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Computer modeling of crack propagation in concrete retaining walls: A case study

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Abstract. Concrete retaining walls are the most common types of geotechnical structures for controlling instable slopes resulting from lateral pressure. In analytical stability, calculation of the concrete retaining walls is regarded as a rigid mass when its safety is required. When cracks in these structures are created, the stability may be enforced and causes to defeat. Therefore, identification, creation and propagation of cracks are among the important steps in control of lacks and stabilization. Using the numerical methods for simulation of crack propagation in concrete retaining walls bodies are among the new aspects of geotechnical analysis. Among the considered analytical methods in geotechnical appraisal, the boundary element method (BEM) for simulation of crack propagation in concrete retaining walls is very convenient. Considered concrete retaining wall of this paper is Pars Power Plant structured in south side in Assalouyeh, SW of Iran. This wall's type is RW6 with 11 m height and 440 m length and endurance of refinery construction lateral forces. To evaluate displacement and stress distributions ($\sigma_{1,max}/\sigma_{3,min}$), the surrounding, especially in tip and its opening crack BEM, is considered an appropriate method. By considering the result of this study, with accurate simulation of crack propagation, it is possible to determine the final status of progressive failure in concrete retaining walls and anticipate the suitable stabilization method.

Keywords: geotechnique; retaining walls; concrete; crack propagation; simulation; fracture

1. Introduction

An important principle in geotechnical engineering is the application of protection strategies for environmental stabilization (soil, rock, and multiplex mass instability). One of these mentioned issues is related to the slopes stabilization existed in construction sites, facilities, metropolitan area, edge road, tunnels, portals, etc. (Cheng and Lau 2014). An appropriate stabilization solution forum stable slope in such cases is the use and designing of the retaining walls (Ge *et al.* 2009). According to definition "a retaining wall is any constructed wall that holds back soil, liquid or other materials, where there is an abrupt change in elevation" (Brooks 2010). Utilization of retaining walls has numerous advantages; some important advantages are as follows (Clayton *et al.* 2014, Coleman 2015, Prelini 2015):

- Using more simple materials for construction
- Low and affordable economical costs
- Flexibility for various projects
- Ability to upgrading and reinforced
- Requires less specialized personnel than other methods

Generally, the retaining walls can be categorized into 4 groups including gravity wall, Semi gravity wall, Counterfort wall and Cantilever wall (Azarafza and Asghari-Kaljahi 2016). These are shown in Fig. 1.

Concrete is one of the most important tools in construction of retaining walls in geotechnical structures. Concrete is composition consisting of coarse aggregate and fluid cement which hardens over time (Setareh and Darvas 2006). Retaining walls are used to control lateral pressure of backfill materials. Lateral earth pressure variations and the process of applying lateral forces comprehensively have been explained by Rankine and Coulomb theories (Chu 1991, Paik and Salgado 2003, Evangelista *et al*. 2010, Iskandern *et al*. 2013). However, these cases often focus on the ingredients behind retaining walls and very few cases were focused on retaining walls bodies themselves. In this paper, attempts have been made to investigate and simulate a crack generation and propagation in concrete retaining walls body by using numerical modeling. The results can be used in determining failure in concrete.

2. Crack propagation

In solid mechanics, ideal materials are defined as homogeneous, continuous and isotopes (Srinath 2009, Bower 2010, Shaowei *et al*. 2016). The existence of any deviations in such a situation causes severe change in material behaviors (Anderson 2004, Broek 2008, Perez 2016). In order to analyze and characterize the behavior-related conditions, various methods were used, including analytical methods, numerical methods, fourier method, integral transform method and complex variable method (Irwin 1957, Sanford 2002). The numerical methods that are used for behavior analysis includes finite element method (FEM), finite difference method (FDM), discrete elements

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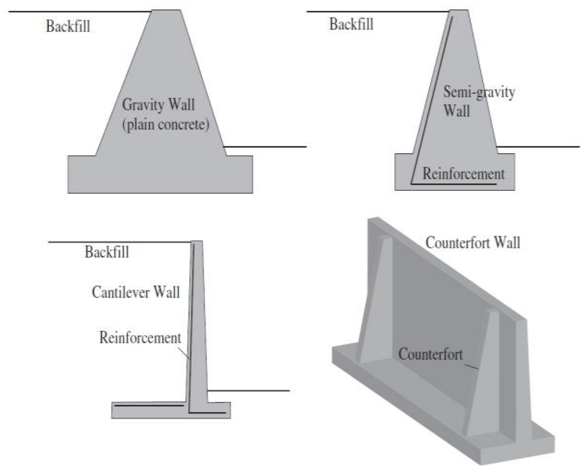

Fig. 1 Types of retaining walls (Azarafza and Asghari-Kaljahi 2016)

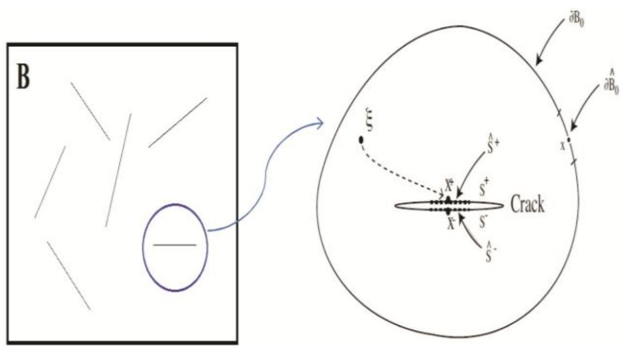

Fig. 2 A single crack geometry (Mukherjee 2001)

method (DEM), boundary element method (BEM), boundary integral method (BIM), dual boundary element method (DBEM) (Rossi and Richer 1987, Rossi and Wu 1992, Tailhan *et al*. 2010, Marji and Dehghani 2010, Marji 2013, 2014). Dealing with ideal materials is implausible in geo-engineering. Discontinue rock environment is an example that changes its behavior and causes a lot of problems for structures (Scavia 1990, Azarafza 2013, Azarafza *et al*. 2013, 2014a, b, c, 2015, 2016).

Boundary integral equations (BIE) and the boundary element method (BEM) based on BIEs, are the most appropriate methods for numerical analysis of a large variety of problems in science and engineering. For crack propagation simulation BIE approaches, linear elasticity and linear elastic fracture mechanics theory (LEFM) are used in the main body of boundary element method. The starting point (for calculation) is from fracture point (FP) as integral involves traction sums and crack opening displacements when the ordering point near a crack surface in the limit to the boundary (LTB) is implicated. According to Fig. 2, the total surface of "*B*" is defined as cracked environment

$$\partial B = \partial B_0 \cup S^{+} \cup S^{-} \tag{1}$$

Where S^+ and S^- are coincidentally upper and lower geometrical surfaces of a crack in the *B* environment, these surfaces are just the same opposition normals at any pair of twin points (x^+, x^-) are defined. By defining an internal point "ξ" near *x* on outward of ∂B and with respect to the concrete is rigid mass ($\xi \in \partial B$) (Rizzo 1967)

$$u_k(\xi) = \int_{\partial B} [U_{ik}(\xi, y)\,\tau_i(y) - T_{ik}(\xi, y)\,u_i(y)]\,dS(y) \tag{2}$$

$$U_{ik} = \frac{1}{16\pi(1-\upsilon)Gr}[(3-4\upsilon)\delta_{ik} + r_{,i}r_{,k}] \tag{3}$$

$$T_{ik} = -\frac{1}{8\pi(1-\upsilon)r^2}\left[\{(1-2\upsilon)\delta_{ik} + 3r_{,i}r_{,k}\}-\frac{\partial r}{\partial n} + (1-2\upsilon)(r_{,i}n_k - r_{,k}n_i)\right] \tag{4}$$

Where u_k and τ_k are the components of the displacement and traction respectively, δ_{ik} is Kronecker delta and the normal "*n*" is boundary of field around point "*y*".

$$r_{,i} = \frac{\partial_r}{\partial_{ri}} = \frac{y_i - \xi_i}{r} \tag{5}$$

With Placement in Eq. (2)

$$u_k(\xi) = \int_{\partial B} [U_{ik}(\xi, y)\,\sigma_{ij}(y) - \Sigma_{ijk}(\xi, y)\,u_i(y)]\,e_j \cdot dS(y) \tag{6}$$

$$\Sigma_{ijk} = E_{ijmn}\frac{\partial U_{km}}{\partial y_n} = -\frac{1}{8\pi(1-\upsilon)r^2}[(1-2\upsilon)(r_{,i}\delta_{jk} + r_{,j}\delta_{ik} - r_{,k}\delta_{ij}) + 3r_{,i}r_{,j}r_{,k}] \tag{7}$$

Where σ is the stress tensor, $\tau_i=\sigma_{ij}n_j$ and $T_{ik}=\Sigma_{ijk}n_j$ ($e_j \cdot dS(y)=n_j(y)dS(y)$), *E* is the elasticity tensor. If the Eq. (2) taking the limit ($\xi \rightarrow x$)

$$\begin{aligned} u_k(x) &= \lim_{\xi \to x} \int_{\partial B} [U_{ik}(\xi, y)\,\tau_i(y) - T_{ik}(\xi, y)\,u_i(y)]\,dS(y) \\ &= \oint_{\partial B} U_{ik}(x, y)\,\tau_i(y) - T_{ik}(x, y)\,u_i(y)]\,dS(y) \end{aligned} \tag{8}$$

In Eq. (8) the $\oint$ indicates the integral finite part. However, for BIE displacement (According to Eq. (8))

$$\begin{aligned} &\int_{\partial B_0} [U_{ik}(x, y)\,\tau_i(y) - T_{ik}(x, y)(u_i(y) - u_i(x)]\,dS(y) \\ &+ \int_{S^+} [U_{ik}(x, y)\,q_i(y) - T_{ik}(x, y)\upsilon_i(y)]\,dS(y) = 0 \end{aligned} \tag{9}$$

Where *q* is sum of the tractions across a crack ($q_i(y)= \tau_i(y^+)\tau_i(y^-)$) and υ is crack opening displacement ($v_i(y)=v_i(y^+)-v_i(y^-)$) (Mukherjee 2001).

Using concrete properties as rigid masses and linear displacement modes validity, (Lutz *et al*. 1992, Mukherjee 2000, Mukherjee *et al*. 2000), so we have

$$\int_{\partial B_0} \Big\{ D_{ijk}(x,y)[\sigma_{kh}(y)-\sigma_{kh}(x)]n_h(y) - S_{ijk}(x,y)$$

$$[u_k(y)-u_k(x)-u_{k,l}(x)(y_l-x_l)]\Big\}\, dS(y) \qquad (10)$$

$$+\int_{S^+}[D_{ijk}(x,y)q_k(y)-S_{ijk}(x,y)\upsilon_k(y)]dS(y)=0$$

In FP displacement tensor BIE with $x \rightarrow x^+ \in S^+$ is

$$u_k(x^+)=\int_{\partial B_0}[U_{ik}(x,y)\tau_i(y)-T_{ik}(x^+,y)u_i(y)]dS(y)$$
$$+\oint_{S^+}[U_{ik}(x^+,y)q_i(y)-T_{ik}(x^+,y)\upsilon_i(y)]dS(y) \qquad (11)$$

$$x^+ \in \hat{S}^+ \Rightarrow$$
$$u_k(x^+)=\int_{\partial B_0}[U_{ik}(x^+,y)\tau_i(y)-T_{ik}(x^+,y)u_i(y)]dS(y)$$
$$+\int_{S^+-\hat{S}^+}[U_{ik}(x^+,y)q_i(y)-T_{ik}(x^+,y)\upsilon_i(y)]dS(y)+\int_{\hat{S}^+}U_{ik}(x^+,y)q_i(y)\,dS(y) \qquad (12)$$
$$-\int_{\hat{S}^+}T_{ik}(x^+,y)[u_i(y)-u_i(x)]\,dS(y)-u_i(x)\oint_{\hat{S}^+}T_{ik}(x^+,y)dS(y)$$

Also, in FP stress tensor BIE with $x \rightarrow x^- \in S^-$ is

$$\sigma_{ij}(x^+)=\int_{\partial B_0}[D_{ijk}(x^+,y)\tau_k(y)-S_{ijk}(x^+,y)u_k(y)]dS(y)$$
$$+\oint_{S^+}[D_{ijk}(x^+,y)q_k(y)-S_{ijk}(x^+,y)\upsilon_k(y)]dS(y) \qquad (13)$$

$$x^+ \in \hat{S}^+ \Rightarrow$$
$$\sigma_{ij}(x^+)=\int_{\partial B_0}[D_{ijk}(x^+,y)\tau_k(y)-S_{ijk}(x^+,y)u_k(y)]dS(y)$$
$$+\int_{S^+-\hat{S}^+}[D_{ijk}(x^+,y)q_k(y)-S_{ijk}(x^+,y)\upsilon_k(y)]dS(y)$$
$$+\int_{\hat{S}^+}D_{ijk}(x^+,y)[s_{kl}(y)-s_{kl}(x)]n_l\,dS(y) \qquad (14)$$
$$-\int_{\hat{S}^+}S_{ijk}(x^+,y)[\upsilon_k(y)-\upsilon_k(x)-d_{kn}(x)(y_n-x_n^+)]dS(y)$$
$$-\upsilon_k(x)\oint_{\hat{S}^+}S_{ijk}(x^+,y)\,dS(y)$$
$$+d_{mn}(x)\oint_{\hat{S}^+}[E_{klmn}D_{ijk}(x^+,y)n_l(y)-S_{ijm}(x^+,y)(y_n-x_n^+)]dS(y)$$

Thus with solving the finite differential equations the stress field and displacement element tensors at the crack tip can be calculated (Fig. 3).

As the crack is created in concrete body, it is expanded under in-situ stress from backfill lateral earth pressures. Empirically, the crack grows parallel with in-situ $\sigma_{1,\max}$ perpendicular with $\sigma_{3,\min}$ in field or UCS experiment. This event is computed by θ angle and θ=45+ϕ/2 (Mogi 2006). Propagation phenomenon and crack grows (Kinked crack)

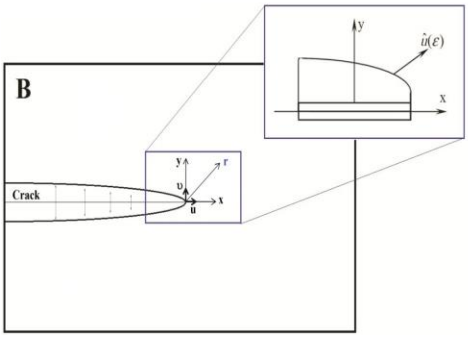

Fig. 3 The general displacement in crack tip (Adapted from Crouch and Starfield 1983)

are formulated by Marji and Dehghani (2010). These researchers said that the crack tip is kinked after the crack starts its propagation in the direction of crack initiation angle θ and crack will propagate incrementally by an amount Δb in θ direction. In addition, they have shown kinked crack geometry as Fig. 4 and stated that crack grows indirectly and other directions for modeling the kinked crack do as follows:

• The ordering points out side of the kink point are moved to the crack kink

• To obtain displacement at crack kink from both sides of kink, the adjacent two sides of displacement are extrapolated.

• The kink affection in both sides are averaged

3. Numerical simulation of this study

The studied retaining wall is conditioned in south side of Pars Power Plant in Assalouyeh, SW of Iran. This case is overlooking the 125 corridor with 11 m average height and 440 m length is designed as concrete gravity wall. This wall’s stability is very important for main transportation artery. The location of studied retaining wall is shown in Fig. 5. This wall’s type is RW6, the maximum height of the wall by taking 1.8 m base height and one meter alternative overhead soil in wall behind is about 12.6 m. Fig. 6 shows the studied retaining wall geometry.

The retaining wall is studied as homogenous concrete wall and linear elasticity material behavior. The materials used in the concrete modeling are described in Table 1.

Crack propagation in rigid body can be generated as curves or kinks (Shou and Crouch 1995, Felekoğlu and Keskinateş 2016, Haeri and Sarfarazi 2016, Sarfarazi and Haeri 2016), by using quadratic displacement discontinuity elements with only one special crack tip element at each crack end and following relationships where the crack displacement propagation can be simulated in solid media (Cotterell and Rice 1980, Marji *et al*. 2006)

$$G=\frac{1-\upsilon^2}{E}(K_I^2+K_{II}^2)\ and\ K_I=\sigma\cos\frac{\alpha}{4}\left(\frac{\pi r\sin\frac{\alpha}{2}}{1+\sin^2\frac{\alpha}{2}}\right)^{\frac{1}{2}}$$
$$,\ K_I=\sigma\sin\frac{\alpha}{4}\left(\frac{\pi\sin\frac{\alpha}{2}}{1+\sin^2\frac{\alpha}{2}}\right)^{\frac{1}{2}} \qquad (15)$$

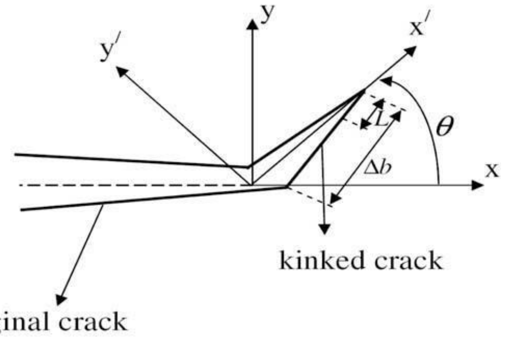

Fig. 4 The geometry of kinked crack (Marji and Dehghani 2010)

Fig. 5 The location of the studied retaining wall

Fig. 6 The geometry of studied retaining wall

Table 1 Mechanical properties of the concrete

Description	Parameter	Value	Unit
Crack length	2d	10	mm
Unit weight	ρ	2400	Kg/cm^3
Modulus of elasticity	E	38.1	GPa
Poisson's ratio	υ	0.2	-

In upper Equation, G is shear modulus, K_{I} and K_{II} are stress intensity factors and α is circular arc of crack.

For the simulation of in pre-processing step, a solid model of concrete was prepared, parameters were specified, boundary conditions were imposed and crack was introduced. In the next step, the BEM meshes of body were generated and equations for the displacements were formulated and solved (linear elasticity in solid media). Finally, the errors were determined and investigated to see if the desired accuracy has been achieved, or further adaptation were necessary after subsequent adaptation. In Fig. 7, the simulation cycle flowchart is presented. The results of the crack modeling for the studied case are presented in Figs. 8 to 14.

4. Conclusions

Numerical analysis in the Boundary Element Method and Boundary Integral Method base has good application in the solution of stress and displacement fields near to the crack tips. The crack propagation assessment during the generation, expansion, growth and maturity stages is an appropriate approach in the crack development analysis. Cracks monitoring in the solid media shows where the most prone area to failure in body is, how failure modes and what the stage orders. According to the results of this research, in order to monitor crack propagation (step by step) from generation to maturation in concrete, there has been tried to identify and declare crack development and release pattern. As the simulation results have been evident in the early stages, the crack was extended linearly after generation under in-situ stress field. In the next stage, mass process takes curves by normal stress and shear strength affection in concrete. After creation of crack inthe concrete body on crack tip, the stress concentration is generated. This stress concentration causes the growth of crack along the $\sigma_{1,\max}$ and extension in perpendicular axis and along the $\sigma_{3,\min}$.

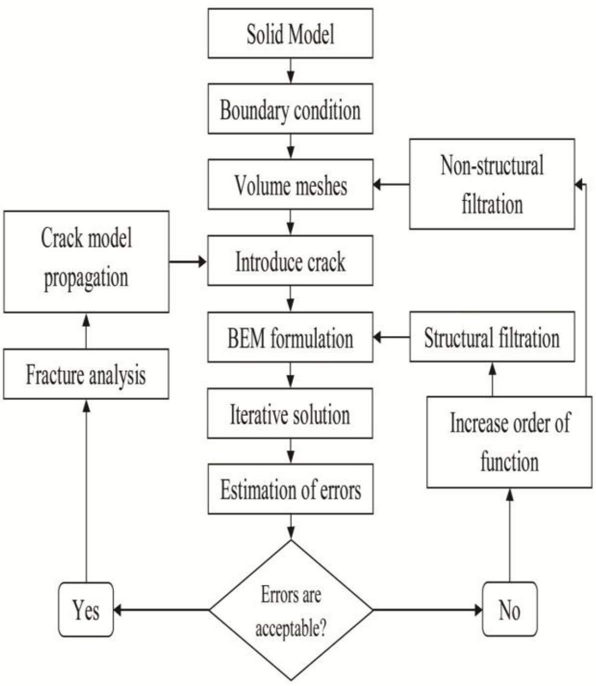

Fig. 7 The flowchart of crack propagation in concrete by BEM

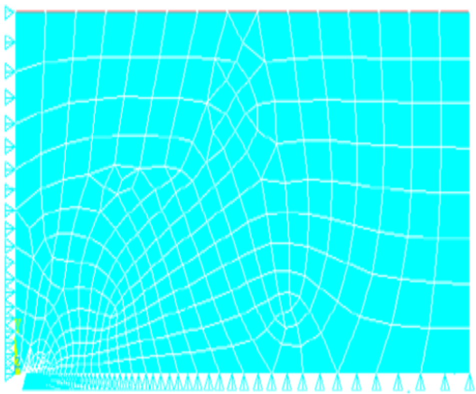

Fig. 8 Concrete body geometry

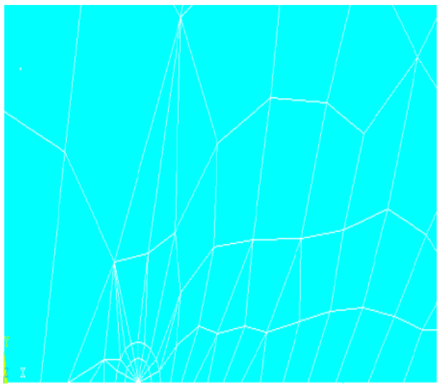

Fig. 9 Crack tip geometry

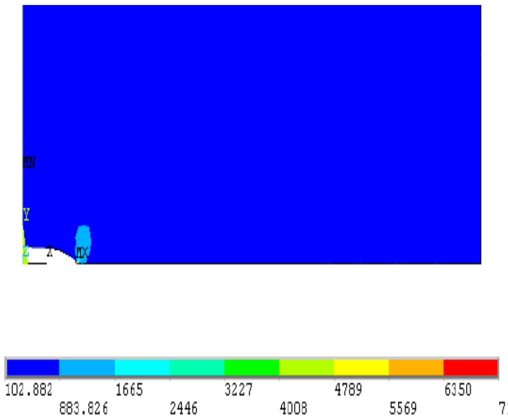

Fig. 10 Geometry of crack generation in the concrete body (Early stage)

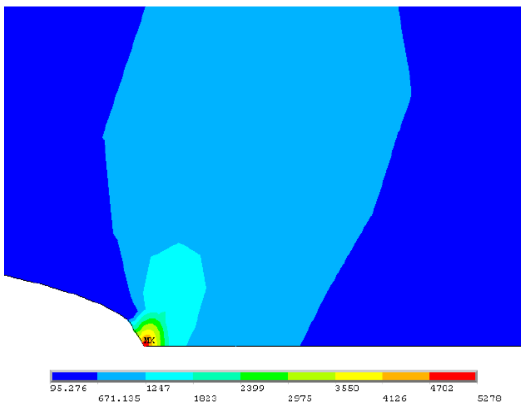

Fig. 11 Crack propagation model (Final stage)

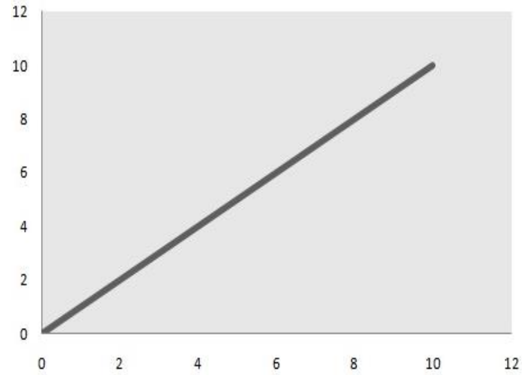

Fig. 12 Initial crack in concrete (crack path status)

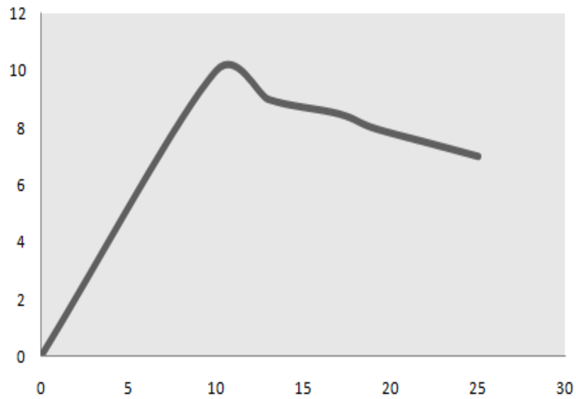

Fig. 13 Crack propagation and development (crack growth)

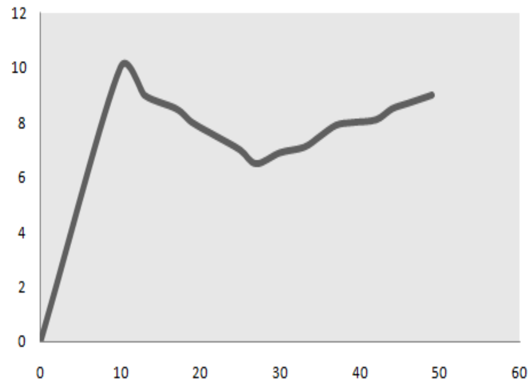

Fig. 14 Crack propagation and development (crack maturity)

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