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Deformation-based Strut-and-Tie Model for reinforced concrete columns subject to lateral loading

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Abstract. This paper presents a Strut-and-Tie Model for reinforced concrete (RC) columns subject to lateral loading. The proposed model is based on the loading path for the post-yield state, and the geometries of struts and tie are determined by the stress field of post-yield state. The analysis procedure of the Strut-and-Tie Model is that 1) the shear force and displacement at the initial yield state are calculated and 2) the relationship between the additional shear force and the deformation is determined by modifying the geometry of the longitudinal strut until the ultimate limit state. To validate the developed model, the ultimate strength and associated deformation obtained by experimental results are compared with the values predicted by the model. Good agreements between the proposed model and the experimental data are observed.

Keywords: deformation; lateral loading; reinforced concrete column; strut-and-time model; ultimate strength

1. Introduction

Over the past decades, various Strut-and-Tie Models have been proposed and developed for the design of reinforced concrete (RC) structure (To *et al*. 2000, Tjhin and Kuchma 2004, Hong *et al*. 2011, Kim *et al*. 2012, Chun 2014 and Kassem 2015). The cyclic analysis with computer program using Strut-and-Tie elements was introduced and proposed by To *et al*. (2000). Using the Strut-and-Tie model, hysteretic responses of doubly reinforced concrete beams subject to cyclic loading were predicted. Tjhin and Kuchma (2004) developed the Strut-and-Tie Models for load-deformation analysis of D-regions to present a design procedure satisfying the strength and serviceability requirements. For RC interior beam-column joints, the deformation-based Strut-and-Tie Model was developed by Hong *et al*. (2011). The shear force and moment from adjacent beams and columns and the strain distribution along the beam bars within the joint were mainly

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considered to predict the shear strength degradation of RC interior beam-column joints. More recent studies regarding Strut-and-Tie Models have also been conducted for various applications (Kim *et al*. 2012, Chun 2014 and Kassem 2015).

If the layout of Strut-and-Tie Model for load-deformation analysis is based on the stress distribution in elastic state, the prediction of strength and response for the ultimate limit state cannot be credited. On the contrary, if the layout of the Strut-and-Tie Model is based on the stress distribution of ultimate limit state, the initial stiffness and yield displacement of the structural component cannot be accurately provided. Therefore, it is reasonable that Strut-and-Tie Models are constructed according to the design objectives: i.e., 1) models for serviceability requirement based on the elastic state and 2) models for deformation capacity estimation based on the ultimate limit state.

Strut-and-Tie Model is a linearly simplified representation of the actual load path at a defined loading stage not the entire loading stages. Thus the behavior characteristics of the truss components cannot represent the real response and the fixed geometries of struts and ties cannot be valid for the entire loading stages. Appropriate selection of truss geometries, effective width of struts, constitutive models of struts and ties, and effective strength of struts, ties and nodes must be ensured to get the reasonable results of load-deformation analysis of Strut-and-Tie Models. Basic assumptions usually employed in the load-deformation analysis are listed as follows: 1) The configuration of Strut-and-Tie Model is same for the entire loading states; 2) The primary modes of failure are yielding of ties and crushing of struts or nodal zones. In case of indeterminate truss, yielding of ties reduces the stiffness of the structure, while the failure of the structure occurs by crushing of struts or nodal zone; 3) Stress-strain relationships of struts and ties are defined. Though the stress-strain characteristics for concrete and steel are generally employed in struts and ties, respectively, they can be modified so as to consider the local interactions between the concrete and steel bars; 4) Deformation in nodal zones is ignored; and 5) Small deformation is assumed.

In this paper, deformation-based Strut-and-Tie Model for reinforced concrete columns subject to lateral loading is first proposed. Due to the deformability requirements, the developed model is based on loading path of post-yield state, and the geometries of struts and ties are determined by the stress field of the post-yield state. Second, in order to validate the model, experimental data of the intermediate and short columns subject to lateral loading are compared with the estimations by the proposed Strut-and-Tie model.

2. Determination of deformation capacity of RC columns

2.1 Strut-and-Tie model

Fig. 1(a) shows a typical RC intermediate and short columns subject to shear force V and axial force N. The length of the column between the inflection point and the end face is denoted by L. Since the post-yield deformation of the column is mainly controlled by the end region, the column is divided into the rigid region and deformable region. Fig. 1(b) illustrates the stress field in the deformable region. The Strut-and-Tie Models are constructed based on the stress field, as shown in Fig. 1(c) and 1(d), and the other region above the deformable region is assumed as a rigid body. In Fig. 1(c) and 1(d), the element forces T and C are the tension force in tie element and the compression force in strut elements, respectively, and subscripts l, tr, and d denote the longitudinal, transvers, and diagonal element, respectively. The strut C_{d1} represents the fan-shaped stress field and the strut C_{d2} represents the uniformly distributed compression stress field. The angle θ is the diagonal strut angle, which is assumed to be the inclined cracking angle at the initial yielding of main bars in the bottom end and is expressed in Eq. (1).

$$\theta = \tan^{-1}\left[\frac{2L}{2A_{sl}f_{yl}+N}\left(\frac{A_{sh}E_s\varepsilon_{cr}}{s}+E_c\varepsilon_{cr}b\right)\right] \tag{1}$$

where A_{sl} is the cross-sectional area of the longitudinal reinforcing bar; f_{yl} is the specified yield strength of longitudinal reinforcing bar; A_{sh} is the cross-sectional area of transverse reinforcement per layer; E_s is the Young's modulus of steel; ε_{cr} is the crack strain of concrete; s is the spacing of transverse reinforcement; and b is the width of the strut.

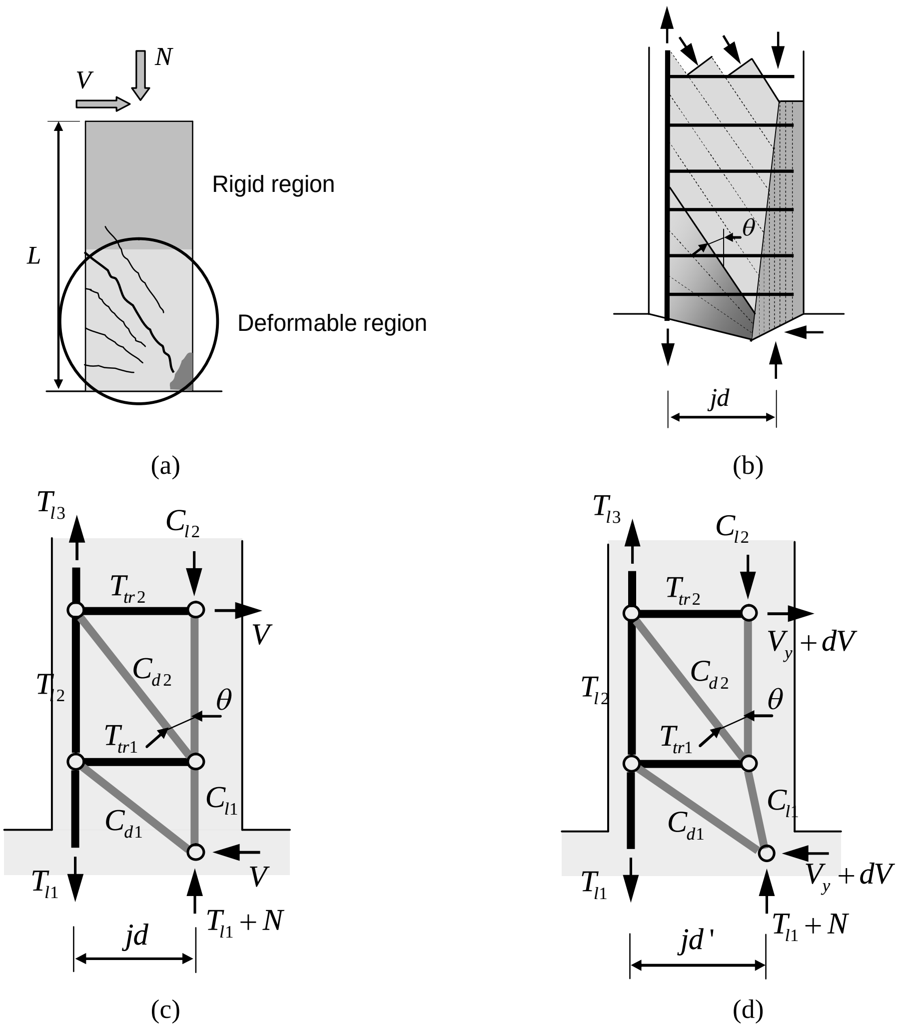


Fig. 1 (a) Rigid and deformable regions of RC column; (b) stress fields of RC column; (c) initial yield state; and (d) post yield state

The failure sequence of a reinforced concrete member is that flexural reinforcement at the end yields first and subsequent failure of any other component results in member failure. Therefore, two models are presented: 1) Strut-and-Tie Model in elastic state representing the stress flows in initial yield state; and 2) the model in post-yield state representing the ultimate state, as shown in Fig. 1(c) and 1(d) respectively.

2.2 Stress-strain relationship of components

For the load-deformation analysis of the Strut-and-Tie model, the stress-strain relationships of the Strut-and-Tie components are appropriately employed. Therefore, the stiffness, strength and post-yield behavior of each component are defined in this section.

2.2.1 Longitudinal tie components

The longitudinal tie element is subjected to uniaxial tension with transverse cracks in the flexural tension region. It is assumed that the area of the tie element is the cross-sectional area of longitudinal reinforcing bars A_{sl}, and the element length l_e is $(jd\cot\theta)/2$ for T_{l1} and $jd\cot\theta$ for T_{l2} and T_{l3}, as exhibited in Fig. 1. Because the deformation of RC columns depends on the elongation of the tie element, the constitutive models need to be precise. The deformation of the tie element can be determined with crack spacing and crack width, which are estimated by the bond stress-slip relationship between reinforcing bars and cover concrete.

Minimum crack spacing S_{min} can be estimated by the following equation.

$$S_{min} = \frac{A_{c,eff}\, f_{ct}}{n\pi d_b\, f_b} \tag{2}$$

where $A_{c,eff}$ is the effective cross-sectional area of concrete strut; f_{ct} is the tensile strength of concrete; n is the number of reinforcing bars in tension chord; d_b is the diameter of a reinforcing bar; and f_b is the average bond strength.

For the convenience of calculation, $A_{c,eff}$ is simply selected as $A_g/3$, where A_g is the gross cross-sectional area. Assuming that f_b/f_{ct} = 2 and S = 1.5S_{min} gives a simple estimation of the crack spacing along the longitudinal tie element in Eq. (3).

$$S_l = \frac{A_g}{4n\pi d_b} \tag{3}$$

Fig. 2 demonstrates a free body diagram of a differential element. The equilibrium and compatibility conditions for a differential element give a differential equation for the bond-slip relationship as follows

$$\frac{d^2\delta}{dx^2} = K_s f_b \tag{4}$$

where δ is the relative displacement between the reinforcing bar and surrounding concrete; x is the distance from non-slip point along the bar; and the constant K_s is expressed in Eq. (5).

$$K_s = 4\left(1 - \frac{E_s A_s}{E_c A_{c,eff}}\right) \Big/ \left(d_b E_s\right) \tag{5}$$

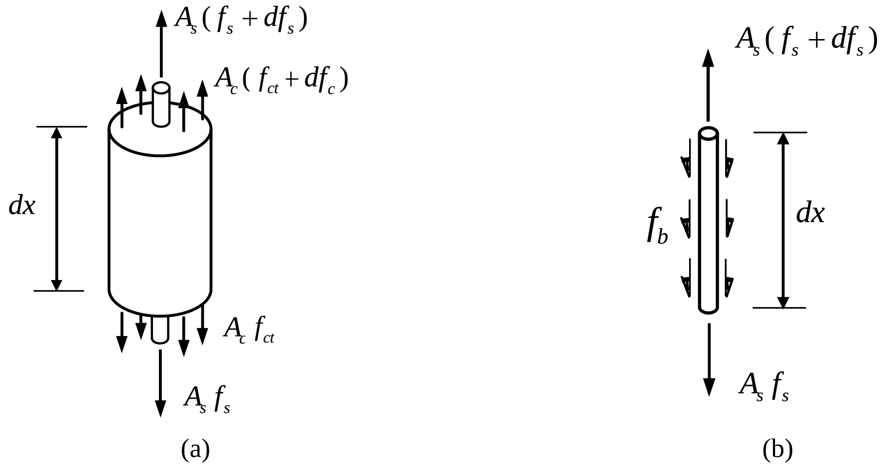


Fig. 2 Differential tension chord element: (a) forces acting on a tension chord with length of *dx* and (b) equilibrium condition

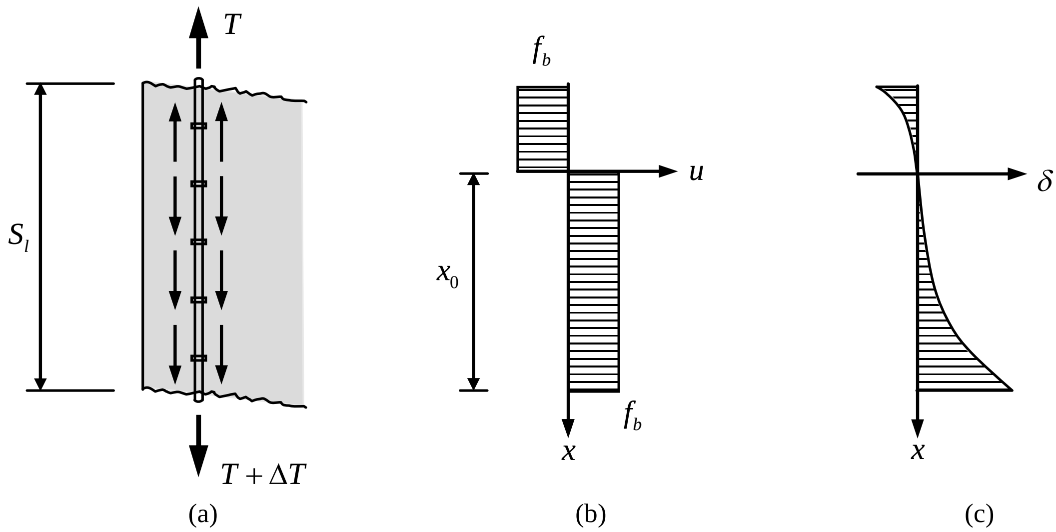


Fig. 3 Relationship between bond stress and slip within a single crack: (a) forces acting on a crack spacing; (b) bond stress distribution; and (c) slip distribution

Fig. 3(a) displays the forces acting on a longitudinal tie element between two adjacent cracks. Distribution of local bond stress f_b is assumed to be a constant value which equals to local bond strength, as exhibited in Fig. 3(b). Equating of the sum of local bond stress to the vertical components of the diagonal strut forces gives the length x_0 between a crack and a non-slip point

$$x_0 = \frac{S_l}{2}\left(\frac{u}{f_b}+1\right) \tag{6}$$

where u is a global bond stress developed by shear force in Eq. (7).

$$u = V/(n\pi d_{bl}\, jd) \tag{7}$$

and f_b is a local bond strength that is derived from the bond strength depending on steel strain.

$$f_b = f_{ct}\left(2 - f_{sl}/f_{yl}\right) \tag{8}$$

Using the boundary conditions of $\delta = 0$ at $x = 0$, $\varepsilon_s = f_{sl} / E_s$ at $x = x_0$, and $\varepsilon_c = 0$ at $x = x_0$, Eq. (4) yields as follows

$$\delta(x_0) = \frac{f_{sl}}{E_s} x_0 - \frac{1}{2} K_s f_b x_0^2 \tag{9}$$

A slip in the opposite direction can be calculated in the same way to obtain a crack width. The slip is distributed in Fig. 3(c), and the crack width w can be calculated by adding the two of the end slips.

$$w = \frac{f_{sl}}{E_s} S_l - \frac{1}{2} K_s f_b \left(x_0^2 + (S_l - x_0^2)\right) \tag{10}$$

Note that the second term of the right side in Eq. (10) expresses the reduction of deformation by a tension stiffening effect. Using the relations of $\varepsilon_l = w / S_l$ and $\delta_{T_l} = l_e \varepsilon_l$, the force-deformation relationship for a longitudinal tie element in elastic state is determined as follows

$$\delta_{T_l} = \frac{l_e}{S_l}\left(\frac{f_{sl}}{E_s} S_l - \frac{1}{2} K_s f_b \left(x_0^2 + (S_l - x_0^2)\right)\right) \tag{11}$$

For yielding conditions, bond failure and yielding of bars are considered as follows

1) At bar yielding condition

$$T_l = A_{sl} f_{yl} \tag{12}$$

2) At bond failure

$$u = f_b \quad \text{or} \quad x_0 = S_l \tag{13}$$

The post-yield behavior of the longitudinal ties is assumed to be perfectly plastic. The deformation after yielding should be determined by compatibility with that of longitudinal strut element.

2.2.2 Longitudinal strut components

A longitudinal strut component is defined as a flexural compression component subject to uniaxial compression force. To define the relationship between load and deformation of the element, a linear stress-strain relationship is assumed to have the secant modulus of cylinder concrete at the ultimate, $E_c/2$.

The area of the longitudinal strut is assumed to be 2/3 of the area of flexural compression zone which is an elastic triangular compression block area at the initial yielding of the longitudinal tension tie

$$A_{Cl} = \frac{2}{3} bc \tag{14}$$

where b is the width of the strut and c is the length of the concrete compression block, which is determined by the compatibility condition between the compressive strain of concrete triangular block and the tensile strain of steel bar at the initial yielding in Eq. (15).

$$c = \frac{-(A_{sl} f_{yl} + N) + \sqrt{(A_{sl} f_{yl} + N)^2 + 2(A_{sl} f_{yl} + N) d \varepsilon_y b E_c}}{\varepsilon_y b E_c} \tag{15}$$

The lever arm length jd, which is a distance between longitudinal tie and longitudinal strut, can be calculated by Eq. (16).

$$jd = d - \frac{c}{3} \tag{16}$$

where d is the effective column depth.

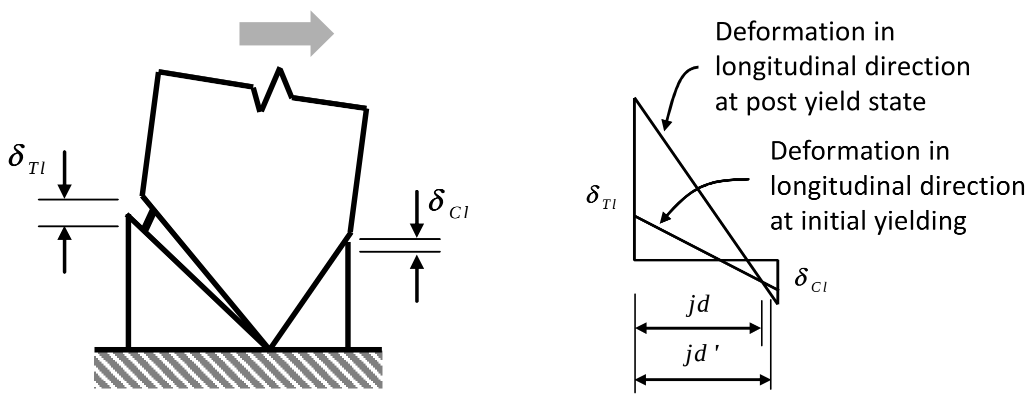
In the model for the post-yield state in Fig. 1(d), lever arm length jd is changed to carry the additional shear dV. The changed lever arm length jd' is determined from the compatibility condition between the deformation of the yielding longitudinal tie and the deformation of concrete strut (Fig. 4). The deformation of longitudinal tie element after yielding depends on that of longitudinal strut element and changed lever arm length. Thus, the relationship of the deformation of longitudinal tie increased by additional shear and changed lever arm length is given by

$$\delta_{T_{l1}} = \frac{3jd' - 2d}{3(d - jd')} \delta_{C_{l1}} \tag{17}$$

where $\delta_{C_{l1}}$ is the deformation of longitudinal strut.

The additional shear force dV is expressed by equilibrium of longitudinal strut in terms of lever arm length jd at the initial yielding and the changed lever arm length jd' at the ultimate state as

$$dV = \frac{2(jd' - jd)}{jd \cot\theta} \left[\left(\frac{L}{jd} - \frac{\cot\theta}{2} \frac{jd}{jd'} \right) V_y + \frac{N}{2} \right] \tag{18}$$



(a) (b)

Fig. 4 Relationship between deformations of longitudinal tie component and strut: (a) deformation mechanism and (b) compatibility condition between longitudinal components

2.2.3 Transverse tie components

Transverse tie components are supposed to carry the member shear force. General shear failure of flexural members occurs by the failure of transverse tie component in critical section as shown in Fig. 5(a). According to the current ACI 318 design equation for shear strength of RC flexural member (ACI 2014), the shear force is assumed to be carried by the contribution of transverse reinforcement and that of web concrete as follows

$$V_n = V_s + V_c \tag{19}$$

Fig. 5(b) presents the shear mechanism in a diagonal crack at shear failure. It is assumed that all of the transverse bars yield. The deformation of the transverse tie element is expressed in terms of diagonal crack spacing and width using the equation $\varepsilon_{tr} = w_d / S_d$ as follows

$$\delta_{T_{tr}} = \frac{w_d\, jd}{S_d} \tag{20}$$

where w_d is diagonal crack width and S_d is diagonal crack spacing. The diagonal crack spacing S_d is approximated in Eq. (21).

$$S_d = \frac{S_x S_l}{S_x \sin\theta + S_l \cos\theta} \tag{21}$$

where $S_x = bs/(4n\pi d_{tr})$ is the horizontal component of diagonal crack spacing. Neglecting the bond effect between the transverse reinforcing steel and concrete, the steel stress can be determined in terms of crack width. The shear force carried by concrete is due to friction along the crack face. This force acting on the diagonal crack is calculated using the equation proposed in the modified compression field theory (MCFT) by Vecchio *et al.* (1986). If there is no effect of flexural deformation, the crack width at the maximum shear resistance w_{dy} can be determined from yield stress f_{yh} for the transverse bar stress f_{sh}. Therefore, the strength and corresponding deformation of the transverse tie element is calculated as follows

$$T_{try} = b\, jd \left(f_{yh}\rho_h \cot\theta + \tau_{c,\ \mathrm{at}\ w_{dy}} \right) \tag{22}$$

$$\delta_{T_{tr}y} = \frac{w_{dy}\, jd}{S_d} \tag{23}$$

It is assumed that the relationship between force and deformation before the deformation in Eq. (23) is linear, neglecting large shear stiffness before cracking. The shear strength is considerably affected by member's flexural behavior. Flexural deformation components in a crack width are depicted by tensile deformation at longitudinal tie $\delta_{st} = \varepsilon_{T_l} S_d / \cos\theta$, and compressive deformation longitudinal strut $\delta_{sc} = \varepsilon_{C_l} S_d / \cos\theta$, where ε_{T_l} and ε_{C_l} denote the longitudinal strain $\varepsilon_{T_{l2}}$, $\varepsilon_{T_{l3}}$ and $\varepsilon_{C_{l1}}$, $\varepsilon_{C_{l2}}$ in the cases of T_{tr1} and T_{tr2}, respectively. The longitudinal deformation components give the additional crack width so that all of the transverse reinforcing bars at the crack face yield. Fig. 5(c), 5(d) and 5(e) show the change in yield crack width.

Note that the deformation of the tie element is treated as a positive value in tension and that of the strut element is positive in compression. Using this crack width at maximum shear resistance, Eqs. (22) and (23) give transverse tie element behavior with flexural deformation.

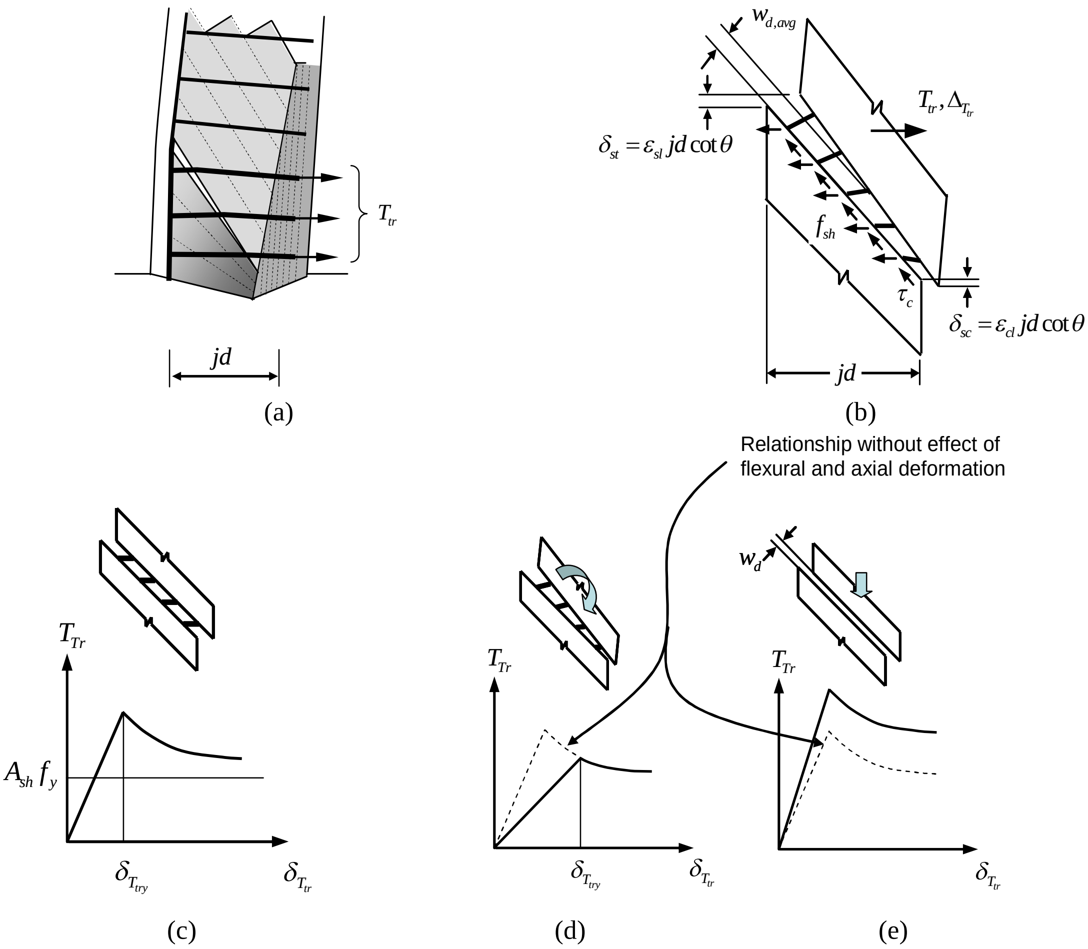


Fig. 5 Force-deformation relationship of transverse tie: (a) critical transverse tie in stress field; (b) equilibrium and deformed shape; (c) force-deformation relationship without longitudinal element's deformation; (d) effect of flexural rotation; and (e) effect of axial deformation of column

2.2.3 Diagonal strut element

A diagonal strut element is a discrete representation of a diagonally cracked compression field subject to uniaxial compression. The strength of the compression strut decreases as the transverse tensile strain increases. The constitutive equation proposed in the MCFT is used for the stress strain relationship of the diagonal strut

$$\sigma_d = f_{2,\max}\left[2\left(\frac{\varepsilon_2}{\varepsilon_c'}\right)-\left(\frac{\varepsilon_2}{\varepsilon_c'}\right)^2\right] \tag{25}$$

where $f_{2,\max} = \dfrac{f_c'}{0.8+170\varepsilon_1} \le f_c'$ (MPa); σ_d and ε_2 denote the compressive stress and strain respectively; ε'_c = 0.002; and $\varepsilon_1 = \Delta_{T_{tr}} / jd$.

The area of diagonal strut C_{d2} is calculated by $bjd\cos\theta$ from the geometry of the stress field. However, since the diagonal strut C_{d1} represents a fan-shaped region, the area cannot be determined from the stress field. For simplicity, the area of the diagonal strut C_{d1} is assumed as the mean of the area as $(bjd\cos\theta)/2$.

2.3 Equilibrium and forces on components

Since general truss models depict the ultimate limit state, they can be treated as determinate structures. In Fig. 1(c), the proposed model is also determinate structures in which component forces can be determined with equilibrium condition. The forces of the components can be expressed in terms of external shear force V and axial force N as follows

1) Longitudinal tie components

$$T_{l1} = V\frac{L}{jd} - \frac{N}{2} \tag{26}$$

$$T_{l2} = V\left(\frac{L}{jd} - \frac{1}{2}\cot\theta\right) - \frac{N}{2} \tag{27}$$

$$T_{l3} = V\left(\frac{L}{jd} - \frac{3}{2}\cot\theta\right) - \frac{N}{2} \tag{28}$$

2) Longitudinal strut components

$$C_{l1} = V\left(\frac{L}{jd} - \frac{1}{2}\cot\theta\right) + \frac{N}{2} \tag{29}$$

$$C_{l2} = V\left(\frac{L}{jd} - \frac{3}{2}\cot\theta\right) + \frac{N}{2} \tag{30}$$

3) Transverse tie component

$$T_{tr} = V \tag{31}$$

4) Diagonal strut components

$$C_{d1} = V\sqrt{1 + \frac{\cot^2\theta}{4}} \tag{32}$$

$$C_{d2} = V / \sin\theta \tag{33}$$

After yielding of longitudinal tie T_{l1}, the Strut-and-Tie Model in Fig. 1(c) cannot explain the increasing of shear force dV that is carried by the longitudinal strut. Fig. 1(d) shows a changed Strut-and-Tie Model in post-yield state, which can address the additional shear transfer mechanism by the change of the strut angular orientation. The forces acting on the each component by the additional shear at post-yield state are determined as follows

1) Longitudinal tie components

$$T_{l1} = A_{sl} f_{yl} \tag{34}$$

$$T_{l2} = V_y \left(\frac{L}{jd} - \frac{jd}{2jd'} \cot\theta \right) - \frac{N}{2} \tag{35}$$

2) Longitudinal strut components

$$C_{l1,\mathrm{ver}} = V_y \left(\frac{L}{jd} - \frac{jd}{2jd'} \cot\theta \right) + \frac{N}{2} \tag{36}$$

$$C_{l1,\mathrm{hor}} = dV \tag{37}$$

3) Transverse tie components

$$T_{tr1} = V_y \tag{38}$$

$$T_{tr2} = V_y + dV \tag{39}$$

4) Diagonal strut components

$$C_{d1} = V_y \sqrt{1 + \frac{\cot^2\theta}{4}} \tag{40}$$

$$C_{d2} = (V_y + dV)/\sin\theta \tag{41}$$

where *jd’* is the modified lever arm length in post-yield state.

2.4 Ultimate deformation of Strut-and-Tie model

From the above force-deformation relationships of the components, deformation of each component can be determined under given shear force. Lateral deformation of a column at the point of inflection is determined by combining truss element deformation with joint rotation. The truss deformation is represented by the lateral displacement and the rotation at point of $(3jd\cot\theta)/2$ from the bottom end, as displayed in Fig. 6(a). The lateral displacement of the Strut-and-Tie Model Δ_{truss} is calculated as follows

$$\begin{aligned} \Delta_{\mathrm{truss}} = {} & \delta_{T_{l1}} \left(\frac{3\cot\theta}{2} \right) + \delta_{T_{l2}} \cot\theta + \delta_{C_{l1}} \cot\theta + \delta_{C_{d1}} \sqrt{1 + \frac{\cot^2\theta}{2}} \\ & + \delta_{C_{d2}} \frac{1}{\sin\theta} + \delta_{T_{tr1}} + \delta_{T_{tr2}} \end{aligned} \tag{42}$$

The rotation by truss deformation Θ_{truss} is calculated as follows

$$\Theta_{\mathrm{truss}} = \left(\delta_{T_{l1}} + \delta_{T_{l2}} + \delta_{C_{l1}} + \delta_{C_{l2}} \right) \Big/ jd \tag{43}$$

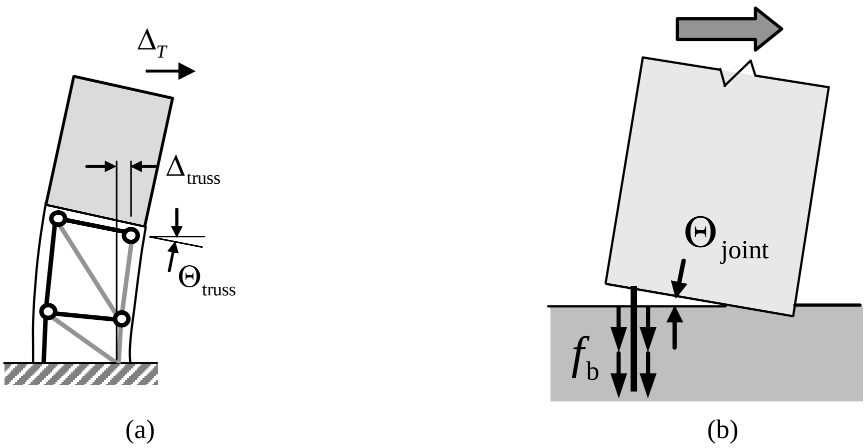


Fig. 6 (a) Deformation by truss deformation and (b) deformation by joint shear mechanism

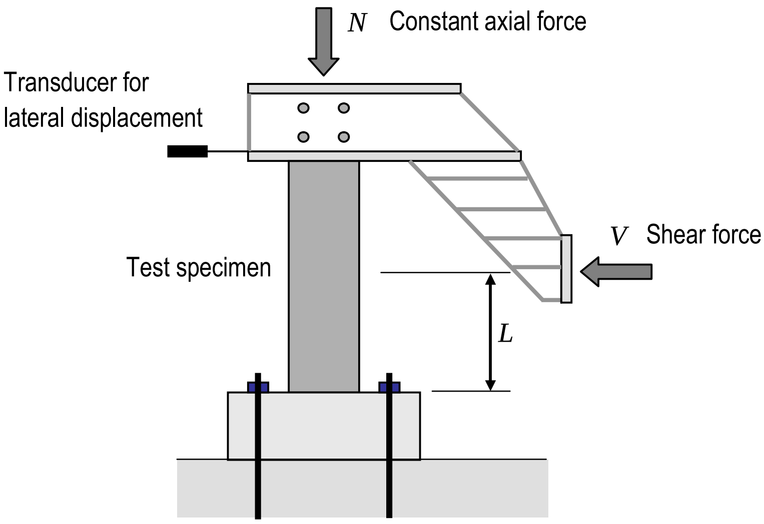


Fig. 7 Test setup of column specimen

Member deformation must include the effect of a joint rotation determined by the slip of the anchored reinforcing bars and concrete block contraction. By assuming a joint region as an elastic state at failure, the rotation is simply estimated. The joint deformation is dominantly affected by the shear mechanism, as exhibited Fig. 6(b). At the member face of the joint, however, this deformation can be expressed by end rotation due to the extrusion of anchored tension bars and the shortening of the compression concrete block. By assuming the joint to be elastic, the joint rotation is calculated as follows

$$\Theta_{\text{joint}} = \left(\frac{1}{E_s} - \frac{K_s d_{bl}}{8}\right)\frac{d_b}{4 f_b}\left(\frac{T_{l1}}{A_{sl}}\right)^2 \frac{1}{4jd - 3d} \tag{44}$$

Therefore, the member deformation at inflection point is obtained as the following

$$\Delta_{\mathrm{T}} = \Delta_{\mathrm{truss}} + \left(L - \frac{3\,jd\cot\theta}{2} \right) \Theta_{\mathrm{truss}} + L\,\Theta_{\mathrm{joint}} \tag{45}$$

In addition, the plastic hinge rotation Θ_p can also be determined by dividing Δ_T by member length L.

2.5 Analysis procedure of Strut-and-Tie model

The proposed Strut-and-Tie Model is employed to analyze the relationship between the lateral load and lateral displacement of RC columns at the post-yield state. First, the shear force and displacement at the initial yield state are calculated and then the relationship between the additional shear force and the deformation is determined by modifying the geometry of the longitudinal strut until the ultimate limit state. A specific set of procedures for determining the ultimate deformation analysis is proposed as follows

Step 1: Assuming the yielding of longitudinal bars in tension, determine the geometric properties of the Strut-and-Tie model, θ, c and jd.

Step 2: Calculate the yield strength V_y, the shear forces derived from T_{l1} by substituting f_{yl} for f_{sl}.

Step 3: Calculate all the element forces in terms of V_y by equilibrium, and all the element deformations from the constitutive laws.

Step 4: Calculate the yield deformation of the member.

Step 5: As the lever arm length jd' increases gradually, find the additional shear force dV and the state of failure of the other element. The longitudinal tie element extends while maintaining its strength before any of the other elements yields. Then, calculate the member deformation at that state by repeating Step 4. Note that this deformation is the limited ductility of members.

Step 6: After yielding of shear component at Step 5, the descending branch of load-deformation curves can be obtained. Repeat Step 5 and Step 6 with increasing the deformation of the transverse tie element. This represents strength degradation after shear failure.

3. Comparisons with test data

To verify the proposed model, the ultimate strength and the associated deformation obtained by the Strut-and-Tie Models are compared with those of the experimental results. The data include two rectangular columns of authors' own study and seven rectangular columns of Priestley *et al.* (1994a, 1994b). All of the specimens are intermediately short columns having the shear span ratio of 1.5 to 2. Most of the test results show the shear failure mode after the yielding of the longitudinal main reinforcing bars. Fig. 7 exhibits the conceptual test setup, where constant axial force is exerted on the top of the specimen and lateral force is imposed on the mid-point of the column. In Fig. 7, the shear span is the half of the column length. In Table 1, the dimensions and sectional properties of tested columns are provided and the information of reinforcement and material properties are summarized in Table 2. The results of the test and those obtained by the proposed model are presented in Table 3, where the comparisons of the ultimate strength and deformation are included. In Fig. 8, the comparison results of the strength and the deformation at the failure are illustrated. The comparison of strength shows a relatively good correlation between the experimental data and the prediction results.

Table 1 Dimensions and properties of test specimens

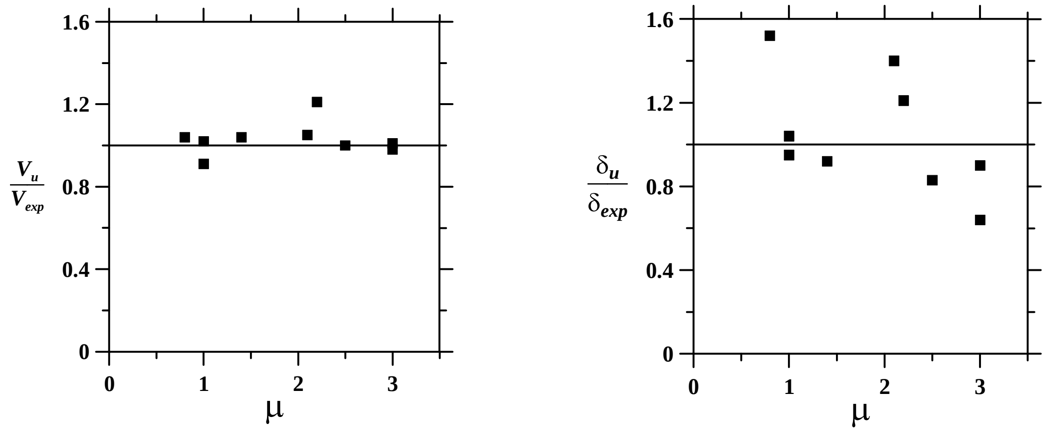
Specimen		b (mm)	h (mm)	L (mm)	N (kN)
Authors	PT1	400	400	800	372.40
	PT2	400	400	800	372.40
Priestley *et al.*	R1A	406	533	1066	505.68
	R3A	406	533	1066	505.68
	R5A	406	533	800	505.68
	C1A	488	488	1220	565.46
	C3A	488	488	1220	1713.04
	C5A	488	488	1220	565.46
	C7A	488	488	915	569.38

Table 2 Material properties and reinforcement information

		Concrete	Flexural reinforcement		Shear reinforcement		
		f_c' (MPa)	A_{sl} (mm^2)	f_{yl} (MPa)	A_{sh} (mm^2)	f_{yh} (MPa)	s (mm)
Authors	PT1	28	2280	408	157	408	200
	PT2	28	1700	408	157	408	200
Priestley *et al.*	R1A	35	3140	324	63	359	127
	R3A	34	3140	469	63	324	127
	R5A	32	3140	469	63	324	127
	C1A	31	3140	324	63	359	127
	C3A	34	3140	324	63	324	127
	C5A	36	3140	469	63	324	127
	C7A	31	3140	469	63	324	127

Table 3 Comparisons of strength and deformation with test results

		Test result			Proposed model		Comparison	
		μ	V_{exp} (kN)	δ_{exp} (mm)	V_u (kN)	δ_u (mm)	V_u/V_{exp}	δ_u/δ_{exp}
Authors	PT1	2.1	422	11.20	443	15.68	1.05	1.4
	PT2	2.2	324	7.20	392	8.71	1.21	1.21
Priestley *et al.*	R1A	3.0	566	14.92	572	13.43	1.01	0.9
	R3A	1.4	627	10.66	652	9.81	1.04	0.92
	R5A	0.8	748	5.60	778	8.51	1.04	1.52
	C1A	2.5	574	13.42	574	11.14	1.00	0.83
	C3A	3.0	734	10.98	719	7.03	0.98	0.64
	C5A	1.0	614	8.54	559	8.11	0.91	0.95
	C7A	1.0	792	7.32	808	7.61	1.02	1.04



(a) (b)

Fig. 8 Comparisons with test results: (a) strength and (b) deformation at failure

4. Conclusions

The Strut-and-Tie Model for RC columns subject to lateral loading are defined based on the stress field of the column and then the deformation capacity of the Strut-and-Tie Model is calculated by combining the deformations of the components at the ultimate limit state. Based on the comparison between the experimental results with the predictions obtained by the proposed Strut-and-Tie model, the following conclusions are drawn

(1) For the simple analysis focusing on the ultimate limit state, the analysis procedures for estimating the ultimate strength and associated deformation of reinforced concrete columns using Strut-and-Tie Models are proposed. Since those methods require only the properties and dimensions of simple truss elements, those can reduce the effort to analyze the response of structural concrete requiring the deformation problems.

(2) The relatively good correlations between the predictions by the proposed model and the experimental data are demonstrated.

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