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Damage detection in beams and plates using wavelet transforms

S. Rajasekaran† and S. P. Varghese‡

Infrastructure Engineering, PSG College of Technology, Coimbatore, Tamilnadu, India
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Abstract. A wavelet based approach is proposed for structural damage detection in beams, plate and delamination of composite plates. Wavelet theory is applied here for crack identification of a beam element with a transverse on edge non-propagating open crack. Finite difference method was used for generating a general displacement equation for the cracked beam in the first example. In the second and third example, damage is detected from the deformed shape of a loaded simply supported plate applying the wavelet theory. Delamination in composite plate is identified using wavelet theory in the fourth example. The main concept used is the breaking down of the dynamic signal of a structural response into a series of local basis function called wavelets, so as to detect the special characteristics of the structure by scaling and transformation property of wavelets. In the light of the results obtained, limitations of the proposed method as well as suggestions for future work are presented. Results show great promise of wavelet approach for damage detection and structural health monitoring

Keywords: damage detection; wavelet analysis; finite difference-composite plates-finite element method-delamination.

1. Introduction

Detection and identification of structural damage is a vital part of the monitoring and servicing of structural systems during their lifetime. Structural damage in normal lifetime may include corrosion, fatigue, ageing or even impact of natural forces. Future intelligent structures would demand high system performance, structural safety, integrity and low maintenance cost. To meet these challenges, structural health monitoring has emerged as a reliable, efficient and economical approach to make a diagnosis to the structural health conditions and to make maintenance decision. The objective of the detection and identification of structural damage is to construct a qualitative or quantitative description of the deterioration in the system. Cracks in the structural elements cause some local variation in the stiffness that affect the static or dynamic behavior of the structure considerably. Change in deflection, frequency patterns and Eigen function of the natural vibration and dynamic stability occur because of existence of such crack. Methods allowing an early detection and localization of cracks have been the subject of intensive investigation for the past two decades.

Methods were developed by which the damage could be detected by finding the changes in Eigen frequencies. However, the effect of a small crack may not be evident from changes to Eigen

† Professor, Corresponding Author, E-mail: drrajasekaran@gmail.com
‡ Graduate Student

frequencies of the structures or from the first several Eigen functions. In the Eigen value analysis, abrupt changes of Eigen function occur in higher order modes and the results obtained for the higher order modes are generally inaccurate because of the limits of mathematical and computational model. Some of the present day developing tools and trends for the damage detection include wavelet theory, simulation verification using modal data, spatial wavelet packet signature analysis and single input–single output measurements. In this paper, effort has been taken to identify damage on structural elements such as beams and plates using the discrete wavelet transforms.

2. State of the art

2.1. Liew and Wang (1998)

Their research presents the first attempt of an application of the wavelet theory for crack identification of structures. As a case study, they have considered the crack identification using wavelet theory for a simply supported beam with non-propagating open crack. A mathematical model of the cracked beam is derived and the wavelet using sine and cosine expression in the space domain is proposed for solution. For comparison purpose, the simply supported cracked beam is solved using both eigen theory as well as the wavelet method of analysis. The results show that crack identification using wavelet analysis is accomplished easily whereas it can hardly be detected by the traditional eigen value analysis. They arrived at the conclusion that wavelet analysis can be applied easily to investigate the eigen function rather than the tedious application of eigen theory.

Also, they highlighted that wavelet analysis will not encounter the mathematical and computational limitations as shown by Eigen value analysis.

2.2. Hou, Noori and Amand (2000)

A wavelet based approach is proposed for structural damage detection and health monitoring in this paper. Here characteristics of representative vibration signals under the wavelet transformation are examined. The methodology is then applied to simulation data generated from a simple structural model subjected to a harmonic excitation. This model consists of multiple breakable springs, some of which may suffer irreversible damage when the response exceeds a threshold value or the number of cycles of motion is beyond their fatigue life. In cases of abrupt or accumulated damage, its occurrence can be determined in the details of wavelet decomposition of these data.

Real time acceleration data of San Fernando earthquake is also analyzed with this technique as a verification for their work.

2.3. Corbin, Hera and Hou

Their research presents an application of wavelet analysis for damage detection and locating damage regions in structures. Simulation data were generated for a three degree of freedom mass spring dashpot model, a cantilever beam and a Finite element model for buildings used in ASCE benchmark data studies (Hera and Hou 2004). Both harmonic and random excitations are employed. Damages were introduced either by breakage of springs for an excessive response or removal of certain stiffness of structures at a specified moment. Wavelets were used to analyze the simulation data and the results were used to determine the moment when the damage occurred and hence in localizing the damage.

2.4. Douka, Loutridis and Trochidis (2003)

In their research the fundamental vibration mode of a cracked beam is analyzed using continuous wavelet transform and both the location and size of crack are estimated. The position of crack is located by sudden change in the spatial variation of the transformed response. To estimate the size of the crack, an intensity factor is defined to relate the size of the crack to the coefficients of the wavelet transform. An intensity factor law is established which allows accurate prediction of crack size. The viability of the proposed method is investigated both analytically and experimentally in the case of a cantilever beam containing a transverse surface crack. Results are discussed and suggestions for future work are also presented.

2.5. Ovanesova and Suarez (2004)

Their paper presents application of the wavelet transform to detect cracks in beams and plane frame. The method requires the knowledge of only the response of the damaged structures, i.e., no information about the original undamaged structure is required. In addition, it is shown that the procedure can detect the crack by using a response signal from dynamic or static loads. The results of the simulation show that if a suitable wavelet is selected, the method is capable to extract damage information from the response signal in a simple, robust and reliable way.

2.6. Wavelets

Wavelet is a theoretical formalism that was initiated by a French seismologist (Morlet and Grossman 1984) in context of quantum physics. But it was initiated by Joseph Fourier (1807) with his theories of frequency analysis. After 1807, the first mention of wavelet came into mathematical field through the thesis of Haar in 1909. Others who gave pioneering contributions to wavelet theory include Daubechies (1996), Meyer (1993) Mallat (1988), Coifman and Wickerhauser. (see tutorial on Polikar on wavelets in the web site given)

A wavelet is a function with two important properties namely oscillation and short duration. A function $\Psi(x)$ is a wavelet if and only if its Fourier transform $\Psi(u)$ satisfies the condition between $-\infty$ to $+\infty$, $\int \Psi(u)\, du = 0$. It also must satisfy not only zero mean condition but also the admissibility condition. The domain between $-\infty$ to $+\infty$ is called full domain. For practical purposes, it is required that wavelet is concentrated in a limited interval, or in other words, it has compact support. Wavelet analysis starts by selecting a basic wavelet function that can be a function of space x or time t. This basic wavelet function called the mother wavelet is dilated by 'a' and translated in space by 'b' to generate a set of basis functions as follows.

$$\Psi_{a,b}(x) = \frac{1}{\sqrt{a}}\Psi\left(\frac{x-b}{a}\right) \tag{1a}$$

and wavelet transform is defined as

$$C(a,b) = \int_{-\infty}^{\infty} f(X)\frac{1}{\sqrt{|a|}}\Psi^*\left(\frac{x-b}{z}\right)dx \tag{1b}$$

The above function is centered at b and the spread proportional to a. Wavelet transform correlates the function $f(x)$ with $\psi_{a,b}(x)$ where a, b are real numbers and a must be positive and * denotes complex conjugate of the parent wavelet $\Psi(x)$ through the convolution of signal and the scaled parent wavelet.

It is to be noted that a wavelet associated with mother wavelet $\Psi(x)$ is generated by two operations : - dilation and translation. The translation parameter b indicates the location of the moving wavelet window in the wavelet transform. Shifting the wavelet window indicates examining a signal in the neighbourhood of the current location. The dilation parameter a indicates the width of wavelet window. A smaller value indicates a high resolution filter i.e., the signal is examined through narrow wavelet window in a smaller scale. The signal $f(x)$ may be reconstructed by an inverse wavelet transform of $C(a, b)$ as

$$f(x) = \frac{1}{C_\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(a,b)\,\Psi\left(\frac{x-b}{a}\right)\frac{1}{a^2}da\,db \tag{2a}$$

where

$$C_\psi = \int_{-\infty}^{\infty}\frac{|F_\psi(\omega)|^2}{|\omega|}d\omega \tag{2b}$$

The mother wavelet needs to satisfy the admissibility condition such that $C_\psi<\infty$ to ensure the existence of inverse wavelet transform.

In a continuous wavelet transform (CWT) a large number of wavelet coefficients $C(a, b)$ are generated during analysis. CWT is highly redundant and it is not necessary to use the full domain to reconstruct the original signal $f(x)$. So instead of continuous dilation and translation, in a practical signal processing, a discrete version of wavelet transform is often employed. Dilation is defined as $a = 2^j$ and the translation parameter as $b = k\,2^j$, $j, k \in Z$ where Z = a set of positive integers. For a special choice of $\Psi(x)$ the corresponding discretized wavelet $\Psi_{j,k}$ is defined as

$$\{\Psi_{j,k}(x)\} = 2^{-j/2}\,\Psi(2^{-j}x-k) \quad j,k\in Z \tag{3a}$$

constitute an orthogonal base for $L^2(R)$.

This sampling of coordinates is referred as dyadic sampling because consecutive values of discrete scales differ by a factor of 2. So using dyadic scale one can define the discrete wavelet transforms (DWT) as

$$C_{j,k} = 2^{-j/2}\int_{-\infty}^{\infty} f(x)\,\Psi(2^{-j/2}x-k)dx = \int_{-\infty}^{\infty} f(x)\,\Psi_{j,k}(x)dx \tag{3b}$$

When the wavelet transform is only available for small scale of $a<a_o$ and if one wants to recover it, one has to complement the information for $a>a_o$. To obtain this information, another function called scaling function is introduced.

$$D(a_0,b) = \int_{-\infty}^{\infty} f(x)\,\Phi_{(a,b)}(x)dx \tag{4}$$

Scaling function does not exist for all wavelet. If the dyadic scale for a and b is applied at a level N, we get one set of coefficients as

$$cD_N(k) = \int_{-\infty}^{\infty} f(x)\,\Psi_{N,k}(x)d \tag{5}$$

This set is called detail coefficients. When we use the scaling function, we get

$$cA_N(k) = \int_{-\infty}^{\infty} f(x)\,\Phi_{N,k}(x)dx \tag{6}$$

This set of coefficients are called approximate coefficients. The discrete version of the reconstructed signal is

$$f(x) = \sum_{j=-\infty}^{N}\left(\sum_{k=-\infty}^{\infty} cD_j(k)\,\Psi_{j,k}(x)\right) + \left(\sum_{k=-\infty}^{\infty} cA_N(k)\,\Phi_{j,k}(x)\right) \tag{7}$$

Above equation tells that the original function can be expressed as the sum of its approximation at Nth level plus the sum of all the details up to that level

$$f(x) = A_N(x) + \sum_{j \le N} D_j(x) \tag{8}$$

2.7. WHY….?

Wavelet transform (WT) (Christian 2003) is capable of providing the time and frequency information simultaneously, hence giving a time-frequency representation of the signal. WT was developed to overcome resolution related problems of the Short time Fourier transform. The actual physical concept involved in wavelet transform is, we pass the time-domain signal from various high pass and low pass filters, which filters out either high frequency or low frequency portions of the signal. This procedure is repeated, every time some portion of the signal corresponding to some frequencies being removed from the signal. Suppose we have a signal which has frequencies up to 1000 Hz. In the first stage we split up the signal in to two parts by passing the signal from a high pass and a low pass filter (filters should satisfy some certain conditions, so-called admissibility condition) which results in two different versions of the same signal: portion of the signal corresponding to 0-500 Hz (low pass portion), and 500-1000 Hz (high pass portion). Then, we take either portion (usually low pass portion) or both, and do the same thing again. This operation is called decomposition. Assuming that we have taken the low pass portion, we now have 3 sets of data, each corresponding to the same signal at frequencies 0-250 Hz, 250-500 Hz, 500-1000 Hz. Then we take the low pass portion again and pass it through low and high pass filters; we now have 4 sets of signals corresponding to 0-125 Hz, 125-250 Hz, 250-500 Hz, and 500-1000 Hz. We continue like this until we have decomposed the signal to a pre-defined certain level. Then we have a bunch of signals, which actually represent the same signal, but all corresponding to different frequency bands. We know which signal corresponds to which frequency band, and if we put all of them together and plot them on a 3-D graph, we will have time in one axis, frequency in the second and amplitude in the third axis. This will show us which frequencies exist at which time.

There are different wavelets that can be created. Wavelets are composed of a family of basis functions that are capable of describing signals in a localized time and frequency format. The sets of basis functions have compact support meaning all their energy is localized to a finite space in time. Wavelet analysis starts with a basis wavelet function called the mother wavelet or generating wavelet or prototype wavelet. This prototype wavelet is then shifted and scaled to generate a family of ortho-normal basis function.

3. Damage analysis of beams

3.1. Mathematical modeling

Following the procedure of Liew and Wang (1998), a uniformly simply supported Euler–Bernoulli beam of unit length was considered. The governing differential equation of the beam and its boundary conditions are

$$\frac{\partial^2 y(x,t)}{\partial t^2}+A\frac{\partial^4 y(x,t)}{\partial x^4}=0 \qquad \begin{array}{l} y(0)=y''(0)=0 \\ y(1)=y''(1)=0 \end{array} \tag{9}$$

where A = material constant = EI / M; I = moment of inertia; M = mass; E = modulus of elasticity and y, the transverse deflection of the beam. y_1 and y_2 are the deflections of the beam to the left and right portions of the crack of the beam. It is assumed that the crack only changes the stiffness of the beam and mass remains unchanged.

To construct the mathematical model for the uniform simply supported cracked beam, we assume that y_i be the displacement of the point "i" at the position of the crack. The beam is divided into two sections, as shown in Fig. 1**,** where section I is the left part of the crack and section II is the right part of the crack. An initial imperfection of sine variation of the beam is considered and the response of the member after 0.1s time interval is calculated using finite difference method. Assuming $C = A\Delta t^2 = 1$; the relation connecting initial and final displacement is as

$$\{y\}_{0.1} = \{y\}_0 - c[K]\{y\}_0 \tag{10}$$

where $[K]$= stiffness matrix; $\{y\}_o$ =initial displacement (see Fig. 2). In the stiffness matrix $[K]$, θ is the non-dimensional parameter giving the ratio of the depth of the crack to the depth of the beam. The response displacement values are given as the input for the MATLAB program. Applying discrete wavelet transform using Daubechies 4, one will be able to get the approximate and detailed coefficients. Various wavelet families are tried and Daubechies 4 gives quite accurate results. These detailed coefficients are plotted across the length of the beam. Sudden variation or kinks can be noted at the point of damage. It is to be noted that Liew and Wang (1998) used sin 2 $\pi x/L$, cos 2 $\pi x/L$ as two sets of wavelet expressions whereas the authors have considered Daubechies 4 wavelet family.

In this work, the beam has been divided into equal divisions. Then the response values are calculated and applied as input for the program. Initial work was done on the beam divided into 12

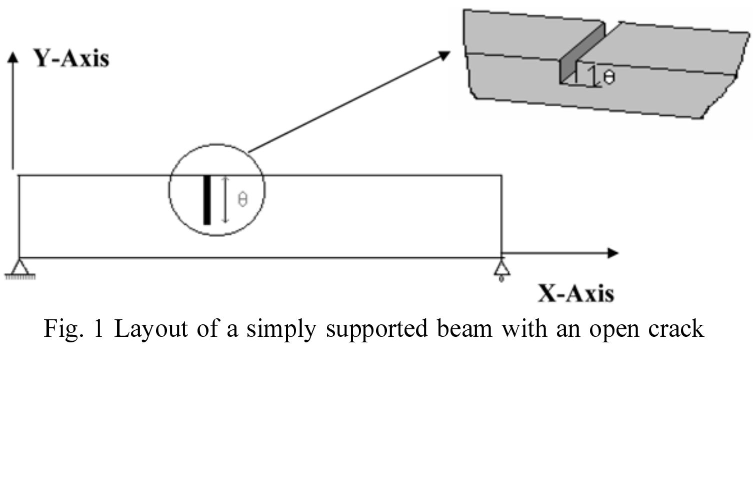


Fig. 1 Layout of a simply supported beam with an open crack

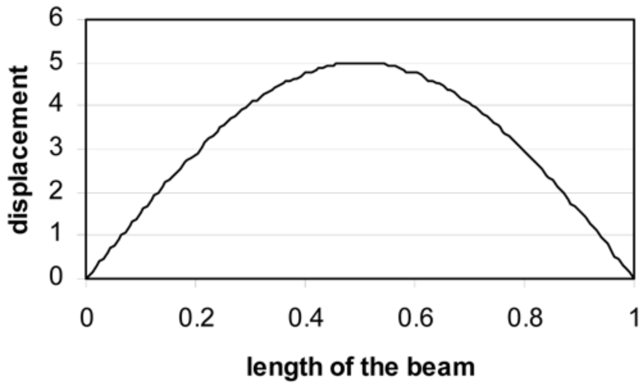


Fig. 2 Deflection response at t=0.1 sec

parts. It was extended to 30, 50 and 100 divisions. The coefficient values which are available from the wavelet transforms are plotted along the length of the beam. These coefficients are the detailed part of the original signal. Presence of variation from the normal graph shows the presence of a crack.

3.1.1. Numerical example 1

As an example, damage is induced at 0.2 Divisions: 100. In this case, both initial displacement and the deflected shape are plotted on the same layout (Fig. 2). Since the beam is divided into 100 parts, it is not possible to visually judge the position of the damage by mere comparison of these two graphs.

From the wavelet coefficients of Fig. 3, location of damage was identified as around 0.2, because a large kink was noted on the graph at 0.2. The wavelet used was db4 and only first level of decompositions had to be performed

3.2. Comparative study of different wavelets for a simply supported beam

For this work a comparative study has been carried out with symlets and coiflets. From the Figs. 4 and 5, one can evidently see that variation in the spectrum at the damaged position. But along

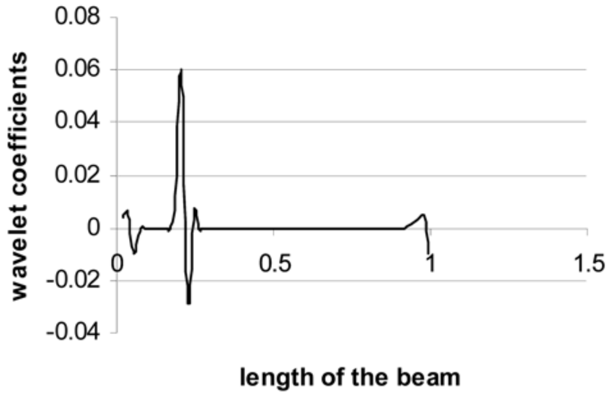


Fig. 3 Wavelet coefficients (db1) plotted along the length (one level of decomposition – example-1)

with these kinks, we also get other variations of equal importance. So predicting the location of the damage based on these wavelets would be difficult in practical applications. So, a conclusion can be drawn from the above comparative study that all kinds of wavelets cannot be used for all applications. This conclusion has also been arrived at by Ovanesova and Suarez (2004). Depending upon the type of application, wavelet family will have to be changed. Also, application and the type of output determine the order of the wavelet to be applied.

3.3. Study of change in coefficient value as depth of crack changes

A comparative study of the depth of the crack and the value of 10^{th} detailed coefficient is made in Fig. 6. Depth of crack was varied from a value of .05 to 0.8 and the corresponding values of the coefficients are found out. It is seen that as the value of crack is increased from 0.05 to 0.1 there is a steep increase in the coefficient value. Once the depth increases beyond 0.1 the value falls till the depth is 0.2 and then there is a slight decrement in the value of coefficient. From this, it is evident that for very small cracks or irregularities the detailed coefficient has high value. However, wavelet will be able to predict the zones of both small and deep cracks.

4. Wavelet transform in two dimensions

Until now one-dimensional wavelet transforms has been discussed. To transform images one has to use two-dimensional wavelets (Arivazhagan and Ganesan 2003, Rao and Bopardikar 1998, Seungcheol and Roman 2004) or apply the one-dimensional transform to the rows and columns of the image successively as separable two-dimensional transform. In most of the applications, where wavelets are used for image processing and compression, the latter choice is taken, because of the low computational complexity of separable transforms.

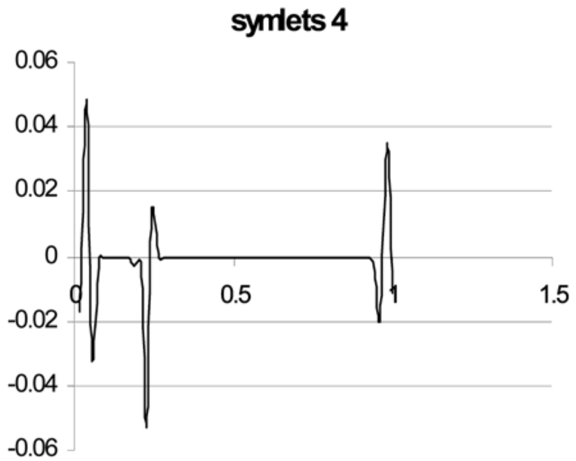


Fig. 4 Wavelet coefficient (symlets 4) plotted against the length – one level decomposition (example – 1) (x- axis : - section along the length y- axis:- wavelet coefficient)

Fig. 5 Wavelet coefficient plotted against length of the beam (level- 1) example. 1 x-axis :- section along the length y-axis: - wavelet coefficient

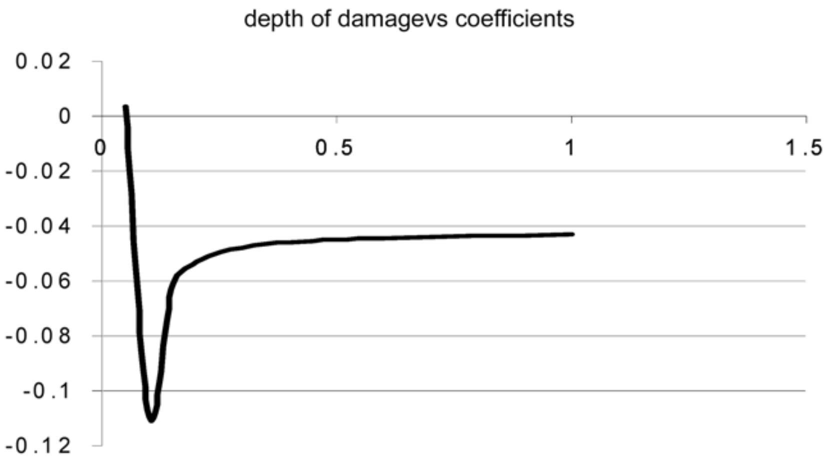


Fig. 6 Variation of wavelet coefficient with respect to crack depth (example. 1)- x-axis – crack depth y-axis – wavelet coefficient

Let $f(x, y)$ be a square integrable function in the two variables x and y and $\Phi(t)$ be a one dimensional scaling function. Consider a two dimensional function

$$S_{\phi\phi}(x,y) \Rightarrow \phi(x)\phi(y) \tag{11}$$

Let $f_0(x, y)$ be the projection of $f(x, y)$ on the linear vector space. Then

$$f_0(x,y) = \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} a_0(i,j)s_{\phi\phi}(x-i,y-j)$$
$$\text{where,} \quad a_0(i,j) = <f(x,y), \ \ s_{\phi\phi}(x-i,y-j)> \tag{12}$$

Consider

$$f_{-1}(x,y) \Rightarrow \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} a_{-1}(i,j), \ \ s_{\phi\phi}(2x-i,2y-j) \tag{13}$$

Substituting Eq. (11) in Eq. (10), we have

$$f_{-1}(x,y) \Rightarrow \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} a_{-1}(i,j)\phi(2x-i)\phi(2y-i) \tag{14}$$

Let

$$f_{-1}(x,j) \Rightarrow \sum_{i=-\infty}^{\infty} a_{-1}(i,j)\phi(2x-i) \tag{15}$$

Then,

$$f_{-1}(x,y) \Rightarrow \sum_{i=-\infty}^{\infty} f_{-1}(x,j)\phi(2y-j) \tag{16}$$

This can be split into two orthogonal functions; that is, it can be expressed as the sum of two functions namely

$$f_0(x,j) \Rightarrow \sum_{n=-\infty}^{\infty} a_{j,0}(n)\,\phi(x-n) \tag{17}$$

$$g_0(x,j) \Rightarrow \sum_{n=-\infty}^{\infty} b_{j,0}(n)\,\psi(x-n) \tag{18}$$

So,

$$f_{-1}(x,j) = f_0(x,j) + g_0(x,j) \tag{19}$$

Putting Eq. (19) in Eq. (16)

$$f_{-1}(x,y) = \sum_{j=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_{j,0}(n)\,\phi(x-n)\,\phi(2y-j) + \sum_{j=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} b_{j,0}(n)\,\psi(x-n)\,\phi(2y-j) \tag{20}$$

One can define

$$f_{-1}(n,y) \Rightarrow \sum_{n=-\infty}^{\infty} a_{j,0}(n)\,\phi(2y-j) \tag{21}$$

$$g_{-1}(n,y) \Rightarrow \sum_{n=-\infty}^{\infty} b_{j,0}(n)\,\psi(2y-j) \tag{22}$$

Then,

$$f_{-1}(x,y) = \sum_{n=-\infty}^{\infty} f_{-1}(n,y)\,\phi(x-n) + \sum_{n=-\infty}^{\infty} g_{-1}(n,y)\,\psi(x-n) \tag{23}$$

$$f_{-1}(n,y) = A_0(n,\ y) + B_0(n,\ y) \tag{24}$$

$$g_{-1}(n,y) = C_0(n,\ y) + D_0(n,\ y) \tag{25}$$

Where:

$$A_0(n,\ y) \Rightarrow \sum_{p=-\infty}^{\infty} a_0(n,\ p)\,\phi(y-p) \tag{26}$$

$$B_0(n,\ y) \Rightarrow \sum_{p=-\infty}^{\infty} b_0(n,\ p)\,\psi(y-p) \tag{27}$$

$$C_0(n,\ y) \Rightarrow \sum_{p=-\infty}^{\infty} c_0(n,\ p)\,\phi(y-p) \tag{28}$$

$$D_0(n,\ y) \Rightarrow \sum_{p=-\infty}^{\infty} d_0(n,\ p)\,\psi(y-p) \tag{29}$$

Again defining,

$$S_{\phi\psi} = \phi(x)\,\psi(y) \tag{30}$$

$$S_{\psi\phi} = \psi(x)\,\phi(y) \tag{31}$$

$$S_{\psi\psi} = \psi(x)\,\psi(y) \tag{32}$$

$$S_{\phi\phi} = \phi(x)\,\phi(y) \tag{33}$$

Putting everything together, we get:

$$f_{-1}(x,y) = \sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} a_0(n,\ p)s_{\phi\phi}(x-n,\ y-p) + \sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} b_0(n,\ p)s_{\phi\psi}(x-n, y-p) + \sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} c_0(n,\ p)s_{\psi\phi}(x-n,\ y-p) + \sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} d_0(n,\ p)s_{\psi\psi}(x-n,\ y-p) \tag{34}$$

The first term in the equation forms the approximate part of the output while the other three terms make up for the details along horizontal, vertical and diagonal directions. Hence, one can write the working of 2-D transform as

$$S = CA + CH + CV + CD$$

where the terms represent signal, approximate coefficients, horizontal coefficients vertical coefficients and diagonal coefficients respectively. For more details refer (Arivazhagan and Ganesan 2003). The physical description of the application of two- dimensional wavelet is very much similar to its one dimensional wavelets. The signal is made to pass through a low pass and a high pass filter. The output hence obtained is combined to give the transformed image as shown in Fig. 7. The input image is replaced by four images. Each of these four images requires only one-fourth the numbers of pixels as the original. In layman's terms, this can be depicted as shown in Fig. 8. The original signal is subjected to a pair of filters; both low pass and high pass. Two residual outputs are obtained from each of these filters. Four different combinations are possible with these four outputs namely LL, HL, LH and HH. Viz: -(Low Low, High Low, Low High, High High) as shown in Fig. 8. Each of these combinations has different interpretations.

LL- the upper left quadrant consists of all coefficients, which were filtered by the analysis low pass filter along the row and then again along the column. This sub block represents the approximated version of the original signal at half its resolution.

HL/LH- the upper right block and the lower left block were filtered along the rows and columns with low pass and high pass filter alternatively. LH block contains the vertical edge while the HL block contains the horizontal edges clearly.

HH- the lower right quadrant was derived analogously to that of the upper left quadrant but with the row and column being filtered by high pass filter. this block can be interpreted as the area where edges of the original image along the diagonal direction.

The next level of decomposition will be on the upper left quadrant. The sub bands at the next higher level will be named as LL^1, HL^1, LH^1 and HH^1. Multi resolution scheme after one and two levels of decomposition is shown in Fig. 9.

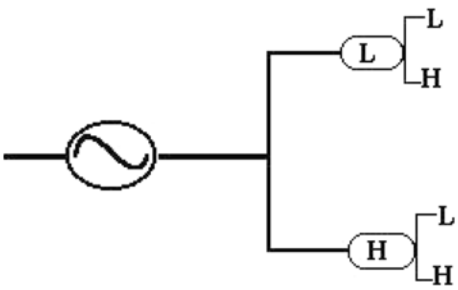


Fig. 7 Schematic representation of 2-D wavelet transform

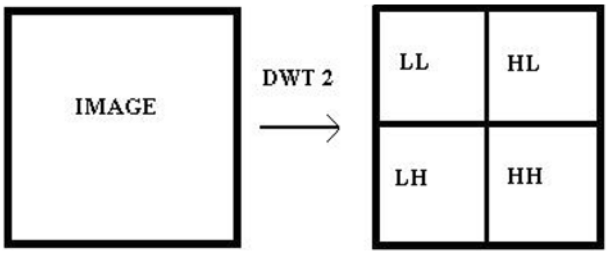
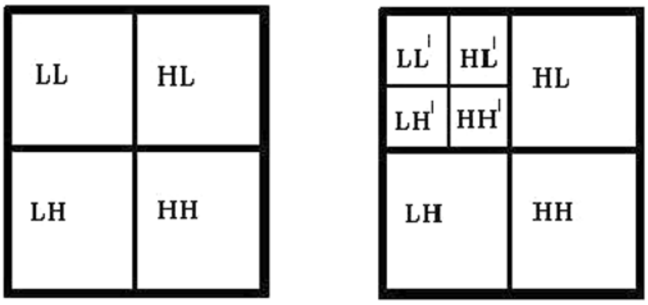


Fig. 8 2-D Wavelet transform of an image



a b

Fig. 9 Multi-resolution scheme after (a) one level and (b) two levels of decomposition

4.1. Numerical example 2

A simply supported isotropic plate of size 1 m × 1 m subjected to udl of 1 kN/sq.m was analyzed by a standard analysis package namely STAAD-Pro. The element was mesh up into 2500 elements by a 50 by 50 mesh. Damage was induced at (0.6, 0.2) from the bottom left support of the plate on the required element by reducing the modulus of elasticity. The nodal displacements was obtained from the analysis. Fig 10 shows the deformed shape of the plate due to loading.

4.1.1. Discussion

Analysis of the plate was performed and the wavelet coefficient is plotted as shown in Fig. 11. The values obtained as response was given as an input for wavelet transforms. The wavelet used was db1 and only first level of decompositions had to be performed. The output spectrum in Fig. 11. shows the plot of detailed coefficients and irregularity at the damage induced point as shown.

4.2. Numerical example 3

In this example damage was induced at (0.26, 0.42) from top left support of the plate of example 1. From the deformed shape one cannot make out the damage. Fig. 12 shows the output obtained after applying wavelet transforms. All the four coefficients are plotted as seen in Fig. 12. We can see a variation in the HH plot of the output. A zoomed view of this plot is shown in Fig. 13. Location of damage can be seen as a red pixel amidst blue pixel

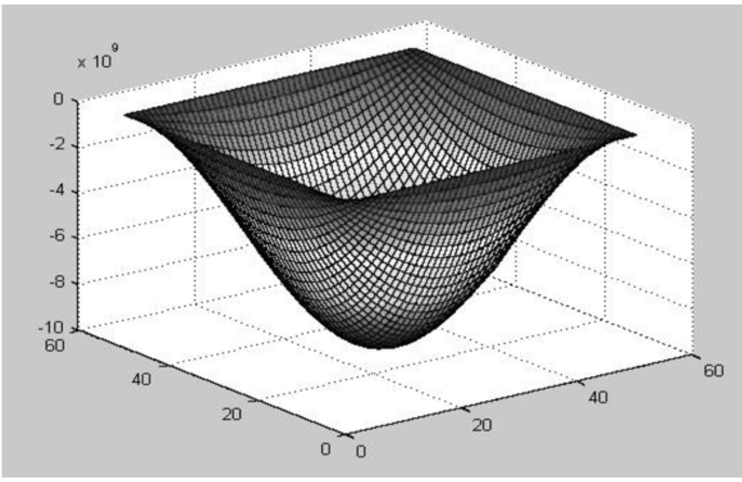


Fig. 10 Deformed shape *Z*- axis :- wavelet coefficient

4.3. Numerical example 4

Damage diagnosis of 8 layered simply supported composite plate of carbon fibres (0/45/60/90) sy is carried out. Composites are becoming an essential part of today's material because they offer advantages such as low weight, corrosion resistance, high fatigue strength, faster assembly etc. They

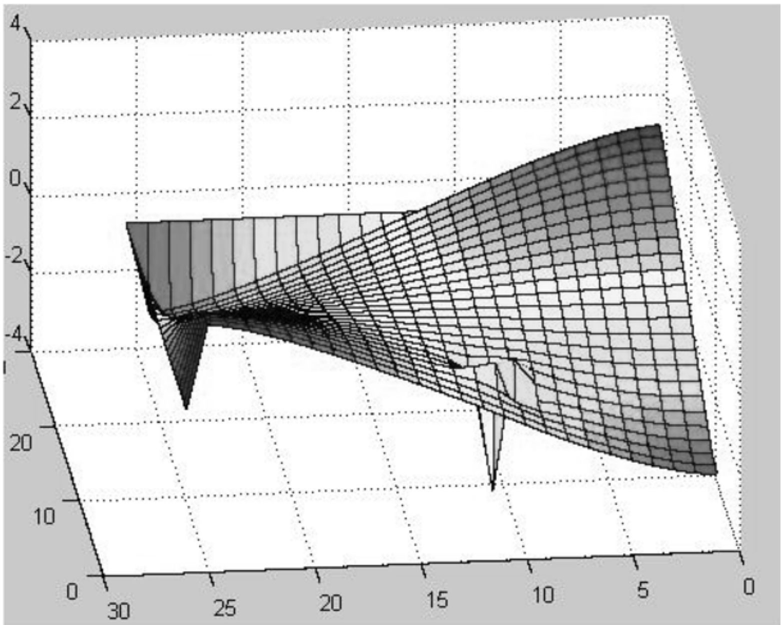


Fig. 11 Output spectrum showing the damage (grid 14,5)=(0.6,0.2) *Z*- axis:- wavelet coefficient

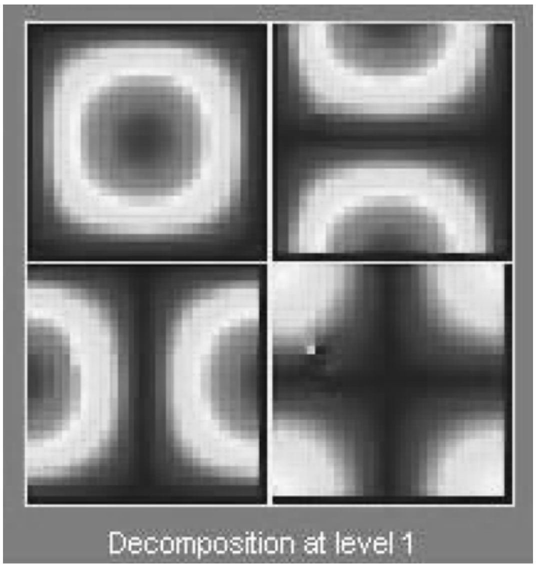


Fig. 12 Output showing the approximate and the detail coefficients

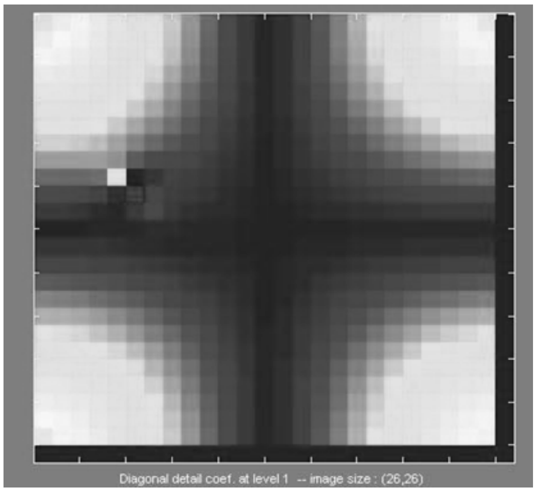


Fig. 13 Full size view of diagonal detail coefficient showing the damage

are used as material in making aircraft structures, space vehicles to home building. A composite consists of combining two or more constituents such as reinforcing phase and matrix. The reinforcing phase is in the form of fibres, particles or flakes and matrix is continuum.

Impact of a composite plate reduces the strength of laminate and also initiates delamination in composites. Delamination becomes more problematic since many times from visual examination one cannot detect the damage due to delamination. In this example delamination in a composite plate is induced by removing a layer of ply at the required section. Analysis is carried out using FEASTC (1993).

4.3.1. Discussion

Analysis of the plate was performed using FEASTC (1993) and the deformed shape of the element was obtained. The values obtained as response was given as an input for wavelet transforms. The wavelet used was db1 and only 1 level of decompositions had to be performed. Damage was induced on the plate by removing the plies at the required place. Three such outputs are plotted. Fig. 14 shows the plot of diagonal coefficient for a plate where seven plies were removed from the center. The pop up at the center of the plot depict the presence of damage. Fig. 15 shows the plot of diagonal coefficient for a plate where six plies were removed from the center. The uplift at the center of the plot helps us to identify the damage. Fig. 16 shows the plot of diagonal coefficient for a plate where five plies were removed at the center. The variation at the center of the plot isolates that location from the remaining portion of the slab and hence points out the location of damage.

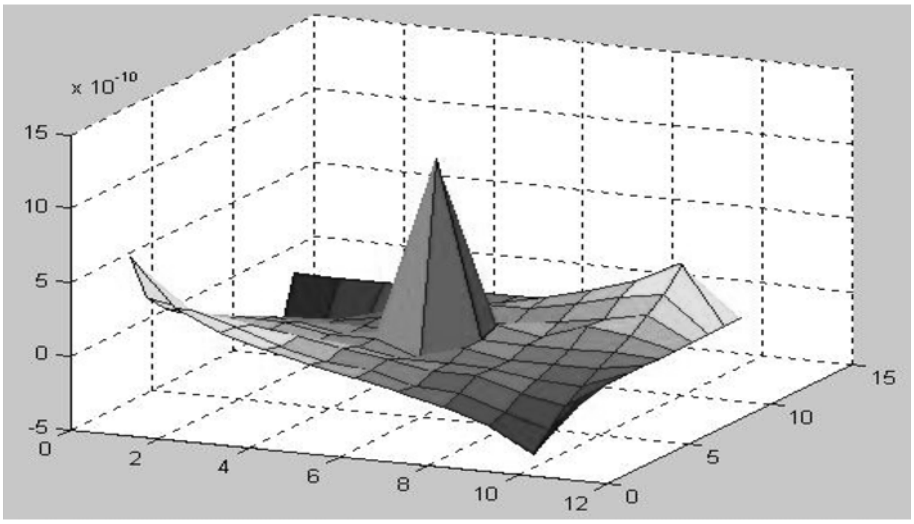


Fig. 14 Diagonal detail coefficients plotted showing Danage (7 plies removed)-z-axis :- wavelet coefficient xy- plane of the plate

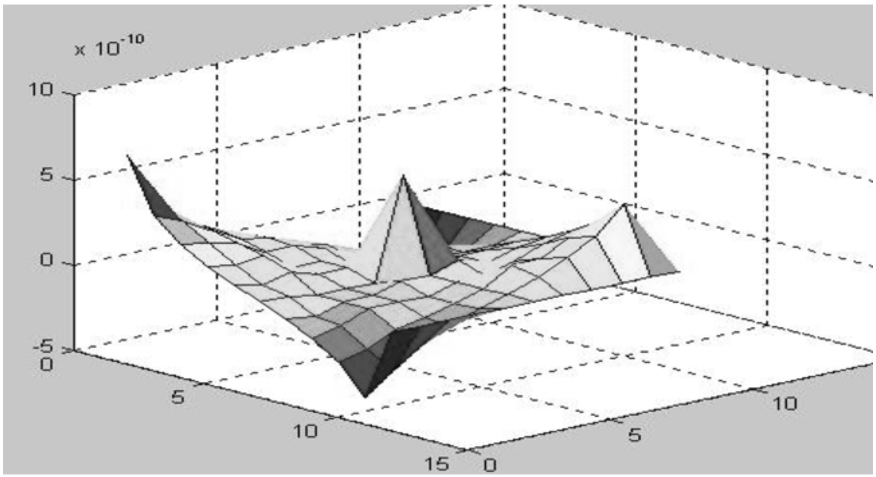


Fig. 15 Diagonal detail coefficients plotted showing damage (6 plies removed) z-axis:- wavelet coefficient; xy – plane of the plate.

4.4. Variation of coefficient with damage

Variation of the number of undamaged plies with the value of detailed coefficient is as shown in Fig. 17.

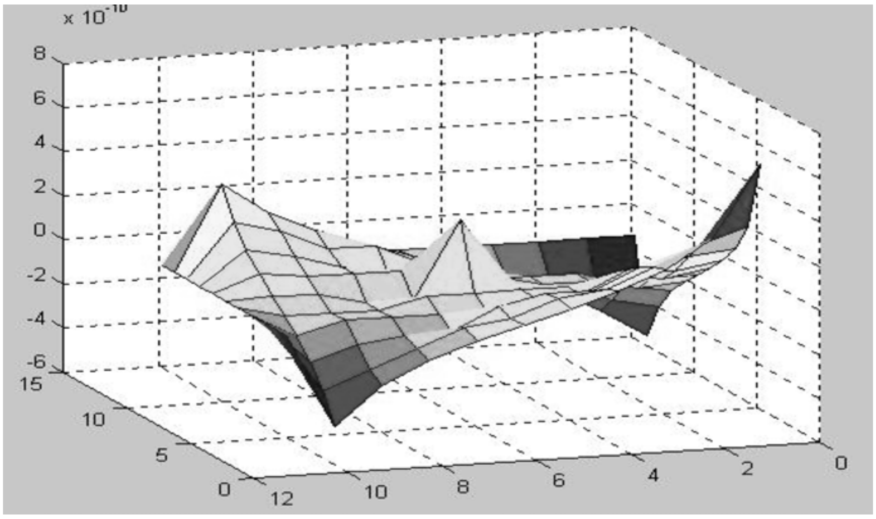


Fig. 16 Diagonal detail coefficients plotted showing damage (5 plies removed) *z*- axis:- wavelet coefficient; *xy* – plane of the plate

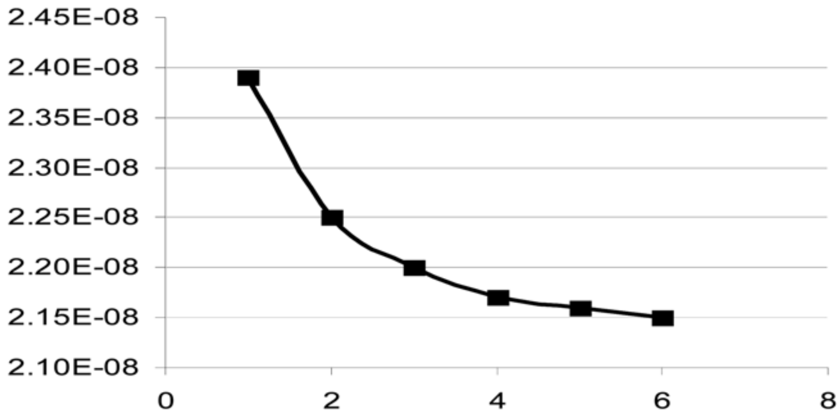


Fig. 17 Variation of wavelet coefficient (*y* axis) with the number of undamaged plies (*x* axis)

5. Conclusions

The wavelet method of analysis has been applied to the structure and some of the hidden features like crack location, delamination in composites etc of the structure could be identified. The results of the simulation of the four examples show that if a suitable wavelet is selected the method is capable to extract damage information form the response of the signal in a simple, robust and reliable way. The following advantages can be substantiated with evidences from the works carried out as well as from the literatures that has been studied. They include.

- Wavelet analysis can be applied easily to investigate the structural health as compared to the first principle methods
- No problem relating to mathematical and computing limitations will be encountered

• Even very small variations can be found out ,if the proper wavelet is selected

It can also be found out that fourth order Daubechies wavelets give good results for beam examples. This may be because the fourth order wavelets give the fourth derivative of the signal and permits additional investigation of the signal for all scales of interest. This can also be evidently supported by the works of Douka, *et al.* (2003). Using 2-D wavelets the damage of isotropic plate elements and composite plates are predicted with much success. STAAD-Pro and FEASTC were used for analysis. Damage was induced by reducing the modulus of elasticity of the required element for isotropic plates and delamination in case of composite plates. Wavelet used for prediction was db1. Of the three coefficients obtained, diagonal details show the location of damage.

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