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# Combined effect of CFRP–TSR confinement on circular reinforced concrete columns

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**Abstract.** The use of external carbon-fiber-reinforced polymer (CFRP) wraps is one of the most effective techniques existing for the confinement of the circular concrete columns. Currently, several researches have been made to develop models for predicting the behavior of this type of confinement. The disadvantage of the most models, is to not take into account the contribution of the transverse steel reinforcements (TSR) effect, However, very limited models have been recently developed that considers this combined effect and gives less accurate results. This paper presents the development of a new model for the axial behavior of circular concrete columns confined by combining external CFRP warps-and-internal TSR (hoops or spirals) based on the existing experimental data. The comparison between the proposed model and the experimental results showed good agreement comparing to the several existing models. Moreover, the expressions of estimating the ultimate strength and the corresponding strain are simple and precise, which make it easy to use in the design applications.

**Keywords:** reinforced concrete columns; CFRP; stress-strain model; confinement; TSR

## 1. Introduction

The need for strengthening deficient existing reinforced concrete (RC) structures is suggested by many reasons. For years, engineers have been studying ways to retrofit or strengthen existing deficient RC columns to meet new code requirements, especially in earthquake prone areas (Youssef *et al*. 2007). Various methods for strengthening and rehabilitation of RC structures have been developed in the past several decades (Nam *et al.* 2016). Recently, the use of externally-bonded fiber-reinforced polymer (FRP) reinforcement for the strengthening or repair of reinforced concrete structures has become a popular technology (Fanggi and Ozbakkaloglu 2015). The FRP composites have been used successfully for rehabilitation and repair of deficient reinforced-concrete structures such as buildings, bridges, etc (Morsy and Mahmoud 2013). One of the important applications of the FRP strengthening technology is on the enhancement of RC column load-carrying capacity through the provision of confining FRP warps (Ozbakkaloglu 2013). The column wrapping technique is particularly effective for circular columns as the strength and ductility of concrete in circular section can be substantially increased through lateral confinement. FRP is characterized by high strength fibers embedded in polymer resin (Lu *et al*. 2015). FRP offers such advantages as high strength and stiffness, low density, chemical stability, high durability, and ease of installation. The most common type of FRP in industry is made with carbon, aramid or glass fibers (Zhang *et al*. 2016).

In this context, several studies have been conducted on the compressive behavior of circular concrete columns confined externally by carbon fiber reinforced polymer (CFRP) (Sadeghian and Fam 2015). Consequently, several stress-strain models for predicting the compressive axial behavior have been proposed (Saadatmanesh *et al.* 1994, Samaan *et al*. 1998, Lam and Tang 2003, Wei and Wu 2012, Ozbakkaloglu *et al*. 2013). However, these models do not consider the confining effect of internal transverse steel reinforcement (TSR). From here, other works have appeared recently that consider the confinement under the combined effect of the external CFRP warps and internal TSR. Harajli *et al*. (2006) proposed a new model for circular and rectangular concrete columns confined by both CFRP warps and TSR. Eid and Paultre (2008) developed a complicated analytical model requiring numerous parameters to predict stress-strain behavior of circular concrete columns. The developed model is characterized by good accuracy compared to the experimental data. Lee *et al*. (2010) developed a new empirical model to predict the behavior of circular concrete columns confined by both CFRP and steel spiral. Chastre and Silva (2010) proposed a model for circular concrete columns confined by both CFRP and steel ties using stress-strain relationship of Richard and Abbott (1975). Recently, Hu and Seracino (2013) proposed a constitutive model to predict the ultimate strength and its corresponding strain, the first ascending branch was proposed based on the Popovics (1973) model, while the second branch which takes into account the combined effect of CFRP and TSR was deduced from the integration of (Mander *et al*. 1988) and (Teng *et al*. 2007) models.

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In the same context, Teng *et al*. (2014) proposed a constitutive model using Jian and Tang (2007) equation and modified (Mander *et al*. 1988) expressions to predict the ultimate stress and its corresponding strain for concrete confined by both CFRP and TSR. Recently, Shirmohammadi *et al*. (2015) proposed a constitutive model for circular concrete sections confined by both CFRP and TSR using (Thorenfeldt *et al*. 1987) equation, where the ultimate stress and its corresponding strain are calibrated with using a several existing experimental data. However, the proposed model includes several parameters which make the behavior very different.

Furthermore, few studies are currently available that consider the confining effects of both external carbon fiber reinforced polymer (CFRP) warps and transverse steel hoops or spiral reinforcement (TSR) to describe the compressive axial behavior of circular concrete columns; as well as the complexity of the proposed model equations to predict the ultimate strength and its corresponding strain. Therefore, it is essential to give more attention for this combined effect, and calibrated the existing experimental data with a thorough analysis, in order to develop a new external CFRP wrapping and TSR (hoops or spirals) confinement model for circular concrete columns, in particular to predict the compressive axial behavior more simply and accurately.

## 2. Existing stress-strain models

Before discussing the development of the new model, some existing stress-strain models of confined concrete by both external CFRP warps and internal TSR are presented.

### *2.1 Model developed by Chastre and Silva (2010)*

Chastre and Silva (2010) proposed a bilinear stress-strain model for circular concrete section confined under the combined effect of CFRP and TSR. This model is based on the experimental results of twenty five circular concrete columns confined by both CFRP warps and steel hoops under monotonic axial compression load.

The stress-strain relationship is based on an expression of four parameters ($E_1$, $E_2$, $f_o$, $n$) previously proposed by Richard and Abbott (1975). This relationship is given as follows

$$f_c = \frac{(E_1 - E_2)\varepsilon_c}{\left[1 + \left(\frac{(E_1 - E_2)\varepsilon_c}{f_o}\right)^n\right]^{1/n}} + E_2\varepsilon_c \tag{1}$$

Where $f_c$ and $\varepsilon_c$ = axial stress and strain of concrete, respectively; $E_1$ and $E_2$= irst and second slopes, respectively; $f_0$=reference plastic stress at the intercept of the second slope with the stress axis; and $n$=a curve-shaped parameter that mainly controls the curvature in the transition zone.

The ultimate compressive strength $f_{cu}$ and its corresponding strain $\varepsilon_{cu}$ are provided as follows

$$f_{cu} = f_D + 5{,}29 f_{lu} \tag{2}$$

$$\varepsilon_{cu} = 17{,}65 \times \varepsilon_{co}\left(\frac{f_{lu}}{f_D}\right)^{0{,}7} \tag{3}$$

$$f_{le} = f_{lf} + f_{ls} \tag{4}$$

Where $f_D$=compressive strength of the concrete column; $f_{lf}$=lateral confinement pressure provided by the CFRP warp; $f_{ls}$=lateral confinement pressure provided by transverse steel reinforcement applied to the concrete core; $f_{le}$=total confining pressure from steel hoops or spirals $f_{ls}$ and FRP warp $f_{lf}$; and $\varepsilon_{co}$=axial strain in unconfined concrete corresponding to $f_{co}$.

It noted to this model is only dealt the TSR as hoops, without taking account the effect of the TSR as spirals, from here the use of this model is not generalized, although the simplicity of their expressions.

### *2.2 Model developed by Lee et al. (2010)*

Based on experimental results, Lee *et al*. (2010) developed an empirical model for circular concrete confined by both CFRP warps and steel spirals. The proposed equations are given as follows

$$f_c = \begin{cases} E_c\varepsilon_c + (f_{co} - E_c\varepsilon_{co}) \times \left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2 & for \quad 0 \le \varepsilon_c \le \varepsilon_{co} \\ f_{co} + (f_{cs} - f_{co}) \times \left(\frac{\varepsilon_c - \varepsilon_{co}}{\varepsilon_{cs} - \varepsilon_{co}}\right)^{0{,}7} & for \quad \varepsilon_{co} \le \varepsilon_c \le \varepsilon_{cs} \\ f_{cs} + (f_{cu} - f_{cs}) \times \left(\frac{\varepsilon_c - \varepsilon_{cs}}{\varepsilon_{cu} - \varepsilon_{cs}}\right)^{0{,}7} & for \quad \varepsilon_{cs} \le \varepsilon_c \le \varepsilon_{cu} \end{cases} \tag{5}$$

And

$$\varepsilon_{cs} = \begin{cases} \varepsilon_{cu}\left\{0{,}85 + 0{,}03 \times \left(\frac{f_{lf}}{f_{ls}}\right)\right\} & for \quad f_{lf} \ge f_{ls} \quad and \quad f_{cs} = 0{,}95 \times f_{cu} \\ 0{,}7 \times \varepsilon_{cu} & for \quad f_{lf} < f_{ls} \quad and \quad f_{cs} = f_{cu} \times \left(\frac{\varepsilon_{cs}}{\varepsilon_{cu}}\right)^{0{,}4} \end{cases} \tag{6}$$

Where $f_{cs}$ and $\varepsilon_{cs}$=compressive stress and strain of confined concrete at yielding of steel spiral, respectively.

To estimate the ultimate confined compressive strength $f_{cu}$ and the ultimate strain $\varepsilon_{cu}$, Lee *et al*. (2010) modified the equations proposed previously by Lam and Teng (2003), for the concrete ultimate strain $\varepsilon_{cu}$ they are introduced two parameters, $k_s$ and $k_f$. The estimated expressions are given as follows

$$f_{cu} = f_{co} \times \left(1 + \frac{2(f_{lf} + f_{ls})}{f_{co}}\right) \tag{7}$$

$$\varepsilon_{cu} = \varepsilon_{co} \times \left(1{,}75 + 5{,}25\left(\frac{k_f f_{lf} + k_s f_{ls}}{f_{co}}\right) \times \left(\frac{\varepsilon_{frp.r}}{\varepsilon_{co}}\right)^{0{,}45}\right) \tag{8}$$

Where $\varepsilon_{frp.r}$=rupture strain of CFRP.

Unlike to Chastre and Silva (2010) model, the model of Lee *et al*. (2010) is only dealt the TSR as spirals, without taking account the effect of the TSR as hoops, for this reason the use of this model is not generalized, in addition to the complexity of the proposed expressions.

### 2.3 Model developed by Teng et al. (2014)

Teng *et al*. (2014) have proposed a stress-strain model for circular concrete columns confined by both external CFRP warps and internal steel hoops or spiral. The part of external CFRP is based on the relations proposed previously by Jian and Tang (2007), however, the contribution of the transverse steel reinforcement (TSR) is considered by the relationship of (Mander *et al*. 1988). The proposed stress-strain relationship is given by the following equation

$$\frac{f_c}{f_{cu}} = \frac{\left(\frac{\varepsilon_c}{\varepsilon_{cu}}\right) r}{r - 1 + \left(\frac{\varepsilon_c}{\varepsilon_{cu}}\right)^r} \tag{9}$$

Where

$$r = \frac{E_c}{E_c - \frac{f_{cu}}{\varepsilon_{cu}}} \tag{10}$$

To estimate the ultimate confined compressive strength $f_{cu}$ and the ultimate strain $\varepsilon_{cu}$, Teng, Lin *et al*. (2014) introduced the parameter of interaction $\rho_f$ between the CFRP and TSR. The proposed $f_{cu}$ and $\varepsilon_{cu}$ are given as follows

$$\frac{f_{cu}}{f_{co}} = 1 + 3{,}5\frac{f_{lf}}{f_{ls}} + 3{,}12\left[\frac{f_{lsy}}{f_{co}(1 + 0{,}202\rho_f^{0,145})}\right]^{0,736} \tag{11}$$

$$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1 + 3{,}9\left(\frac{f_{cu}}{f_{co}} - 1\right)^{1,2} \tag{12}$$

The parameter $\rho_f$ is given as follow

$$\rho_f = \frac{E_{frp} t_{frp} s d_c}{k_e E_s A_{st} D} \tag{13}$$

Where $E_{frp}$ and $t_{frp}$=elastic modulus and thickness of the CFRP warps, respectively; $E_s$ and $A_{st}$=elastic modulus and cross-sectional area of a steel hoops or spirals, respectively; $d_c$=diameter of center line of steel hoops or spirals; $s$=vertical center-to-center spacing of steel hoops or spirals; $k_e$=confinement effectiveness coefficient to account for confinement non uniformity over the column height; and $D$=diameter of the column.

Though, Teng *et al*. (2014) identified explicitly in their model, the interaction between CFRP and TSR by introducing the coefficient $\rho_f$ doubled the effect of interaction. Consequently, the values obtained by this model are very large compared with the other works, plus the complexity of their relationships.

### 2.4 Model developed by Shirmohammadi et al. (2015)

Shirmohammadi *et al*. (2015) have proposed a stress-strain constitutive model for confined circular concrete columns by both CFRP and steel hoops or spiral based on stress-strain relationship of Thorenfeldt and Tomaszewicz (1987). This relationship is given by the following equation

$$\frac{f_c}{f_{cu}} = \frac{\left(\frac{\varepsilon_c}{\varepsilon_{cu}}\right) r}{r - 1 + \left(\frac{\varepsilon_c}{\varepsilon_{cu}}\right)^{kr}} \tag{14}$$

And

$$r = \frac{E_c}{E_c - \frac{f_{cu}}{\varepsilon_{cu}}} \tag{15}$$

Where $k$=0.8 is Thorenfeldt parameter. The proposed ultimate strength $f_{cu}$ and ultimate strain $\varepsilon_{cu}$ are expressed as follows

$$\frac{f_{cu}}{f_{co}} = 1{,}1 + 2{,}5\left(\frac{f_{lf}}{f_{co}}\right)^{0,8} \times \left(\frac{f_{ls}}{f_{co}}\right) + 3{,}5\left(\frac{f_{ls}}{f_{co}}\right)^{0,2}\left(\frac{d_c^2}{D^2}\right)^4 \tag{16}$$

$$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 2 + 6{,}5\left(\frac{f_{lf}}{f_{co}}\right)^{0,7} \times \left(\frac{f_{ls}}{f_{co}}\right)^{0,7} + 6\left(\frac{f_{ls}}{f_{co}}\right)^{0,04}\left(\frac{s}{0{,}5 \times D}\right)^{-0,8} \times \left(\frac{E_{frp}}{f_{co}}\right)^{0,5} \tag{17}$$

The proposed model takes explicitly into account the combined effect of CFRP and TSR, but their expressions are largely complicated.

## 3. Experimental database

An experimental database of 34 specimens confined by the both CFRP warps and steel hoops or spirals is created. This database is collected from the studies of (Lee *et al*. 2010, Benzaid *et al*. 2010, Wang *et al*. 2012). Table 1 presents all parameters collected such as: the two important parameters in the stress-strain model $f_{cu}$ and $\varepsilon_{cu}$, the axial strain of unconfined concrete $\varepsilon_{co}$, the mechanical properties of the CFRP (the thickness $t_{frp}$, the elasticity modulus $E_{frp}$, the tensile strength of CFRP $f_{frp}$), the steel yield strength $f_{ys}$, diameter $d_s$ of steel (hoops or spirals). This table contains also four variable parameters such as: the compressive strength of unconfined concrete $f_{co}$ varies between 24.5 and 61.81 MPa, the diameter of specimens $D$ varies between 150 and 305 mm, the number of CFRP layers varies between 1 and 3 and the steel spacing $s$ varies between 20 and 140 mm.

Table 1 Experimental database

| Authors and years | Specimens | $D$ (mm) | $f_{co}$ (MPa) | $\varepsilon_{co}$ $\times 10^{-3}$ | CFRP Type | | | TSR | | | | $f_{cu}$ (MPa) | $\varepsilon_{cu}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $t_{frp}$ (mm) | $E_{frp}$ (GPa) | $f_{frp}$ (MPa) | type | $f_{ys}$ (MPa) | $s$ (mm) | $d_s$ (mm) | | |
| Lee *et al.* (2010) | S2F1 | 150 | 36,20 | 2,40 | 0,11 | 250 | 4510 | SP | 1200 | 20 | 5 | 72,87 | 0,0390 |
| | S2F2 | 150 | 36,20 | 2,40 | 0,22 | 250 | 4510 | SP | 1200 | 20 | 5 | 92,68 | 0,0360 |
| | S2F3 | 150 | 36,20 | 2,40 | 0,33 | 250 | 4510 | SP | 1200 | 20 | 5 | 108,01 | 0,0340 |
| | S2F4 | 150 | 36,20 | 2,40 | 0,44 | 250 | 4510 | SP | 1200 | 20 | 5 | 115,72 | 0,0380 |
| | S2F5 | 150 | 36,20 | 2,40 | 0,55 | 250 | 4510 | SP | 1200 | 20 | 5 | 150,80 | 0,0430 |
| | S4F1 | 150 | 36,20 | 2,40 | 0,11 | 250 | 4510 | SP | 1200 | 40 | 5 | 60,00 | 0,0190 |
| | S4F2 | 150 | 36,20 | 2,40 | 0,22 | 250 | 4510 | SP | 1200 | 40 | 5 | 74,77 | 0,0230 |
| | S4F3 | 150 | 36,20 | 2,40 | 0,33 | 250 | 4510 | SP | 1200 | 40 | 5 | 88,80 | 0,0290 |
| | S4F4 | 150 | 36,20 | 2,40 | 0,44 | 250 | 4510 | SP | 1200 | 40 | 5 | 104,15 | 0,0300 |
| | S4F5 | 150 | 36,20 | 2,40 | 0,55 | 250 | 4510 | SP | 1200 | 40 | 5 | 123,64 | 0,0360 |
| | S6F1 | 150 | 36,20 | 2,40 | 0,11 | 250 | 4510 | SP | 1200 | 60 | 5 | 50,37 | 0,0170 |
| | S6F2 | 150 | 36,20 | 2,40 | 0,22 | 250 | 4510 | SP | 1200 | 60 | 5 | 68,52 | 0,0250 |
| | S6F4 | 150 | 36,20 | 2,40 | 0,44 | 250 | 4510 | SP | 1200 | 60 | 5 | 99,44 | 0,0340 |
| | S6F5 | 150 | 36,20 | 2,40 | 0,55 | 250 | 4510 | SP | 1200 | 60 | 5 | 114,64 | 0,0360 |
| Benzaid *et al.* (2010) | I,RCC,1,1L | 160 | 29,50 | 3,77 | 1 | 34 | 450 | HP | 235 | 140 | 8 | 50,59 | 0,0160 |
| | I,RCC,1,3L | 160 | 29,50 | 3,77 | 3 | 34 | 450 | HP | 235 | 140 | 8 | 70,83 | 0,0220 |
| | I,PCC,1,1L | 160 | 25,93 | 3,77 | 1 | 34 | 450 | HP | 235 | 140 | 8 | 39,63 | 0,0130 |
| | I,PCC,1,3L | 160 | 25,93 | 3,77 | 3 | 34 | 450 | HP | 235 | 140 | 8 | 66,14 | 0,0150 |
| | II,RCC,2,1L | 160 | 58,24 | 3,00 | 1 | 34 | 450 | HP | 235 | 140 | 8 | 79,18 | 0,0930 |
| | II,RCC,1,3L | 160 | 58,24 | 3,00 | 3 | 34 | 450 | HP | 235 | 140 | 8 | 101,48 | 0,0140 |
| | II,PCC,1,1L | 160 | 49,46 | 1,70 | 1 | 34 | 450 | HP | 235 | 140 | 8 | 52,75 | 0,0020 |
| | II,PCC,1,3L | 160 | 49,46 | 1,70 | 3 | 34 | 450 | HP | 235 | 140 | 8 | 82,91 | 0,0070 |
| | III,RCC,2,1L | 160 | 63,01 | 2,70 | 1 | 34 | 450 | HP | 235 | 140 | 8 | 74,43 | 0,0030 |
| | III,RCC,2,3L | 160 | 63,01 | 2,70 | 3 | 34 | 450 | HP | 235 | 140 | 8 | 94,71 | 0,0080 |
| | III,PCC,1,1L | 160 | 61,81 | 2,80 | 1 | 34 | 450 | HP | 235 | 140 | 8 | 62,68 | 0,0030 |
| | III,PCC,1,3L | 160 | 61,81 | 2,80 | 3 | 34 | 450 | HP | 235 | 140 | 8 | 93,19 | 0,0100 |
| Wang *et al.* (2012) | C1H1L1C | 305 | 24,50 | 2,00 | 0,167 | 244 | 4340 | HP | 397 | 80 | 6 | 43,10 | 0,0196 |
| | C1H1L2M | 305 | 24,50 | 2,00 | 0,334 | 244 | 4340 | HP | 397 | 80 | 6 | 52,20 | 0,0268 |
| | C1H2L1M | 305 | 24,50 | 2,00 | 0,167 | 244 | 4340 | HP | 397 | 40 | 6 | 47,00 | 0,0232 |
| | C1H2L2M | 305 | 24,50 | 2,00 | 0,334 | 244 | 4340 | HP | 397 | 40 | 6 | 62,10 | 0,0330 |
| | C2H1L1M | 204 | 24,50 | 2,00 | 0,167 | 244 | 4340 | HP | 397 | 120 | 6 | 52,10 | 0,0231 |
| | C2H1L2M | 204 | 24,50 | 2,00 | 0,334 | 244 | 4340 | HP | 397 | 120 | 6 | 66,10 | 0,0341 |
| | C2H2L1M | 204 | 24,50 | 2,00 | 0,167 | 244 | 4340 | HP | 397 | 60 | 6 | 52,20 | 0,0253 |
| | C2H2L2M | 204 | 24,50 | 2,00 | 0,334 | 244 | 4340 | HP | 397 | 60 | 6 | 69,50 | 0,0341 |

Style type = Spiral (SP) or Hoop (HP)

## 4. Proposed axial stress-strain model

### 4.1 Effective lateral confinement pressure

The lateral confinement pressure is produced in confined concrete when the member is loaded such that the concrete starts to dilate and expands laterally. The value of such pressure depends on the geometry of the confined member and the amount and mechanical properties of confinements materials provided (see Fig. 1).

According to ACI 440 (2008), the lateral confinement pressure ($f_{lf}$) resulting from the external CFRP warps for circular columns is given by the following equation

$$f_{lf} = \frac{2 \times t_{frp} \times f_{frp}}{D} \tag{18}$$

Where $t_{frp}$=thickness of the CFRP warps, $f_{frp}$=tensile strength of CFRP and $D$=diameter of the column.

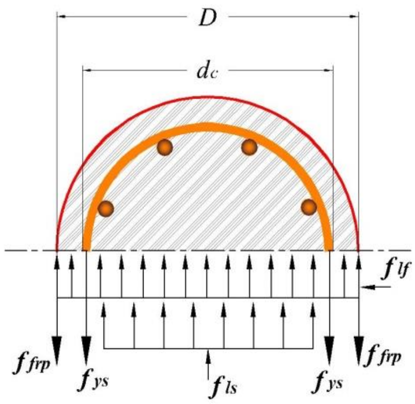


Fig. 1 Mechanism of confinement for confined concrete by CFRP and TSR

The results reported in Table 1 indicated that the confinement of circular concrete columns is not only depending to CFRP but also to the TSR. The confinement by these two materials simultaneously (CFRP and TSR) produces a combined effect. The lateral confinement pressure that considers the combined effect of CFRP and TSR expressed by Teng *et al*. (2014) is given by

$$f_{le} = f_{lf} + k_e f_{ls} \tag{19}$$

Where $f_{lf}$ is given by Eq. (18) and $f_{ls}$=lateral confinement pressure provided by transverse steel reinforcement applied to the concrete core (see Fig. 1). It is given as follows

$$f_{ls} = \frac{2 \times A_{st} \times f_{ys}}{s \times d_c} \tag{20}$$

Where $A_{st}$=cross-sectional area of a steel hoops or spirals; $f_{ys}$=steel yield strength; $d_c$=diameter of center line of steel hoops or spirals; $s$=vertical center-to-center spacing of steel hoops or spirals; and $k_e$=confinement effectiveness coefficient to account for confinement non uniformity over the column height. For the circular concrete columns confined by steel hoops or spirals, the original expression proposed by Mander *et al.* (1988) for the coefficient $k_e$ is given as follows

$$k_e = \begin{cases} \dfrac{\left(1 - \dfrac{s'}{2d_c}\right)^2}{(1-\rho_{cc})} & \textit{for circular hoops} \\[2ex] \dfrac{\left(1 - \dfrac{s'}{2d_c}\right)}{(1-\rho_{cc})} & \textit{for circular spirals} \end{cases} \tag{21}$$

Where $s'$=clear spacing between hoops or spiral reinforcement; and $\rho_{cc}$ is the longitudinal steel ratio relative to the confined concrete core.

### *4.2 Proposed axial ultimate strength $f_{cu}$*

The axial ultimate strength $f_{cu}$ is very important parameter on the stress-strain model as it considers the lateral confinement pressure effect. For the circular concrete columns confined by CFRP, the axial ultimate strength is largely higher than the unconfined concrete strength $f_{co}$. Generally, the stress-strain curve of confined concrete by CFRP is represented by a second ascending branch. The relationship between the axial ultimate strength of confined circular concrete by CFRP and the parameters that affect it will be considered as a strength model. Currently, many researchers have formulated the ratio $\frac{f_{cu}}{f_{co}}$ and $\frac{f_{lf}}{f_{co}}$ as a linear function, according to Lam and Teng, Lin (2003), this relationship is given as follows

$$\frac{f_{cu}}{f_{co}} = 1 + k_1 \frac{f_{lf}}{f_{co}} \tag{22}$$

Where $k_1$ is the confinement effectiveness coefficient; and $f_{co}$ is the compressive strength of unconfined concrete. Furthermore, the results presented in Table 1 indicated that the TSR confinement effect should not be ignored in the presence of the CFRP confinement. From the general Eq. (22) for the confined concrete by FRP, a new relationship can be developed for ($f_{cu}$) by adding the effect of the confinement of TSR. These coefficients and factors involved are calibrated from the regression analysis of the experimental data of 34 specimens illustrated in Table 1.

The proposed expression for the ultimate strength ($f_{cu}$) is expressed as follows

$$\frac{f_{cu}}{f_{co}} = 1 + 2{,}2\left(\frac{f_{lf}}{f_{co}}\right)^{0{,}94} + 1{,}1\left(\frac{k_e f_{ls}}{f_{co}}\right)^{0{,}76} \tag{23}$$

The Fig. 2 shows a good linear correlation between the experimental and the proposed ratio $\frac{f_{cu}}{f_{co}}$ in Eq. (23) of 26 specimens. The correlation coefficient ($R^2$) in this figure is equal to 0.90, which proves the accuracy of our proposal.

### *4.3 Proposed axial ultimate strain $\varepsilon_{cu}$*

The axial ultimate strain $\varepsilon_{cu}$ is also an important parameter in the stress-strain model of confined concrete. Similar to the axial ultimate strength $f_{cu}$ expression provided in Eq. (23), several researchers as Lam and Teng (2003) proposed an expression for the axial ultimate strain as follows

$$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1 + k_2 \frac{f_{lf}}{f_{co}} \tag{24}$$

Where $\varepsilon_{co}$=axial strain in unconfined concrete corresponding to $f_{co}$; and $k_2$=confinement effectiveness coefficient.

In order to account the effect of the combined confinement of CFRP-TSR in the general Eq. (24) of confined concrete by FRP, a new expression for the ultimate strain ($\varepsilon_{cu}$) can be inspired for confined concrete under the combined effect of CFRP and TSR confinement. These coefficients and factors that form are predicted from the regression analysis of the experimental data shown in Table

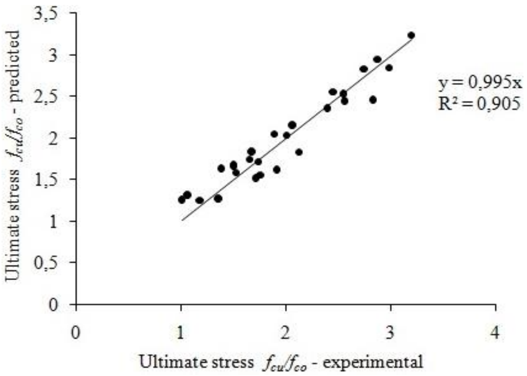


Fig. 2 Comparison between experimental and predicted ultimate stress

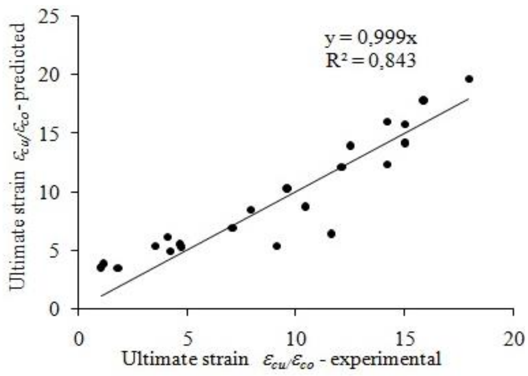


Fig. 3 Comparison between experimental and predicted ultimate strain

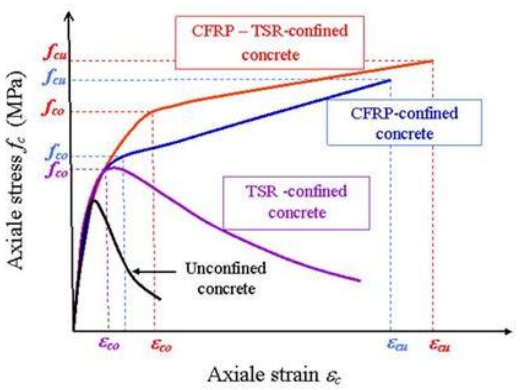


Fig. 4 Proposed stress strain curve for CFRP-TSR confined concrete

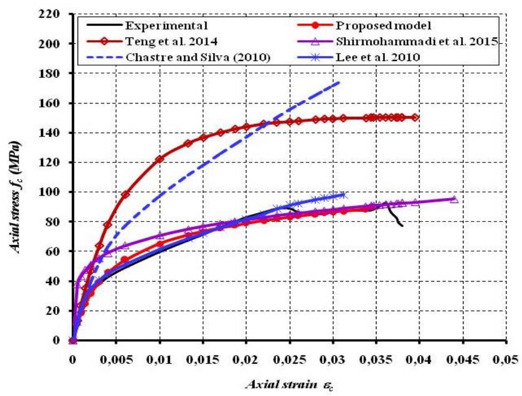


Fig. 5 Comparison between confinement models and stress-strain curve of specimen S2F2 (Lee *et al*. 2010)

1. The proposed expression is written as follows

$$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 2 + 10\left(\frac{f_{lf}}{f_{co}}\right) + 15\left(\frac{k_e f_{ls}}{f_{co}}\right)^{0,7} \tag{25}$$

The Fig. 3 shows a very acceptable linear correlation between the experimental and the proposed ratio $\frac{\varepsilon_{cu}}{\varepsilon_{co}}$ in Eq. (25) of 21 specimens.

### *4.4 Stress-strain curve*

In order to develop the axial stress-strain model for circular concrete columns confined by the combined external CFRP warps and internal TSR (hoops or spirals), the following modified model of Thorenfeldt *et al*. (1987) is used

$$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 2 + 10\left(\frac{f_{lf}}{f_{co}}\right) + 15\left(\frac{k_e f_{ls}}{f_{co}}\right)^{0,7} \tag{25}$$

$$\frac{f_c}{f_{cu}} = \frac{\left(\frac{\varepsilon_c}{\varepsilon_{cu}}\right)r}{r - 1 + \left(\frac{\varepsilon_c}{\varepsilon_{cu}}\right)^{0,8r}} \tag{26}$$

And

$$r = \frac{E_c}{\left[E_c - \left(\frac{f_{cu}}{\varepsilon_{cu}}\right)\right]} \tag{27}$$

Several researchers used different formulas to calculate the concrete modulus of elasticity $E_c$. In this paper, the following expression given by ACI 318 (2011) is used

$$E_c = 4700\sqrt{f_{co}} \tag{28}$$

The proposed model presented in Fig. 4 was developed based on this value of $E_c$.

## 5. Validation of proposed model

The new proposed model of confined circular concrete columns under the combined external CFRP warp and internal TSR (hoops or spiral) effect will be compared with different experimental and numerical results in order to validate our proposal. The Figs. 5-12 illustrate a confrontation between the new proposed model presented in this paper and the models of (Lee *et al*. 2010, Chastre and Silva 2010, Teng *et al*. 2014, Shirmohammadi *et al*. 2015), as well as the experimental results of eight specimens S2F2, S2F3 and S4F2 from the research of Lee *et al.* (2010) and C1H1L1C, C1H1L2M, C1H2L1M, C2H2L1M and C2H2L2M from the research of Wang *et al.* (2012).

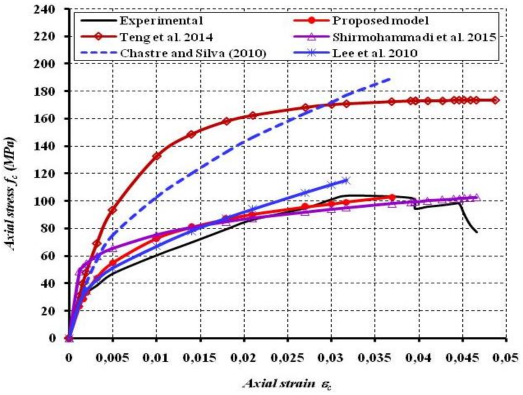


Fig. 6 Comparison between confinement models and stress-strain curve of specimen S2F3 (Lee *et al*. 2010)

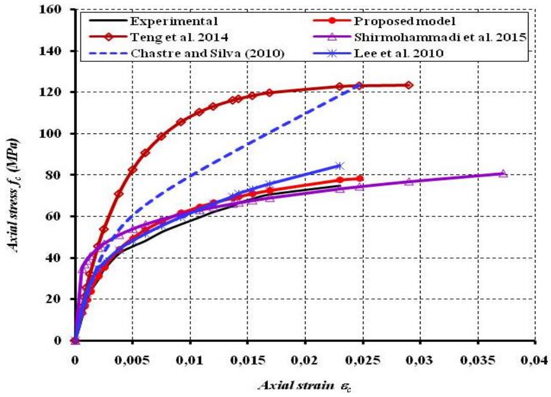


Fig. 7 Comparison between confinement models and stress-strain curve of specimen S4F2 (Lee *et al*. 2010)

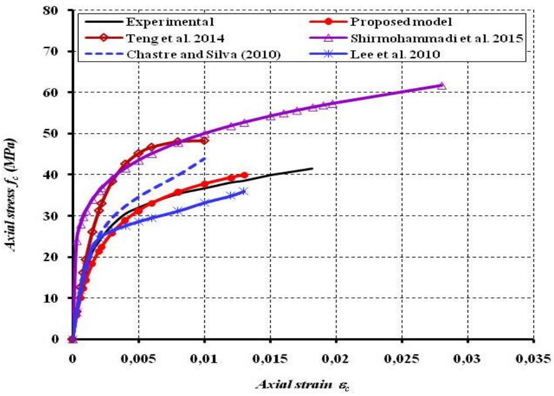


Fig. 8 Comparison between confinement models and stress-strain curve of specimen C1H1L1C (Wang *et al*. 2012)

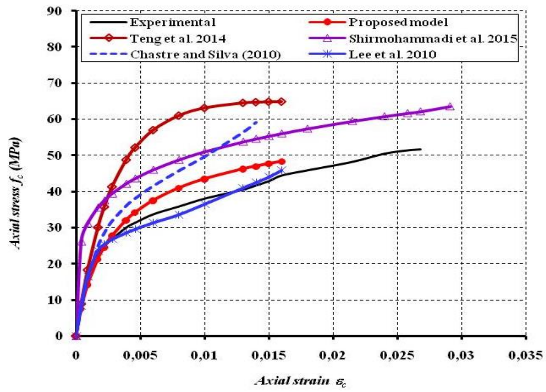


Fig. 9 Comparison between confinement models and stress-strain curve of specimen C1H1L2M (Wang *et al*. 2012)

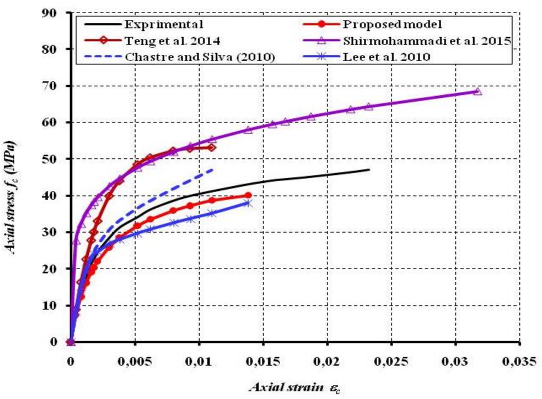


Fig. 10 Comparison between confinement models and stress-strain curve of specimen C1H2L1M (Wang *et al*. 2012)

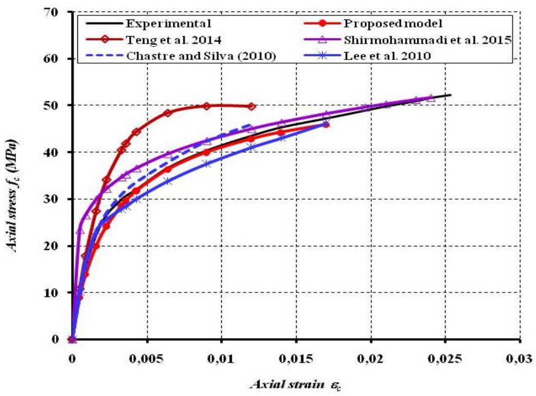


Fig. 11 Comparison between confinement models and stress-strain curve of specimen C2H2L1M (Wang *et al*. 2012)

The comparison of the proposed stress-strain curve with the experimental curves of Lee *et al*. (2010) and other models illustrated in Figs. 5-7 clearly shows the performance of our proposed model. It is clear that our curve coincides exactly with the experimental curves, as it approaches to (Lee *et al.* 2010, Shirmohammadi *et al.* 2015) curves, although, the expressions of our model are very simplified than those two models. From these figures, we can note also that our model gives a very advantageous result compared to (Chastre and Silva 2010, Teng *et al*. 2014) models, where the Figs. 5-7 show that these last two models are largely overestimate the ultimate strength and strain values compared with the experimental results.

These observations will be largely validated by the comparison between our model and Wang *et al*. (2012) experimental curves illustrated in Figs. 8-12. Indeed, the points of our model approach good with the experimental curves of Wang *et al.* (2012) and the model of Lee *et al.*

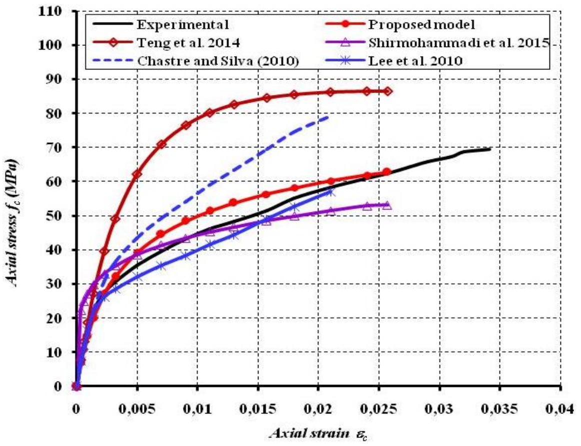


Fig. 12 Comparison between confinement models and stress-strain curve of specimen C2H2L2M (Wang *et al*. 2012)

(2010), nevertheless, the ultimate values ($f_{cu}$ and $\varepsilon_{cu}$) offered by our model remain always lower than the experimental values, which offers a considerable safety margin. Moreover, the ultimate strengths of (Chastre *et al*. 2010, Teng *et al*. 2014) far exceed the experimental results.

However, the model of Shirmohammadi *et al*. (2015) shows a very variable behavior compared to other models, due to the complexity of their strength and strain expression forms. Consequently, our model shows a very simple form with a very acceptable correlation and it gives more reliable results which approach to the experimental results.

## 5. Conclusions

The use of external carbon-fiber-reinforced polymer (CFRP) wrapping is one of the effective strengthening methods for structural members such as reinforced concrete columns. In the design of this application, it is necessary to develop a CFRP-confined reinforced concrete model to evaluate their stress-strain behavior. However, it is also necessary to consider the confinement contribution of the internal transverse steel reinforcement (TSR). In addition, few models that consider the confined concrete under the combined external CFRP wrapping and internal TSR effect have been developed over the past 2 decades.

In this paper, a new axial stress- strain model has been developed based on experimental database to predict the axial compressive behavior of confined circular concrete columns under combined effect of external CFRP wrapping and internal TSR (hoops or spirals). The Comparison between the proposed model and the experimental results shows a good agreement compared to the several models previously presented. Additionally, the expressions proposed to estimate the ultimate strength and its corresponding strain are simple and precise which make them easy to use in the applications design.

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