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# Improved strut-and-tie method for 2D RC beam-column joints under monotonic loading

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**Abstract.** In the previous analytical studies on 2D reinforced concrete (RC) beam-column joints, the modified compression field theory (MCFT) and the strut-and-tie method (STM) are usually employed. In this paper, the limitations of these analytical models for RC joint applications are reviewed. Essentially for predictions of RC joint shear behaviour, the MCFT is not applicable, while the STM can only predict the ultimate shear strength. To eliminate these limitations, an improved STM is derived and applied to some commonly encountered 2D joints, viz., interior and exterior joints, subjected to monotonic loading. Compared with the other STMs, the most attracting novelty of the proposed improved STM is that all critical stages of the shear stress-strain relationships for RC joints can be predicted, which cover the stages characterized by concrete cracking, transverse reinforcement yielding and concrete strut crushing. For validation and demonstration of superiority, the shear stress-strain relationships of interior and exterior RC beam-column joints from published experimental studies are employed and compared with the predictions by the proposed improved STM and other widely-used analytical models, such as the MCFT and STM.

**Keywords:** improved strut-and-tie method; 2D RC beam-column joint; shear stress-strain relationship; monotonic loading

## 1. Introduction

Progressive collapse is defined as the disproportionate collapse of a structure caused by the failure or damage of a relatively small part. A specific description of the phenomenon provided by General Services Administration (2003) is "Progressive collapse is a situation where local failure of a primary structural component leads to the collapse of adjoining members which, in turn, leads to additional collapse." As the transfer regions of internal forces, the deformation capacity of beam-column joints ensures the structural integrity against progressive collapse.

In the relatively small volume inside the RC beam-column joints, there is a highly nonlinear region due to the composite action of steel reinforcement and concrete and local stress variations within joints, which brings about difficulties when analysing the behaviour of RC beam-column

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joints. In fact, the shear strength of RC beam-column joints is still under extensive study nowadays. Unlike the assume rigid joints in the most of the available numerical studies, Shafaei *et al*. (2014) emphasised the effect of joint flexibility due to slip of the beam longitudinal reinforcement and shear deformation of joint panel when assessing the behaviour of existing non-seismically detailed RC frames. Niroomandi *et al*. (2014) numerically quantified the degree of influence of two structural parameters, i.e. joint aspect ratio and beam longitudinal reinforcement ratio, on the shear failure of RC exterior joints without transverse reinforcement. Wong and Kuang (2014) proposed a theoretical model to predict the shear strength of RC interior joints by modifying the rotating-angle softened-truss model and the modified compression field theory (MCFT) based on the deep beam analogy. Sengupta and Li (2013) proposed an analytical approach to predict the hysteresis behaviour, such as stiffness and strength degradation and pinching, of RC joints with limited transverse reinforcement. Masi *et al*. (2013) performed experimental tests and numerical simulations to focus on cyclic behaviour and failure mode on full-scale RC exterior joints under different axial loadings.

In practice, component-based joint models are usually employed as an approximation to model the complex deformation behaviour. For 2D RC beam-column joints, various component-based approaches have been proposed (Alath and Kunnath 1995, Youssef and Ghobarah 2001, Lowes and Altoontash 2003, Altoontash 2004, Bao *et al*. 2008, Birely *et al*. 2012). As an example of a typical interior joint (Lowes and Altoontash 2003) as shown in Fig. 1, the idea of component-based beam-column joint model is to differentiate the characteristics of critical regions in the joint and treat each spring as an independent and functional component. The shear-panel component is employed to simulate the strength and stiffness loss due to shear failure of the joint core subjected to pure shear. In order to conduct efficient finite element analysis of framed structures, the complete load-deformation properties of the shear-panel component should be calibrated by reliable analytical models.

## 2. Review of existing analytical studies

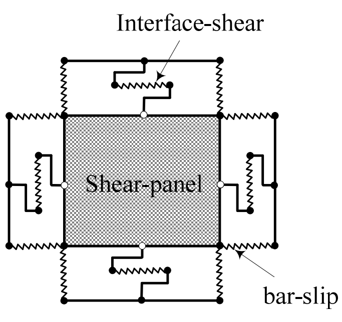



Fig. 1 Component-based beam-column joint model

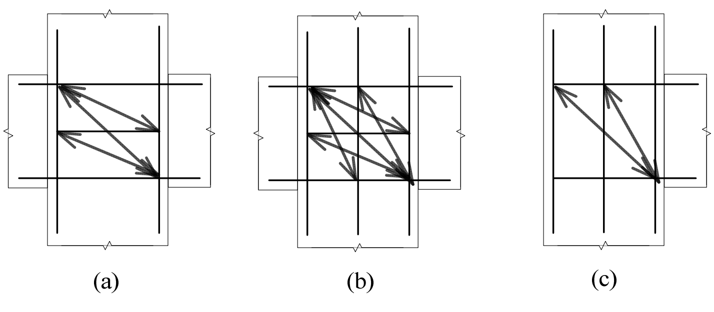



Fig. 2 Different strut configurations used in previous researches on the STM

To predict the shear deformation behaviour of the shear-panel component in the RC joint (Fig. 1), the MCFT (Vecchio and Collins 1986, 1993) has been widely employed (Youssef and Ghobarah 2001, Lowes and Altoontash 2003, Altoontash 2004, Shin and Lafave 2004, Mitra and Lowes 2007, Bao *et al*. 2008). However, it should be noted that some researchers (Shin and Lafave 2004, Mitra and Lowes 2007) reported that the analytical form of MCFT is inappropriate to predict the RC joint shear behaviour with low joint transverse reinforcement ratios. In fact, the MCFT is incapable of predicting the shear strength accurately, even if the joint panel is effectively confined as stipulated in the ACI 352R-02 (2002), which was clearly demonstrated by comparisons based on an extensive database (Kim and LaFave 2009).

Theoretically speaking, the intrinsic limitation of the MCFT stems from the assumption of uniform steel reinforcement when predicting the in-plane shear behaviour of 2D reinforced concrete elements. For beam-column joints, the assumed distributed longitudinal steel reinforcement is markedly different from conventional horizontal joint core hoops, because the latter in the form of hoops can provide more efficient and dependable diagonal compression struts to resist the horizontal joint shear force. This discrepancy has been confirmed by an experimental study (Wong *et al*. 1990). Therefore, the MCFT is not suitable for the shear strength predictions of RC joints and excluded in the present study.

As an alternative to the MCFT, the concept of strut-and-tie method (STM) is widely utilized in the design of deep beams, shear walls and beam-column joints where there are clear force paths or discrete struts joining the loading point to the support. However, the behaviour of beam-column joints is too complex to be modelled realistically with a simple STM based on plasticity theory and, thus, empirical approaches were proposed to develop an essentially descriptive STM for beam-column joints (Vollum and Newman 1999). Fundamentally, the proposed STM consists of the strut configurations and the corresponding load transfer mechanisms. In many previous studies on STM, several strut configurations have been proposed as shown in Fig. 2 (the arrow indicates the concrete strut), such as one direct and two horizontally indirect struts (Vollum and Newman 1999) (Fig. 2(a)), one direct and four indirect struts (Hwang and Lee 1999, 2000) (Fig. 2(b)) and one direct and one indirect struts (Park and Mosalam 2012a, 2012b) (Fig. 2(c)). The model with one direct and one indirect struts (Park and Mosalam 2012a, 2012b) is proposed for exterior beam-column joints without transverse reinforcement, while the first and second models with appropriate load transfer mechanisms are more general and can be applied to many types of 2D beam-column joints by adjusting the transferred forces. For example, the second model (Hwang and Lee 1999, 2000) has been adopted (Favvata *et al*. 2008) to predict the shear strengths of exterior RC beam-column joints under seismic loading.

Table 1 Strength and weakness of the previous analytical models

| Model | Strength | Weakness |
|---|---|---|
| MCFT | 1. well developed based on continuum mechanics<br>2. appropriate to predict the shear behaviour of RC members with uniform distributed reinforcement | Not suitable to joint panel of RC joints due to steel reinforcement detailing |
| STM | 1. simple in terms of force paths<br>2. widely utilized in predicting shear strength of shear members | Only the ultimate shear strength is achieved, while complete load-displacement response is needed when analysing RC joints in progressive collapse |

It should be noted that in the previous studies on beam-column joints using the concept of STM (Hwang and Lee 1999, Vollum and Newman 1999, Hwang and Lee 2000, Favvata *et al*. 2008, Park and Mosalam 2012b), most of researchers were only interested in the predictions of joint shear strengths under seismic loading. However, prior to the crushing of a concrete strut, concrete will crack and transverse reinforcement may also yield. Therefore, if one wishes to simulate realistic joint shear behaviour, besides the prediction of ultimate shear strength, it is important for the analytical model to simulate the critical stages of development of concrete cracking and transverse reinforcement yielding. To date, only Park and Mosalam (2012a) and Mitra and Lowes (2007) estimated the joint shear stress-strain relationship based on the concept of STM. However, the proposed relationship by Park and Mosalam (2012a) is oversimplified and exclusively focused on the predictions of shear strengths for exterior beam-column joints without transverse reinforcement. On the other hand, the study by Mitra and Lowes (2007) is based on the response of concrete strut described by the concrete model from Mander *et al*. (1988), nevertheless, the proposed relationship by Mitra and Lowes (2007) is only validated in terms of the ultimate shear strength rather than the complete shear stress-strain relationship.

Based on the above review on strength and weakness of the previous analytical models, a summary is made for clarity in Table 1 from the point view of beam-column joints in progressive collapse which is usually treated as a monotonic process. In order to eliminate the limitations of the previous analytical models, such as MCFT and STM, an improved STM for RC shear panels is proposed in the next section.

## 3. Improved STM for 2D RC beam-column joints

As the most attracting novelty of this paper, an improved STM is proposed to predict not only the ultimate shear strengths but also complete shear stress-strain responses of RC beam-column joints with concrete cracking/crushing and transverse reinforcement yielding/hardening subjected to monotonic loading. The proposed improved STM incorporates average stress and strain fields and load transfer mechanisms to simulate the nonlinear shear deformation of RC beam-column joints subjected to monotonic loading. In the proposed model, several critical stages are identified as follows: (a) prior to concrete cracking, (b) prior to stirrup yielding, (c) stirrup has yielded but prior to crushing of concrete strut, and (d) after crushing of concrete strut. Throughout these stages, equilibrium, compatibility and constitutive laws for concrete and steel reinforcement are satisfied either in terms of average stress and strain criteria (stages (a) and (b)) or in terms of participation distributions of horizontal and vertical ties (stages (c) and (d)). This makes the proposed model considerably rational compared to other existing research findings.

Regarding compatibility condition, it is noteworthy that the joint region in the stages (a) and (b) is assumed to be a stress continuum of where Mohr's circle is applicable. However, when severe crack openings occur after the stage (b), the compatibility condition is constructed conservatively for the discrete stress and strain fields based on the concept of STM.

When deriving the proposed improved STM, important structural effects due to RC joint characteristics should be incorporated as follows. Due to the presence of tensile strain perpendicular to the strut direction, the compressive behaviour of concrete in the joint region is different from that in the standard cylinder test under uniaxial compression (Kashiwazaki and Noguchi 1996).

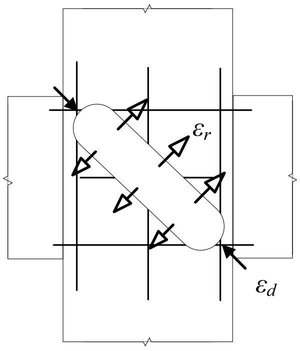



Fig. 3 Concrete compression softening phenomenon in beam-column joints

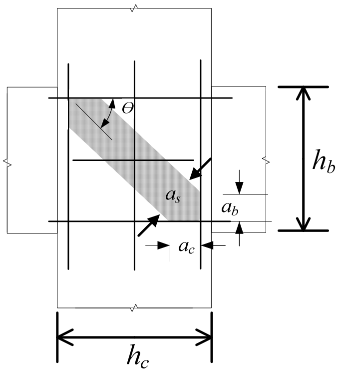



Fig. 4 Effective area of the concrete strut

This is known as concrete compression softening phenomenon and has been observed in deep beams (Arabzadeh *et al*. 2009, Hong and Ha 2012) and shear walls (Vecchio and Collins 1986, 1993, 1998). In the study of RC joints, a similar concept should be taken into account for beam-column joints as shown in Fig. 3. Besides, the confinement effect from the joint transverse reinforcement (Scott *et al*. 1982, Foster and Gilbert 1996, Tsonos 2007) should also be accounted for in the proposed improved STM to predict the joint deformation behaviour of RC structures with joint transverse reinforcement. Due to the presence of transverse reinforcement, the concrete compressive strength and the maximum compressive strain are enhanced, which will influence the ductility of the beam-column joints. In addition, as reported in many previous studies (Bakir and Boduroğlu 2002, Park and Mosalam 2012b), there are many geometric, material and loading parameters to be considered, which will complicate the analytical model study. Hence, important parameters will be identified as the dominant parameters to shear strength predictions of RC beam-column joints in the proposed improved STM, that is, joint aspect ratio, joint stirrup ratio, column reinforcement ratio, beam longitudinal reinforcement ratio, concrete cylinder strength, and column axial stress.

### *3.1 Equilibrium*

In this analytical model based on the concept of STM, the effective area of the concrete strut has to be determined before proceeding to subsequent stages. As shown in Fig. 4, the width $a_s$ of the diagonal concrete strut can be approximated as

$$a_s = \sqrt{(a_b)^2 + (a_c)^2} \tag{1}$$

where $a_b$ and $a_c$ are the depth of the compression zone in the beam and the column cross-sections, respectively. However, due to inevitable concrete crushing at the small beam compression zone of normally reinforced beams, the contribution of $a_b$ to the strut dimension can be neglected (Zhang and Jirsa 1982). On the other hand, for typical strong-column-and-weak-beam design, the adjacent column of the joint usually does not reach its nominal moment of resistance prior to that of the adjacent beam. Therefore, previous studies (Zhang and Jirsa 1982, Paulay and

Priestley 1992) recommended the depth of the compression zone in the adjacent column $a_c$ to be the depth of the flexural compression zone for an elastic column, empirically defined as follows (Hwang and Lee 1999, 2000, Mitra 2007).

$$a_c = \left(0.25 + 0.85\frac{N}{A_g f_c'}\right) h_c \tag{2}$$

where $N$ is the applied column axial load, $f_c'$ is the concrete cylinder strength, $A_g = b_c h_c$ is the gross cross-sectional area with $b_c$ and $h_c$ as the width and height of the column cross-section, respectively, as shown in Fig. 5.

The width $b_s$ of the concrete strut is reasonably taken as the confined thickness $b_b$ (Fig. 5) inside a beam-column joint, and the effective area of the concrete strut is given as

$$A_{strut} = a_c b_b = \left(0.25 + 0.85\frac{N}{A_g f_c'}\right) h_c b_s \tag{3}$$

It should be noted that as a common defect, the shear strength predictions based on STM are highly dependent on the dimensions $a_c$ and $b_b$ of the concrete struts. Furthermore, the effect of column axial load $N$ on the shear strength of RC joints has not been completely understood, according to reported studies (Pantazopoulou and Bonacci 1992, Vollum and Newman 1999, Bakir and Boduroğlu 2002, Park and Mosalam 2012b). In the analytical model by Pantazopoulou and Bonacci (1992), the joint shear strength decreases with increasing column axial load. Vollum and Newman (1999) summarized their known test data and concluded that the joint shear strength is reasonably independent of column axial load unless a hinge is formed in the upper column end of the beam-column joint without stirrups. Based on considerable scattered experimental data, Bakir and Boduroğlu (2002) also arrived at a similar conclusion that the column axial load does not influence the joint shear strength of monotonically-loaded exterior beam-column joints. A more balanced conclusion was drawn by Park and Mosalam (2012b) that a high column axial load will actually benefit the joint shear strength for a weak-column-and-strong-beam design. However, for the strong-column-and-weak-beam design, the effect of a high column axial load may not be significant. According to their comparison study (Park and Mosalam 2012b), the joint shear strength is not affected by the column axial load up to $0.2A_g f_c'$. Thus, the effect of column axial load on the shear strength of RC joints is conservatively considered as shown in Eq. (3). In the proposed improved STM, in order to reflect the effect of constraint conditions on the shear panels in different joint types (interior and exterior), the effective area of the concrete strut is determined by Table 2 based on Eq. (3), in which the critical strut area is defined as

$$A_{strut,cr} = 0.325\, h_c b_b \tag{4}$$

where the value 0.325 is conservatively taken as the average of 0.25 from the references (Zhang and Jirsa 1982, Paulay and Priestley 1992) and 0.40 from the reference (Vollum and Newman 1999).

Based on the evidence observed in the numerical and experimental studies (Bakir and Boduroğlu 2002, Haach *et al.* 2008), cracks of joint concrete form and propagate along the diagonal direction of the joint region. Therefore, as shown in Fig. 4, the direction of principal stress is simply assumed to be determined from the joint geometry as

$$\tan\theta = \frac{h_b}{h_c} \tag{5}$$

Table 2 Effective area of the concrete strut in the proposed improved STM for interior and exterior joints

| Joint type | Interior | Exterior |
|---|---|---|
| $A_{strut}$ | $a_c b_s$ if $a_c b_s \geq A_{strut,cr}$ | $0.5 a_c b_s$ if $0.5 a_c b_s \geq A_{strut,cr}$ |
| | $A_{strut,cr}$ if $a_c b_s < A_{strut,cr}$ | $\frac{1}{2}(0.5 a_c b_s + A_{strut,cr})$ if $0.5 a_c b_s < A_{strut,cr}$ |

*$A_{strut,cr}$ is defined in Eq. (4)

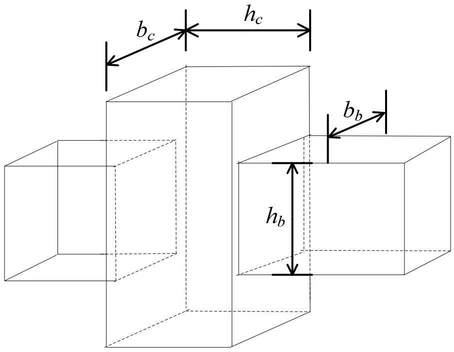



Fig. 5 Typical dimensions of a 2D beam-column joint

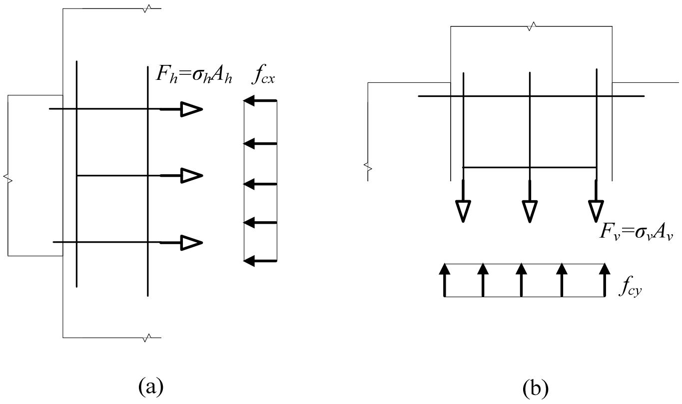



Fig. 6 Equilibrium across the horizontal and vertical cross-sections

where $h_b$ is the cross-sectional height of the adjacent beam and $h_c$ is the cross-sectional height of the adjacent column, as shown in Fig. 5. However, it should be noted that this assumption made in Eq. (5) is only valid for the joints with concentrated cracks along the joint diagonal direction, rather than those with smeared cracks.

Since the shear panel region is idealized to be subjected to pure shear throughout the loading stage as explain in Fig. 1, the vertical joint shear force $V_{jv}$ and horizontal joint shear force $V_{jh}$ can be approximately related (Mitra and Lowes 2007) by

$$\frac{V_{jv}}{V_{jh}} = \frac{h_b}{h_c} = \tan\theta \tag{6}$$

where the subscripts $h$ and $v$ indicate the directions of transverse reinforcement and longitudinal column bars, respectively, which will be used in the later derivation. This relationship between horizontal and vertical joint shear forces is kept the same throughout the loading history.

Based on the concept of STM, the load transfer mechanism idealized by Hwang and Lee (1999, 2000, 2002) and Hwang *et al*. (2000) as shown in Fig. 2(b) is adopted in the proposed analytical model, because this STM configuration is the most general in terms of load transfer path and is applicable to both interior and exterior types of beam-column joints (Hwang and Lee 1999, 2000, Favvata *et al*. 2008). For ease of derivation, the work on the computation of compressive stress of the concrete strut by Hwang and Lee (1999, 2000) is quoted in Eq. (7) through Eq. (15). The compressive stress of the concrete strut obtained from the load decomposition (Hwang and Lee

1999, 2000) can be written as

$$\sigma_d = \frac{1}{A_{strut}}\left\{F_D + \frac{\cos\left(\theta-\tan^{-1}\left(\frac{h_b}{2\,h_c}\right)\right)}{\cos\left(\tan^{-1}\left(\frac{h_b}{2\,h_c}\right)\right)}F_h + \frac{\cos\left(\tan^{-1}\left(\frac{2\,h_b}{h_c}\right)-\theta\right)}{\sin\left(\tan^{-1}\left(\frac{2\,h_b}{h_c}\right)\right)}F_v\right\} \tag{7}$$

where the forces $F_D$, $F_h$ and $F_v$ are idealized from the diagonal, horizontal and vertical mechanisms, respectively, and can be given as

$$F_D = \frac{1}{\cos\theta}\frac{R_d}{R_d+R_h+R_v}V_{jh} \tag{8}$$

$$F_h = \frac{R_h}{R_d+R_h+R_v}V_{jh} \tag{9}$$

$$F_v = \frac{1}{\cot\theta}\frac{R_v}{R_d+R_h+R_v}V_{jh} \tag{10}$$

while coefficients $R_d$, $R_h$ and $R_v$ are obtained as follows

$$R_d = \frac{(1-\gamma_h)(1-\gamma_v)}{1-\gamma_h\gamma_v} \tag{11}$$

$$R_h = \frac{\gamma_h(1-\gamma_v)}{1-\gamma_h\gamma_v} \tag{12}$$

$$R_v = \frac{(1-\gamma_h)\gamma_v}{1-\gamma_h\gamma_v} \tag{13}$$

with the empirical relationships (Jennewein and Schäfer 1992, Schäfer 1996)

$$\gamma_h = \frac{2\tan\theta-1}{3}\ for\ 0 \le \gamma_h \le 1 \tag{14}$$

$$\gamma_v = \frac{2\cot\theta-1}{3}\ for\ 0 \le \gamma_v \le 1 \tag{15}$$

Once the yielding of horizontal tie (joint transverse reinforcement) or vertical tie (column longitudinal reinforcement) occurs, the shear resisting mechanism within the RC joint region will be redistributed and the corresponding values of $\gamma_h$ or $\gamma_v$ will be assigned as zero afterwards.

In addition to the equilibrium of the nodal zone or concrete strut as discussed in Eq. (7) through Eq. (15), equilibrium across the horizontal and vertical cross-sections should be achieved by equilibrating the respective stress of steel reinforcement and concrete as shown in Fig. 6. Similar to the conservative assumptions (Pantazopoulou and Bonacci 1992, Bakir and Boduroğlu 2002) ignoring concrete tension stiffening effect, equilibrium condition in terms of average stress of both steel reinforcement and concrete can be expressed as

$$f_{cx} = -\frac{F_h}{h_b b_b} \tag{16}$$

$$f_{cy} = -\frac{F_v}{h_c b_c} \tag{17}$$

where the forces $F_h$ and $F_v$ due to joint transverse reinforcement and column longitudinal reinforcement can be obtained from Eqs. (9) and (10), while $f_{cx}$ and $f_{cy}$ are the average horizontal and vertical stresses of concrete, respectively. The terms $h_b$, $b_b$, $h_c$ and $b_c$ are the typical dimensions the adjacent beam and column of a beam-column joint, as shown in Fig. 5.

By using the Mohr's circle in stress with the assumption of continuous stress field, the joint shear stress can be determined as

$$\tau_{cxy} = (f_{c1} - f_{cx}) \tan\theta \tag{18}$$

$$\tau_{cxy} = \frac{f_{c1} - f_{cy}}{\tan\theta} \tag{19}$$

The principal compressive stress $f_{c2}$ can be given as

$$f_{c2} = f_{c1} - \tau_{cxy}\left(\tan\theta + \frac{1}{\tan\theta}\right) \tag{20}$$

Thus, the principal tensile stress $f_{c1}$ can be determined in Eq. (21) from Eqs. (18)-(20), while from Eqs. (19) and (20), the principal tensile stress $f_{c1}$ can be given in Eq. (22).

$$f_{c1} = \frac{(1+\tan^2\theta) f_{cx} - f_{c2}}{\tan^2\theta} \tag{21}$$

$$f_{c1} = \left[\left(1 + \frac{1}{\tan^2\theta}\right) f_{cy} - f_{c2}\right] \tan^2\theta \tag{22}$$

### *3.2 Constitutive law for reinforced concrete*

The concrete compressive strain can be calculated with consideration of compression softening effect (Vecchio and Collins 1986, 1993, Zhang and Hsu 1998) and confinement effect due to stirrups in the joint core (Scott *et al*. 1982, Foster and Gilbert 1996, Tsonos 2007). The Kent and Park model (Park *et al*. 1972, Park *et al*. 1982) is adopted for the stress-strain relationship for confined concrete struts inside the beam-column joints. For the ascending curve prior to attainment of ultimate compressive strength, the compressive stress $\sigma_d$ is given (Park *et al*. 1972, Park *et al*. 1982) as

$$\sigma_d = f_{d,max}\left[2\left(\frac{\varepsilon_d}{\varepsilon_0}\right) - \left(\frac{\varepsilon_d}{\varepsilon_0}\right)^2\right] \tag{23}$$

where $\varepsilon_d$ is principal compressive strain, $f_{d,max}$ is the modified ultimate compressive strength and $\varepsilon_0$ is the corresponding strain. In addition, $f_{d,max}$ and $\varepsilon_0$ are given (Vecchio and Collins 1986, Foster and Gilbert 1996) for concrete cylinder strength between 20 MPa (2.9 ksi) and 100 MPa (14.5 ksi) in Eqs. (24) and (25) , respectively.

$$f_{d,max} = \frac{f_c'}{0.8 - 0.34\frac{\varepsilon_r}{\varepsilon_0}} \tag{24}$$

$$\varepsilon_0 = -0.002 - 0.001\left(\frac{f_c' - 20}{80}\right) \tag{25}$$

where $\varepsilon_r$ is the principal tensile strain.

For the descending portion after the ultimate compressive strength, which will strongly influence ductility of RC beam-column joints, the concrete compressive stress is given (Park *et al*. 1972, 1982) as

$$\sigma_d = f_{d,max}[1 - Z_m(\varepsilon_d - \varepsilon_0)] \tag{26}$$

where the descending gradient $Z_m$ and the ultimate concrete compressive strain $\varepsilon_u$ (Tsonos 2007) are given in Eqs. (27) and (28), respectively.

$$Z_m = \frac{f_c^{'} - f_{c,res}}{\varepsilon_0 - \varepsilon_u} \tag{27}$$

$$\varepsilon_u = -0.004 - 0.9\frac{f_{yh}}{300} \tag{28}$$

where $f_{yh}$ is the yield strength of transverse reinforcement in MPa and the residual stress $f_{c,res}$ for crushed concrete is taken as $0.2f_c^{'}$. On the other hand, the concrete tensile stress (Vecchio and Collins 1986) is empirically given by

$$\sigma_r = \begin{cases} E_c\varepsilon_r & for\ \varepsilon_r \le \varepsilon_{cr} \\ \frac{f_t}{1+\sqrt{200\,\varepsilon_r}} & for\ \varepsilon_r > \varepsilon_{cr} \end{cases} \tag{29}$$

where $E_c$ is the Young's modulus of concrete and $f_t$ is the concrete tensile strength.

As for steel reinforcement, the stress-strain relationship is assumed to be bi-linear with the stress corresponding to the junction point as the yield strength and the maximum stress as the fracture criterion.

### *3.3 Compatibility condition*

In the first two stages (a) prior to concrete cracking and (b) prior to stirrup yielding, it is reasonable to assume continuous stress and strain fields (Wang *et al*. 2012) and the joint shear strain can be determined by Mohr's circle. This assumption is similar to the one made in the MCFT (Vecchio and Collins 1986, 1993) throughout the loading history to attain an arbitrary strain along a certain direction and the joint shear strain. In this study, this assumption holds until the yielding of stirrups or the crushing of concrete struts. As a result, the average horizontal and vertical strain of steel in the first two stages (a) prior to concrete cracking and (b) prior to stirrup yielding can be given as

$$\varepsilon_h = \frac{\sigma_h}{E_{sh}} = \frac{F_h}{E_{sh}A_h} \tag{30}$$

$$\varepsilon_v = \frac{\sigma_v}{E_{sv}} = \frac{F_v}{E_{sv}A_v} \tag{31}$$

where $\sigma$ is the average stress of steel, $E_s$ is the Young's modulus and $A$ is the cross-sectional area of the steel reinforcement. The subscripts $h$ and $v$ indicate the horizontal and vertical directions, respectively.

According to Mohr's circle in strain, one obtains

$$\frac{\gamma_{hv}}{2} = \frac{\varepsilon_h - \varepsilon_d}{\tan\theta} = (\varepsilon_r - \varepsilon_h)\tan\theta \tag{32}$$

$$\varepsilon_h + \varepsilon_v = \varepsilon_r + \varepsilon_d \tag{33}$$

where $\theta$ is the direction of the joint diagonal, $\varepsilon_h$ is the average horizontal strain, $\varepsilon_v$ is the average vertical strain, $\gamma_{hv}$ is the shear strain at the joint panel, $\varepsilon_r$ and $\varepsilon_d$ are the principal tensile strain and principal compressive strain along the direction of the joint diagonal, respectively.

After reorganizing the expressions, the tensile strain that lies orthogonal to the joint diagonal in the joint plane can be determined from Eqs. (34) and (35).

$$\varepsilon_r = \varepsilon_h + (\varepsilon_h - \varepsilon_d)\cot^2\theta \tag{34}$$

$$\varepsilon_r = \varepsilon_v + (\varepsilon_v - \varepsilon_d)\tan^2\theta \tag{35}$$

After stage (b), the compatibility above can be no longer maintained in the joint region, since severe crack opening with transverse reinforcement yielding invalidates the continuum condition. In order to obtain a conservative estimate in the critical stage (c) prior to the crushing of concrete strut with the yielding of transverse reinforcement, the contribution due to transverse reinforcement hardening is neglected and the constitutive relationship of transverse reinforcement is assumed to be elasto-perfectly-plastic. Therefore, the average horizontal strain of the joint stirrup after yielding cannot be accurately calculated based on the stress-strain relationship and has to be determined empirically.

Similar difficulties were encountered by Altoontash (2004) when analysing the beam-column joints without transverse reinforcement. To solve the problem, 45% of the beam or column longitudinal reinforcement at the joint perimeter was taken by Altoontash (2004) as the effective transverse reinforcement, based on a limited calibration with seven specimens to best fit the measured joint shear strength.

In the proposed improved STM, based on the participation distributions of the transverse reinforcement or the longitudinal column bars (Hwang and Lee 2000), it is assumed that ties in both horizontal and vertical mechanisms do not fully yield at the same time and the remaining elastic proportion of ties will contribute to the post-yielding resistance and, thus, mobilize further shear deformation. As shown in Fig. 7, the areas $0.5\,A_h$ and $0.5\,A_v$ in the horizontal and vertical mechanisms are assumed to fully participate in the shear resistance prior to the occurrence of yielding of ties and, therefore, the remaining elastic portion in terms of both area and strength will contribute to the post-yielding shear resistance. Based on average stress and strain, the equivalent hardening modulus $H_s$ after yield strength can be obtained according to Fig. 7 by formulating.

$$\begin{aligned} A_h H_{sh} &= (0.25A_h \times 2) \times E_{sh} \times 50\% \\ A_v H_{sv} &= (0.25A_v \times 2) \times E_{sv} \times 50\% \end{aligned} \tag{36}$$

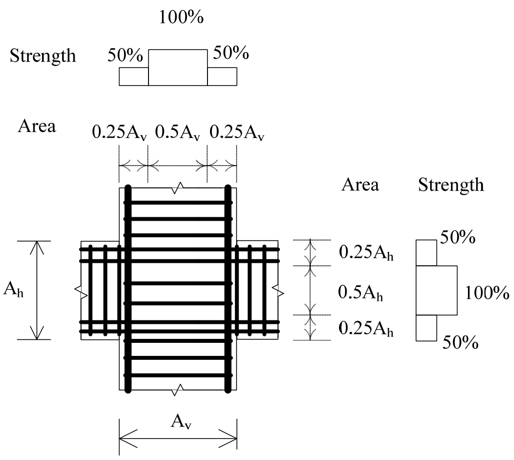



Fig. 7 Participation distribution of reinforcement

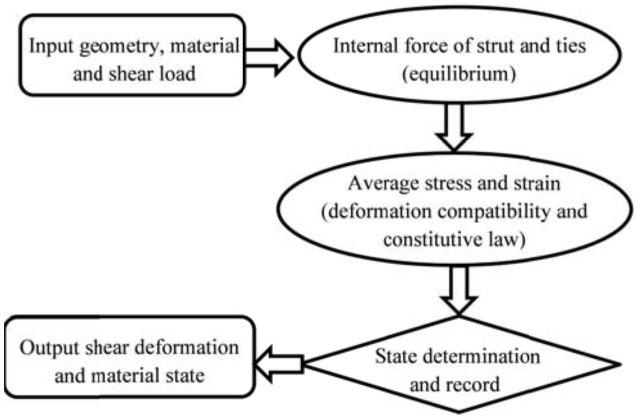



Fig. 8 Flowchart of the proposed improved STM

Thus, $H_s = 0.25\,E_s$ can be obtained in the equivalent stress-strain relationship at the cross-sectional level based on full $A_h$ and $A_v$. The subscripts $h$ and $v$ indicate the horizontal and vertical directions, respectively. Nevertheless, it should be noted that the occurrence of stirrup yielding is not inevitable, because in some cases, the stirrups do not yield and consequently, there will be no stage (c) at all in the shear deformation history. For instance, for joint specimens with sufficient transverse reinforcement, the deformation of the joint is directly controlled by the crushing of concrete struts as indicated in stage (d). If the criterion $\sigma_d > f_{d,max}$ is satisfied, then the ultimate shear strength can be captured and in the last stage, the evolution of compressive strain of concrete strut takes over in stage (d). The empirical expressions for average horizontal and vertical strains can be written as

$$\varepsilon_h = \frac{\frac{F_h}{A_h} - f_{yh}}{H_{sh}} + \frac{f_{yh}}{E_{sh}} \tag{37}$$

$$\varepsilon_v = \frac{\frac{F_v}{A_v} - f_{yv}}{H_{sv}} + \frac{f_{yv}}{E_{sv}} \tag{38}$$

where $f_y$ is the yield strength, $E_s$ is the Young's modulus, $H_s$ is the hardening modulus and $A$ is the cross-sectional area of the steel reinforcement. The subscripts $h$ and $v$ indicate the horizontal and vertical directions, respectively. For convenience of readers to understand the proposed approach and facilitate the implementation in existing finite element programs, a concise flowchart of the proposed improved STM is illustrated in Fig. 8. Additionally, in order to demonstrate how these assumptions and formulae above work together, a detailed numerical solution procedure is given in the Appendix. The solution procedure of the proposed improved STM has been successfully implemented into a finite element program FEMFAN3D for analysing progressive collapses of RC framed structures (Long *et al*. 2012).

## 4. Evaluation of the proposed improved STM

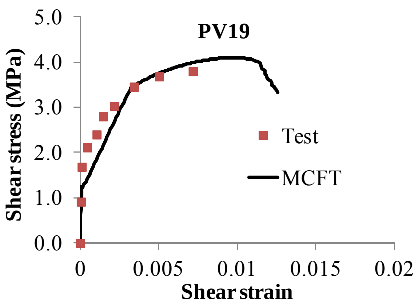
In the context of progressive collapse, the scope of the present paper is to focus on the joint behaviour subjected to monotonic shear loading and, therefore, joint experiments under monotonic loading is preferably employed to validate the proposed improved STM (labelled as 'ISTM' in the later figures). However, in terms of joint shear failure, the joint experiments under monotonic loading with complete load-displacement results reported are extremely rare, compared with joint experiments under seismic loading. In the present study, two series of interior and exterior joints are reasonably selected from available experimental studies. The predictions based on the proposed improved STM are compared with corresponding experimental results and other published analytical models (the models based on MCFT (Vecchio and Collins 1986) and the STM (Hwang and Lee 1999, 2000). Firstly, the implemented original MCFT (Vecchio and Collins 1986) has been verified against the experimental results on RC shear panels (Vecchio and Collins 1986, Maekawa 2003) as shown in Fig. 9, while the implemented STM has been verified by comparisons with the published shear strength predictions (Hwang and Lee 1999, 2000) on RC beam-column joints (Megget 1974, Lee *et al*. 1977, Alameddine 1990, Kaku and Asakusa 1991) as shown in Table 3. Clearly, the implemented MCFT gives good predictions of RC shear panels with uniform transverse and longitudinal reinforcement, whereas the implemented STM in Table 3 gives acceptable results compared with the original model (Hwang and Lee 1999, 2000). Thus, the credibility of MCFT and STM as programmed by the authors are very reliable. These two models will be used later on.

### *4.1 Interior joints*

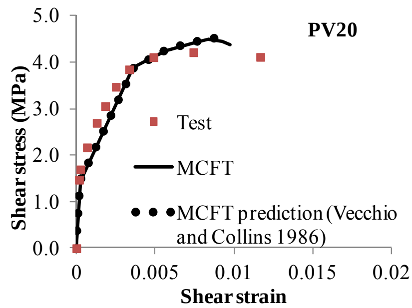
There are fairly limited numbers of publications on interior RC beam-column joint tests with complete load-deformation responses under monotonic loading. As a reasonable compromise, the experimental studies by Noguchi *et al*. (1988, 1992) are chosen to validate the applications of the proposed improved STM for 2D RC interior joints. The dimensions and reinforcement details of the specimens are shown in Fig. 10. The material properties of concrete and steel reinforcement employed in the interior joints are given in Tables 4 and 5, respectively.

Table 3 Verifications of the implemented STM

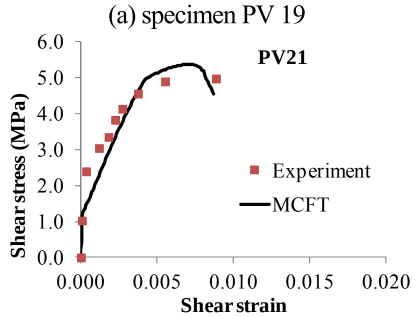
| Specimen ID (Reference) | STM(Hwang and Lee 1999, 2000) (kN) | Implemented STM (kN) |
|---|---|---|
| Unit A (Megget 1974) | 419 | 420 |
| 6 (Lee *et al*. 1977) | 155 | 155 |
| LL8 (Alameddine 1990) | 724 | 722 |
| HH11 (Alameddine 1990) | 937 | 937 |
| 2 (Kaku and Asakusa 1991) | 300 | 300 |
| 4 (Kaku and Asakusa 1991) | 349 | 347 |
| 6 (Kaku and Asakusa 1991) | 210 | 209 |
| 14 (Kaku and Asakusa 1991) | 261 | 262 |
| 15 (Kaku and Asakusa 1991) | 233 | 234 |



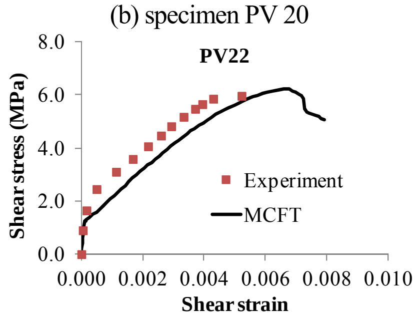

(a) specimen PV 19




(b) specimen PV 20




(c) specimen PV 21




(d) specimen PV 22

Fig. 9 Verifications of the implemented MCFT

Table 4 Concrete properties of the series of interior joints

| Specimen | OKJ-1, 2, 4, 5 | OKJ-3 | No.2, 4 |
|---|---|---|---|
| Cylinder strength (MPa) | 70.0 | 107.0 | 70.6 |
| Young's modulus (MPa) | 35100 | 40300 | 35100 |
| Poisson's ratio | 0.2 | 0.2 | 0.2 |
| Critical compressive strain | 0.00296 | 0.00286 | 0.00296 |

Table 5 Steel reinforcement properties of the series of interior joints

| Bar size | D6 | D13 |
|---|---|---|
| Young's modulus | 186 GPa | 182 GPa |
| Poisson's ratio | 0.3 | 0.3 |
| Yield stress | 718 MPa | 955 MPa |
| Maximum stress | 767 MPa | 1140 MPa |

(a) Dimensions of the interior joint (b) Steel reinforcement details of the interior joint

Fig. 10 Dimensions and reinforcement details of the interior joints

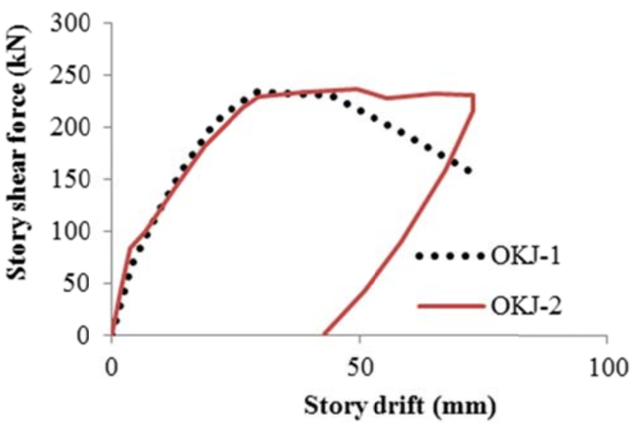


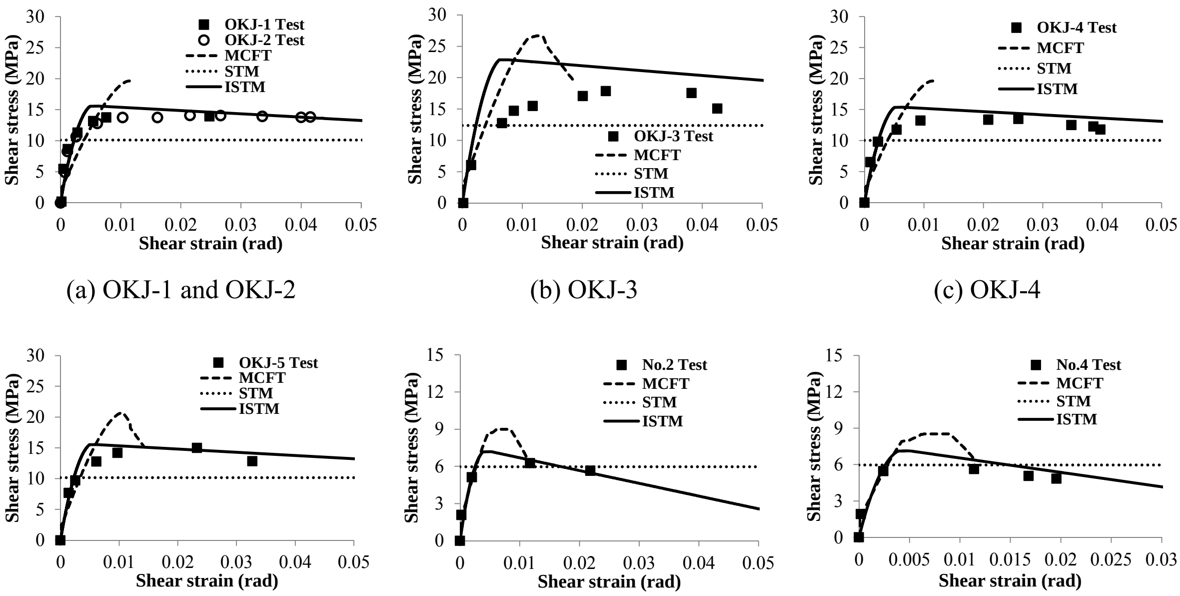

Fig. 11 Experimental result comparison of interior joints OKJ-1 and OKJ-2

It was reported that shear failure with or without yielding of beam longitudinal reinforcement in the joint panel was observed in all the specimens. Even though most of the specimens in these series of tests were conducted under cyclic loading, one of the specimens was tested under both cyclic and monotonic loading, indicated as OKJ-1 and OKJ-2, respectively. With the backbone curves obtained from the published test results, a comparison of experimental load-deformation relationships of these two specimens is given in Fig. 11.

It is obvious that the effect of moderate loading reversal does not affect the deformation behaviour until at a much later stage after attaining the peak strength. Therefore, the experimental data from these series of tests under similar moderate loading reversals can be reasonably adopted to validate the proposed improved STM under monotonic loading. Besides, the predictions from the MCFT (Vecchio and Collins 1986) and the STM (Hwang and Lee 1999, 2000), both of which have been verified, are also compared with the experimental results as shown in Figs. 12(a)-12(f).

Therefore, it is safely concluded that the MCFT predictions are too conservative for ductility and tend to overestimate the ultimate shear strengths of interior RC beam-column joints, while the STM predictions (Hwang and Lee 1999, 2000) underestimate the ultimate shear strengths for OKJ series (Figs. 12(a)-12(d)) and specimens No. 2 and No. 4 (Figs. 12(e) and 12(f)). By contrast, the proposed improved STM (labelled as 'ISTM') is capable of reasonably predicting both the ductility and the ultimate shear strengths of interior RC beam-column joints.

It is noteworthy that since there was sufficient confinement from transverse reinforcement and longitudinal column bars, no yielding of confining reinforcement occurred and concrete struts constituted the main shear resisting mechanism. For the descending part of the curve, the post-peak concrete behaviour is fairly accurately reflected, which is governed by the ratio and yield strength of transverse reinforcement, and maximum concrete compressive strain (Scott *et al.* 1982).




(a) OKJ-1 and OKJ-2 (b) OKJ-3 (c) OKJ-4

(d) OKJ-5 (e) No.2 (e) No.4

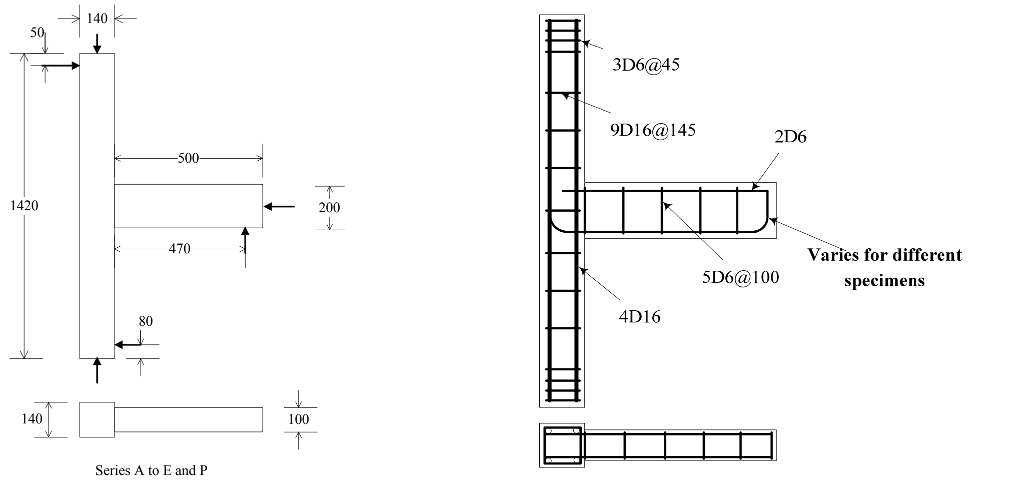
Fig. 12 Comparison of load-deformation relationships of RC interior beam-column joints

### *4.2 Exterior joints*

A series of 2D RC exterior joints under monotonic loading were tested by Taylor (1974) with variations in beam steel reinforcement, column axial load, beam thrust, concrete strength and beam depth. The dimensions and steel reinforcement details are illustrated in Figs. 13(a) and 13(b). Concrete cover to the main steel is 22 mm. The Young's modulus of concrete is determined by $3900\sqrt{f_c'}$ proposed by Pang and Hsu (1996). As for steel reinforcement, the material properties for different series are given in Table 6.

Exterior beam-column joints in all series (P, A, D, E and F) have been studied and it is found that the predictions, based on the proposed improved STM, for the load-deformation relationships are satisfactory in terms of ductility and ultimate shear strength compared with the predictions by MCFT and STM. Because of limitations of space, only series P, A and D are discussed in detail as shown in Figs. 14-16, which show that the proposed improved STM is able to predict well the critical stages in exterior RC beam-column joints with a wide range of parameters, viz., the stages prior to concrete cracking, transverse reinforcement yielding and concrete strut crushing of shear panels. It is noteworthy that markedly different from the other specimens, there is no yielding of transverse reinforcement in specimen D3/41/06 as shown in Fig. 16(d) and the joint deformation is directly controlled by the crushing of concrete strut as indicated in stage (d) because of the low concrete cylinder strength (Taylor 1974).

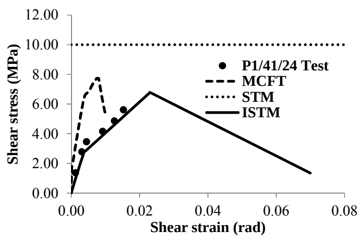
Similar to the conclusions for interior joint, the MCFT predictions for exterior RC beam-column joints are generally too conservative for ductility. As shown in Fig. 16 for series D, the terminations of the MCFT predictions result from shear failure of RC joints and there is a descending stage in the shear stress-strain response, which, however, is not so significant due to the small ductility. Since the convergence in the post-peak stage is difficult to attain for the MCFT, there is no descending stage after the shear capacity for several specimens. Nevertheless, it should be noted that the predictions by the MCFT with only transverse reinforcement is far from the experimental results.



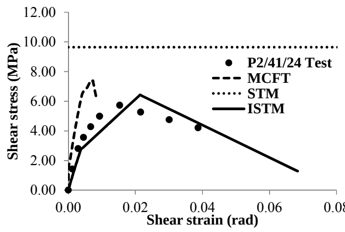

(a) Dimensions of the exterior joints

(b) Steel reinforcement details of the exterior joints

Fig. 13 Dimensions and reinforcement details of the exterior joints

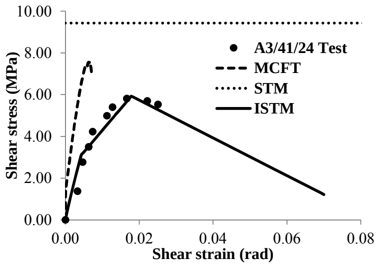



(a) P1/41/24

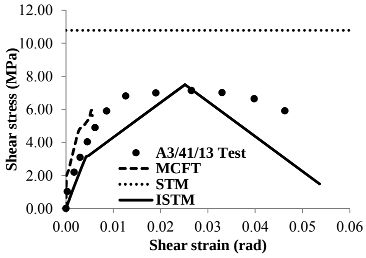



(b) P2/41/24

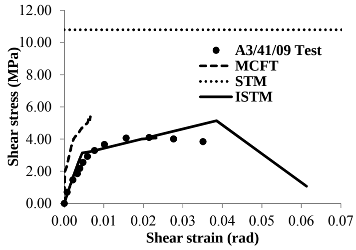
Fig. 14 Load-deformation relationships of series P joints



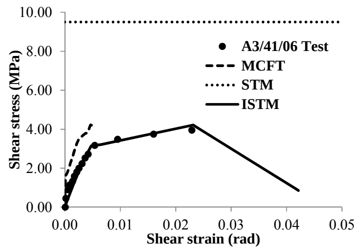

(a) A3/41/24




(b) A3/41/13




(c) A3/41/09




(d) A3/41/06

Fig. 15 Load-deformation relationships of series A joints

Table 6 Steel reinforcement properties of the series of exterior joints

| | Series P | Series A to F |
|---|---|---|
| Young's modulus | 200 GPa | 200 GPa |
| Poisson's ratio | 0.3 | 0.3 |
| Yield stress | 410 MPa | 460 MPa |
| Maximum stress | 515 MPa | 578 MPa |

Thus, 45% of the beam longitudinal reinforcement is artificially converted as joint transverse reinforcement in the predictions above and assumed to contribute to the joint confinement as proposed by Altoontash (2004).

Based on the validations above, the proposed improved STM is generally capable of predicting the critical stages (including the stages prior to concrete cracking, transverse reinforcement yielding and concrete strut crushing) of shear panels in both interior and exterior RC beam-column joints. In addition, the shear stress-strain relationships with consideration of concrete compression softening phenomenon and transverse confinement effect can be obtained.

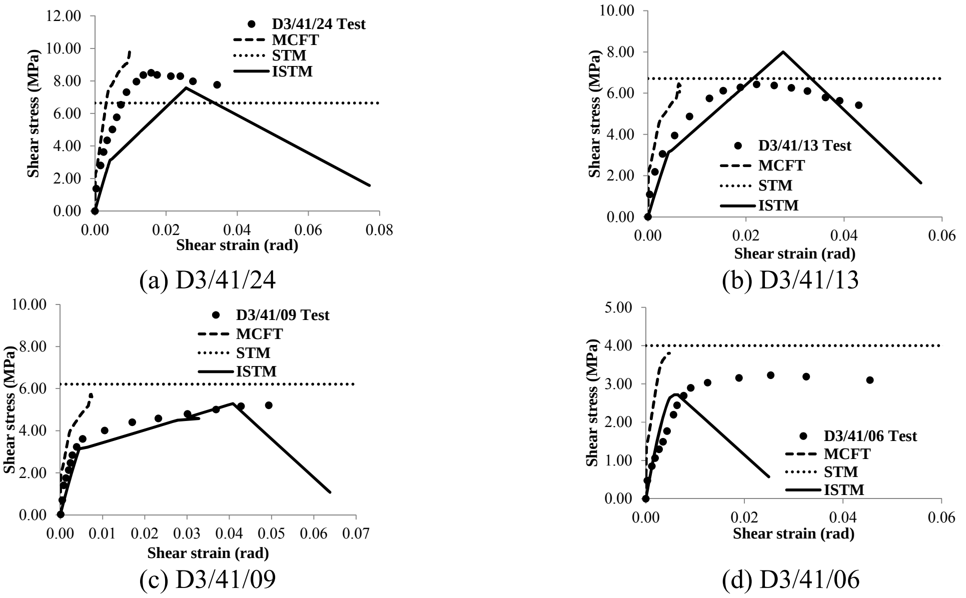



Fig. 16 Load-deformation relationships of series D joints

## 5. Conclusions

An improved strut-and-tie method (STM) for reinforced concrete (RC) beam-column joint is proposed and is applicable to interior and exterior types of 2D joints subjected to monotonic loading. The proposed improved STM satisfies compatibility, equilibrium and constitutive laws for both concrete and steel reinforcement.

The most appealing advantage of the proposed improved STM is its capability of predicting all the critical stages including the stages prior to concrete cracking, transverse reinforcement yielding and concrete strut crushing. The approach also provides complete shear stress-strain relationships with consideration of concrete compression softening phenomenon due to tensile strain and confinement effect of transverse reinforcement to the concrete core inside the RC joints. According to previous theoretical and experimental studies, several important parameters are taken into account in the proposed improved STM for RC joints, such as joint aspect ratio, joint stirrup details, column reinforcement ratio, beam longitudinal reinforcement ratio, concrete cylinder strength, and column axial stress.

With validations against experimental studies and other available analytical models (the models based on MCFT and STM) considering the variations in beam and column longitudinal steel reinforcement, transverse reinforcement, column axial load, concrete strength and joint aspect ratio, the proposed improved STM is capable of providing stable and reliable predictions on the complete shear stress-strain relationships of 2D RC interior and exterior beam-column joints subjected to monotonic shear loading.

At last, it should be pointed out that by integrating state-of-the-art formulae in literature to quantify joint geometry and material properties, the objective of the present study is to propose a practical yet rational approach to assess the monotonic shear resistance of 2D RC beam-column joints in progressive collapse. Therefore, the proposed methodology is exclusively applicable to the joint behaviour under monotonic shear loading, which is likely to overestimate the joint resistance under the scenario of cyclic loading, such as seismic action, due to the degradation of strength and stiffness of structures.

It should also be noted that even though the proposed improved STM is intended to overcome the limitations of existing STM/MCFT for shear resistance of 2D RC beam-column joints in progressive collapse, the proposed STM has also limitations embedded in the empirical formulae adopted for developing the modified STM. For example, Eqs. (2) and (3) are applicable for beam-column joints designed according to strong column- weak beam philosophy. Moreover, due to the adopted concrete constitutive law in Eq. (23), this improved STM is limited to normally reinforced concrete joints with concrete compressive strengths ranging from between 20 MPa (2.9 ksi) to 100 MPa (14.5 ksi).

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## Notations

$a_b$ : depth of the compression zone in the beam
$a_c$ : depth of the compression zone in the column
$a_s$ : width of the concrete strut
$b_b$ : joint thickness
$b_c$ : width of the column cross-section
$b_s$ : width of the concrete strut
$f_c^{'}$ : concrete cylinder strength
$f_t$ : concrete tensile strength
$f_{c\mathrm{x}}$ and $f_{c\mathrm{y}}$ : average horizontal and vertical stresses of concrete
$f_{c1}$ and $f_{c2}$ : principle tensile and compressive stresses
$h_b$ : depth of the beam cross-section
$h_c$ : height of the column cross-section
$A_g$ : gross cross-sectional area
$A_h$ : joint horizontal cross-sectional area
$A_v$ : joint transverse cross-sectional area
$A_{str}$ : effective area of the concrete strut
$A_{strut,cr}$: critical area of the concrete strut
$F_D$ : idealized forces for the diagonal mechanism
$E_c$ : Young's modulus of concrete
$E_s$ : Young's modulus of steel
$F_h$ : idealized forces for the horizontal mechanism
$F_v$ : idealized forces for the mechanism
$H_s$ : hardening modulus
$N$ : column axial load
$R_d$, $R_h$ and $R_v$ : coefficients in load transfer mechanism
$V_{jh}$ : horizontal joint shear force
$V_{jv}$ : vertical joint shear force
$\gamma_{hv}$ : shear strain at the joint panel
$\gamma_h$ and $\gamma_v$ : coefficients in the reduced statically indeterminate mechanisms
$\varepsilon_h$ and $\varepsilon_v$ : average horizontal and vertical strains
$\varepsilon_r$ and $\varepsilon_d$ : principle tensile and compressive strains
$\theta$ : direction of principle stress
$\sigma_h$ and $\sigma_v$ : average horizontal and vertical stresses
$\sigma_r$ and $\sigma_d$ : tensile and compressive stresses of the concrete strut
$\tau_{c\mathrm{xy}}$ : joint shear stress

## Appendix: Solution procedure of the improved STM

The aim of the assumptions and the empirical formulae introduced in the improved strut-and-tie method is to build average stress and strain fields and load transfer mechanisms with satisfying equilibrium, compatibility and constitutive laws for concrete and steel reinforcement throughout all the critical stages in the shear panels of RC beam-column joints subjected to monotonic loading. To demonstrate how these assumptions and formulae work together, a numerical solution procedure is given as follows.

The solution procedure is separated into 2 parts as shown in Figs. A1 and A2. The first part describes an equilibrium analysis based on the load transfer path of the improved STM with consideration of yielding of transverse reinforcement and longitudinal column bars. The second part is the average stress and strain analysis based on respective concrete and steel reinforcement constitutive laws and compatibility conditions with consideration of concrete compression softening effect and confinement effect due to transverse reinforcement.

In Figs. A1 and A2, several indicators are employed to represent the different stages of RC beam-column joints. The "*Type*" indicator is an integer with 0 for stage (a) prior to concrete cracking, 1 for stage (c) with transverse reinforcement yielding and prior to crushing of concrete strut, and 2 for stage (c) with longitudinal column bars yielding and prior to crushing of concrete strut. The "*Sign*" indicator denotes the shear loading direction and a value of 1 represents an increase of the applied shear load prior to concrete strut crushing, while a value of -1 indicates a decrease in the applied shear load in stage (d) after crushing of strut. The "*iLow*" is an indicator to differentiate between different cases of sufficient and insufficient beam longitudinal reinforcement, since the former will enhance the confinement effect of concrete struts and, therefore, weaken the compression softening effect due to existence of tensile strain orthogonal to the predetermined joint region crack.

The crushing criterion of the concrete strut is determined from the condition of $\sigma_d > f_{d,max}$. Once the error term defined by $\|(f_{d,max} - \sigma_d)/f_{d,ma}\|$ is less than a given tolerance $Tol.$ (which is assigned as $10^{-5}$ in the present study), the ultimate shear strength is calculated and the compressive strain of concrete strut becomes the dominant criterion for the joint shear strain in stage (d) after crushing of the concrete strut.

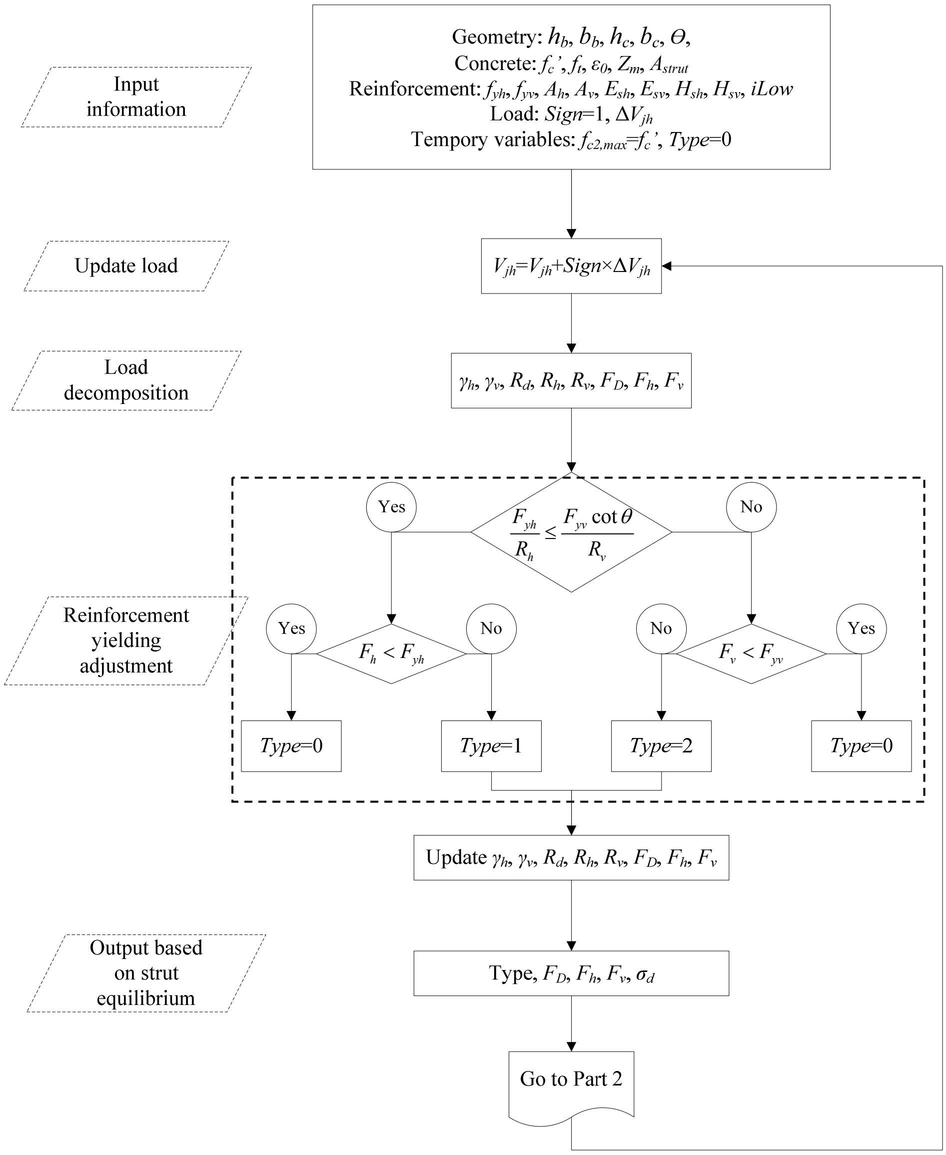


Fig. A1 Part 1 of solution procedure: Equilibrium analysis

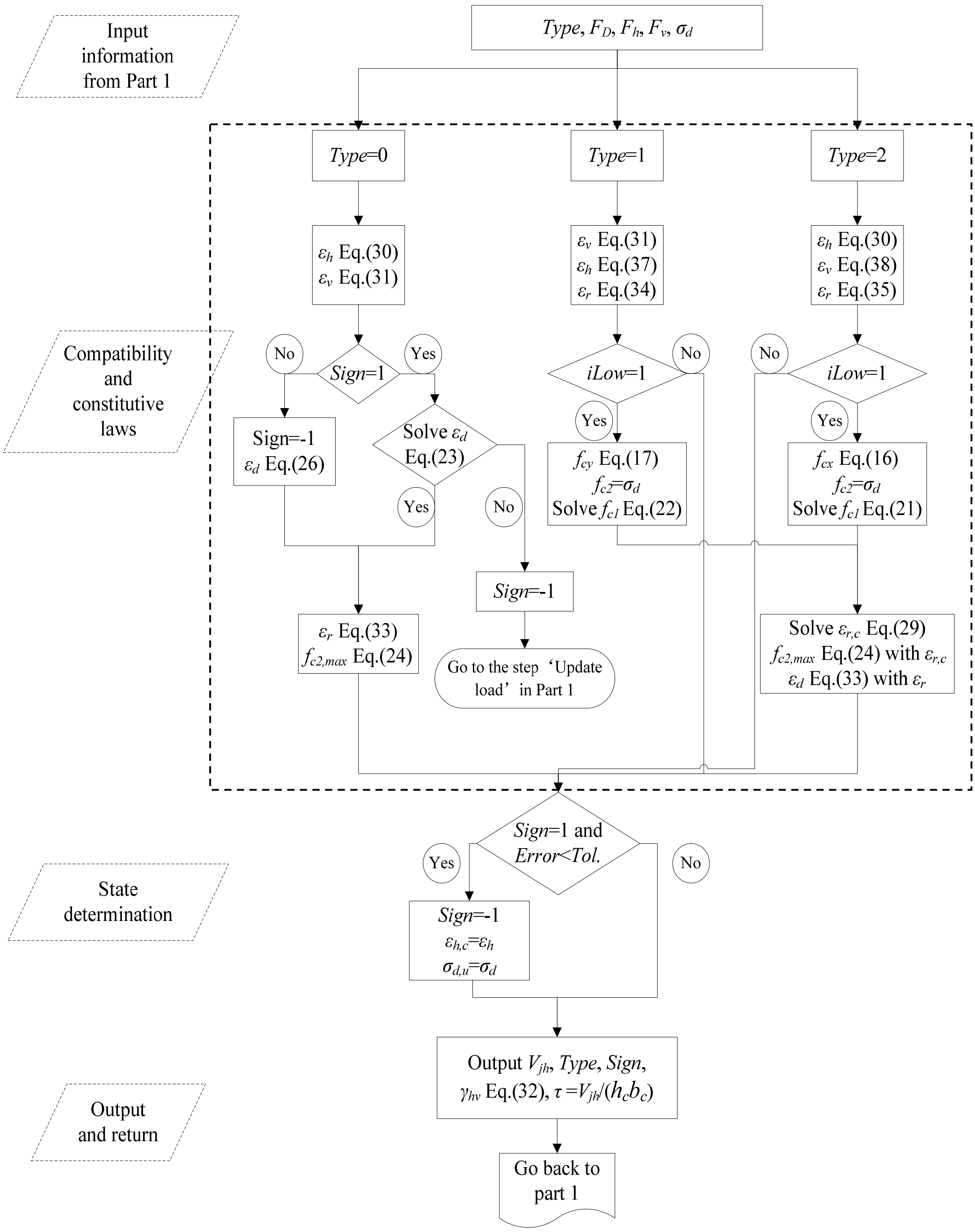


Fig. A2 Part 2 of solution procedure: Average stress and strain analysis