Using multi-type sensor measurements for damage detection of shear connectors in composite bridges under moving loads

Xingyu Fan¹, Jun Li^{*1,2}, Hong Hao¹ and Zhiwei Chen³

¹Centre for Infrastructural Monitoring and Protection, School of Civil and Mechanical Engineering, Curtin University, Kent Street, 6102, Bentley, Western Australia
²State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian, China
³Department of Civil Engineering, Xiamen University, Xiamen, China

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Abstract. This paper proposes using the multi-type sensor vibration measurements, such as from a relative displacement sensors and a traditional accelerometer for the damage detection of shear connectors in composite bridge under moving loads. Hilbert-Huang Transform (HHT) spectra of these responses will be fused with a data fusion approach i.e., Dempster-Shafer method, to detect the damage of shear connectors. Experimental studies on a composite bridge model in the laboratory are conducted to demonstrate the effectiveness and performance of using the proposed approach in detecting the damage of shear connectors in composite bridges. Both undamaged and damaged scenarios are considered. The detection results with the data fusion of multi-type sensor measurements show a more reliable and robust performance and accuracy, avoiding the false identifications.

Keywords: composite bridge; shear connector; damage detection; relative displacement; data fusion; Hilbert-Huang Transform

1. Introduction

Engineering structures are designed to carry anticipated loads safely while operating in varying environmental conditions. Deterioration and damage may occur over their operational lifespans. Civil structures are continuously exposed to environmental conditions and various external dynamic loads, such as traffic, earthquakes, winds, blast and impact etc. If the early damage or anomalies are not detected timely, they could cause catastrophic failure of structures and result in a significant loss. Structural health monitoring is usually referred as the measurement of the operating and loading environment and the critical responses of structures to track and evaluate the symptoms of operational incidents, anomalies, and deterioration or damage that may affect operation, serviceability or safety (Aktan et al. 2000, Li and Hao 2016). In this field, long term monitoring (Nguyen et al. 2015), structural damage detection (Brownjohn 2007, Zhu et al. 2014, Weng et al. 2014), novel optimal sensor placement approaches (Yi et al. 2016), force identification (Lai et al. 2016), deterioration detection of boundary conditions (Wang et al. 2016) etc, have been widely studied.

One of the typical bridge forms on Australia highways is composite bridge consisting of steel or precast reinforced concrete (RC) girders with cast-in-situ RC slab. The girders and slab are connected together by distributed steel shear connectors. The shear connection between slab and girders subjects to fatigue and possible corrosion damage, as well as possible overstressing owing to increased traffic weights and volume. Deterioration or break of the shear connection in some regions of the bridge structure causes a loss in the composite action so that the bridge slab and girder respond to traffic loadings independently, resulting in a significant decrease of the overall rigidity and ultimate resistance of the bridge (Dilena and Morassi 2004).

Numerous vibration based damage detection methods have been developed for the condition assessment of shear connectors in composite bridges. Xia et al. (2007) proposed a local detection method by directly comparing the frequency response functions of simultaneously measured vibrations on the slab and girder. Wavelet based Kullback-Leibler distance (Zhu et al. 2012) and wavelet packet energy (Ren et al. 2008) have also been proposed for connectors. damage identification of shear Α transmissibility based damage index (Li et al. 2014a) has been proposed to reflect the local change of shear connector conditions and detect the possible damage of shear connectors. Li et al. (2014b) proposed a damage detection approach by using the wavelet packet energy percentage change of cross correlation functions to identify the damage of shear connectors under ambient vibrations.

Various sensors may be used in the structural health monitoring systems for monitoring the structural conditions, such as global sensors (accelerometers, laser displacement sensors) to measure global vibration responses, and local sensors (strain gauges) to measure local responses. Different sensors may have different pros and cons for various structural health monitoring purposes. Multi-type of

^{*}Corresponding author, Senior Lecturer E-mail: junli@curtin.edu.au



Fig. 1 Wheatstone bridge circuit for the relative displacement sensor



Fig. 2 Design and installation of the developed relative displacement sensor

sensors for vibration measurements could provide more comprehensive information and advanced features for better monitoring the structural conditions. Since damage or failure of shear connectors in composite bridge will most likely result in shear slippage, the induced relative displacement between girders and slab, if the applied load is large enough to overcome the friction force between the girders and slab, will be significant. As relative displacement is directly related to the shear connector conditions, it is believed to be more sensitive than other response quantities such as acceleration and deformation. Wang and Hao (2013) used laser displacement sensors to obtain the relative displacements between slab and girder and found out that this is a promising index for the damage detection of shear connections. The laser displacement sensors usually measure the absolute displacements and are required to be mounted on a fixed foundation and placed very close to the target surface, which may bring significant difficulties in the experimental installation. Besides, the laser displacement sensors are normally costly. Other noncontact displacement measurement methods and equipment have been developed in recent years based on, e.g., laser Doppler vibrometer (Nassif et al. 2005), GPS (Yi et al. 2013) and vision-based methods (Fukuda et al. 2010). Those technologies are developed to measure the absolute structural displacement. To obtain the relative displacement of structures, multiple numbers of equipment will be required. A relative displacement sensor based on strain measurement techniques has been developed for directly measuring the relative displacement and its applications for structural health monitoring has been investigated (Li et al. 2015).



Fig. 3 Sensitivity of a relative displacement sensor

This paper mainly investigates the use of multi-type vibration measurements such as those with the newly developed relative displacement sensor and traditional accelerometer for damage detection of shear link conditions in composite bridges with a data fusion strategy. Experimental studies on a composite bridge model in the laboratory are conducted to perform the damage detection by combining the measured relative displacement and acceleration responses with Dempster-Shafer method and time-frequency analysis, i.e., with Hilbert-Huang Transform.

2. Relative displacement sensor

An innovative relative displacement sensor has been developed to measure the relative displacement between two points on a structure (Li et al. 2015). The main idea of the developed relative displacement sensor will be briefly reviewed herein. Four strain gauges are stuck on a square component connecting two pads, which are used to install the sensor on the testing structure. These four strain gauges are connected as a Wheatstone bridge circuit. The sensitive component of the sensor is a square metallic block of 15 mm×15 mm. The metal plate is 1 mm thick, thin enough to reduce the effect of thickness on the accuracy of the developed sensor. The sensor plate is made of aluminum, which means that the stiffness of the developed relative displacement sensor is low to avoid affecting the local stiffness of structure. Two pads at the ends are used to fix the sensor to structures with four 8 mm diameter screws. The sensor measures shear distortion of the metal block owing to the relative displacement of the two locations that the two end pads of the sensor are fixed to, which occurs along the horizontal direction as shown in Fig. 1.

Compared with a civil engineering structure, the developed sensor is of a very small size and stiffness. Therefore, placing such sensors on civil structures is not likely to significantly change the natural dynamic properties of structures. Owing to the negligible mass of the sensor as compared to the structure, e.g., a bridge, the dynamic interaction between sensor and bridge may also not need to be considered.

The four arms of the Wheatstone bridge circuit are denoted by the resistors R_1 to R_4 . The output voltage of the



Fig. 4 Data fusion architecture in this study

full bridge is calculated as

$$\frac{v}{U} = \frac{1}{4} \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right)$$
(1)

where v and U are output and input voltages, respectively. ΔR_1 , ΔR_2 , ΔR_3 and ΔR_4 are the resistance variations of the four resistors R_1 to R_4 , respectively.

Eq. (1) can be formed with the relationship between the resistance variations and the strain as

$$v = \frac{1}{4} \cdot k \cdot U \cdot (\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4)$$
(2)

where ε_1 , ε_2 , ε_3 and ε_4 are strains of the four resistors R_1 , R_2 , R_3 , and R_4 , respectively.

A relative displacement d along the *x*-axis will deform the four strain gauges differentially due to the diagonal orientation so that the relative displacement appears as the shear distortion of the plate. For diagonal deformation we have

$$\mathcal{E} = \mathcal{E}_1 = -\mathcal{E}_2 = \mathcal{E}_3 = -\mathcal{E}_4 \tag{3}$$

Substituting Eq. (3) into Eq. (2), the output voltage due to the shear distortion is

$$v = \frac{1}{4} \cdot k \cdot U \cdot (\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4) = k \cdot U \cdot \varepsilon$$
(4)

The output voltage is linearly proportional to the strain ε and hence the relative displacement d for a given input voltage and a constant strain gauge factor. The supplying input voltage for the developed sensor is 2.5 V in the study. An experimental calibration test is necessary to find out the constant *K*, which is a linear sensitivity coefficient between strain and relative displacement, in the following relationship

$$d = K\varepsilon \tag{5}$$

Fig. 2 shows the design of the developed relative displacement sensor. Calibration is required to identify the sensitivity coefficient between the measured strain and the relative displacement. Fig. 3 shows a typical relative displace measurement sensor calibration and sensitivity coefficient value. The symmetric behaviour of the used Wheatston bridge circuit ensures that there is no voltage output for the tension, compression, bending and torsion effects, but only measures the relative displacement between the two connecting pads of the sensor. More details on the features of the relative displacement sensor, calibration and validation test can be referred in (Li *et al.* 2015).

3. Damage detection with multi-type sensor measurements

3.1 Hilbert-huang transform

Hilbert-Huang Transfrom (HHT) is a powerful signal processing analysis tool which can process nonlinear and nonstationary signals. One significant component of this method is the signal decomposition. Ensemble Empirical Mode Decomposition (EEMD), as an improved signal decomposition method over the traditional Empirical Mode Decomposition (EMD), can be used to decompose a signal into a finite number of components, call as Intrinsic Mode Functions (IMFs). Performing Hilbert transform of IMFs describes the energy distribution in time-frequency scales. The main theoretical background of EEMD and Hilbert transform are briefly reviewed in this paper.

EMD is able to decompose any signal into some IMFs. A signal w(t) is expressed as the sum of n IMFs and a residue,

$$w(t) = \sum_{j=1}^{n} c_{j}(t) + r_{n}(t)$$
(6)

where $c_j(t)$ is the *j*th IMF and $r_n(t)$ is the signal residue. IMFs are simple oscillatory functions with varying amplitudes and frequencies. EMD is implemented with a sifting procedure (Huang *et al.* 1998).

When EMD is applied to decompose a signal, mode mixing problem may be observed (Lei *et al.* 2009). To overcome this problem, EEMD is developed by defining the true IMF components as the mean of an ensemble of trails. Each trail consists of the decomposition results of the signal plus a white noise of finite amplitude. The procedure of the EEMD method can be found in (Wu and Huang 2009).

When all the IMFs are obtained, Hilbert transform is performed to obtain the instantaneous amplitudes and frequencies. Hilbert transform of an extracted IMF $c_i(t)$ is

$$y(t) = \frac{1}{\pi} P \int \frac{c_i(\tau)}{t - \tau} d\tau$$
(7)

where *P* is the Cauchy principal value of the singular integral. y(t) is Hilbert transform and the complex conjugate of $c_i(t)$. The analytical signal can be obtained as

$$z(t) = x(t) + iy(t) = A(t)e^{i\varphi(t)}$$
(8)

where *i* is the imaginary unit. A(t) and $\varphi(t)$ are instantaneous amplitude and phase angle, respectively. They are denoted as

$$A(t) = \sqrt{x^{2}(t) + y^{2}(t)}$$
(9)

$$\varphi(t) = \arctan\left(\frac{y(t)}{x(t)}\right)$$
 (10)

Hilbert transform gives the information of instantaneous frequency with the first derivative of instantaneous phase angle, which is expressed as

$$\omega(t) = \frac{\mathrm{d}\varphi(t)}{\mathrm{d}t} \tag{11}$$

The obtained instantaneous amplitudes and frequencies as a function of time can form a contour map as Hilbert-Huang Transform (HHT) spectrum, which can be used to detect the structural system characteristic change. In this study, HHT is employed to analyse the measured responses from a composite bridge under moving loads. The conditions of shear connectors are monitored by examining the HHT spectrum of the measured responses. If there is significant abruption and variation in the identified instantaneous frequencies, that means the condition of shear connectors is changed.

3.2 Data fusion

Data fusion combines the measured information from different types of sensors to produce a more specific and comprehensive dataset of the observed model. When applying for structural health monitoring, the objective is to provide a better and more accurate prediction of structural conditions.

Dempster-Shafer theory (Shafer 1976) is one of the most popular and widely used data fusion methods, which is used to deal with the measured information with the assigned measures of "belief". Each data set is assigned a basic probability assignment function, m(x). Particularly when two information sources S_1 and S_2 are used for data fusion, with their associated assigned functions as m1 and m2, the fused information is given as

$$m^{1,2}(\bar{S}) = \frac{\sum_{s_1 \bigcap s_2 = \bar{S}} m_1(S_1) \times m_2(S_2)}{1 - Q}$$
(12)

where Q represents a measure of the degree of conflict between the two sources, which is expressed as

$$Q = \sum_{S_1 \bigcap S_2 = \phi} m_1(S_1) \times m_2(S_2)$$
(13)

The responses from multi-sensors, such as accelerometers and the developed relative displacement sensors, will be are analyzed by using EEMD and HHT for identifying the condition of shear connectors. The measured acceleration and relative displacement are analyzed by using EEMD to obtain the IMFs. HHT spectra are then obtained from the extracted major IMFs. Their HHT spectra will be fused with the abovementioned Dempster-Shafer method. $m_1(S_1)$ and $m_2(S_1)$ can be defined as the output instantaneous amplitude of Hilbert spectrum of Sensors 1 and 2. The data fusion architecture is described and shown



(a) The composite bridge model in the laboratory



(b) Installed relative displacement sensor



(c) The placed sensor locations and the moving direction of the vehicle

Fig. 5 Experimental setup and sensor placement

in Fig. 4. The fused output will also be a spectrum by combining the HHT spectrum from both two sensors.

4. Experimental investigations

Experimental studies on a composite bridge model in the laboratory are conducted to demonstrate the effectiveness and performance of using the proposed approach in detecting the damage of shear connectors in composite bridges under moving loads with multi-sensor vibration measurements.

4.1 Experimental setup

A composite bridge model was constructed in the



Fig. 6 The experimental bridge-vehicle system



Fig. 7 Measured relative displacement and acceleration from the moving load test under the damaged state

laboratory with a reinforced concrete slab supported on two steel girders. Sixteen identical shear connectors were mounted with equal spacing in each girder to link the slab and steel girders. The bridge model was located on two steel frames which were fixed to the laboratory strong floor as shown in Fig. 5(a). The slab constructed with Grade 40 concrete was connected to two 150UB14 universal steel girders by using shear connectors. The design of the composite bridge model allows for simulating the failure of specific shear connectors as well as for resetting to the undamaged state. Steel bolts screwing into metric nuts cast in the slab were used to connect the slab and girder. The nuts were welded onto the reinforcement bar in the slab before pouring. If all bolts are engaged in the nuts and tightened, the structure condition corresponds to the undamaged state. The damage of shear connectors is introduced into the structure by unscrewing several specific metric bolts to simulate the failure of shear connectors.

A relative displacement sensor and an accelerometer were placed on the bridge model. Fig. 5(b) shows the developed relative displacement sensor prototype installed on the bridge. One end of the sensor is fixed on the steel girder with two screws, and the other end on concrete slab. Fig. 5(c) shows the locations of the placed relative displacement sensor and accelerometer. A National



Fig. 8 Damage detection results with only measured relative displacement and acceleration respectively



Fig. 9 Damage detection result with fused multi-type sensor measurements

Instruments (NI) data acquisition system was setup and used for data recording.

4.2 Data fusion for damage detection

A simple vehicle model was fabricated in the lab by using a steel beam with two concrete blocks on top to simulate the mass. A roller system was designed and the crane was used to pull the vehicle to travel on the top of the bridge model with a constant speed as the pulling force of the crane is steady and stable. A fabricated track was placed on the top of the concrete slab to make sure the vehicle is easy to move on a predetermined travelling path. Fig. 6 shows the experimental setup to carry out the dynamic test of the composite bridge under a moving load.

The relative displacement sensor and accelerometer are denoted as RD1 and A1, respectively. They are placed in the center between Shear Connector SC1 and SC2. Dynamic moving load test was conducted with shear connector SC3 removed. The sampling rate is set as 2000Hz. Only the measured data from the damaged state were used for the damage detection. The responses from RD1 and A1 are analyzed, respectively, by using HHT for identifying the condition of shear connectors. Then their HHT spectrum



Fig. 10 Condition assessment results with measurements from the undamaged structure



Fig. 11 Condition assessment results with fused multi-type sensor measurements from the undamaged structure under moving loads

will be fused with the presented Dempster-Shafer method. The detection results before and after the data fusion will be compared to demonstrate the effectiveness of the proposed data fusion method. The Dempster-Shafer method will be combined with time-frequency analysis method, i.e., HHT used in this study to give a fused output of the damage detection results of shear link conditions in the composite bridge model.

Figs. 7(a) and (b) respectively show the measured time domain relative displacement and acceleration responses under the moving load test with shear connector SC3 damaged. Figs. 8(a) and (b) show the damage detection results with measured relative displacement and acceleration, respectively, by using HHT. The detection result with the relative displacement may indicate the damage location, which is close to the true damage location as shown in the HHT spectrum. Since the relative displacement is very small, only about 0.4 mm and the oscillation in the measured relative displacement is not distinct, the detected high energy region in the HHT spectrum is not so obvious at the true damage location. The detection result with the acceleration only may be sensitive

to changes, including the change in loading conditions from moving vehicle owing to varying bridge road surface conditions. There are some false identification results especially at 1m location due to the surface roughness on the track that induces the vehicle impact force on the bridge model.

Fig. 9 shows the damage detection result with the fused information by combing the HHT spectrum obtained from relative displacement and acceleration measurements based on the presented Dempster-Shafer method and timefrequency analysis, i.e., HHT. Optimal selection of weights for different multi-type sensor measurements may be conducted based on the reliability or confidence level of the sensors. However, in this study, the same weight has been assigned to both the measured relative displacement and acceleration, which means m(RD1)=m(A1)=1. Therefore, the obtained instantaneous amplitude of the HHT spectrum will be taken as the input to the Dempster-Shafer method. It can be observed in Fig. 9 that the identified damage location agrees well with the true damage location, and no false identification is generated. Compared with the results as shown in Fig. 8(a), it can be found out that the detection with fused information gives a high contrast and more sensitive performance to the shear link damage.

4.3 Data fusion for undamaged scenario

Measured relative displacement and acceleration from the undamaged state are also analyzed here. They are processed with HHT separately and fused together for the condition assessment. Figs. 10(a) and (b) show the detection results with relative displacement and acceleration respectively. It is shown that when only the relative displacement and acceleration are used individually, some false identification results are observed even not very distinct. Those oscillations around 1m location also proves the same observed false identification due to the surface roughness conditions on the track as shown in Fig. 8(b). Fig. 11 shows the condition assessment results with the HHT spectrum by fusing the relative displacement and acceleration information. It is observed a better result is obtained with no false identification, indicating the data fusion provides a more reliable condition monitoring result.

5. Conclusions

This paper explores using the multi-type sensor vibration measurements, such as from newly developed relative displacement sensors and traditional accelerometers for the damage detection of shear connectors in composite bridge. A data fusion approach based on Dempster-Shafer theory and time-frequency analysis, i.e., HHT in this study, is presented to fuse the multi-type sensor measurements and detect the damage of shear connectors. Experimental studies demonstrate that the damage detection with the proposed approach by combining the measured relative displacement and acceleration gives a more robust and confident result with a higher sensitivity to the damage of shear connectors. The results for the undamaged scenario also show a better and more robust results to remove the false identification results.

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