Computers and Concrete, *Vol. 16, No. 3 (2015) 357-380*
DOI: http://dx.doi.org/10.12989/cac.2015.16.3.357

Strut-tie model for two-span continuous RC deep beams

H.S. Chae[a] and Y.M. Yun*

Department of Civil Engineering, Kyungpook National University,
80 Daehak-ro, Buk-gu, Daegu 702-701, Republic of Korea

(Received September 2, 2011, Revised January 19, 2015, Accepted February 11, 2015)

Abstract. In this study, a simple indeterminate strut-tie model which reflects complicated characteristics of the ultimate structural behavior of continuous reinforced concrete deep beams was proposed. In addition, the load distribution ratio, defined as the fraction of applied load transferred by a vertical tie of truss load transfer mechanism, was proposed to help structural designers perform the analysis and design of continuous reinforced concrete deep beams by using the strut-tie model approaches of current design codes. In the determination of the load distribution ratio, a concept of balanced shear reinforcement ratio requiring a simultaneous failure of inclined concrete strut and vertical steel tie was introduced to ensure the ductile shear failure of reinforced concrete deep beams, and the primary design variables including the shear span-to-effective depth ratio, flexural reinforcement ratio, and compressive strength of concrete were reflected upon. To verify the appropriateness of the present study, the ultimate strength of 58 continuous reinforced concrete deep beams tested to shear failure was evaluated by the ACI 318M-11's strut-tie model approach associated with the presented indeterminate strut-tie model and load distribution ratio. The ultimate strength of the continuous deep beams was also estimated by the experimental shear equations, conventional design codes that were based on experimental and theoretical shear strength models, and current strut-tie model design codes. The validity of the proposed strut-tie model and load distribution ratio was examined through the comparison of the strength analysis results classified according to the primary design variables. The present study associated with the indeterminate strut-tie model and load distribution ratio evaluated the ultimate strength of the continuous deep beams fairly well compared with those by other approaches. In addition, the present approach reflected the effects of the primary design variables on the ultimate strength of the continuous deep beams consistently and reasonably. The present study may provide an opportunity to help structural designers conduct the rational and practical strut-tie model design of continuous deep beams.

Keywords: continuous deep beam; indeterminate strut-tie model; load distribution ratio; ultimate strength

1. Introduction

Continuous reinforced concrete deep beams are fairly common structural elements. They are

*Corresponding author, Professor, E-mail: ymyun@knu.ac.kr
[a]Ph.D., Research Associate, E-mail: hschae11@knu.ac.kr

http://www.techno-press.org/?journal=cac&subpage=8 ISSN: 1598-8198 (Print), 1598-818X (Online)

used as load distribution elements such as transfer girders, pile caps, and foundation walls, often receiving many small loads and transferring them to a small number of reaction points. Continuous reinforced concrete deep beams differ from either simply supported reinforced concrete deep beams or continuous reinforced concrete shallow beams. In continuous deep beams, the regions of high shear and high moment coincide and failure usually occurs in these regions. In simply supported deep beams, the region of high shear coincides with the region of low moment. the failure mechanisms of continuous and simply supported deep beams are different. Despite the different failure mechanisms, the current design codes of practice for shear in continuous deep beams are based entirely on tests of simply supported deep beams because there have not been theoretical and experimental studies on continuous deep beams.

A strut-tie model approach, known as a design method for structural concrete with disturbed regions, has been accepted in the current design codes including the BS8110 (1997), CSA (2005), NZS 3101 (2006), FIB (2010), AASTHO-LRFD (2010) and ACI 318M-11 (2011). And, the approach has mainly been applied to the analysis and shear design of simply supported reinforced and prestressed concrete deep beams (Hwang *et al.* 2000, Yun 2000, Foster and Malik 2002, Hwang and Lu 2002, Matamoros and Wong 2003, Yun 2005, 2006, Quintero-Febres *et al.* 2006, Park and Kuchma 2007, Tjhin and Kuchma 2007, Ashour and Yang 2008, Yun and Kim 2008, Kim and Yun 2011, Chetchotisak *et al.* 2014). However, even though excluding the subject of continuous deep beams, an appropriate strut-tie model that represents a true load transfer mechanism for simply supported deep beams and reflects the effects of the primary design variables on shear behavior has not been provided. Though the studies about the strut-tie model analysis and design of continuous deep beams were conducted by Alshegeir (1992) and MacGregor (1997), a simple determinate truss type of strut-tie model which seems to be incapable of representing appropriate load transfer mechanisms of continuous deep beams was presented.

In this study, a simple internally and externally indeterminate strut-tie model reflecting all characteristics of the ultimate strength and complicated nonlinear structural behavior was proposed for the design of continuous deep beams. In addition, a load distribution ratio, defined as the fraction of applied load carried by one of ties in the internally indeterminate model, was proposed to help structural engineers employ the strut-tie model approaches of the current design codes in practice by transforming the internally indeterminate model into an internally determinate model. In the determination of the ratio, numerous finite element material nonlinear analyses of a single type of internally indeterminate strut-tie model with changeable primary design variables were conducted to reflect the effects of primary design variables, and a concept of balanced shear reinforcement ratio requiring a simultaneous failure of inclined concrete strut and vertical steel tie of an internally indeterminate strut-tie model was introduced as well to ensure the ductile shear strength design of continuous deep beams. The appropriateness of the present study was examined through the strength analysis of 58 continuous deep beams tested to shear failure by using the ACI 318M-11's strut-tie model approach.

2. Strut-tie models and load distribution ratios of previous studies

The development of strut-tie models for continuous deep beams has not been the subject of much attention, and any indeterminate strut-tie models for the beams have not been proposed yet. However, a few determinate and indeterminate strut-tie models for simply supported deep beams have been suggested. The CSA (2005) and AASHTO-LRFD (2010) have suggested a basic concept of a strut-tie model that satisfies equilibrium and constitutive relationships, and they have allowed the design of simply supported deep beams with the determinate strut-tie model shown in Fig. 1(a). This has influenced the ACI 318M-11 (2011) to allow the same model for simply supported deep beams with the requirement that the angle between a concrete strut and a tie be greater than 25 degrees. When the requirement on the angle is considered, the strut-tie model shown in Fig. 1(a) can be used for simple deep beams with a shear span-to-effective depth ratio a/d of less than 1.93 ($a/z = 2.14$, $z = 0.9d$, $d = 0.9h$, $h =$ depth). Thus, the simply supported deep beams with $a/d \geq 1.93$ can be designed by using the determinate strut-tie model shown in Fig. 1(b) (ACI 445 2002).

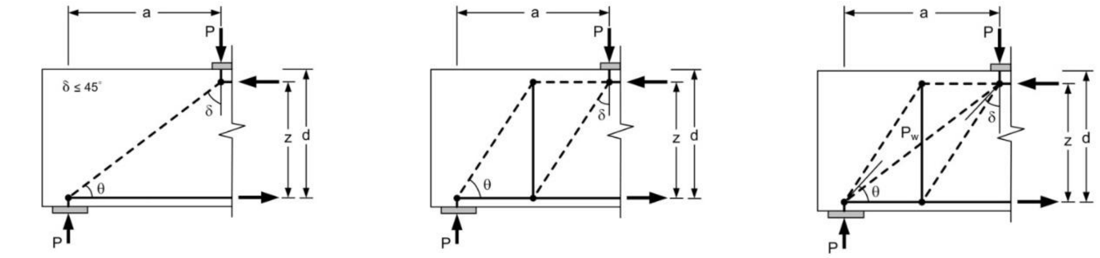
FIB (2010) suggested the determinate and indeterminate strut-tie models of Figs. 1(a)-1(c) for simply supported deep beams, representing respectively an arch load transfer mechanism (hereinafter, arch mechanism) for $a/z \leq 0.5$, a truss load transfer mechanism (hereinafter, truss mechanism) for $a/z \geq 2.0$, and a combination of arch and truss mechanisms for $0.5 < a/z < 2.0$. As the strut-tie model in Fig. 1(c) is the first-order indeterminate truss structure, a load distribution ratio was proposed to calculate the cross-sectional forces of struts and ties by simply employing the force equilibrium equations at nodes. With the load distribution ratio α of Eq. (1), varying linearly as a function of a/z, the cross-sectional force of a vertical steel tie P_w in the truss mechanism of Fig. 1(a) is directly obtained from the following equation

$$\alpha = \frac{P_w}{P} = \frac{2a/z - 1}{3 - N_{sd}/P} \tag{1}$$

where P is a vertically applied load and N_{sd} is a horizontally applied axial load.

Similar to the FIB's strut-tie models, Foster and Gilbert (1998) suggested the determinate and indeterminate strut-tie models of Figs. 1(a)-1(c) for simply supported deep beams, respectively for use in the ranges of $a/z \leq 1$, $a/z \geq \sqrt{3}$, and $1 \leq a/z \leq \sqrt{3}$. The load distribution ratio α for the indeterminate strut-tie model was proposed as follows

$$\alpha = \frac{P_w}{P} = \frac{a/z - 1}{\sqrt{3} - 1} \tag{2}$$



(a) Determinate Strut-Tie Model representing Arch Mechanism ($a/z \leq 0.5$)

(b) Determinate Strut-Tie Model representing Truss Mechanism ($a/z \geq 2.0$, $a/d \geq 1.8$)

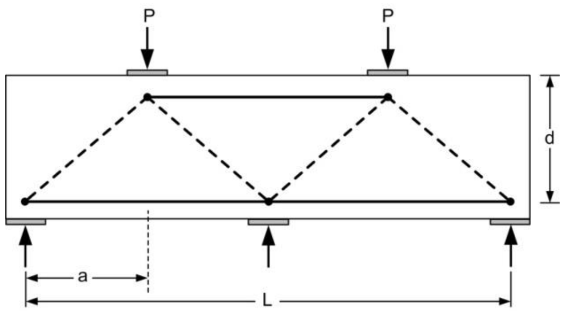
(c) Indeterminate Strut-Tie Model representing Combined Arch and Truss Mechanisms ($0.5 < a/z < 2.0$)

Fig. 1 Strut-tie models for simply supported deep beams

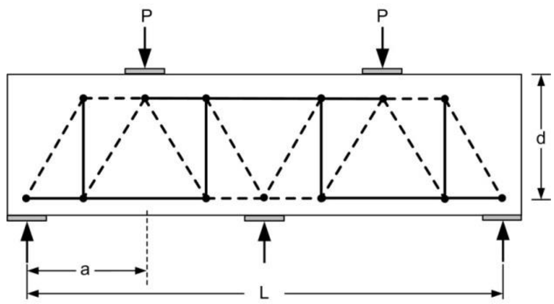
Kim and Yun (2011) proposed a single type of indeterminate strut-tie model of Fig. 1(c) for whole range of simply supported deep beams, and they proposed a load distribution ratio α as follows

$$
\begin{aligned}
\alpha(\%) &= \frac{P_w}{P}\times 100 = \beta(f_c' - 40) + \frac{200-40(\rho/\rho_b)}{a/d}\ \ln\left(\frac{a/d}{1.1-0.25(\rho/\rho_b)}\right) && for \quad \frac{a}{d} < \eta \\
\alpha(\%) &= \frac{P_w}{P}\times 100 = \beta(\frac{a}{d} - \eta) + \left(61.5 - 2\frac{\rho}{\rho_b}\right) && for \quad \frac{a}{d} \geq \eta
\end{aligned}
\tag{3}
$$

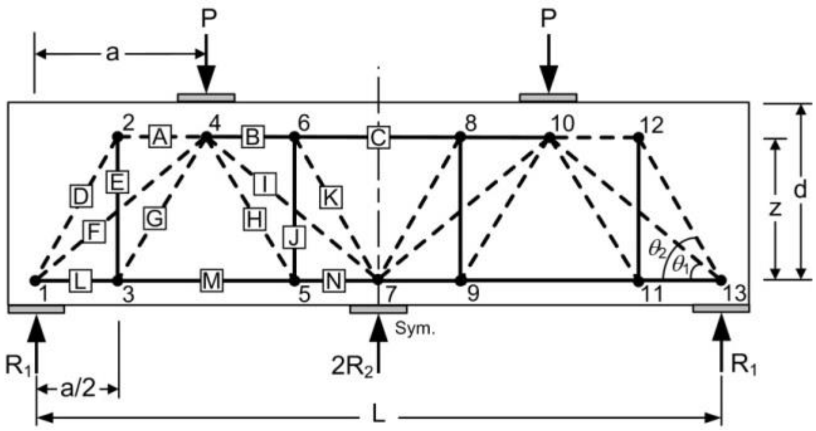
where ρ_b is the balanced flexural reinforcement ratio, η is the value of a/d that decides the type of governing failure mechanism between the arch and truss mechanisms, and β is the parameter that considers the variation of the load distribution ratio according to primary design variables. Detail explanations of the parameters are given in the reference.



(a) Externally indeterminate strut-tie model representing arch mechanism



(b) Externally indeterminate strut-tie model representing truss mechanism



(c) Internally and externally indeterminate strut-tie model representing combined arch and truss mechanisms

Fig. 2 Indeterminate strut-tie models for two-span continuous deep beams

3. Strut-tie model and load distribution ratio of present study

3.1 Indeterminate Strut-Tie model

The ultimate behavior of continuous deep beams is highly nonlinear in accordance with the design variables including the shear span-to-effective depth ratio, flexural and shear reinforcement ratios, load and support conditions, and material properties, as the case of simply supported deep beams. Therefore, similar types of the strut-tie models suggested for simply supported deep beams, such as an internally determinate strut-tie model of Fig. 2(a) in which an external concentrated load is directly transferred to the supports by an inclined strut to represent an arch mechanism, an internally determinate strut-tie model of Fig. 2(b) in which an external concentrated load is transferred to the supports by the combination of inclined struts and a vertical tie to represent a truss mechanism, and an internally indeterminate strut-tie model of Fig. 2(c) representing a combination of arch and truss mechanisms, can be extended to continuous deep beams. In this study, an internally and externally indeterminate strut-tie model of Fig. 2(c) is proposed for the rational design of the continuous deep beams with $a/d \geq 1.0$ by considering the effects of primary design variables on the ultimate strength and behavior and by satisfying the fundamental concept that the load acting on top of a continuous deep beam must be transferred to supports by concrete and reinforcing bars. In the proposed model, the role of horizontal shear reinforcing bars is not reflected upon because, according to the research by Rogowsky *et al.* (1986) and Ashour (1997), the effect of horizontal shear reinforcing bars on shear strength is not significant when the deep beams with $a/d \geq 1.0$ do not contain plenty of horizontal reinforcing bars.

3.2 Reaction distribution ratio

To perform the strut-tie model design of structural concrete by using current design codes of practice, the indeterminate strut-tie model of Fig. 2(c) needs to be transformed to a determinate strut-tie model. In this study, the indeterminate strut-tie model was transformed to an externally determinate strut-tie model by using the reaction distribution ratio. In design practice, the reactions at exterior and interior supports of a two span continuous deep beam are generally determined by a linear elastic analysis. They are $5P/16$ and $11P/16$, respectively, where P is an external load acting at each mid-span of the deep beam. However, the reactions at ultimate state are different as shown in Fig. 3, where the exterior reactions obtained from Bernoulli's beam theory and the tests of Rogowsky *et al.* (1986) and Ashour (1997) are plotted. In this study, linear elastic finite element analyses of the strut-tie model of Fig. 2(c), with a varying shear span-to-depth ratio of 1.0~3.0 and assumptions of $z = 0.9d$ and unit axial stiffness of all struts and ties, were performed. The reactions obtained from the analyses showed to be very similar to experimental ones, as shown in Fig. 3. Thus, through the curve fitting of the analysis results, an equation of reaction distribution ratio that traces the experimental support reactions satisfactory was developed as follows

$$\gamma = \frac{R_1}{P} = 1 - \frac{R_2}{P} = 0.011\left(\frac{a}{d} - 3\right)^2 + 0.34 \tag{4}$$

where, R_1 and R_2 are the reactions at exterior and interior supports, and a and d are the shear span and effective depth of a two span continuous deep beams.

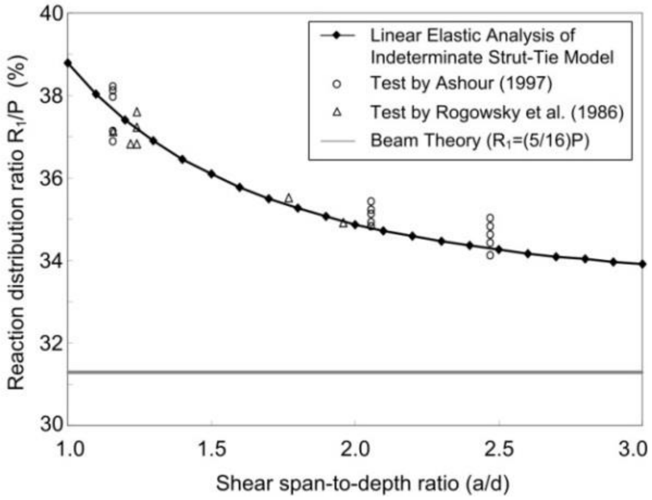


Fig. 3 Reaction distribution ratio of externally indeterminate strut-tie model for two-span continuous deep beams

3.3 Load distribution ratio

In the present study, a load distribution ratio is determined by conducting a finite element material nonlinear analysis of the internally indeterminate strut-tie model. A state of simultaneous failure of the inclined concrete strut and vertical steel tie, defined as a state of balanced shear reinforcement ratio, is used as a condition for determining the load distribution ratio. It was assumed that the horizontal strut and ties A, B, C, L, M and N that could guarantee ductile structural behavior of deep beams by using the concept of balanced flexural reinforcement ratio and the struts and tie D, F, G, and E placed at the exterior shear span of a continuous deep beam of Fig. 2(c) had no direct relations with the failure of the deep beam. Instead, the struts and tie H, I, K, and J placed at the interior shear span of the deep beam were regarded as the main elements composing a shear resistant mechanism and were assumed to fail ahead of the other elements at ultimate state. At the balanced shear reinforcement ratio, a simultaneous failure of concrete strut I and steel tie J (denoting a failure of arch mechanism) or a simultaneous failure of concrete strut H (or K) and steel tie J (denoting a failure of truss mechanism) was assumed to occur. To determine the load distribution ratio at the state of balanced shear reinforcement ratio, the finite element material nonlinear analysis of the internally indeterminate strut-tie model was conducted by changing the magnitude of the applied load P and the amount of the vertical shear reinforcement area $A_{J\,tie}$, according to the procedure shown in Fig. 4. In Fig. 4, the maximum value of P, $P_{\max}$, was determined from the flexural strength of deep beams, and the maximum value of $A_{J\,tie}$, $A_{J\,tie,\,\max}$, is the vertical shear reinforcement area required for $P_{\max}$. The initial values of P and $A_{J\,tie}$, $P_{initial}$ and $A_{J\,tie,\,initial}$, respectively, were chosen as 1% and 0.5% of their maximum values. With the load distribution ratio, an optimum design of a continuous deep beam may be ensured by deciding the cross-sectional areas of reinforcing bars at a state of the simultaneous failure. Additionally, the ductile structural behavior caused by the yield of the steel tie before the crushing of the inclined concrete strut may be assured in design practice by using a smaller load distribution ratio than the one obtained at a state of the simultaneous failure.

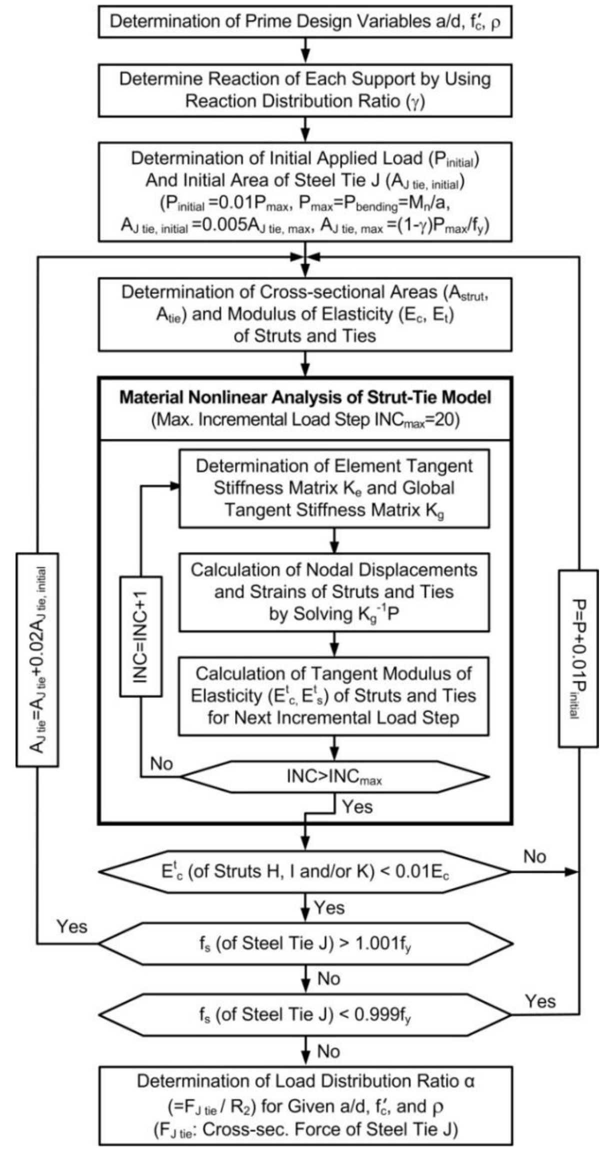


Fig. 4 Algorithm for determining load distribution ratio of internally indeterminate strut-tie model for two-span continuous deep beams

Since the load distribution ratio of the present study was determined by the nonlinear analysis of internally indeterminate truss structure, the axial stiffness of struts and ties *EA* (*E*=modulus of elasticity, *A*=cross-sectional areas) associated with the stress states of struts and ties must be considered. In this study, the cross-sectional areas of struts and ties were decided as the maximum areas of struts and ties that they could contain, as the method of conventional strut-tie model approaches. As shown in Fig. 5, the cross-sectional areas of struts A, B and C placed at the biaxial compression region were decided by multiplying the width of the strut w_s (which is the same as the depth of the equivalent rectangular stress block) by the beam thickness b, as in Eq. (5)

$$w_{A,B,C\,strut} = \beta_1 c = \frac{f_y A_s}{0.85 f_c' b} = \frac{f_y \xi \rho_b d}{0.85 f_c'} \tag{5}$$

where, β_1 is the coefficient of the equivalent rectangular stress block, c is the distance from the top of the beam to the neutral axis, $A_s\,(=\xi\rho_b bd)$ is the cross-sectional area of flexural reinforcement, d is the effective depth of the beam, ρ_b is the balanced flexural reinforcement ratio, and ξ is the variable of flexural reinforcement (in the case of maximum flexural reinforcement ratio ρ_{max}, $\xi = 0.75$). The cross-sectional areas of inclined struts D, F, G, H, I, and K placed at the shear span were decided by multiplying the beam thickness b by the smaller width of the strut and nodal zone boundary, as expressed in the following

$$w_{D\ strut} = w_{L\ tie}\cos\theta_2 + l_{b,1}\sin\theta_2 \tag{6a}$$

$$w_{F\ strut} = \min\left(w_{L\ tie}\cos\theta_1 + l_{b,1}\sin\theta_1,\ w_{A\ strut}\cos\theta_1 + 0.5 l_{b,4}\sin\theta_1\right) \tag{6b}$$

$$w_{G\ strut} = w_{A\ strut}\cos\theta_2 + 0.5 l_{b,4}\sin\theta_2 \tag{6c}$$

$$w_{H\ strut} = w_{B\ tie}\cos\theta_2 + 0.5 l_{b,4}\sin\theta_2 \tag{6d}$$

$$w_{I\ strut} = \min\left(w_{N\ tie}\cos\theta_1 + 0.5 l_{b,7}\sin\theta_1,\ w_{B\ tie}\cos\theta_1 + 0.5 l_{b,4}\sin\theta_1\right) \tag{6e}$$

$$w_{K\ strut} = w_{N\ tie}\cos\theta_2 + 0.5 l_{b,7}\sin\theta_2 \tag{6f}$$

where, $w_{D\,strut}$ and $w_{L\ tie}$ are the widths of strut D and tie L, $\theta_i\,(i=1,2)$ is the angle between inclined struts and horizontal axis, and $l_{b,i}$ is the width of the bearing (or loading) plate of nodal zone i. In the present study, the width of the bearing or loading plate l_b was determined to satisfy the ACI 318M-11's (2011) strength requirement of nodal zone, as expressed in the following

$$l_b = \frac{P(or\,R)}{0.85\beta_n f_c' b} \tag{7}$$

where, β_n is the coefficient of the effective strength of nodal zone. For nodal zones 1 and 4 which are classified as CCT nodal zone, the values of 0.8 was taken as the coefficient. For nodal zone 7 which is classified as CCC nodal zone, the values of 1.0 was taken as the coefficient. The cross-sectional areas of horizontal ties B, C, L, M, N placed at the top and bottom of the beam were decided as $A_{tie} = \xi\rho_b bd$, the cross-sectional area of flexural reinforcing bars. The cross-sectional area of vertical tie J was obtained by changing its area repeatedly in order to reach the state of simultaneous failure of the inclined concrete strut and vertical steel tie.

For the finite element material nonlinear analysis of the internally indeterminate strut-tie model, the tangential modulus of elasticity of a concrete strut, as expressed in Eq. (8), was evaluated by differentiating the stress-strain relationship of Pang and Hsu (1995) with the strain of a concrete strut

$$E_c{}^t = E_c\left[1-\frac{\varepsilon_c}{\zeta\varepsilon_0}\right] \qquad \text{for} \quad \varepsilon_c/\zeta\varepsilon_0 \le 1$$
$$E_c{}^t = -E_c\left[\frac{\varepsilon_c/\zeta\varepsilon_0 - 1}{(2/\zeta - 1)^2}\right] \qquad \text{for} \quad \varepsilon_c/\zeta\varepsilon_0 > 1 \tag{8}$$

where, ε_c is the compressive strain of a concrete strut, ζ is the softening coefficient of concrete, and ε_0 is the compressive strain that corresponds to the peak compressive stress of a concrete strut defined as $\varepsilon_0 = 2f_c'/E_c$ where E_c is the initial modulus of elasticity of concrete (for $f_c' \le 30MPa$, $E_c = 4700\sqrt{f_c'}$; for $f_c' > 30MPa$, $E_c = 3300\sqrt{f_c'} + 7700$). Following the ACI 318M-11's suggestion for the effective strength of concrete struts, the softening coefficient of $\zeta = 0.85$ was employed for concrete strut A located at the biaxial compression region, and $\zeta = 0.638(= 0.85\beta_s = 0.85\times 0.75)$ was employed for bottle-shaped concrete struts D, F, G, H, I, and K located at the biaxial compression-tension region. In the nonlinear analysis, it was assumed that the failure of nodal zones did not occur. The tangential modulus of elasticity of a steel tie, as expressed in Eq. (9), was evaluated by assuming a bi-linear stress-strain relationship of steel

$$E_s{}^t = E_s \qquad \text{for} \quad \varepsilon_s \le \varepsilon_y$$
$$E_s{}^t = 0.001E_s \qquad \text{for} \quad \varepsilon_s > \varepsilon_y \tag{9}$$

where E_s is the initial modulus of elasticity of steel.

Fig. 6 shows the load distribution ratios of the internally indeterminate strut-tie model of Fig. 2(c) with geometric properties of a = 200~1200mm, b = 100mm, d = 400mm, L = 800~4800mm and design variables of a/d =0.5~3.0, ρ/ρ_b=0.15~0.75, f_c'= 20~70MPa, f_y =400MPa. The ratios were determined according to the algorithm of Fig. 4. It is shown that the applied load transferred by the arch mechanism becomes greater as in the case of Foster and Gilbert (1998) when the shear span-to-effective depth ratio a/d decreases, and the load transferred by the truss mechanism increases when the ratio a/d increases. However, unlike the results of earlier studies by the FIB (2010) and Foster and Gilbert (1998) where 100% of the applied load is transferred by the truss mechanism when the ratio a/d is greater than 1.80 and 1.56, respectively, the present study reveals that more than 25% of the applied load is still carried by the arch mechanism when the ratio a/d is greater than 2.50. This indicates that the shear-resistant capacity by the concrete struts making up the arch mechanism exists although the ratio a/d increases, as proven to be true in the previous studies of simply supported deep beams (Leonhardt 1965, Park and Paulay 1975, Kim *et al.* 2003). Fig. 6 also shows that the range of a/d where deep beams fail due to the failure of the arch mechanism decreases because the load-carrying capacity of the arch mechanism improves by the increase of the flexural reinforcement ratio. This result is similar to the previous studies (Zsutty 1971, Okamura and Higai 1980, Niwa *et al.* 1986, Bazant 1997, ACI 318-99 1999) expressing that the load transferred by the arch mechanism in simply supported deep beams increases as the flexural reinforcement ratio increases.

Through the curve fittings of Fig. 6, an equation of load distribution ratio associated with the primary design variables was developed in order that structural engineers could employ the

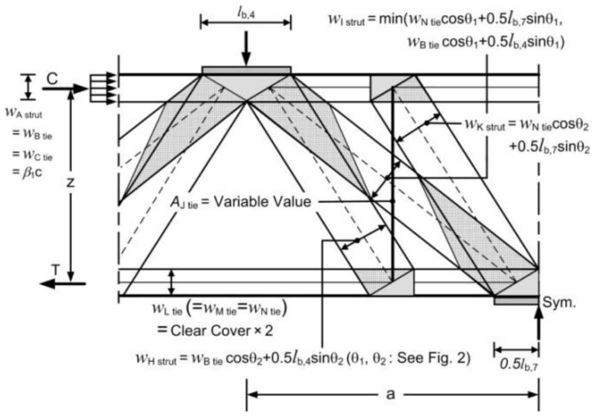
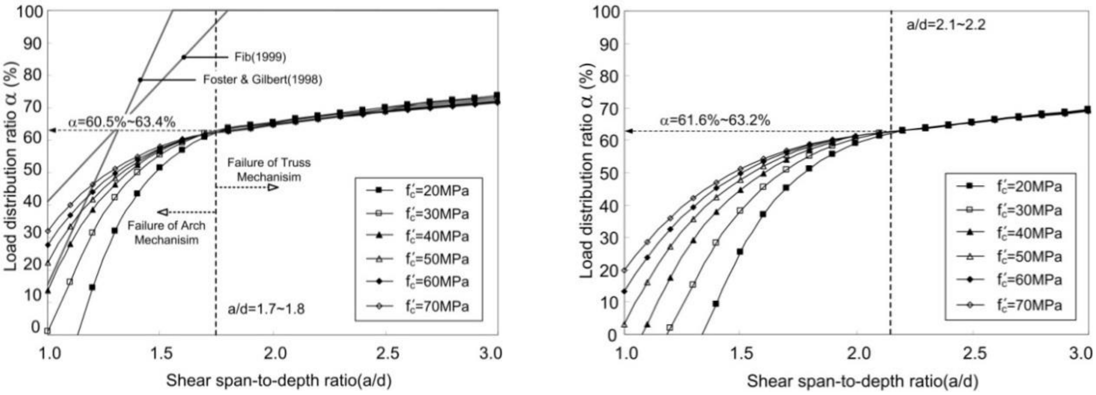


Fig. 5 Maximum cross-sectional widths (Areas) of struts and ties in indeterminate strut-tie model for two-span continuous deep beams



(a) $\rho = 0.75\rho_b$ (b) $\rho = 0.15\rho_b$

Fig. 6 Load distribution ratios of internally indeterminate strut-tie model associated with primary design variables

equation directly to the design of continuous deep beams. The developed equation is as follows

$$\alpha = \frac{P_w}{(1-\gamma)P} = \frac{25}{f_c'}\left(\frac{a}{d} - \eta\right) + 0.6 \qquad for \ \frac{a}{d} < \eta$$
$$\alpha = \frac{P_w}{(1-\gamma)P} = 0.1\left(\frac{a}{d} - \eta\right) + 0.6 \qquad for \ \frac{a}{d} \geq \eta \tag{10}$$

where, γ is the reaction distribution ratio and ρ_b is the balanced flexural reinforcement ratio of the beam. η, expressed in terms of ρ / ρ_b, is defined as follows

$$\eta = 1.85 - \frac{1}{3}\left(\frac{\rho}{\rho_b}\right) \tag{11}$$

Table 1 Geometries and material properties of continuous RC deep beams tested to failure

Investigators	No. of Beams	b_w (mm)	d (mm)	h (mm)	f_c' *(MPa)*	f_y *(MPa)*	a/d	ρ (%)	ρ/ρ_b
Rogowsky *et al.* (1986)	16	200	445-975	500-1000	14.5-46.8	363-594	1.16-2.47	0.46-1.13	0.173-0.855
Ashour (1997)	8	120	226-609	425-625	22.0-39.2	347-480	1.19-2.02	0.33-1.02	0.138-0.546
Subedi (1998)	3	50-75	370-570	400-600	44.7-56.5	340-527	1.35-1.47	0.53-1.47	0.309-0.502
Asin (1999)	13	150	550-950	1000	28.2-37.1	569-586	1.26-2.18	0.32-0.95	0.196-0.464
Yang *et al.* (2007)	12	160	565	600	32.1-68.2	483-562	1.06	0.95	0.314-0.463
Yang *et al.* (2007)	6	160	355-653	470-720	32.1-68.2	562	1.06-1.13	0.97-1.10	0.320-0.534
Total	58	50-200	226-975	400-1000	14.5-68.2	347-594	1.06-2.47	0.32-1.47	0.138-0.855

4. Validity evaluation

In this study, the ultimate strength of 58 two-span continuous reinforced concrete deep beams tested to shear failure was evaluated by the ACI 318M-11's strut-tie model approach associated with the proposed indeterminate strut-tie model and load distribution ratio. The ultimate strength of the deep beams was also estimated by using the experimental shear strength models (Zsutty 1971, ACI 318-99 1999, EC2 2004), the theoretical shear strength models (CEB-FIP 1993, AASHTO-LRFD 2010), and the strut-tie model design codes (FIB 2010, AASHTO-LRFD 2010, ACI 318M-11 2011). The ultimate strength evaluated by each method and classified according to the primary design variables was compared to verify the appropriateness of the proposed indeterminate strut-tie model and load distribution ratio.

4.1 Experimental results

The 58 two-span continuous reinforced concrete deep beams with $1.0 \leq a/d \leq 3.0$, tested to shear failure by Rogowsky *et al*. (1986), Ashour (1997), Subedi (1998), Asin (1999), and Yang *et al.* (2007a, 2007b) were selected to prove the validity of the present study. The ranges of the shear span-to-depth ratio, flexural reinforcement ratio, and compressive strength of concrete of the selected deep beams are $1.06 \leq a/d \leq 2.47$, $0.138 \leq \rho/\rho_b \leq 0.855$, and $14.5MPa \leq f_c' \leq 68.2MPa$. All of the deep beams were tested under the two-point concentrated loadings. The characteristics of the materials and geometries of the beams are listed briefly in Table 1.

4.2 Strength evaluation by conventional approaches

The evaluation of the ultimate strength by using the experimental shear strength models of the ACI 318-99 (1999), EC2 (2004), and Zsutty (1971) was conducted employing the following Eqs. (12)-(14), respectively.

$$\begin{cases} V_n = \xi_1 \xi_2 b_w d + \rho_v \left(\dfrac{1 + l_n / d}{12} \right) f_{vy} b_w d + \rho_{vh} \left(\dfrac{11 - l_n / d}{12} \right) f_{vhy} b_w d \quad (MPa) \qquad for\ \dfrac{a}{d} \le 2 \\ \quad \xi_1 = 3.5 - 2.5 \dfrac{M_u}{V_u d} \le 2.5,\ \xi_2 = 0.16 \lambda \sqrt{f_c'} + 17 \rho_w \dfrac{V_u d}{M_u},\ \xi_1 \xi_2 \le 0.5 \sqrt{f_c'} \\ V_n = \xi_2 b_w d + \rho_v f_{vy} b_w d \quad (MPa) \quad \left(\xi_2 \le 0.29 \lambda \sqrt{f_c'}\ ,\ \dfrac{M_u}{V_u d} \le 1 \right) \qquad for\ \dfrac{a}{d} > 2 \end{cases} \tag{12}$$

$$\begin{cases} V_n = \dfrac{1}{\beta} A_v f_{yd} \dfrac{z \cot\theta}{s} \le \dfrac{1}{\beta} \upsilon_1 f_{cd} b_w z \dfrac{1}{\cot\theta + \tan\theta}\ (MPa) \qquad for\ \ A_v \ne 0 \\ V_n = \dfrac{0.18}{\beta \gamma_c} k (100 \rho f_c')^{1/3} b_w d \ge \dfrac{0.035}{\beta} k^{3/2} \sqrt{f_c'} b_w d \qquad for\ \ A_v = 0 \\ \beta = \dfrac{a}{2d},\ \ 0.25 \le \beta \le 1,\ \ \upsilon_1 = 0.6(1 - f_c' / 250),\ \ \cot\theta = (\dfrac{\upsilon_1 f_c'}{\rho_w f_{yd}} - 1)^{1/2},\ \ 1 \le \cot\theta \le 2.5 \end{cases} \tag{13}$$

$$\begin{cases} V_n = 2.2 \left(f_c' \rho \dfrac{d}{a} \right)^{1/3} b_w d + \rho_v f_{vy} b_w d \quad (MPa) \qquad for\ \ \dfrac{a}{d} \ge 2.5 \\ V_n = \left(2.5 \dfrac{d}{a} \right) \times 2.2 \left(f_c' \rho \dfrac{d}{a} \right)^{1/3} b_w d + \rho_v f_{vy} b_w d \quad (MPa) \qquad for\ \ \dfrac{a}{d} < 2.5 \end{cases} \tag{14}$$

In the above equations, a, d, b_w, f_c', and λ are the shear span length, effective depth, web width, strength of concrete, and factor reflecting the lower tensile strength of lightweight concrete, respectively. ρ_w $(= A_s / b_w d$, A_s =area of flexural reinforcing bars), ρ_v $(= A_v / b_w s$, A_v =area of vertical shear reinforcing bars within a distance s), and ρ_{vh} $(= A_{vh} / b_w s_h$, A_{vh} =area of horizontal shear reinforcing bars within a distance s_h) are the ratios of flexural reinforcement, vertical shear reinforcement, and horizontal shear reinforcement, respectively. f_{vy} and f_{vhy} are the yield strength of the vertical and horizontal reinforcing bars, respectively. M_u, V_u, and l_n in Eq. (12) are the factored moment at section, factored shear at section, and clear span measured face-to-face of supports, respectively. In Eq. (13), the factor k is defined as $1 + \sqrt{200 / d}$, where the unit of effective depth d is mm. The factor k should not exceed 2.0. In the strength analysis, the value of the strength reduction factor γ_c was taken as 1.0, and the design strengths of steel and concrete, f_{yd} and f_{cd}, were taken as the yield strength of steel f_y and the strength of concrete f_c', respectively.

The ultimate strength of the deep beams by the specifications of the CEB-FIP (1993) and AASHTO-LRFD (2010) that were based on a variable truss model and the modified compression field theory of Vecchio and Collins (1986), respectively, was estimated by employing the following Eqs. (15) and (16).

$$\begin{cases} V_n = A_v f_{yd} \dfrac{z \cot\theta}{s} \le f_{cd2} b_w z \dfrac{\cot\theta}{1+\cot^2\theta} \quad (MPa) \qquad for\ A_v \ne 0 \\ \quad f_{cd2} = 0.60(1 - f_c'/250) f_{cd}\ ,\ 1 \le \cot\theta = \sqrt{\left(\dfrac{f_{cd2}}{\rho_w f_{yd}} - 1\right)} \le 3 \\ V_n = 0.12\left(1+\sqrt{200/d}\right)^3 \sqrt{\left(\dfrac{100 A_s}{b_w d} f_c'\right)} b_w d \quad (MPa) \qquad for\ A_v = 0 \end{cases} \tag{15}$$

$$\begin{cases} V_n = 0.083\beta\sqrt{f_c'} b_w d_v + A_v f_y \dfrac{d_v \cot\theta}{s} \le 0.25 f_c' b_w d_v \\ \varepsilon_s = \dfrac{|M_u|/d + |V_u|}{E_s A_s},\ \ \beta = \dfrac{4.8}{1+7500\varepsilon_s},\ \ \theta = 29^o + 3500\varepsilon_s \end{cases} \tag{16}$$

In Eq. (15), A_s, d, z and ρ_w ($= A_v / b_w s$, A_v =area of vertical shear reinforcing bars within a distance s, b_w=web width) are the area of flexural reinforcing bars, effective depth in mm, lever arm between the compression and tension, and ratio of vertical shear reinforcement, respectively. The design strength of steel and concrete, f_{yd} and f_{cd}, were taken as the yield strength of steel f_y and the compressive strength of concrete f_c'. In Eq. (16), d_v, the lever arm between the compression and tension, was taken as $0.9d$, and β and θ represent the factors indicating ability of diagonally cracked concrete to transmit tension and the angle of inclination of diagonal compressive stresses, respectively.

The ultimate strength by the strut-tie model specifications of FIB (2010), AASHTO-LRFD (2010), and ACI 318M-11 (2011) was evaluated by the methods of ACI 445 (2002) and PCA (2004) that examines the requirements of the effective strength of the concrete struts and nodal zones specified in each code. In the implementation of the AASHTO-LRFD and ACI 318M-11 specifications, the internally determinate strut-tie model reflecting an arch mechanism was used for the beams with $a/z < 2.14$ to satisfy the requirement for an angle of 25 degrees between the strut and the tie. Also, the internally determinate strut-tie model reflecting a truss mechanism was used for the beams with $a/z \ge 2.14$. In the application of the FIB (2010), the internally determinate strut-tie models of Figs. 2(a) and 2(b) reflecting arch and truss mechanisms were used for the beams with $a/z \le 0.5$ and $a/z \ge 2.0$. Also, the internally indeterminate strut-tie model of Fig. 2(c) reflecting the combined arch and truss mechanisms was used for the beams with $0.5 < a/z < 2.0$. The support reactions in the strut-tie models of Figs. 2(a)-2(c) were determined from Eq. (4). Since the same procedure as the one illustrated in the following section “Strength Evaluation by Present Approach” was employed, the illustration of the strength evaluation procedure by using an internally indeterminate strut-tie model is omitted here.

4.3 Strength evaluation by present approach

For the evaluation of the ultimate strength of the deep beams by the present approach, the ACI 318M-11's strut-tie model approach associated with the indeterminate strut-tie model of Fig. 2(c), reaction distribution ratio of Eq. (4), and load distribution ratio of Eq. (10) was employed. Since

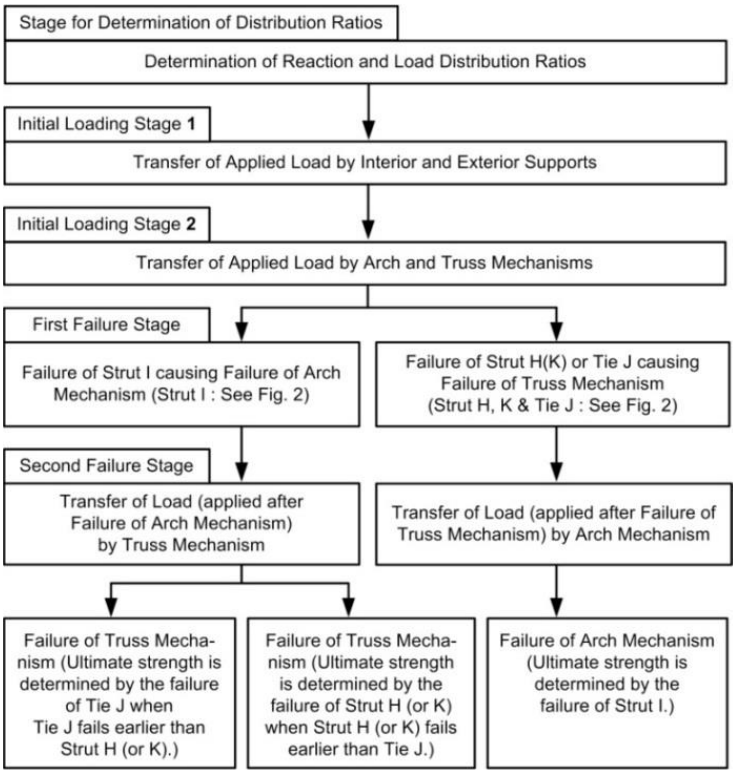


Fig. 7 Strength evaluation procedure of two-span continuous deep beams using indeterminate strut-rie model

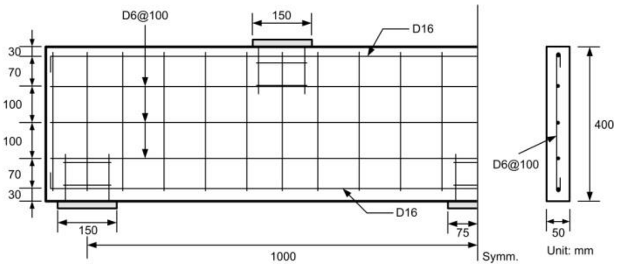
the indeterminate strut-tie model reflects both the arch and truss load transfer mechanisms at the same time, the ultimate strength of a deep beam failing in shear was decided according to the sequential failure of both the load transfer mechanisms. The flowchart for evaluating the ultimate strength by using the indeterminate strut-tie model is given in Fig. 7.

In the following, the detailed procedure for evaluating the ultimate strength of the deep beams by the present approach is illustrated with Beam 1CB2, one of the deep beams tested by Subedi (1998). The reinforcement layout and the selected indeterminate strut-tie model of the beam are shown in Figs. 8(a) and 8(b). In the model, the horizontal ties at the upper and lower chords were placed at a distance of clear cover from the top and bottom of the beam. Based on the primary design variables (a/d = 1.351, f_c' = 56.5 MPa, ρ/ρ_b = 0.312) of the beam, the reaction distribution ratio γ and load distribution ratio α of the indeterminate strut-tie model were determined from Eqs. (4), (10) and (11) as follows

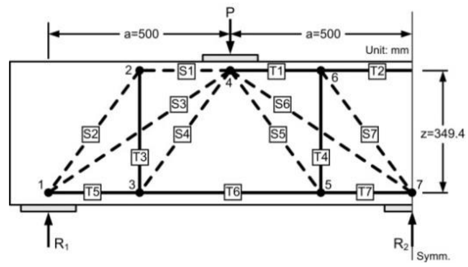
$$\gamma = 0.011 \times (1.351 - 3)^2 + 0.34 = 0.370 = 37.0\,(\%)$$

$$\eta = 1.85 - \frac{1}{3} \times 0.312 = 1.746$$

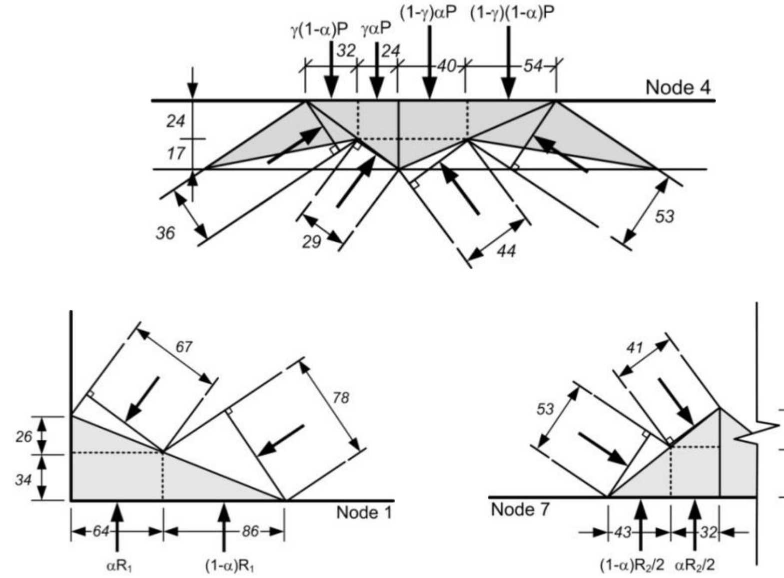
$$\alpha = \frac{25}{56.5}\left(1.351 - 1.746\right) + 0.6 = 0.425 = 42.5\,(\%)$$



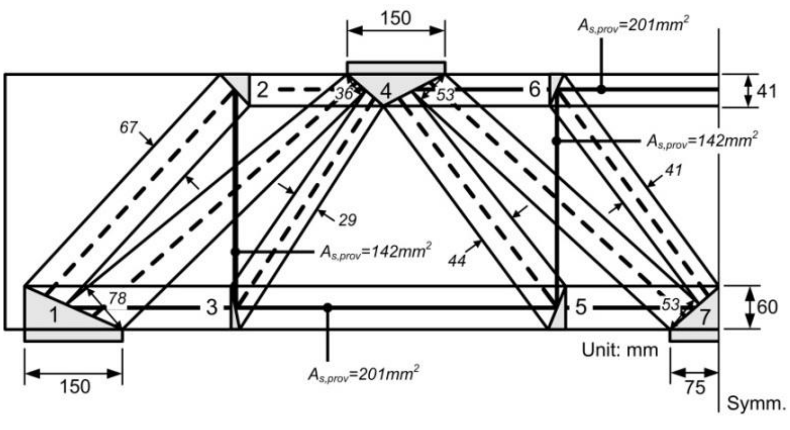
(a) Reinforcement details of beam 1CB2



(b) Indeterminate strut-tie model for beam 1CB2

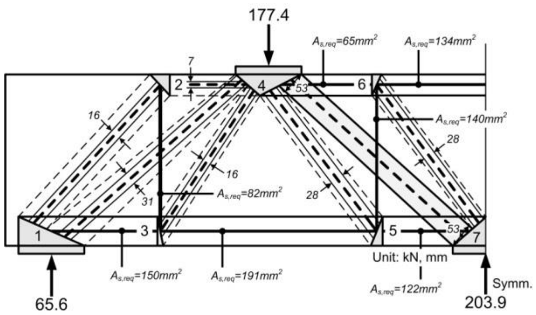


(c) Maximum widths of struts determined by reaction/load distribution ratios and nodal zone shapes

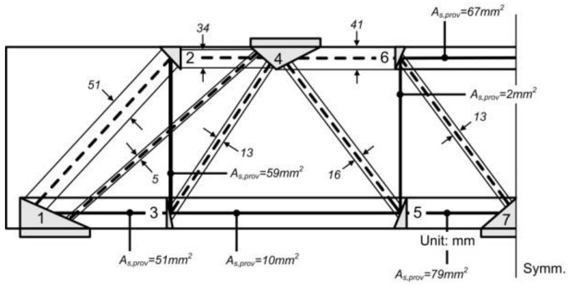


(d) Maximum cross-sectional widths (areas) of struts and ties

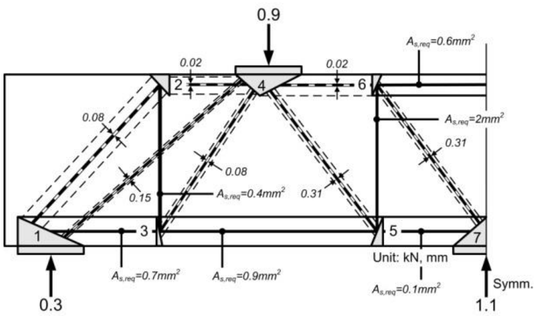
Fig. 8 Indeterminate strut-tie model of continuous deep beam 1CB2 for implementing present approach



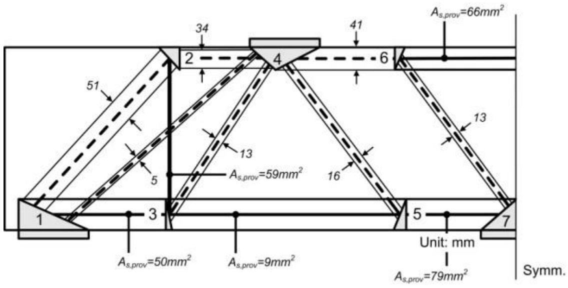
(a) Required cross-sectional widths (areas) of struts and ties at the first failure



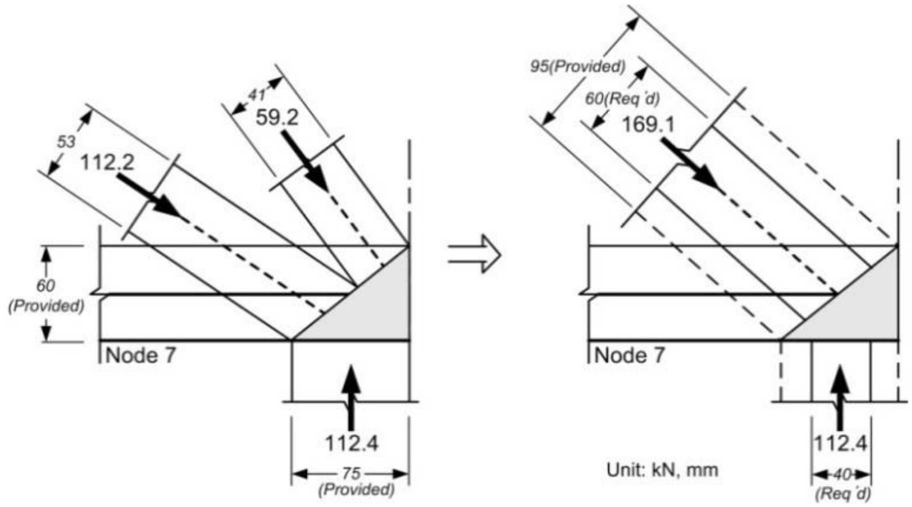
(b) Remaining capacity of struts and ties after the first failure



(c) Required cross-sectional widths (areas) of struts and ties at the second failure



(d) Unstable strut-tie model after the second failure



(e) Strength verification of nodal zone at node 7

Fig. 9 Strength evaluation of continuous deep beam 1CB2 by present approach

Table 2 Strength evaluation of beam 1CB2 by present approach

(a) Struts and ties at the first failure

Element No.	Element Type	β_s	f_c' (MPa)	f_{cu} (MPa)	F_{req} (kN)	w_{req} (mm)	w_{prov} (mm)	w_{prov}/w_{req}	Fail/Safe
S1	Strut	1.00	56.50	56.50	20.3	7	41	5.752	O
S2	Strut	0.75	56.50	42.38	34.8	16	67	4.060	O
S3	Strut	0.75	56.50	42.38	66.8	32	36	1.130	O
S4	Strut	0.75	56.50	42.38	34.8	16	29	1.777	O
S5	Strut	0.75	56.50	42.38	59.3	28	44	1.556	O
S6	Strut	0.75	56.50	42.38	113.8	54	53	0.986	X
S7	Strut	0.75	56.50	42.38	59.3	28	41	1.457	O
Element No.	**Element Type**	β_t	f_y (MPa)	f_{cu} (MPa)	F_{req} (kN)	$A_{s,req}$ (mm^2)	$A_{s,prov}$ (mm^2)	$A_{s,prov}/A_{s,req}$	**Fail/Safe**
T1	Tie	1.00	493.00	493.00	32.5	66	201	3.049	O
T2	Tie	1.00	493.00	493.00	67.0	136	201	1.478	O
T3	Tie	1.00	340.00	340.00	28.3	83	142	1.698	O
T4	Tie	1.00	340.00	340.00	48.3	142	142	0.997	O
T5	Tie	1.00	493.00	493.00	75.0	152	201	1.321	O
T6	Tie	1.00	493.00	493.00	95.3	193	201	1.040	O
T7	Tie	1.00	493.00	493.00	60.8	123	201	1.631	O

Effective Strength of Strut $f_{cu} = \beta_s f_c'$; Effective Strength of Tie $f_{cu} = \beta_t f_y$; F_{req} = Cross-sectional Force under Experimental Failure Load; Required Strut Width $w_{req} = F_{req}/bf_{cu}$; Required Tie Area $A_{s,req} = F_{req}/f_{cu}$; w_{prov} = Strut Width Provided from Beam Geometry; O: Safe; X: Fail

(b) Struts and ties at the second failure

Element No.	Element Type	β_s	f_c' (MPa)	f_{cu} (MPa)	F_{req} (kN)	w_{req} (mm)	w_{prov} (mm)	w_{prov}/w_{req}	Fail/Safe
S1	Strut	1.00	56.50	56.50	20.3	7	34	4.766	O
S2	Strut	0.75	56.50	42.38	34.8	16	51	3.075	O
S3	Strut	0.75	56.50	42.38	66.8	32	5	0.145	X
S4	Strut	0.75	56.50	42.38	34.8	16	13	0.791	X
S5	Strut	0.75	56.50	42.38	139.5	66	16	0.242	X
S7	Strut	0.75	56.50	42.38	139.5	66	13	0.200	X
S8	Strut	1.00	56.50	56.50	14.1	5	41	8.251	O
Element No.	**Element Type**	β_t	f_y (MPa)	f_{cu} (MPa)	F_{req} (kN)	$A_{s,req}$ (mm^2)	$A_{s,prov}$ (mm^2)	$A_{s,prov}/A_{s,req}$	**Fail/Safe**
T2	Tie	1.00	493.00	493.00	67.0	136	67	0.492	X
T3	Tie	1.00	340.00	340.00	28.3	83	59	0.713	X
T4	Tie	1.00	340.00	340.00	113.4	334	2	0.005	X
T5	Tie	1.00	493.00	493.00	75.0	152	51	0.335	X
T6	Tie	1.00	493.00	493.00	95.3	193	10	0.054	X
T7	Tie	1.00	493.00	493.00	14.1	29	79	2.774	O

F_{req} = Cross-sectional Force under Experimental Failure Load; $w_{prov} = w_{prov}$ (at First Failure)-0.986× w_{req} (at First Failure); $A_{s,prov} = A_{s,prov}$ (at First Failure)-0.986× $A_{s,req}$ (at First Failure), See Figs. 9(a) and 9(b)

Table 2 Continued (c) Nodal zones

Node No.	Node Type	β_n	f_c' (MPa)	f_{cu} (MPa)	F_{req} (kN)		w_{req} (mm)	w_{prov} (mm)	w_{prov}/w_{req}	Fail/Safe
1	CCT	0.80	56.50	45.20	R	66.0	29	150	5.14	O
					S2	34.5	44	144	3.29	O
					S3	66.2				
					T5	74.3	33	60	1.82	O
4	CCT	0.80	56.50	45.20	V	178.3	79	150	1.90	O
					S1	20.1	9	41	4.64	O
					S3	66.2	44	65	1.48	O
					S4	34.5				
					S5	59.2	75	97	1.30	O
					S6	112.2				
					T1	32.0	14	41	2.92	O
7	CCC	1.00	56.50	56.50	V	112.4	40	75	1.89	O
					S6	112.2	60	95	1.58	O
					S7	59.2				

Effective Strength of Nodal Zone $f_{cu} = \beta_n f_c'$; F_{req} = Cross-sectional Force under 99.1% of Experimental Failure Load; R= Support Reaction; V= Applied Shear Force (= 99.1% of Experimental Failure Load)); w_{prov} = Node Width Provided from Beam Geometry; Required Node Width $w_{req} = F_{req} / bf_{cu}$

After determining the reactions $R_1 (= \gamma P$, P = external load applied vertically) and the cross-sectional force of the vertical steel tie $P_w (= \alpha P)$, the maximum widths (or areas) of the struts and ties shown in Fig. 8(d) were determined by considering the shapes of nodal zones, the reaction distribution ratio, and load distribution ratio, as shown in Fig. 8(c). The shapes of nodal zones were constructed based on the ACI 318M-11 that considers the geometry of strut-tie model and the size of loading and bearing plates.

Conducting the strength prediction in accordance with the flowchart of Fig. 7, the initial failure of the indeterminate strut-tie model, as explained in Fig. 9(a) and Table 2(a), was caused by the concrete strut S6 of an arch mechanism at a load of 177.4kN (98.6% of its experimental failure load). After the initial failure, the indeterminate strut-tie model became the determinate one that was still able to transfer a fraction of the applied load to the supports by other struts and ties of a truss mechanism, as shown in Fig. 9(b). After the initial failure, the element T1 having a tensile cross-sectional force before the initial failure was renamed as concrete strut S8 since it was under compression. When an additional load of 0.9kN (0.5% of its experimental failure load) was applied, the second failure of the strut-tie model occurred due to the steel tie T4, as shown in Fig. 9(c) and Table 2(b). After the second failure, the strut-tie model became an unstable truss structure that could not carry any additional load, as shown in Fig. 9(d). At a load of 178.3kN(=177.4+0.9, 99.1% of its experimental failure load) that the indeterminate strut-tie model could carry to the utmost limit, the strength of nodal zones was examined by the ACI 318M-11's strut-tie model approach, as shown in Fig. 9(e). As checked in Fig. 9(e) and Table 2(c), the strength of the nodal zones was sufficient to transfer the strut and tie forces through the nodal zones. Therefore, 99% of the experimental failure load of Beam 1CB2 was predicted as the ultimate strength of the beam by the present approach.

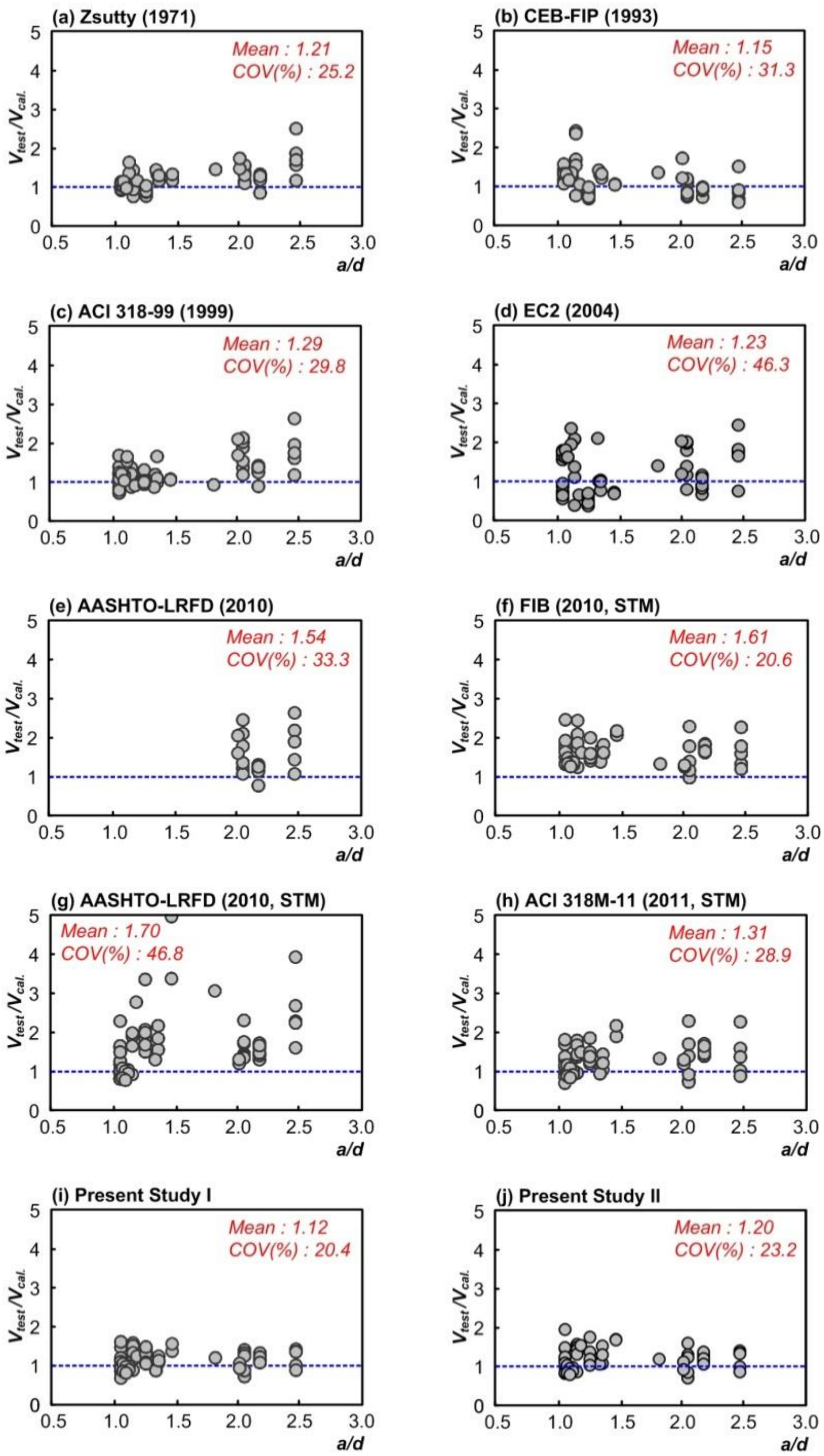


Fig. 10 Ultimate strength classified by shear span-to-effective depth ratio

Table 3 Ultimate strength evaluated by present and conventional approaches

(a) Ultimate strength

Investigators		Conventional approach ($V_{test}/V_{cal.}$)					Strut-Tie model approach ($V_{test}/V_{cal.}$)				
		Zsutty (1971)	CEB-FIP (1993)	ACI 318-99 (1999)	EC2 (2004)	AASHTO -LRFD (2010)	FIB (2010)	AASHTO -LRFD (2010)	ACI 318M-11 (2011)	Present study I	Present study II
Rogowsky *et al.* (1986)		1.40	1.16	1.54	1.53	1.73	1.62	1.92	1.37	1.16	1.19
Ashour (1997)		1.37	1.31	1.27	1.26	1.82	1.48	1.88	1.23	1.09	1.22
Subedi (1998)		1.27	1.12	1.10	0.79	-	1.98	3.34	1.72	1.31	1.51
Asin (1999)		1.05	0.85	1.17	0.72	1.15	1.64	1.75	1.47	1.20	1.26
Yang *et al.* (2007)		1.02	1.29	1.15	1.20	-	1.68	1.24	1.16	1.08	1.19
Yang *et al.* (2007)		1.18	1.27	1.29	1.82	-	1.35	0.93	0.97	0.92	0.92
Total	Mean	1.21	1.15	1.29	1.23	1.54	1.61	1.70	1.31	1.12	1.20
	STDEV	0.31	0.36	0.38	0.57	0.51	0.33	0.80	0.38	0.23	0.28
	COV(%)	25.2	31.3	29.8	46.3	33.3	20.6	46.8	28.9	20.4	23.2

Present Study I: Strength evaluations by using Fig. 2(c) model, Eq. (4), Eq. (10), and ACI 318M-11 strength parameters;
Present Study II: Strength evaluations by using Fig. 2(c) model, Eq. (4), Eq. (10), and FIB strength parameters

(b) Ultimate strength classified by shear span-to-effective depth ratio

Design variables		Conventional approach ($V_{test}/V_{cal.}$)					Strut-Tie model approach ($V_{test}/V_{cal.}$)			
		Zsutty (1971)	CEB-FIP (1993)	ACI 318-99 (1999)	EC2 (2004)	AASHTO -LRFD (2010)	FIB (2010)	AASHTO -LRFD (2010)	ACI 318M-11 (2011)	Present study I
$a/d \leq 2.0$	Mean	1.10	1.25	1.13	1.18	-	1.64	1.67	1.26	1.13
(38*)	COV(%)	19.0	28.6	22.0	49.6	-	19.2	52.4	27.5	21.1
$a/d > 2.0$	Mean	1.42	0.95	1.59	1.35	1.54	1.55	1.77	1.40	1.12
(20)	COV(%)	24.9	29.1	26.4	39.6	33.3	23.5	36.0	30.4	19.5

*: number of specimens

(c) Ultimate strength classified by concrete strength

Design variables		Conventional approach ($V_{test}/V_{cal.}$)					Strut-tie model approach ($V_{test}/V_{cal.}$)			
		Zsutty (1971)	CEB-FIP (1993)	ACI 318-99 (1999)	EC2 (2004)	AASHTO -LRFD (2010)	FIB (2010)	AASHTO -LRFD (2010)	ACI 318M-11 (2011)	Present study I
$f_c' \leq 35MPa$	Mean	1.16	1.16	1.26	1.18	1.47	1.62	1.65	1.35	1.16
(32*)	COV(%)	20.3	30.7	28.4	45.3	34.7	20.4	34.7	21.0	18.6
$f_c' > 35MPa$	Mean	1.28	1.13	1.32	1.31	1.62	1.59	1.77	1.25	1.08
(26)	COV(%)	28.7	32.8	31.7	46.8	33.0	21.3	57.3	37.6	22.3

*: number of specimens

Table 3 Continued (d) Ultimate strength classified by flexural reinforcement ratio

Design variables		Conventional approach ($V_{test}/V_{cal.}$)					Strut-tie model approach ($V_{test}/V_{cal.}$)			
		Zsutty (1971)	CEB-FIP (1993)	ACI 318-99 (1999)	EC2 (2004)	AASHTO -LRFD (2010)	FIB (2010)	AASHTO -LRFD (2010)	ACI 318M-11 (2011)	Present study I
$\rho/\rho_b \le 0.4$	Mean	1.23	1.16	1.28	1.27	1.71	1.57	1.72	1.23	1.09
(33*)	COV(%)	29.0	37.7	32.0	49.2	31.2	22.6	45.2	33.3	22.8
$\rho/\rho_b > 0.4$	Mean	1.19	1.13	1.30	1.19	1.39	1.67	1.68	1.41	1.17
(25)	COV(%)	18.7	20.2	27.3	41.7	33.7	17.8	49.8	22.2	17.0

*: number of specimens

4.4 Results of strength evaluation

The ultimate strength of the 58 deep beams evaluated by the present and conventional approaches is summarized in Table 3. The ratios of the experimental failure strength to the evaluated strength by the experimental shear strength models of Zsutty (1971), ACI 318-99 (1999), and EC2 (2004) were 1.21, 1.29, and 1.23, respectively, underestimating the experimental failure strength. The coefficients of variation by the experimental models were 25.2%, 29.8%, and 46.3%. The ratios of the experimental failure strength to the evaluated strength by the theoretical shear strength models of CEB-FIP (1993) and AASHTO-LRFD (2010) were 1.15 and 1.54, respectively. The coefficients of variation by the theoretical models were 31.3% and 33.3%, relatively large leading to questions about their applicability to structural design. The strut-tie model approaches of the FIB (2010), AASHTO-LRFD (2010), and ACI 318M-11 (2011) also underestimated the experimental failure strength by the ratio of 1.61, 1.70, and 1.31, respectively. The coefficients of variation by the design codes were 20.6%, 46.8%, and 28.9%, respectively. The ratio of the experimental failure strength to the evaluated strength and the coefficient of variation obtained by the present approach associated with the ACI 318M-11's effective strength values of struts and nodal zones were 1.12 and 20.4%. The present approach yielded better results than the conventional approaches, proving the necessity of an appropriate strut-tie model and its corresponding load distribution ratio for the design of continuous deep beams.

To examine the effects of the primary design variables on the ultimate strength and behavior of the deep beams, the strength analysis results were classified and compared according to the primary design variables. The strength analysis results classified in accordance with the shear span-to-effective depth ratio *a/d* are shown in Fig. 10 and Table 3(b). The experimental shear strength model of Zsutty (1971), ACI 318-99 (1999), and EC2 (2004) evaluated the ultimate strength conservatively more and more as the ratio of *a/d* became greater. Also, the coefficient of variation became greater as the ratio of *a/d* became greater. These imply that the experimental shear strength models may not be implemented appropriately in the design of continuous deep beams with relatively large ratio of *a/d*. The theoretical shear strength model of the CEB-FIP (1993) evaluated the ultimate strength critically as the ratio of *a/d* became greater. Unlike the ultimate strength evaluated by the experimental and theoretical shear strength models, the ultimate strength evaluated by the strut-tie model approaches of the current design codes was inaccurate in the entire range of *a/d*. This seems to result from the use of an incorrect load distribution ratio and the use of only a simple determinate arch or truss mechanism. The present approach yielded accurate and consistent strength analysis results throughout the entire range of *a/d* by overcoming the problems existing in other approaches.

The strength analysis results classified in accordance with the concrete strength f_c' are shown in Table 3(c). Also, the strength analysis results classified according to the flexural reinforcement ratio ρ are shown in Table 3(d). The present approach evaluated the ultimate strength of the deep beams comparatively well by properly reflecting the effect of the concrete strength and flexural reinforcement ratio that influences the depth of rectangular stress block, the cross-sectional areas of inclined struts, and the stiffness of steel ties on the nonlinear structural analyses of indeterminate strut-tie models conducted for determination of their load distribution ratios.

5. Conclusions

For the rational strut-tie model design of continuous reinforced concrete deep beams, an appropriate strut-tie model reflecting true load transfer mechanisms of the deep beams must be presented, and the primary design variables influencing the ultimate strength and behavior of the deep beams must be deliberated in the design process as well. In this study, a simple internally and externally indeterminate strut-tie model that reflects the characteristics of the ultimate strength and behavior was proposed for the design of continuous reinforced concrete deep beams. In addition, the equation for the load distribution ratio transforming the proposed indeterminate strut-tie model into an internally determinate model was proposed to help structural designers help structural designers conduct the practical strut-tie model design of the deep beams by using the strut-tie model approaches of current design codes.

To verify the validity of the present study, the ultimate strength of 58 two-span continuous reinforced concrete deep beams tested to shear failure was evaluated by the ACI 318M-11 and FIB strut-tie model approaches associated with the proposed indeterminate strut-tie model and load distribution ratio. The strength analysis results were also compared with those estimated by the experimental shear strength equation, the current design codes of strut-tie model approaches, and the design codes that were based on experimental and theoretical shear strength models.

The present approach evaluated the ultimate strength of the continuous deep beams fairly accurately compared with those by other approaches. In addition, the present approach reflected the effects of the primary design variables on the ultimate strength of deep beams consistently and accurately. The present study may allow the use of an indeterminate strut-tie model with an appropriate load distribution ratio for the rational design of continuous reinforced concrete deep beams, and provides a proper basis for structural design by reflecting the effects of primary design variables on the ultimate strength and behavior of deep beams.

Acknowledgements

This research was supported by a grant 15TBIP-C094425-01-000000 from Technology Business Innovation Program (TBIP) funded by Ministry of Land, Infrastructure and Transport of Korean government.

References

Alshegeir, A. (1992), "Analysis and design of disturbed regions with strut-rie models", Ph.D Dissertation, School of Civil Engineering, Purdue University, West Lafayette, Indiana, USA.

American Association of State Highway and Transportation Officials (2010), *AASHTO LRFD Bridge Design Specifications*, 5th Edition, Washington, D.C., USA.

American Concrete Institute (1999), *Building Code Requirements for Structural Concrete (ACI 318-99) and Commentary (ACI 318R-99),* Farmington Hills, Michigan, USA.

American Concrete Institute (2011), *Building Code Requirements for Structural Concrete (ACI 318M-11) and Commentary*, Farmington Hills, Michigan, USA.

ACI Subcommittee 445-1 (2002), *Examples for the Design for Structural Concrete with Strut-and-Tie Models; SP-208*, Reineck R. H. eds, American Concrete Institute, Michigan, USA.

Ashour, A.F. (1997), "Tests of reinforced concrete continuous deep beams", *ACI. Struct. J.*, **94**(1), 3-12.

Ashour, F. and Yang, K.H. (2008), "Application of plasticity theory to reinforced concrete deep beams: a review", *Mag. Concrete. Res.*, **60**(9), 657-664.

Asin, M. (1999), *The Behavior of Reinforced Concrete Continuous Deep Beams*, Delft University Press, Amsterdam, Netherlands.

Bazant, Z.P. (1997), "Fracturing truss model: size effect in shear failure of reinforced concrete", *J. Eng. Mech. - ASCE*, **123**(12), 1276-1288.

British Standards Institution (1997), *Code of Practice for Design and Construction (BS8110 Part I)*, British Standard, UK.

CSA Technical Committee on Reinforced Concrete Design (2005), *A23.3-04 Design of Concrete Structures*. Canadian Standards Association, Rexdale, Ontario, Canada.

Chetchotisak, P., Teerawong, J., Yindeesuk, S., and Song, J. (2014), "New strut-and-tie-models for shear strength prediction and design of RC deep beams", *Comput. Concrete.*, **14**(1), 19-40.

Comite Euro-International du Beton (1993), *CEB-FIP Model Code 1990*, Thomas Telford Services Ltd.

Concrete Design Committee P 3101 for the Standards Council (2006), *Concrete Structures Standard: Part 1-The Design of Concrete Structures* (NZS 3101-1). Standards New Zealand, Wellington, New Zealand

European Committee for Standardization (2004), *Eurocode 2: Design of Concrete Structures*, Brussels, Belgium.

Foster, S.J. and Gilbert, R.I. (1998), "Experimental studies on high-strength concrete deep beams", *ACI. Struct. J.*, **95**(4), 382-390.

Foster, S.J. and Malik, A.R. (2002), "Evaluation of efficiency factor models used in strut-and-tie model", *J. Struct. Eng. - ASCE*, **128**(5), 569-577.

Hwang, S.J., Lu, W.Y., and Lee, H.J. (2000), "Shear strength prediction for deep beams", *ACI. Struct. J.*, **97**(3), 367-376.

Hwang, S.J. and Lu, W.Y. (2002), "Strength prediction for discontinuity regions by softened strut-and-tie model", *J. Struct. Eng. - ASCE*, **128**(12), 1519-1526.

Comite Euro-International du Beton (2010) *CEP-FIP Model Code 2010*, International Federation for Structural Concrete (*fib*), Lausanne, Switzerland.

Kim, B.H. and Yun, Y.M. (2011), "An indeterminate strut-tie model and load distribution ratio for RC deep beams - (I) model & load distribution ratio", *Adv. Struct. Eng.*, **14**, 1031-1041.

Kim, B.H. and Yun, Y.M. (2011), "An indeterminate strut-tie model and load distribution ratio for RC deep beams - (II) validity evaluation", *Adv. Struct. Eng.*, **14**, 1043-1057.

Kim, W., Jeong, J.P., and Kim, D.J. (2003), "Non-bernoulli-compatibility truss model for RC members subjected to combined action of flexure and shear (I)-its derivation of theoretical concept", *J. Kor. Soc. Civil. Eng.*, **23**(6), 1247-1256.

Kim, W., Jeong, J.P., and Park, D.S. (2003), "Non-bernoulli-compatibility truss model for RC members subjected to combined action of flexure and shear (II)-its practical solution", *J. Kor. Soc. Civil. Eng.*, **23**(6), 1257-1266.

Leonhardt, F. (1965), "Reducing the shear reinforcement in reinforced concrete beams and slabs", *Mag. Concrete. Res.*, **17**(53), 187-198.

MacGregor, J.G. (1997), *Reinforced Concrete: Mechanics and Design*, 3rd Edition, Prentice-Hall, New Jersey.

Matamoros, A.B. and Wong, K.H. (2003), "Design of simply supported deep beams using strut-and-tie models", *ACI. Struct. J.*, **100**(6), 704-712.

Niwa, J., Yamada, K., Yokozawa, K., and Okamura, M. (1986), "Revaluation of the equation for shear strength of R.C.-beams without web reinforcement", *Proceeding of Japan Society of Civil Engineering*, 1986-1988.

Okamura, H. and Higai, T. (1980), "Proposed design equation for shear strength of R.C. beams without web reinforcement", *Proceeding of Japan Society of Civil Engineering*, 131-141.

Pang, X.B. and Hsu, T.T.C. (1995), "Behavior of reinforced concrete membrane elements in shear", *ACI. Struct. J.*, **92**(6), 665-679.

Park, J.W. and Kuchma, D.A. (2007), "Strut-and-tie model analysis for strength prediction of deep beams", *ACI. Struct. J.*, **104**(6), 657-666.

Park, R. and Paulay, T. (1975), *Reinforced Concrete Structures*, John Wiley & Sons, New York.

Portland Cement Association (2004), *AASHTO LRFD Strut-Tie Model Design Examples*, Skokie, Illinois, USA.

Quintero-Febres, C.G., Parra-Montesinos, G., and Wight, J.K. (2006), "Strength of struts in deep concrete members designed using strut-and-tie method", *ACI. Struct. J.*, **103**(4), 577-586.

Rogowsky, D.M., MacGregor, J.G., and Ong, S.Y. (1986), "Tests of reinforced concrete deep beams", *ACI. Struct. J.*, **83**(4), 614-623.

Subedi, N.K. (1998), "Reinforced concrete two-span continuous deep beams", *Proceedings of the Institution of Civil Engineers, Structures and Buildings*, **128**, 12-25.

Tjhin, T.N. and Kuchma, D.A. (2007), "Integrated analysis and design tool for the strut-and-tie model", *Eng. Struct.*, **29**(11), 3042-3052.

Vecchio, F.J. and Collins, M.P. (1986), "The modified compression field theory for reinforced concrete elements subjected to shear", *ACI. J. Proc.*, **83**(2), 219-231.

Yang, K.H., Chung, H.S. and Ashour, A.F. (2007), "Influence of shear reinforcement on reinforced concrete continuous deep beam", *ACI. Struct. J.*, **104**(4), 420-429.

Yang, K.H., Chung, H.S. and Ashour, A.F. (2007), "Influence of section depth on the structural behavior of reinforced continuous deep beams", *Mag. Concrete. Res.*, **59**(8), 575-586.

Yun, Y.M. (2000), "Nonlinear strut-tie model approach for structural concrete", *ACI. Struct. J.*, **97**(4), 581-590.

Yun, Y.M. (2005), "Strut-tie model evaluation of behavior and strength of pre-tensioned concrete deep beams", *Comput. Concrete*, **2**(4), 261-297.

Yun, Y.M. (2005), "Evaluation of ultimate strength of post-tensioned anchorage zones", *J. Adv. Concrete. Technol.*, **3**(1), 149-160.

Yun, Y.M. (2006), "Strength of two-dimensional nodal zones in strut-tie models, *J. Struct. Eng. - ASCE*, **132**(11), 1764-1783.

Yun, Y.M. and Kim, B.H. (2008), "A two-dimensional grid strut-tie model approach for structural concrete", *J. Struct. Eng. - ASCE*, **134**(7), 1199-1214.

Zsutty, T.C. (1971), "Shear strength prediction for separate categories of simple beam tests", *ACI. J. Proc.*, **68**(2), 138-143.

CC