capacity of reinforced concrete corbel beams is also examined.

2. Generalized Strut and Tie model for reinforced concrete corbel beam

In a corbel beam the external load is transferred to internal in such a way that it forms a compressed cracked zone. The compression cracked pattern at rupture is depicted in Fig. 1. The cracked continuum can be idealized as compression strut. This is philosophy leads to the idealization of the reinforced concrete corbel as a tension tie and a compression strut system. This also means that, a strut and tie model assumes crack propagates along the compression strut. Fig. 1 depicts a generalized strut and tie model for a multi layered reinforcement corbel beam subjected to vertical load *V*. The effective span of the corbel beam is a, and width of the base plate is w. The resultant tension forces from multiple layers of main reinforcement and from the shear stirrups are represented by the T_m and T_s respectively. The resultant tension force, T_R, from main reinforcement and shear stirrups are accounted separately and the resultant tension force can be expressed as

$$T_R = T_m + T_s \tag{1}$$

The resultant tension force from the main reinforcement (with *m* layers) is given by

$$T_m = \sum_{i=1}^{m} f_i^{tm} A_i^{sm} \tag{2}$$

Here A_i^{sm} and f_i^{tm} are the area and the tensile strength of the main reinforcement at i^{th} row. Location of the resultant force for the main reinforcement, d_m, can be obtained as

$$d_m = \frac{\sum_{i=1}^{m} T_i d_i}{\sum_{i=1}^{m} T_i} = \frac{\sum_{i=1}^{m} f_i^{tm} A_i^{sm} d_i}{\sum_{i=1}^{m} f_i^{tm} A_i^{sm}} \tag{3}$$

where d_i is the distance of the i^{th} layer of main reinforcement from bottom of the corbel beam.

Similarly, the resultant tension force in the shear stirrups and its location (with *n* layers of stirrups) is given by

$$T_s = \sum_{i=1}^{n} f_i^{ts} A_i^{ss} \tag{4}$$

$$d_s = \frac{\sum_{i=1}^{n} T_i d_i}{\sum_{i=1}^{n} T_i} = \frac{\sum_{i=1}^{n} f_i^{ts} A_i^{ss} d_i}{\sum_{i=1}^{n} f_i^{ts} A_i^{ss}} \tag{5}$$

where A_i^{ss} and f_i^{ts} are the area and the tensile strength of the shear stirrups. Here, d_s is the distance of the resultant tensile force from bottom of the corbel beam.

From Fig. (1), the equilibrium of horizontal and vertical forces can be written as

$$T_R = C_R \sin\theta \tag{6}$$

$$V = C_R \cos\theta \tag{7}$$

where C_R is the resultant compressive force and it can be estimated as follows

$$C_R = f_c' b x \cos\theta \tag{8}$$

where f_c' is the compressive strength of concrete, x is the width of compression strut and b is the width of the corbel beam. From Eqs. (6) and (7), vertical load carrying capacity can be expressed in terms of the tensile loads as

$$V = \frac{T_R}{\tan\theta} \tag{9}$$

The vertical load capacity can be also expressed as the sum of the tension forces from the main and shear stirrups as

$$V = \frac{T_R}{\tan\theta} = \frac{T_m}{\tan\theta_m} + \frac{T_s}{\tan\theta_s} \tag{10}$$

where θ_m and θ_s are the inclination of the compression struts with respect to the vertical and it can be obtained as

$$\tan\theta_m = \frac{a + \frac{x}{2}\cos\theta}{d_m - \frac{x}{2}\sin\theta} \tag{11}$$

$$\tan\theta_s = \frac{a + \frac{x}{2}\cos\theta}{d_s - \frac{x}{2}\sin\theta} \tag{12}$$

By substituting Eqs. (11) and (12) in Eq. (10)

$$\frac{T_R}{\tan\theta} = T_m \frac{d_m - \frac{x}{2}\sin\theta}{a + \frac{x}{2}\cos\theta} + T_s \frac{d_s - \frac{x}{2}\sin\theta}{a + \frac{x}{2}\cos\theta} \tag{13}$$

By re-arranging the terms, Eq. (13) can be re-written as

$$\frac{x}{2}(\cos\theta + \sin\theta\tan\theta) = \frac{\tan\theta\,(T_m d_m + T_s d_s)}{T_R} - a \tag{14}$$

Here, $d = \frac{T_m d_m + T_s d_s}{T_R}$ is the depth of the resultant tension force. The width of the compression strut can be expressed from Eq. (14) as

$$x = 2\,\frac{d\tan\theta - a}{(\cos\theta + \sin\theta\tan\theta)} \tag{15}$$

From Eqs. (6), (8) and (15), one can obtain,

$$(T_R - f_c'\,b\,d)\tan^2\theta + f_c'\,b\,a\,\tan\theta + T_R = 0 \tag{16}$$

Using Eqs. (9), (15) and (16) one can estimate the vertical load carrying capacity of the reinforced concrete corbel beams. It is worth mentioning that the maximum value of θ is limited by the geometrical constraints and it is given by

$$\tan\theta_{\max} = \frac{a + \frac{w}{2}}{d} \tag{17}$$

In Eq. (16), value of θ is obtained in such a way that, compression strain/stress (Fig. 1) and tension strain/stress on the tension tie are inter connected. In other words, a change in the capacity of the tension tie will lead to change in the compression strut in order ensure compatibility between reinforcement (tie) and concrete (strut). To estimate the compressive force on the strut, the width of the compression strut *x*, should be obtained from Eq. (16) and Eq. (12). This is achieved by incorporating compatibility condition. This can be imposed in two ways, as shown below:

(i) It is assumed that the concrete and steel yield at the same time, accordingly value of θ and width of compression strut can be obtained (less than or equal to the maximum allowable value).

(ii) In the second approach, one can assume maximum width of compression strut from Eq. (12) and obtain the value of compressive strain/stress (which is less than or equal to concrete yield strain/stress).

Both (i) and (ii) will yield same load carrying capacity of the corbel beam. It is also worth mentioning that if $\theta \le \theta_{\max}$, then the failure is determined by steel yielding, and the load carrying capacity can be estimated using Eq. (9). Otherwise, the failure is determined by concrete crushing and therefore, load carrying capacity is determined using Eqs. (7) and (8).

3. Simplified Strut and Tie model

It can be seen from the previous section that, strut and tie model obtained by representing the main and shear reinforcements as two different ties results into relatively complex expression and calculation procedure. In this section, a simplified strut and model and easy procedure is presented which is handy for practical purposes. In the simplified procedure (Fig. 2), it is assumed that the compression strut is formed with a prescribed angle $\theta_{\max}$ as given by Eq. (17). Here the effective depth could be obtained as

$$d = \frac{\sum_{i=1}^{m} f_i^{tm} A_i^{sm} d_i + \sum_{i=1}^{n} f_i^{ts} A_i^{ss} d_i}{\sum_{i=1}^{m} f_i^{tm} A_i^{sm} + \sum_{i=1}^{n} f_i^{ts} A_i^{ss}} \tag{18}$$

The vertical load carrying capacity could be obtained using one of the following equations based on the failure mechanism, which can be determined by assuming steel yielding or concrete crushing.

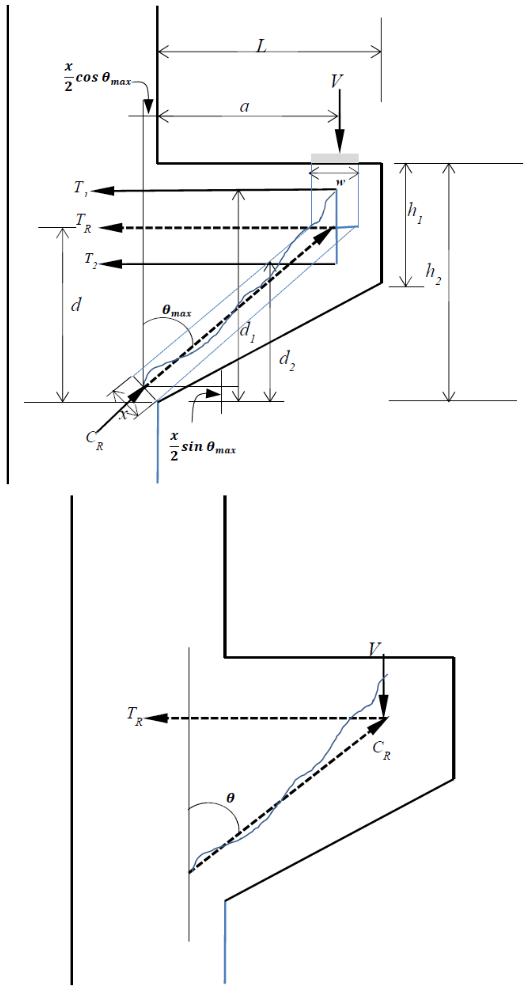

Fig. 2 Simplified strut and tie model

$$V = \frac{T_R}{\tan\theta_{\max}} \tag{19}$$

$$V = C_R\,\cos\theta_{\max} \tag{20}$$

$$C_R = \sigma_{cc}\,b\,x;\quad \sigma_{cc} \le f_c' \tag{21}$$

where $x = w\cos\theta_{\max}$ and $\sigma_{cc} = \frac{T_R}{b\,w\,\cos\theta_{\max}\sin\theta_{\max}}$ is the compressive stress in the concrete strut, which is less than the compressive strength of concrete. It may be noted that in this model it is assumed that steel yielding occurs before concrete crushing. In the case of concrete crushing one can still use Eq. (20) to estimate the load carrying capacity, by substituting $\sigma_{cc} = f_c'$. It may be noted that the angle $\theta_{\max}$ (radians) should not exceed the $\beta = \frac{\pi}{2} - \tan^{-1}\left(\frac{h_2 - h_1}{L}\right)$. Here h_1 and h_2 are the depth at the fixed and free end of the corbel beam respectively, and L is the total span of the corbel beam.

4. Corbel beam subjected to combined vertical and horizontal forces

The free body diagram of a corbel beam with multiple layers of reinforcements represented as resultant tie is presented in the Fig. 3.

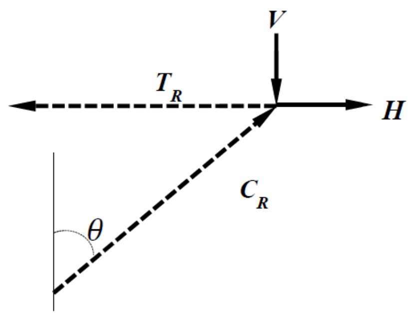

Fig. 3 Free body diagram of simplified strut and tie model of a corbel beam subjected to vertical and horizontal forces

The equilibrium equation is given by

$$T_R = H + C_R \sin\theta \quad (22)$$

$$V = C_R \cos\theta \quad (23)$$

Eq. (22) can be re written by substituting Eq. (23) into Eq. (22)

$$T_R = V(k + \tan\theta) \quad (24)$$

$$V = \frac{T_R}{(k+\tan\theta)} \quad (25)$$

where $k = H/V$, the ratio of horizontal force to vertical force. It may be noted that the ultimate vertical load capacity can be estimated either from Eq. (23) or Eq. (25) depending on whether the failure is due to yielding of reinforcement steel or due to concrete crushing. In other words, if the failure is due to the concrete crushing, the ultimate vertical load carrying capacity can be obtained from Eq. (23). If the failure is due to the steel yielding, the ultimate vertical load carrying capacity can be obtained from Eq. (25). One can employ either the generalized or simplified strut and tie model to evaluate T_R, C_R, and the inclination of the compression strut in Eqs. (23) and (25). It is interesting to note from the Eqs. (23) and (25) that, if the load carrying capacity was limited due to concrete crushing, the failure load is independent of the horizontal load. On the other hand, if the failure is due to concrete crushing, the load carrying capacity is independent of the horizontal load.

In a strut tie model, in general, and in the present work in particular, a linear constitutive relation between the stress/strain relation of concrete and steel is assumed (Yassin *et al.* 2015). Stress/ strain softening coefficients are not considered in the strut and tie model as the effective compression strain/stress in the compression strut is small and nonlinear behavior effectively can be replaced by simple linear relation.

5. Procedure of using simplified Strut and Tie model for estimating load carrying capacity of reinforced corbel beam

Step 1: Obtain T_R and d from Eqs. (1) and (18) respectively.
Step 2: Obtain θ_{max} from Eq. (17).
Step 3: Obtain σ_{cc} from Eq. (21).
Step 4: Check the condition, $\sigma_{cc} \le f_c'$. If this condition is true, estimate the load carrying capacity by using Eq. (25). Else, use Eq. (23) to obtain the load carrying capacity.

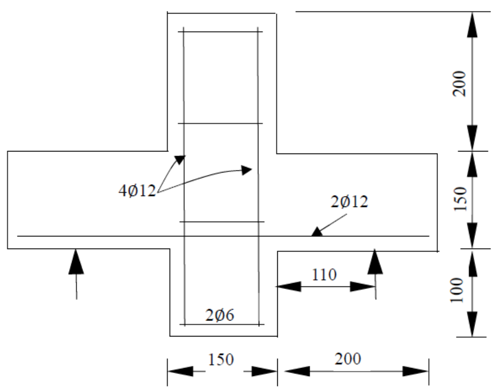

Fig. 4 Geometric details of the corbel beam experiment conducted by Fattuhi (1990)

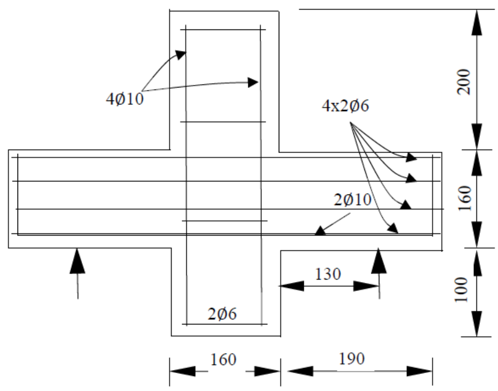

Fig. 5 Geometric details of the corbel beam experiment conducted by Campione *et al*. (2005)

6. Analysis, results, and discussion

The generalized and simplified strut and tie models are benchmarked against the experimental results presented in the literature by various investigators (Fattuhi 1990, Campione *et al*. 2005, Fattuhi and Hughes 1989a). All these cases comply with the ACI code to model as a corbel beam considering the length to depth ratio. The experimental set up by Fattuhi (1990), Campione *et al*. (2005) are depicted in Figs. 4 and 5 respectively. The results predicted from generalized and simplified strut and tie models are presented in Table 1. For the experimental sample (Fattuhi 1990), the generalized strut and tie model predicted the failure load as 112 kN (Table 1) against the experimentally observed value of 109.6 kN. For the same case, simplified strut and tie model predicted 105.9 kN. The two results are matching well with the experimental results. It is interesting to note that simplified model under-predicted the load carrying capacity with respect to generalized model.

The two strut and tie models are compared against the experimental results by Campione *et al*. (2005). The experimental set up is depicted in Fig. 5. The geometric details are given in Table 1. In this case, the corbel beam is

Table 1 Comparison of experimental results with the strut and tie models

Reference Data	a (mm)	h (mm)	b (mm)	d (mm)	a/d	A_s (mm^2)	Experiment (kN)	Generalized Strut and Tie model (kN)	Simplified Strut and Tie model (kN)
Experimental data from Fattuhi (1990)	110	150	160	140	0.79	226.2	109.6	112	105.9
Experimental data from Campione *et al*. (2005)	130	160	160	d_1=140 d_2=110 d_3=80	*0.93*	$A^s{}_1$=213.6 $A^s{}_2$=56.5 $A^s{}_3$=56.5	129.5	131.4	128
Experimental data from Fattuhi and Hughes (1989a) sample T1	89	148	152	128	0.7	157.1	92	99.3	86.6
Fattuhi and Hughes (1989a) sample T2	89	148	152	d_1=128 d_2=74	0.7	$A^s{}_1$=157.1 $A^s{}_2$=157.1	151.4	142.6	136.7
Fattuhi and Hughes (1989a) sample T7	89	148	152	d_1=128 d_2=78	0.7	$A^s{}_1$=226.2 $A^s{}_2$=157.1	153.6	176.2	174.8
Fattuhi and Hughes (1989a) sample T8	89	148	152	d_1=128 d_2=91	0.7	$A^s{}_1$=226.2 $A^s{}_2$=157.1	191	188.6	186.3
Fattuhi and Hughes (1989a) sample T9	89	148	152	d_1=128 d_2=83	0.7	$A^s{}_1$=226.2 $A^s{}_2$=157.1	161	183.1	180.9

reinforced with one row of main reinforcements and three rows of horizontal stirrups. The computed vertical load carrying capacity using the generalized and simplified strut and tie models are 131.4 kN and 128 kN respectively. The corresponding experimental value was 129.5 kN. As observed in the previous case, simplified strut and tie model under-predicted the load carrying capacity compared with generalized model.

Experimental studies from Fattuhi and Hughes (1989a) are analyzed using the strut and tie models. They used samples with different geometric and material properties. The properties of the corbel beams are presented in Table 1. The sample T1 has only main reinforcement, and it was designed without shear stirrups (Table 1). The predicted load carrying capacity using generalized and simplified strut and tie models are 99.3 and 86.6 kN respectively. The corresponding observed value in the experiment was 92 kN. As seen in the previous two cases, the predicted results are in good agreement with experimental results. Samples, T2, T7, T8, and T9 were designed with one row of main reinforcement and one row of shear stirrups. Predicted results using the two strut and models are shown in the Table 1. It can be seen from the Table 1 that for all the cases, results from the strut and models are within 15% error with respect to the experimental results.

It is observed that the simplified model under-predicts the load carrying capacity when compared with the generalized model. This is expected, because in the simplified model, it is assumed that the strut is formed through the maximum angle with respect to the vertical load. Hence this model predicts load carrying capacity that is less than the one predicted using generalized strut and tie model. It is observed that as the number of layers of reinforcements is more than two (including main reinforcements and shear stirrups), the simplified and generalized strut tie model results are quite close to each other. Furthermore, simplified model is more conservative, hence, in most practical cases, it could be used for the analysis of reinforced concrete corbel beams.

6.1 Effect of shear stirrups on the vertical load carrying capacity

It can be seen from Eq. (10) that the horizontal shear stirrups in the reinforced concrete corbel beams influence the vertical load carrying capacity. In fact, as the area of shear stirrups increases, load carrying capacity increases. This was also observed in the experiments by several investigators earlier (Fattuhi and Hughes 1989a). In this paper, the strut and tie model was employed to show the effect of shear stirrups on the load carrying capacity of reinforced concrete corbel beams. In Table 1, the sample T1 is designed without shear stirrups and sample T2 is with one row of shear stirrups but with same amount of main reinforcement. The predicted load carrying capacity using generalized strut and tie model was 99.3 kN for T1 and 142.6 kN for T2. The experimental results for these two cases were 92 kN and 151.4 kN respectively for T1 and T2. Samples T7, T8, and T9 are designed with one row main reinforcement and one row of shear stirrups with same amount of reinforcement for all the cases. The load carrying capacity obtained from the experiment was 153.6 kN, 191 kN and 161 kN respectively for samples T7, T8 and T8 respectively. Similar results were obtained using simplified strut and tie model for these cases. It is interesting to note that the difference in these cases was shear stirrups distribution along the depth of the corbel beam. It is clear that, the area of shear stirrups and its distribution along the depth also affect the load carrying capacity. In fact, as the distance of the shear stirrups from the main reinforcement decreases, the load carrying capacity increases. This observation can also be easily verified from Eq. (13). It will be interesting to see how the crack patterns changes as the stirrup distribution changes. Unfortunately strut and tie model cannot capture the crack patterns and is thus not included in the current study.

6.2 Effect of horizontal force on the vertical load carrying capacity

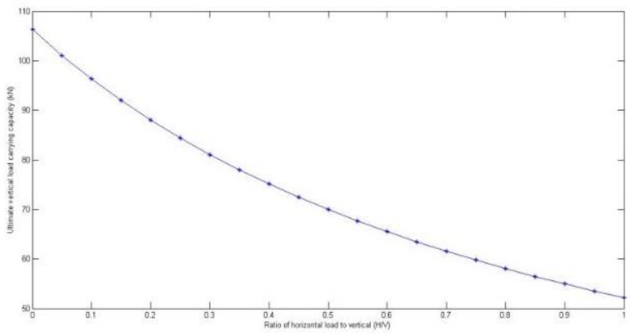

Fig. 6 Vertical load carrying capacity with varying *H*/*V* ratio (Fattuhi 1990)

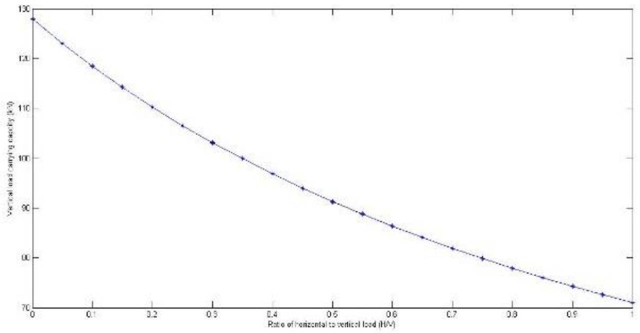

Fig. 7 Vertical load carrying capacity with varying *H*/*V* ratio Campione *et al*. (2005)

The vertical load carrying capacity for a reinforced concrete corbel beam is reduced due to the presence of horizontal load (Eq. (25)). For the purpose of the analysis, two corbel beams experimentally examined by Fattuhi (1990), Campione *et al*. (2005) are used in the present study. Results of the analysis with varying ratios of horizontal to vertical forces obtained from the strut and models are given in the Table 2 and graphically presented in Figs. 6 and 7. It may be noted that the vertical load carrying capacity decreases as the horizontal force component increases. It is worth mentioning that in both cases examined here, the load carrying capacity is limited by steel yielding. Hence, the reduction in load carrying capacity is observed. Comparing Figs. 6 and 7, the slope of the curve is different indicating that the reduction in vertical load carrying capacity depends on reinforcement ratio, and distribution, beam dimensions, concrete and steel properties etc. In the case of combined load application, increase of shear stirrups will not affect the resistance of corbel, if the load carrying capacity was limited due to concrete crushing. Similar observation was made earlier in the experiments conducted by Kriz and Rath (1965).

7. Conclusions

Two strut and tie models (generalized and simplified) are presented for the analysis of reinforced concrete corbel beams. In the generalized strut and tie model, multilayered main reinforcements and shear stirrups are represented as tension ties and accounted separately while estimating the

Table 2 Predicted vertical load carrying capacity of corbels subjected to combined horizontal and vertical load using strut and tie models

H/*P*	Predicted vertical load carrying capacity (kN) (Fattuhi 1990)	Predicted vertical load carrying capacity (kN) (Campione *et al*. 2005)
0	105.9	128
0.1	96.3	118.4
0.2	88.0	110.2
0.3	81.0	103.1
0.4	75.1	96.8
0.5	70.0	91.3
0.6	65.5	86.4
0.7	61.5	81.9
0.8	58.0	77.9
0.9	54.9	74.3
1.0	52.1	71

load carrying capacity of reinforced corbel beams. A simplified strut and tie model is also proposed that could be handy for practical purposes. A step-by-step procedure of using the simplified strut and tie model for estimating load carrying capacity of reinforced corbel beams is also presented. It has been shown that both models predict the load carrying capacity quite accurately. The models were benchmarked against experimental results. It is observed that simplified strut and model is more conservative and it under predicts the load carrying capacity compared to the generalized model.

The behavior of reinforced corbel beam has been examined using the strut and tie models. The effect of horizontal force on the load carrying capacity of reinforced corbel beam is examined using the strut and tie models. It is shown that, as the ratio of horizontal to vertical load increases, the load carrying capacity of the reinforced corbel beam is reduced significantly. The resistance of the corbel beam subjected to combined horizontal and vertical load did not change with increase in shear stirrups if the failure of the corbel is limited by concrete crushing. Furthermore, if the load carrying capacity was limited due to concrete crushing, the failure load is independent of the horizontal load. The strut and tie model shows that the load carrying capacity is affected by the amount and distribution of shear stirrups. This conclusion is verified by comparing the experimental results with predicted values from strut and tie model.

Acknowledgments

The research described in this paper was financially supported by the Kuwait Institute for Scientific Research (KISR). The authors express their gratitude to KISR.

References

Abdul-Razzak, A.A. and Mohammad Ali, A.A. (2011a), "Influence of cracked concrete models on the nonlinear analysis of high strength steel fibre reinforced concrete corbels", *Compos. Struct.*, **93**, 2277-2287.

Abdul-Razzak, A.A. and Mohammad Ali, A.A. (2011b), “Modeling and numerical simulation of high strength fiber reinforced concrete corbels”, *Appl. Math. Model.*, **35**, 2901-2915.

ACI 318M-11 (2011), Building Code Requirements for Structural Concrete (ACI 318M-11) and Commentary, ACI Committee 318M-11, American Concrete Institute, Farmington Hills, MI, USA.

Campione, G. (2009a), “Flexural response of FRP corbels”, *Cement Concrete Compos.*, **31**(3), 204-210.

Campione, G. (2009b), “Performance of steel fibrous reinforced concrete corbels subjected to vertical and horizontal loads”, *J. Struct. Eng.*, **135**(5), 519-529.

Campione, G., La Maendola, L. and Papia, M. (2005), “Flexural behavior of concrete corbels containing steel fibres or wrapped with FRP sheets”, *Mater. Struct.*, **38**(6), 617-625.

EN 1992-1-1 (2008), Eurocode 2: Design of Concrete Structures-Part 1-1: General Rules and Rules for Building.

European Concrete Platform (2008), Worked Examples for Eurocode 2, ASBL, May.

Fatuhi, N.I. (1990), “Column-load effect on reinforced concrete corbels”, *J. Struct. Eng.*, **116**(1), 188-197.

Fatuhi, N.I. (1994), “Strength of FRC corbels in flexure”, *J. Struct. Eng.*, **120**(2), 360-377.

Fatuhi, N.I. and Hughes, B.P. (1989a), “Reinforced steel fiber concrete corbels with various shear-span to depth ratios”, *Struct. J.*, **86**(6), 590-596.

Fatuhi, N.I. and Hughes, B.P. (1989b), “The ductility of RC corbels containing either steel fibers or stirrups”, *ACI Struct. J.*, **86**(6), 644-651.

Foster, S.J., Powell, R.E. and Selim, H.S. (1996), “Performance high strength concrete corbels”, *ACI Struct. J.*, **93**(5), 555-563.

Hagberg, T. (2015), “Do EN 1992-1-1 and the european concrete platform comply with tests? Commentary on the rules for strut-and-tie models using corbels as an example”, *Struct. Concrete*, **16**(3), 418-428.

Hagberg, T.H. (1966), “On the design of brackets”, *Beton und Stahlbetonbau*, **61**(3), 68-72.

Hagberg, T.H. (1983), “Design of concrete brackets: on the application of the truss analogy”, *ACI Struct. J.*, **80**(1), 3-12.

Hermansen, B.R. and Cowan, J. (1974), “Modified shear friction theory for bracket design”, *ACI Struct. J.*, **71**(2), 55-60.

Huwang, S., Lu, W. and Lee, H. (2000), “Shear strength prediction for reinforced concrete corbels”, *ACI Struct. J.*, **97**(4), 543-552.

Kriz, L.B. and Raths, C.H. (1965), “Connections in precast concrete structures-strength of corbels”, *J. Prestres. Concrete Inst.*, **10**(1), 16-61.

Kumar, S. and Barai, S.V. (2010), “Neural networks modeling of shear strength of SFRC corbels without stirrups”, *Appl. Soft Comput.*, **10**(1), 135-148.

Manzoli, O.L., Oliver, J., Diaz, G. and Huespe, A.E. (2008), “Three-dimensional analysis of reinforced concrete members via embedded discontinuity finite elements”, *Struct. Mater. J.*, **1**(1), 58-83.

Mast, R.F. (1968), “Auxiliary reinforcement in concrete connections”, *Proc. ASCE*, **94**(6), 1485-1504.

Mattock, A.H., Chen, K.C. and Soongswang, K. (1976), “The behavior of reinforced concrete corbels”, *J. Prestres. Concrete Inst.*, **10**(1), 52-77.

Mehmet, A. and Freitag, W. (1967), “Test on the load capacity of concrete brackets”, *Der Bauingenieur*, **42**(10), 362-369.

Niedenhoff, H. (1961), “Studies on the structural behavior of brackets and short beams”, PhD Thesis, Technical Hochschule, Karls ruhe.

PCI (2004), *Design Handbook, Design of Connection*, Part 6.8: concrete brackets and corbels, PCI committee on Building code, Chicago 47.

Rejane Martins, F.C., Daniel Alexander, K. and Mounir Khalil, E.D. (2014), “Numerical analysis of reinforced high strength concrete corbels”, *Eng. Struct.*, **74**, 130-144.

Renuka Prasad, H.N., Channakeshava, C., Raghu Prasad, B.K. and Sundara Raja Iyengar, K.T. (1993), “Nonlinear finite element analysis of reinforced concrete corbel”, *Comput. Struct.*, **46**(2), 343-354.

Rezaei, M., Osman, S.A. and Shanmugam, N.E. (2013), “Primary and secondary reinforcements in reinforced concrete corbels”, *J. Civil Eng. Manage.*, **19**(6), 836-845.

Russo, G., Venir, R., Pauletta, M. and Somma, G. (2006), “Reinforced concrete corbels-shear strength model and design formula”, *ACI Struct. J.*, **103**(1), 3-7.

Somerville, G. (1974), “The behavior and design of reinforced concrete corbels”, Shear in Reinforced Concrete, SP-42, ACI 477-502.

Strauss, A., Mordini, A. and Bergmeister, K. (2006), “Nonlinear finite element analysis of reinforced concrete corbels at both deterministic and probabilistic levels”, *Comput. Concrete*, **3**(2-3), 123-144.

Syroka, E., Bobinski, J. and Tejchman, J. (2011), “FE analysis of reinforced concrete corbels with enhanced continuum models”, *Finite Elem. Anal. Des.*, **47**, 1066-1078.

Will, G.T., Uzamerii, S.M. and Sihna, S.K. (1972), “Application of finite element method to the analysis of reinforced concrete beam-column joint”, *Proceedings of Conference on Finite Element Method in Civil Engineering*, Canada.

Yassin, L.A.G., Sayhood, E.K. and Majeed Hasan, Q.A. (2015), “Reinforced concrete corbels-State of the Art”, *J. Mater. Eng. Struct.*, **2**(4), 180-205.

Yong, Y.K. and Balaguru, P. (1994), “Behavior of reinforced high strength-concrete corbels”, *J. Struct. Eng.*, **120**(4), 1182-1201.

Zielinski, Z.A. and Rigotti, M. (1995), “Test on the shear capacity of reinforced concrete”, *J. Struct. Eng.*, **121**(11), 1660-1666.

CC