An improvement on the concrete exothermic models considering self-temperature duration

Zhenyang Zhu^{*1}, Weimin Chen^{2a}, Sheng Qiang^{3a}, Guoxin Zhang^{1a} and Youzhi Liu^{1a}

¹State Key Laboratory of Simulation and Regulation of Water Cycle in River Basin, China Institute of Water Resources and Hydropower Research, Beijing, 100038, P.R. China ²Hydrochina Huadong Engineering Corporation, Hangzhou, 310014, P.R. China ³College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing, 210098, P.R. China

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Abstract. Based on the Arrhenius equations, several hydration exothermic models that precisely calculate the influence of concrete's self-temperature duration on its hydration exothermic rate have been presented. However, the models' convergence is difficult to achieve when applied to engineering projects, especially when the activation energy of the Arrhenius equation is precisely considered. Thus, the models' convergence performance should be improved. To solve this problem and apply the model to engineering projects, the relationship between fast iteration and proper expression forms of the adiabatic temperature rise, the coupling relationship between the pipe-cooling and hydration exothermic models, and the influence of concrete's self-temperature duration on its mechanical properties were studied. Based on these results, the rapid convergence of the hydration exothermic model and its coupling with pipe-cooling models were achieved. The calculation results for a particular engineering project show that the improved concrete hydration exothermic model and the corresponding mechanical model can be suitably applied to engineering projects.

Keywords: hydration exothermic behaviors; self-temperature duration; concrete mechanical properties; convergence; coupling

1. Introduction

Thermal cracks are one of the most common types of cracks in concrete (Emborg 1989, Zhu and Qiang 2011, Liu and Qiang 2011, Zhu and Qiang 2013). Thermal cracking and crack-healing measures have attracted considerable attention (Guo and Wang 2015, Luo and Qiang 2015). Self-temperature duration can affect the thermal and mechanical properties of concrete (Wang and Yan 2013, Park and Kwon 2015, Li and Mao 2016, Kim and Yang 2016). The hydration exothermic behavior of early-age concrete is markedly influenced by its self-temperature duration (Suzuki and Harada 1998, Cervera and Faria 2002, Schutter 2002, Hatte and Thorborg 2003). That is, a higher self-temperature results in a faster hydration exothermic rate.

The adiabatic temperature of concrete can be measured using a concrete adiabatic-temperature testing device. However, the increase in adiabatic temperature tested with devices cannot be directly used in engineering projects, because the influence of the self-temperature duration of an early-age concrete on its hydration exothermic rate should be considered when calculating the temperature field of the concrete under arbitrary boundary conditions. Therefore, the influence of the concrete self-temperature duration on its hydration exothermic rate is an important and difficult problem. Several models consider the effects of self-

(a) when the activation energy of the Affieldus equations is simplified to a constant, convergence may be obtained. However, when precisely considering activation energy and under an arbitrary boundary condition, convergence cannot be achieved. Thus, the convergence performance of those models should be improved.

(b) There is a coupling relationship between the pipecooling model and the hydration exothermic model when calculating the temperature field of mass concrete with cooling pipes. The coupling method should be further studied because pipe cooling is widely used in concrete temperature control.

To solve these problems, this paper investigates on the rapid convergence of the hydration exothermic model, its coupling with pipe-cooling models, and a model of concrete mechanical properties that considers temperature duration.

2. Theoretical basis

2.1 The hydration exothermic model considering temperature duration

temperature duration on the hydration of concrete (Zhu 2003, Zhang and Feng 2004, Zhang and Zhou 2004, Zhu 2014). However, the convergence performance of these models and their corresponding concrete mechanical models should be improved and created for their application to engineering projects: (a) When the activation energy of the Arrhenius

^{*}Corresponding author, Professor

E-mail: 1219921552@qq.com

^aProfessor



Fig. 1 E(a)/R expressed by a cubic spline curve

Hydration degree is the ratio between the quantity of hydrated cementing materials and the total quantity of cementing materials. Based on previous studies on mass temperature and temperature stress field, when mixed with the same materials and with the same hydration degree, concrete elements can be regarded as having the same thermal and mechanical properties. An equal amount of cementing materials can generate an equal amount of hydration heat. Thus, the hydration degree can also be expressed by the ratio between released hydration heat and total quantity of hydration heat. The hydration rate of concrete elements mixed with the same materials and with the same hydration degree satisfies the Arrhenius equation.

Based on the Arrhenius equation, several hydration exothermic models have been presented. In those models, assuming two concrete blocks with the same materials and hydration degree, the hydration exothermic rate of concrete can be expressed as follows (Zhu 2014)

$$\frac{K_b}{K_a} = \exp\left[\frac{E_a}{R}\left(\frac{1}{T_a + 273} - \frac{1}{T_b + 273}\right)\right]$$
(1)

where K_a is the hydration exothermic rate of concrete with temperature T_a , K_b is the hydration exothermic rate of concrete with temperature T_b , E_a is the activation energy (J/mol), and *R* is the gas constant.

In the previous study, the activation energy is usually simplified to a constant. However, Fig. 1 shows that the activation energy may have a nonlinear relationship to the hydration degree (Zhu 2014).

Therefore, when using Eq. (1) to calculate the influence of temperature duration on the hydration rate of concrete, E(a)/R should be determined according to the concrete mix.

2.2 A brief introduction to pipe cooling models for mass concrete temperature control

There are several algorithms to simulate the temperature field of mass concrete with cooling pipes, such as the explicit iterative algorithm and the equivalent algorithm. The explicit iterative algorithm can precisely simulate the water temperature distribution along cooling pipes and the temperature field of concrete near cooling pipes, but it requires a large number of nodes. The equivalent method needs considerably fewer nodes, but it can neither simulate the temperature field of concrete near cooling pipes nor



precisely calculate the water temperature distribution along the cooling pipes. When calculating the temperature and temperature stress field of structures such as dams, the equivalent algorithm is more usually applied because of its computational efficiency. Temperature cracks also occur in thin-walled structures, such as lining structures. Because calculation efficiency is not particularly significant when solving the temperature and temperature stress field of thinwall structures, precision becomes much more important. Moreover, an explicit iterative algorithm can be applied to achieve better precision.

When using the iterative method to solve the temperature of mass concrete with cooling pipes, this method should be used for the unknown water temperature distributions along the cooling pipes. Thus, there is a coupling relationship between the calculation model of pipe cooling and the hydration exothermic model considering self-temperature duration. In this situation, the iteration may be substantially more difficult to accomplish.

3. Improvement on the hydration exothermic model

3.1 Current expression forms of the adiabatic temperature rise

The adiabatic temperature rise of concrete can be tested by adiabatic-temperature testing devices. The traditional adiabatic temperature rise curves can be expressed by the exponential form, composite exponential form, or hyperbolic form.

3.1.1 The adiabatic temperature rise curve expressed by the hyperbolic form

Previous researchers suggested that the adiabatic temperature rise curve could be expressed as follows (Zhu 1999)

$$\theta(\tau_e) = \frac{\theta_0 \tau_e}{n + \tau_e} \tag{2}$$

where *n* is a constant, θ_0 is the terminal adiabatic

temperature value, and τ_e is the age of concrete.

When using the hyperbolic form, Eq. (2) can also be expressed as follows

$$\tau_e(\theta) = \frac{n\theta}{\theta_0 - \theta} \tag{3}$$

3.1.2 The adiabatic temperature rise curve expressed by the composite exponential form

Previous researchers proposed that the composite exponential form could be used to present the cement hydration heat or the concrete adiabatic temperature rise, as follows (Zhu 1999)

$$\theta(\tau_e) = \theta_0 (1 - e^{-a\tau_e^b}) \tag{4}$$

where both *a* and *b* are constants, θ_0 is the terminal adiabatic temperature value, and τ_e is the age of concrete.

When using the exponential form, Eq. (4) can also be expressed as follows

$$\tau_{e}(\theta) = \left[\frac{-\ln(1-\frac{\theta}{\theta_{0}})}{a}\right]^{\frac{1}{b}}$$
(5)

3.2 Problems of the current expression forms of adiabatic temperature rise

Two problems exist for the current expression forms of adiabatic temperature rise. First, the accuracy of the expression for hydration heat release cannot be guaranteed when using the current expression forms of adiabatic temperature rise. The hydration heat release of concrete can be divided into five phases, as shown in Fig. 2. The adiabatic temperature rise curves expressed by the composite exponential or hyperbolic forms can precisely express only the fourth and fifth phases. However, for concrete with a relatively low placement temperature or concrete placed in a cold area, the first three phases can last one or two days, which cannot be neglected.

Secondly, in early-age concrete the self-temperature duration and the hydration exothermic rate play a mutually promoting role. This implies that by increasing the selftemperature of concrete, its hydration exothermic rate can be promoted; in addition, the concrete self-temperature can be increased by an increment of the hydration exothermic rate. Because of this mutually promoting relation, if the correct expression form of the adiabatic temperature rise curve is used, then the iterative convergence can be achieved. The most important condition for convergence is that higher-order terms are not present in the expression form. Taking Eq. (4) as an example, if the constant b is very small (such as 0.25), then Eq. (5) will contain higher-order terms, causing difficulties for achieving convergence. Generally, the hyperbolic form has a better convergence performance compared with the composite exponential form, but it cannot guarantee convergence.

3.3 Expression form of the cubic spline curve

As mentioned above, an expression form of the adiabatic temperature rise curve without higher-order terms can facilitate the realization of convergence. Therefore, if the expression form of the adiabatic temperature rise adopts a cubic spline curve, then the convergence problems should be satisfactorily solved. In addition, a cubic spline curve can be used to express the relationship between age of concrete and E(a)/R in Eq. (1), in such a way that breaking points or second derivative discontinuities can be avoided. This can also facilitate achieving convergence.

3.3.1 An introduction to the cubic spline curve

For a group of data, there are *n* nodes in the interval (a,b), and the curve S(x) is adopted to fit these nodes. Assume that the second derivative of curve S(x) in this interval is continuous. That is, the nodes x_j (*j*=1, 2, 3..., *n*-1) should meet the continuity conditions. Then, S(x) refers to a cubic spline curve. According to the literature (Guan 2012), a cubic spline curve can select several types of boundary conditions, and the natural boundary is used in this paper.

$$S''(x_0) = S''(x_n) = 0 \tag{6}$$

Assume that the cubic spline curve can be presented as follows

$$S(x) = M_{j} \frac{(x_{j+1} - x)^{3}}{6h_{j}} + M_{j+1} \frac{(x - x_{j})^{3}}{6h_{j}} + \left(y_{j} - \frac{M_{j}h_{j}^{2}}{6}\right) \frac{x_{j+1} - x}{h_{j}} + \left(y_{j+1} - \frac{M_{j+1}h_{j}^{2}}{6}\right) \frac{x - x_{j}}{h_{j}} \qquad (7)$$

where M_j and M_{j+1} are coefficients, $h_j = x_{j+1} - x$ Then, M_{1-n} meets the following relationship

where

$$\begin{cases} h_{j-1} + h_{j} \\ h_{j} = 6 \frac{f[x_{j}, x_{j+1}] - f[x_{j-1}, x_{j}]}{h_{j-1} + h_{j}} \end{cases}$$

 $\int \mu_i = \frac{h_{j-1}}{\lambda_i} \quad \lambda_i = 1 - \mu_i$

3.3.2 Adiabatic temperature rise expressed with the cubic spline curve

When using a cubic spline curve, Eq. (7), to express adiabatic temperature rise curves, the hydration heat release and age τ_e can be expressed as follows

$$S(\tau_e) - \theta = 0 \tag{9}$$

Eq. (9) is a univariate cubic equation and can be easily solved.

3.3.3 Convergence performance of the adiabatic temperature rise curve expressed by a cubic spline curve

Assume that the change in the corresponding age caused by the change in the adiabatic temperature rise can be expressed as follows

$$\Delta \tau = \tau(\theta + \Delta \theta) - \tau(\theta) \tag{10}$$

A smaller value of $\Delta \tau$ results in a better convergence performance. Therefore, even if adiabatic temperature rise curves expressed with different forms have slight difference in values, the difference in the convergence performances may be very large. When using a cubic spline curve to express adiabatic temperature rise, τ_e is in a linear relationship with $\theta^{1/3}$, $\theta^{1/2}$, θ . Thus, there is no high-order term when using the cubic spline curve, which guarantees the convergence performance. The experience gained in using this engineering calculation shows that the convergence performance of the adiabatic temperature rise curve expressed by the cubic spline curve is better than other forms, such as the composite exponential or the hyperbolic.

3.4 Iterative method of the hydration exothermic model coupling with pipe cooling method

The iteration method should be applied when using the hydration exothermic model presented in this paper. When using the iterative method to solve the temperature of mass concrete with cooling pipes, this method should also be used for the unknown water temperature distributions along the cooling pipes. The calculation result shows that a reasonable iterative process is very important for convergence of the calculation.

A proper iterative process should be as follows: (a) give the initial temperature of the unknown pipe boundary and the initial concrete hydration rate and then, calculate the initial temperature field; (b) calculate the new temperature of the pipe boundary and the new concrete hydration rate, according to the calculated temperature field; (c) determine whether it is converged; and (d) if the result shows that the calculation is not converged, take the calculated temperature of pipe boundary and the calculated concrete hydration rate as the initial conditions, and then conduct a re-calculation.

4. Concrete mechanical model considering selftemperature duration

The mechanical properties of concrete are also greatly influenced by its self-temperature duration. For the earlyage concrete, a higher self-temperature results in a faster strength development of the concrete. The growth of concrete strength may last for a very long time. However, compared with the early-age concrete, the self-temperature distribution has only a slight influence on the thermal and mechanical properties of the late-age concrete.

The concrete mechanical properties influenced by its self-temperature duration mainly include strength, autologous volume deformation, and elastic modulus. Using the improved hydration exothermic model and after

Table 1 Adiabatic temperature rise, elastic modulus, and autogenous volume deformation

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Itelli	Fitting Formula	A	и	U
Adiabatic Temperature Rise		46°C	0.83	0.5
Elastic Modulus	$A(1-e^{-a\tau_e^b})$	3.7 GPa	0.3	0.5
Autogenous Volume Deformation		-28 µm	-0.25	0.5

calculation of the temperature field, the relationship between the age of concrete under an adiabatic condition and under an arbitrary condition is known. Then, the concrete mechanical property parameters under any arbitrary condition can be calculated based on the concrete mechanical properties under the adiabatic condition.

The mechanical parameters of concrete, such as elastic tensile modulus and tensile strength curve, are always obtained under the standard curing conditions. It is necessary to transform the mechanical parameters under the standard curing conditions into parameters under an adiabatic condition. The relationship between actual age (the age under arbitrary conditions) and relative age (the age under an adiabatic condition) of concrete can be expressed as follows

$$\tau_{e} = \begin{cases} \tau_{e}(\theta) & \tau_{e} \leq 28 \\ t + t_{e,28} - 28 & \tau_{e} \geq 28 \end{cases}$$
(11)

where *t* is the actual age of concrete, t_e is the relative age, and $t_{e,28}$ is the corresponding relative age when actual age is 28 days.

The calculation process can be expressed as follows: (a) calculate the mechanical properties of concrete under an adiabatic condition, according to the mechanical properties of concrete under the standard curing conditions; (b) calculate the concrete heat release under arbitrary conditions, according to the actual age of the concrete and the finite element calculation, thus calculating the relative age of concrete; and (c) calculate the mechanical properties of concrete considering the self-temperature duration.

5. Engineering example

The model presented in this paper is verified by the spillway tunnel of the Baihetan Power Station, which is a 300-m high arch dam in China.

5.1 The mechanical properties of concrete under adiabatic condition

The original data of adiabatic temperature rise, the autogenous volume deformation under the standard curing conditions, and elastic modulus development under the standard curing conditions are presented in Table 1.

When using Eq. (1) to calculate the influence of temperature duration on the hydration rate of concrete, E(a)/R should be determined according to the concrete mix.

However, owing to the lack of relevant test data, the



Fig. 4 Curing and adiabatic temperature rise age



Fig. 5 Autologous volume deformation





Fig. 7 Air and water temperature in the spillway cave



Fig. 8 The element model and a typical section



Fig. 9 Layout of the cooling pipe of a construction section

value of E(a)/R could not be obtained for the concrete used in this study. However, because this case is only used to verify the convergence of the algorithm and the influence of temperature duration on the hydration rate of concrete, the

value of E(a)/R is expressed as shown in Fig. 1. In practical engineering, E(a)/R should use the test data.

The adiabatic temperature rise of the concrete is shown in Fig. 3. The relationship of the concrete mechanical



Fig. 11 Temperature and Stress duration of typical point #1

properties under the adiabatic condition and under the standard curing conditions is shown in Figs. 4-6.

Due to lack of relevant test data, the creep parameters of the concrete are determined according to the recommended value of hydraulic concrete

$$C(t,\tau) = \frac{0.23}{E_0} (1+9.20\tau^{-0.45}) \left[1 - e^{-0.30(t-\tau)} \right] + \frac{0.52}{E_0} (1+1.70\tau^{-0.45}) \left[1 - e^{-0.0050(t-\tau)} \right]$$

where t is time, τ is loading age, and E_0 is the final value of the elastic modulus of concrete.

5.2 Calculation conditions and simulation cases

This section introduces the air temperature, water



Fig. 12 Temperature and stress duration of typical point #2



Fig. 13 Temperature and stress duration of typical point #3

temperature, and ground temperature in the spillway cave of the Baihetan Power Station.

5.2.1 The air temperature, ground temperature, and water temperature conditions



Fig. 14 Temperature and stress duration of typical point #4



Fig. 15 Temperature and stress distributions of side wall 1.5 days after being poured (Case 1)

The value of the ground temperature is 25°C. Fig. 7 shows the air temperature in the spillway cave and water temperature of the river.

5.2.2 The model and typical points

The semi-analytical finite element method (Zhu 2013) is used to calculate the spillway lining temperature. The space between two water pipes is 1 m, as shown in Figs. 8-9. The positions of the typical section and typical points are shown in Figs. 8 and 10.

5.2.3 Construction method

The construction of the spillway tunnel was separated into two stages. For the first stage, the base plate concrete was poured, with 12-m-long sections and with a preparation time of 6 days between the placements of two adjacent



Fig. 16 Temperature and stress distributions of side wall 40 days after being poured (Case 1)



Fig. 17 Temperature and stress distributions of side wall 1.5 days after being poured (Case 2)



Fig. 18 Temperature and stress distributions of side wall 40 days after being poured (Case 2)

sections. In the second stage, that occurred 60 days after the completion of the base plate concrete, the rest of the concrete was poured. Each section of the remaining concrete structure was also 12-m long per section and had a preparation time of 6 days between the placements of two adjacent sections.

5.2.4 Simulation cases

The concrete temperature under arbitrary conditions is generally lower than the temperature under adiabatic conditions in a testing device. Thus, a lower placement temperature and external air temperature result in a more remarkable influence of the concrete self-temperature distribution on the hydration exothermic rate. Therefore, for this verification example, the placement of concrete was performed in winter.

Case 1: Considering the influence of self-temperature duration, the analytical section of the structure was poured in January, with a pouring temperature of 16°C. A pipe-cooling measure was performed after the concrete was poured and lasted 10 days.

Case 2: Without considering the influence of selftemperature duration, while the other conditions are identical to case 1.

5.3 Analysis of the calculation result

According to the calculation results, the two cases have nearly the same efficiency values. As shown in Figs. 11-18, the peak temperature and the primary principal stress of case 1 (considering the influence of self-temperature duration) are lower than in case 2 (without considering the influence of self-temperature duration).

The calculation results show that the self-temperature duration can obviously affect the temperature and stress field of mass concrete structures.

6. Conclusions

In this study, hydration exothermic models were investigated and improved. Rapid convergence of the hydration exothermic model and its coupling with pipecooling models were achieved. Based on the improved exothermic model, a model of concrete mechanical properties that considers concrete self-temperature duration is also presented. The convergence of the improved model was verified through an engineering project example.

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References

- Cervera, M., Faria, J., Oliver, J. and Prato, T. (2002), "Numerical modeling of concrete curing regarding hydration and temperature phenomena", *Comput. Struct.*, 80(18), 1511-1521.
- Emborg, M. (1989), "Thermal stresses in concrete structures at early ages", Ph.D. Dissertation, Lulea University of Technology, Lulea, Sweden.

- Guan, Z. and Lu, J.Y. (2012), *Fundamentals of Numerical Analysis*, Higher Education Press, Beijing, China.
- Guo, Y.C., Wang, X. and Qian, J.S. (2015), "Physical model of drying shrinkage of recycled aggregate concrete", J. Wuhan Univ. Technol.-Mater. Sci. Ed., 30(6), 1260-1267.
- Hatte, J.H. and Thorborg, J. (2003), "A numerical model for predicting the thermomechanical conditions during hydration of early-age concrete", *Appl. Math. Mod.*, 27(1), 1-26.
- Kim, S.J., Yang, K.H. and Lee, K.H. (2016), "Mechanical properties and adiabatic temperature rise of low heat concrete using ternary blended cement", *Comput. Concrete*, **17**(2), 271-280.
- Li, B., Mao, J.Z., Lv, J.F. and Zhou, L.M. (2016), "Effects of micropore structure on hydration degree and mechanical properties of concrete in later curing age", *Eur. J. Environ. Civil Eng.*, 20(5), 544-599.
- Liu, M.Z., Qiang, S. and Zhu, Z.Y. (2011), "Study on crack mechanism for concrete bedding cushion on rock", Adv. Mater. Res., 163, 1291-1295.
- Luo, M., Qian, C.X., Li, R. and Rong, H. (2015), "Efficiency of concrete crack-healing based on biological carbonate precipitation", J. Wuhan Univ. Technol.-Mater. Sci. Ed., 30(6), 1255-1259.
- Park, K.B., Kwon, S.J. and Wang, X.Y. (2015), "Analysis of the effects of rice husk ash on the hydration of cementitious materials", *Constr. Build. Mater.*, **105**, 196-205.
- Schutter, G.D. (2002), "Finite element simulation of thermal cracking in massive hardening concrete elements using degree of hydration based material laws", *Comput. Struct.*, 80(27), 2035-2042.
- Suzuki, Y., Harada, S., Maekawa, K. and Tsuji, Y. (1988), "Evaluation of adiabatic temperature rise of concrete measured with the new testing apparatus", *Doboku Gakkai Rombun H120 Okokushu*, 9, 109-117.
- Wang, J.C. and Yan, P.Y. (2013), "Evaluation of early age mechanical properties of concrete in real structure", *Comput. Concrete*, **12**(1), 53-64.
- Zhang, Z.M., Feng, S.R., Shi, Q.C. and Wang, J. (2004), "Adiabatic temperature rise of concrete based on equivalent time", J. Hohai Univ. (Natur. Sci.), 32(5), 573-577.
- Zhang, Z.M., Zhou, H.J. and Zhao, J.K. (2004), "Influences of temperature on strength of concrete", J. Hohai Univ. (Natur. Sci.), 32(6), 674-679.
- Zhu, B.F. (1999), Thermal Stresses and Temperature Control of Mass Concrete, China Electric Power Press, Beijing, China.
- Zhu, B.F. (2003), "A method for computing the adiabatic temperature rise of concrete considering the effect of the temperature of concrete", J. Hydroelectr. Eng., 20(2), 69-73.
- Zhu, B.F. (2003), "A new computing model for the adiabatic temperature rise of concrete and the method of back analysis", *Wat. Pow.*, **4**, 29-32.
- Zhu, Z.Y., Qiang, S. and Chen, W.M. (2013), "A new method solving the temperature field of concrete around cooling pipes", *Comput. Concrete*, **11**(5), 441-462.
- Zhu, Z.Y., Qiang, S. and Chen, W.M. (2014), "A model for temperature influence on concrete hydration exothermic rate (Part one: Theory and experiment)", J. Wuhan Univ. Technol.-Mater. Sci. Ed., 29(3), 540-545.
- Zhu, Z.Y., Qiang, S., Liu, M.Z. and Wang, H.B. (2011), "Cracking mechanism of long concrete bedding cushion and prevention method", *Adv. Mater. Res.*, **163**, 880-887.

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