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Combined strain gradient and concrete strength effects on flexural strength and ductility design of RC columns

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Abstract. The stress-strain relationship of concrete in flexure is one of the essential parameters in assessing the flexural strength and ductility of reinforced concrete (RC) columns. An overview of previous research studies revealed that the presence of strain gradient would affect the maximum concrete stress developed in flexure. However, no quantitative model was available to evaluate the strain gradient effect on concrete under flexure. Previously, the authors have conducted experimental studies to investigate the strain gradient effect on maximum concrete stress and respective strain and developed two strain-gradient-dependent factors k3 and ko for modifying the flexural concrete stress-strain curve. As a continued study, the authors herein will extend the investigation of strain gradient effects on flexural strength and ductility of RC columns to concrete strength up to 100 MPa by employing the strain-gradient-dependent concrete stress-strain curve using nonlinear moment-curvature analysis. It was evident from the results that both the flexural strength and ductility of RC columns are improved under strain gradient effect. Lastly, for practical engineering design purpose, a new equivalent rectangular concrete stress block incorporating the combined effects of strain gradient and concrete strength was proposed and validated. Design formulas and charts have also been presented for flexural strength and ductility of RC columns.

Keywords: columns; ductility; flexural strength; strain gradient; stress block parameters

1. Introduction

In practical flexural strength design of reinforced concrete (RC) beams and columns, an equivalent rectangular stress block for concrete (Mattock *et al*. 1961; Ibrahim and MacGregor 1996, 1997; Tan and Nguyen 2004) is normally adopted to replace the non-linear concrete stress distribution in the compression zone. Both the actual concrete stress distribution and the respective simplified equivalent rectangular stress block are shown in Fig. 1. In the figure, the equivalent rectangular stress block is defined by two parameters α and β, which are the ratios of average concrete stress developed in flexure to concrete cylinder strength f_c' or cube strength f_{cu} and the depth of equivalent rectangular stress block to neutral axis depth c respectively. The flexural design using the above equivalent stress block has been commonly used in various RC design codes (European Committee for Standardization 2004; Standards New Zealand 2006; ACI

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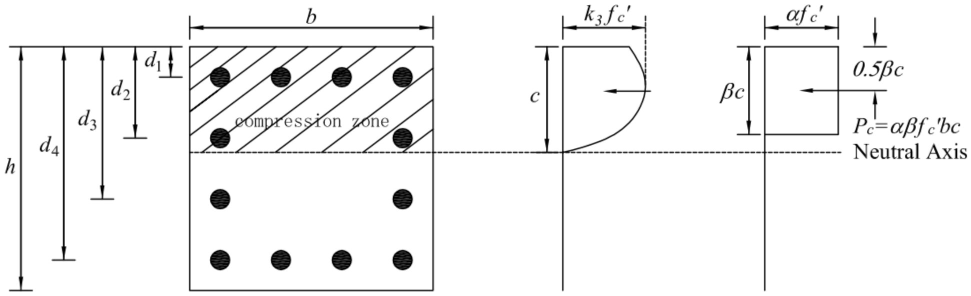


Column section | Actual concrete stress distribution | Equivalent rectangular concrete stress block

Fig. 1 Actual nonlinear concrete stress distribution and simplified equivalent stress block

Table 1 Values of α and β stipulated in various RC design codes

Design code	α		β	
ACI318[a]	0.85	for all f_c'	0.85 0.85 – 0.007 (f_c' – 28)≥0.65	for $f_c' \le 28$ MPa for $f_c' > 28$ MPa
EC2[b]	0.85 0.85 – 0.85[(f_c' – 50)/200]	for $f_c' \le 50$ MPa for $50 < f_c' \le 90$ MPa	0.80 0.8 – [(f_c' – 50)/400]	for $f_c' \le 50$ MPa for $50 < f_c' \le 90$ MPa
NZS[c]	0.85 0.85 – 0.004(f_c' – 55) 0.75	for $0 < f_c' \le 55$ MPa for $55 < f_c' \le 80$ MPa for $f_c' > 80$ MPa	0.85 0.85 – 0.008(f_c' – 30) 0.65	for $0 < f_c' \le 30$ MPa for $30 < f_c' \le 55$ MPa for $f_c' > 55$ MPa

Notes:
[a] ACI Committee 318 [2008]
[b] European Committee for Standardization [2004] based on UK National Annex
[c] Standards New Zealand [2006]

Committee 318 2008). The currently adopted values of α and β in these codes are summarised in Table 1.

By using the values of α and β stipulated in aforementioned RC design codes, the theoretical flexural strength of RC beams and columns can be evaluated. The results of comparison between these theoretical strengths (M_{ACI}, M_{EC} and M_{NZ}) and the respective experimentally measured strengths (M_t) of RC beams and columns obtained by different researchers (Sheikh and Yeh 1990; Watson and Park 1994; Basappa Setty and Rangan 1996; Lloyd and Rangan 1996; Claeson and Gylltoft 1998; Xiao and Martirossyan 1998; Ko *et al*. 2001; Debernardi and Taliano 2002; Ho and

Table 2 Comparison of flexural strengths from various design codes with test results

Specimen code	f_c' (MPa)	$P/A_g f_c'$	Moment (kNm)				$\frac{(1)}{(4)}$	$\frac{(2)}{(4)}$	$\frac{(3)}{(4)}$
			M_{ACI} (1)	M_{EC} (2)	M_{NZ} (3)	M_t (4)			
Beams									
6-30-1[a]	66.6	–	15.9	15.8	15.8	18.4	0.87	0.86	0.86
6-50-1[a]	66.6	–	24.7	24.5	24.5	28.4	0.87	0.86	0.86
T1A1[b]	27.7	–	10.8	10.8	10.8	12.0	0.90	0.90	0.90
T2A1[b]	27.7	–	20.5	20.6	20.5	23.6	0.87	0.87	0.87
C211[c]	85.6	–	350.6	344.4	347.1	390.3	0.90	0.88	0.89
C311[c]	88.1	–	399.1	390.1	394.4	438.1	0.91	0.89	0.90
Columns with low axial load levels									
8[d]	79.1	0.18	21.5	20.2	20.5	22.6	0.95	0.89	0.91
9[d]	79.1	0.15	19.6	18.8	18.9	22.7	0.86	0.83	0.83
HC4-8L19-T10-0.1P[e]	76.0	0.10	155.6	149.0	151.4	166.6	0.93	0.89	0.91
HC4-8L19-T10-0.2P[e]	76.0	0.20	180.4	171.6	175.2	196.6	0.92	0.87	0.89
Columns with medium axial load levels									
2[f]	44.0	0.30	405.1	409.4	405.2	486.0	0.83	0.84	0.83
3[f]	44.0	0.30	405.9	410.2	406.0	479.1	0.85	0.86	0.85
6[d]	79.1	0.27	25.7	23.2	23.6	30.0	0.85	0.77	0.79
7[d]	79.1	0.23	24.9	22.6	23.0	29.8	0.83	0.76	0.77
Columns with high axial load levels									
IVA[g]	58.0	0.52	20.4	19.6	20.1	21.9	0.93	0.89	0.92
31[h]	37.0	0.61	46.5	50.1	46.6	54.0	0.86	0.93	0.86
32[h]	37.0	0.62	46.0	49.5	46.0	51.5	0.89	0.96	0.89
BS-60-06-61[i]	51.1	0.67	385.4	410.2	384.9	417.7	0.92	0.98	0.92
Columns with ultra-high axial load levels									
F-6[j]	27.2	0.75	134.6	135.2	134.6	145.3	0.93	0.93	0.93
D-7[j]	26.2	0.78	119.5	121.3	119.5	133.2	0.90	0.91	0.90
27[h]	33.0	0.75	37.6	41.2	37.6	41.6	0.90	0.99	0.90
28[h]	33.0	0.75	37.6	41.2	37.6	40.6	0.93	1.02	0.93

[a] Ko *et al.* (2001)
[b] Debernardi and Taliano (2002)
[c] Rashid and Mansur (2005)
[d] Basappa Setty and Rangan (1996)
[e] Xiao and Martirossyan (1998)
[f] Watson and Park (1994)
[g] Lloyd and Rangan (1996)
[h] Claeson and Gylltoft (1998)
[i] Ho and Pam (2002)
[j] Sheikh and Yeh (1990)

Note:
–: No axial load is applied to beam specimens.

Pam 2002; Rashid and Mansur 2005) are summarised in Table 2. It can be observed from the results that the average difference between the theoretical strength and measured strength is about: (1) 7% and 9% for RC columns subjected to high ($0.5 < P/A_gf_c' \leq 0.7$) and ultra-high axial load levels ($P/A_gf_c' > 0.7$) respectively. (2) 12% for RC beams without axial load ($P/A_gf_c' = 0$). (3) 11% and 18% for columns with low ($0 < P/A_gf_c' \leq 0.2$) and medium axial load levels ($0.2 < P/A_gf_c' \leq 0.5$) respectively. It is evident that the existing flexural strengths of RC members evaluated by current design codes underestimate the bending capacity of the members. The flexural strength underestimation should be treated with caution as it underestimates the shear demand (Pam and Ho 2001) and violate the design philosophy of "Strong column and weak beam" (Park 2001). More importantly, the difference varies in RC members subjected to different axial loads (and hence strain gradient). It is postulated that the concrete stress developed in flexural members should then depend on strain gradient.

In fact, the effect of strain gradient has been the focus of some of the earlier research studies. Sturman *et al*. (1965) found that a larger maximum concrete stress can be attained for eccentrically-loaded column than concentrically-loaded counterpart due to retardation of micro-cracking formation in concrete. Clark *et al*. (1967) stated that the strain gradient can increase the maximum strain reached prior to crushing. Sargin *et al*. (1971) reported a 25% increase in strain corresponding to the peak stress under eccentric loading and strain gradient effects on the improvement on strength and ductility of confined concrete. Scott *et al*. (1982) reported that the deformability of eccentrically loaded columns was underestimated when the stress-strain curve obtained from concentrically loaded column was adopted. Sheikh and Yeh (1986) reported the ductility enhancement due to existence of strain gradient. The authors have also conducted a series of experimental studies on eccentrically loaded RC columns and found that the maximum concrete stress developed in flexure is influenced by the strain gradient (Peng *et al*. 2012; Ho and Peng 2013). Based on the test results obtained by the authors (Ho and Peng 2013), a tri-linear model for the variation of maximum concrete stress in terms of k_3 (ratio of maximum flexural concrete stress to cylinder strength) against strain gradient was recommended. The flexural strength of RC beams calculated from the above maximum flexural concrete stress model has been compared with the measured strength of over 200 RC beams in other researchers' tests by Chen and Ho (2014). It has been found that the proposed model yields a more accurate strength prediction (the accuracy has been improved by about 6%) than the existing RC design codes. Since it has been realized that the difference between the theoretical and actual flexural strength would even be larger for RC columns, a comprehensive study on the effect of strain gradient on flexural strength of RC columns is thus required.

Apart from accurate flexural strength evaluation, it is also necessary to ensure adequate ductility in the structure. In particular, flexural ductility design of column is important that should not be overlooked in structures where plastic hinge can only be formed in columns, e.g. Bridge piers and buildings with transfer plates. It was reported by researchers (Sturman *et al*. 1965; Clark *et al*. 1967; Karsan and Jirsa 1970; Sargin *et al*. 1971; Sheikh and Yeh 1986, 1992; Ho and Peng 2013; Li 2013) that the presence of strain gradient would influence the ductility of concrete in flexure. Thus, a review on the strain gradient effect on the ductility of RC columns is necessary.

In this study, the authors will carry out a parametric study to investigate the combined effects of strain gradient and concrete strength on the flexural strength and ductility of RC columns with various axial load levels and longitudinal steel ratios using nonlinear moment-curvature analysis. A modified concrete stress-strain model incorporating strain-gradient-dependent factors will be adopted. From the results obtained, it is evident that: (1) The equivalent rectangular concrete stress block parameters α and β vary significantly with both strain gradient and concrete strength. (2) The flexural strength of RC column evaluated with combined effects of strain gradient and concrete strength considered is improved by 19% on average at medium axial load level. (3) Flexural ductility of RC columns is improved with strain gradient considered. Lastly, for practical design purpose, empirical formulas and design charts are developed for flexural strength and ductility design of RC columns with combined effects of strain gradient and concrete strength considered.

2. Nonlinear moment-curvature analysis

2.1 Strain-gradient-dependent concrete stress-strain curve

In this paper, a theoretical study on the combined effects of strain gradient and concrete strength on the flexural strength and ductility of RC columns will be carried out using the uni-axial stress-strain curve proposed by Attard and Setunge (1996), which was proven to be applicable for f_c' = 20 to 130 MPa. The original equation proposed by Attard and Setunge is re-written as follows:

$$\sigma / f_o = \frac{A(\varepsilon / \varepsilon_o) + B(\varepsilon / \varepsilon_o)^2}{1 + (A-2)(\varepsilon / \varepsilon_o) + (B+1)(\varepsilon / \varepsilon_o)^2} \tag{1a}$$

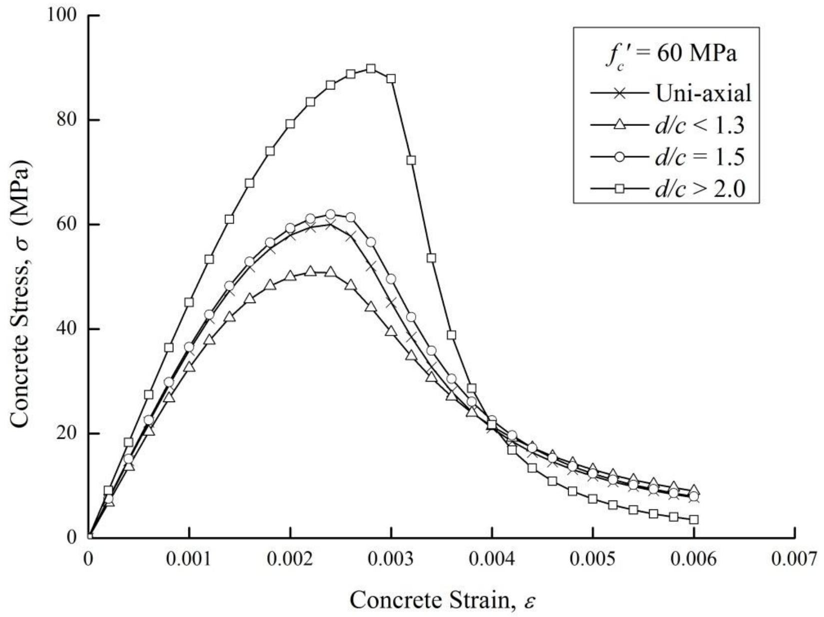


Fig. 2 Stress-strain curves of concrete under uni-axial load and flexure

$$f_o = f_{co}\left(1+\frac{f_r}{0.56\sqrt{f_{co}}}\right)^k \tag{1b}$$

$$\varepsilon_{co} = \frac{4.11(f_{co})^{0.75}}{E_c}\left[1+(17.0-0.06f_{co})\frac{f_r}{f_{co}}\right] \tag{1c}$$

$$E_c = 4370(f_{co})^{0.52} \tag{1d}$$

For RC members in flexure, the nonlinear stress-strain curve of concrete within the compression zone is obtained by applying a factor k_3 to the uniaxial stress-strain curve. It was taken to be 0.85 when strain gradient effect is not considered as proposed by Hognestad (1951) to account for the effects of size, shape and casting position of members. However, in authors' previous experimental studies (Ho and Peng 2011, 2013; Ho et al. 2011), it was found that the ratio of maximum flexural concrete stress to cylinder strength k_3, and that of concrete strain at maximum flexural stress to uni-axial strength k_o, are dependent on strain gradient. The tri-linear empirical formulas of these strain-gradient-dependent parameters k_3 and k_o with strain gradient, which adopts a non-dimensional form in the ratio of effective to neutral axis depth (d/c), were derived (Ho and Peng 2013):

$$k_3 = \begin{cases} 0.85 & \text{for } 0 \le d/c < 1.3 \\ 0.923(d/c) - 0.35 & \text{for } 1.3 \le d/c < 2.0 \\ 1.5 & \text{for } 2.0 \le d/c \end{cases} \tag{2a}$$

$$k_o = \begin{cases} 1.0 & \text{for } 0 \le d/c < 1.3 \\ 0.143(d/c) + 0.814 & \text{for } 1.3 \le d/c < 2.0 \\ 1.1 & \text{for } 2.0 \le d/c \end{cases} \tag{2b}$$

To incorporate strain gradient effect, Eq. (1) is modified to include k_3 and k_o. Firstly, k_3 is applied to the uni-axial concrete stress-strain curve to obtain the respective concrete stress-strain developed in flexure:

$$f_{co} = k_3 f_c' \tag{3a}$$

Secondly, the concrete strain at maximum stress under flexure is obtained by multiplying k_o to the respective strain at uni-axial stress state, i.e.

$$\varepsilon_o = k_o \varepsilon_{co} \tag{3b}$$

where f_{co} is the concrete stress developed in flexure, f_c' is concrete cylinder strength calculated, k_3 is obtained from Eq. (2a), ε_o is the strain of concrete at maximum stress under flexure, ε_{co} is the strain of concrete at maximum uni-axial stress, and k_o is obtained from Eq. (2b). Fig. 2 shows some of the concrete stress-strain curves under uni-axial load and flexure.

2.2 Stress-path-dependent steel stress-strain curve

For longitudinal steel reinforcement, an idealized linear elastic-perfectly plastic stress-strain curve with stress-path dependence property included was adopted. When the strain is increasing, the stress in the steel is given by:

At elastic stage:

$$\sigma_s = E_s \varepsilon_s \tag{4a}$$

After yielding:

$$\sigma_s = f_y \tag{4b}$$

When the strain is decreasing, the stress in the steel is given by:

$$\sigma_s = E_s(\varepsilon_s - \varepsilon_p) \tag{4c}$$

$$\varepsilon_p = \varepsilon_s - \sigma_s / E_s \tag{4d}$$

where ε_p is the residual strain. Fig. 3 shows the adopted stress-strain curve for steel with stress-path dependence considered.

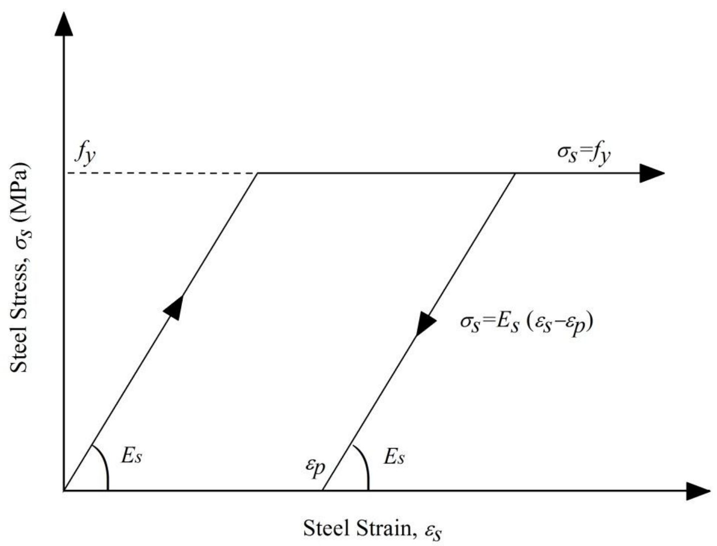


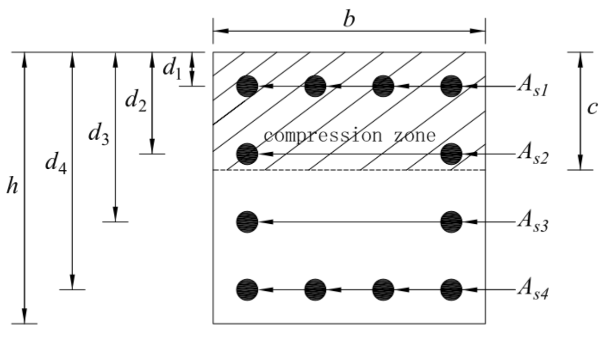
Fig. 3 Stress-strain curve of steel reinforcement with stress-path dependence considered

2.3 Non-linear moment-curvature analysis

In this study, a complete moment-curvature curve is used to investigate the flexural strength and ductility of RC columns. The moment-curvature relation of column section was analyzed by applying prescribed curvatures to the section incrementally starting from zero. At a prescribed curvature and assumed neutral axis depth, the stresses developed in concrete and steel reinforcement can be determined from strain distribution in the section and their respective stress-strain curves as described in previous sections. An iterative procedure of successively adjusting the neutral axis depth was needed until the unbalanced axial force was negligibly small. The resulted neutral axis depth and resisting moment can be therefore evaluated. Such procedure was repeated until the resisting moment increased to the peak and then decreased to half of the peak value.

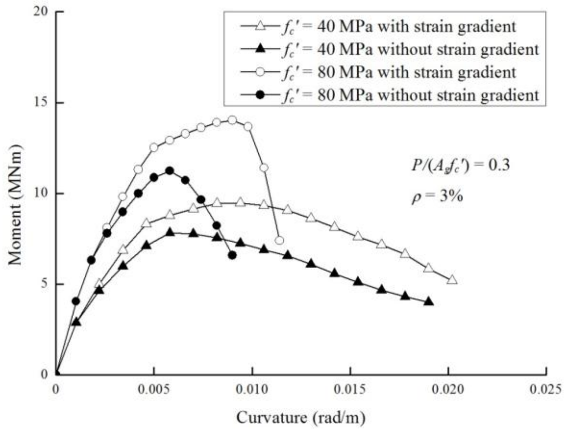
In the parametric study, the sections analysed are the same as the one shown in Fig. 4. The column sections are given constant dimensions of b = 1000 mm, h = 1000 mm, d_1 = 80 mm, d_2 = 360 mm, d_3 = 640 mm and d_4 = 920 mm. The concrete cylinder strength f_c' is varied from 30 to 100 MPa. The longitudinal steel ratio (i.e. ratio of longitudinal steel area A_s to gross area of section A_g) is varied from 1% to 6% and the axial load level P/A_gf_c' applied to the section is varied from 0.1 to 0.8. The steel reinforcement is assumed to have constant yield strength f_y = 460 MPa and elastic modulus E_s = 200 GPa.

Figs. 5(a), 5(b) and 5(c) show the effect of strain gradient on moment-curvature behaviors for column sections with different concrete strengths, longitudinal steel ratios and axial load levels respectively. The strain gradient effect on flexural behaviors of column sections with various concrete strengths (f_c' = 40 and 80 MPa) can be visualized in the moment-curvature curves as shown in Fig. 5(a). It is apparent that strain gradient will improve the flexural strength of column sections. The effect of strain gradient on column sections with different longitudinal steel ratios is shown in Fig. 5(b), which plots the moment-curvature curves of two column sections with longitudinal steel ratio of 2% and 6%. For columns made of same concrete strength at the same axial load level, the relative improvement in flexural strength is slightly larger for section with

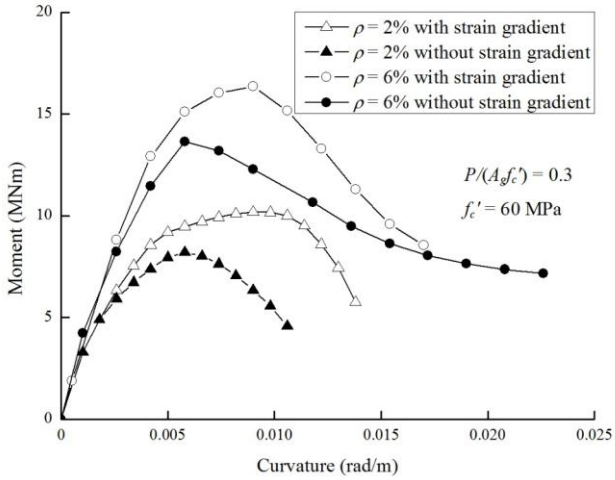


Column section

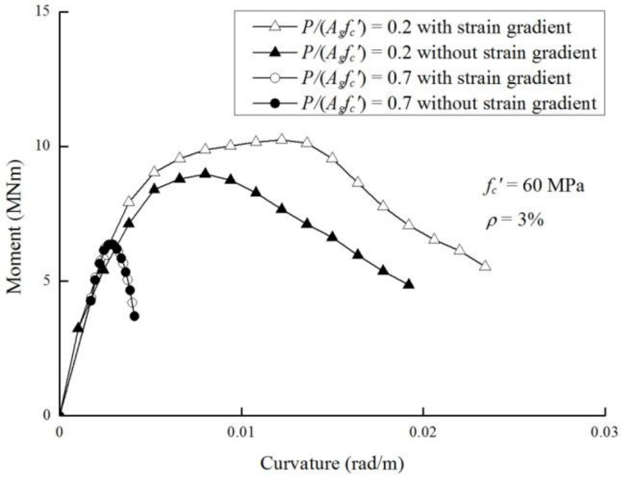
Fig. 4 Column sections analysed in the parametric study



(a) $P/(A_gf_c')$ = 0.3, ρ= 3%



(b) f_c' = 60 MPa, $P/(A_gf_c')$ = 0.3



(c) f_c' = 60 MPa, ρ = 3%

Fig. 5 Moment-curvature curves of RC columns with and without strain gradient effect considered larger longitudinal steel ratio because of larger compression zone. Fig. 5(c) depicts the effect of strain gradient on columns with low and high axial load levels ($P/A_g f_c'$ = 0.2 and 0.7). For columns with low axial load levels, noticeable improvement in flexural strength can be observed.

However, the improvement in flexural behaviors vanishes for column sections with high axial load level since the strain gradient decreases to a considerably low level.

3. Results of analysis

3.1 Effect of strain gradient on flexural strength of RC columns

The effect of strain gradient on flexural strength M of RC columns is studied by plotting flexural capacity M/bh^2 against concrete strength, longitudinal steel ratio and axial load level as shown in Fig. 6. Fig. 6(a) indicates that at medium axial load level of 0.3 and moderate longitudinal steel ratio of 3%, strain gradient improves significantly the flexural strength of RC columns for all concrete strengths. It is also seen in Fig. 6(a) that the improvement of flexural strength is larger for higher concrete strength. In Fig. 6(b), it shows that the strength enhancement increases slightly as the longitudinal steel ratio increases. In Fig. 6(c), it is evident that the strength enhancement initially increases as axial load level increases until at about 0.4 because of larger concrete compression zone, after which drops and vanishes when the axial load level reaches 0.7. It is apparent that considering strain gradient will improve the flexural strength of RC column except when it is subjected to high axial load level.

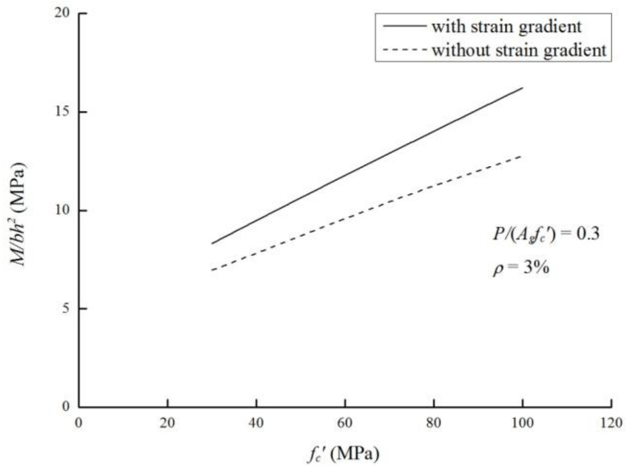
3.2 Effect of strain gradient on flexural ductility of RC columns

In this study, the flexural ductility is expressed in curvature ductility factor μ:

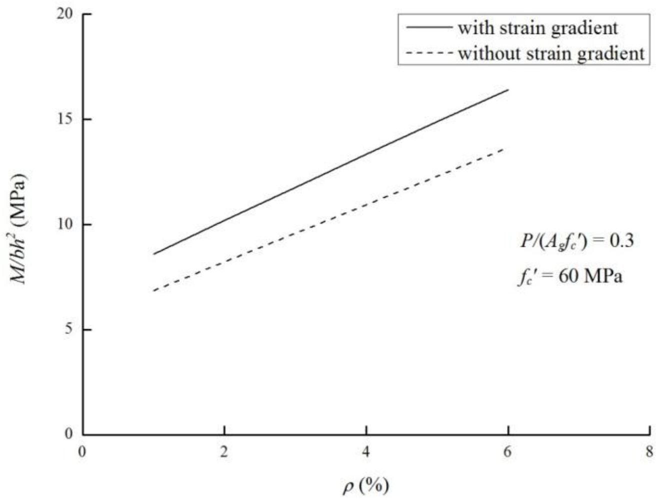
$$\mu = \phi_u / \phi_y \tag{5}$$

where ϕ_u and ϕ_y are the ultimate curvature and yield curvature respectively. The ultimate curvature ϕ_u is taken as the curvature at which the resisting moment has dropped to 80% of the maximum moment capacity in the descending branch of the moment-curvature curve. The yield curvature ϕ_y is defined as the curvature extrapolating from the curvature ϕ_y' at 75% of the maximum moment to the maximum moment point by $\phi_y = \phi_y'/0.75$.

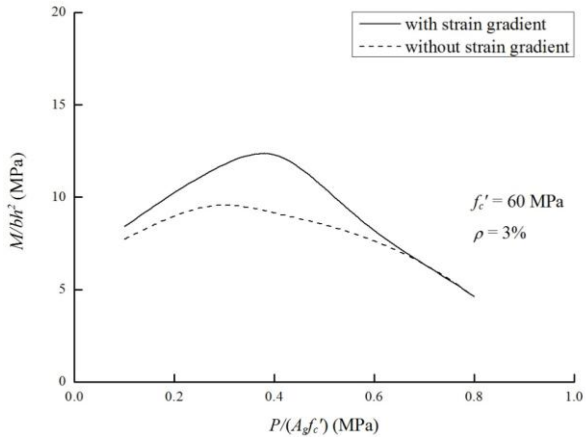
The effect of strain gradient on the flexural ductility of RC columns is studied by plotting the flexural ductility against concrete strength, longitudinal steel ratio and axial load level with and without strain gradient effect considered as shown in Fig. 7. The curvature ductility μ with and without incorporating strain gradient effect is plotted against the concrete strength f_c' at constant axial load level of 0.3 and longitudinal steel ratio of 3% in Fig. 7(a). From Fig. 7(a), it can be seen that strain gradient improves the ductility of RC column. Nevertheless, the improvement decreases as concrete strength increase owing to the reduction in the compression zone. From Fig. 7(b), it is evident that strain gradient improves the ductility of column because of the higher flexural concrete stress developed, except when ρ = 6%. At small ρ, the increase in longitudinal steel ratio will increase the neutral axis depth, and hence the enhancement.



(a) M/bh^2 plotted against f_c' ($P/(A_g f_c')$ = 0.3 and ρ = 3%)

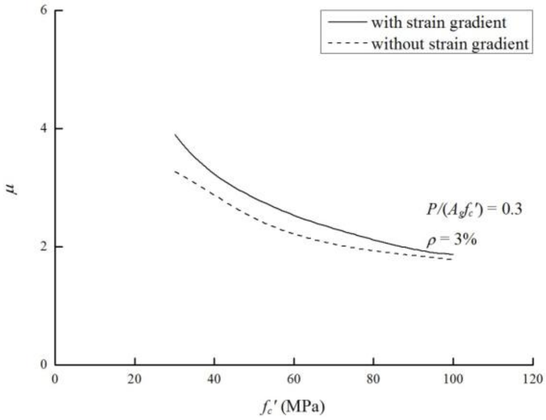


(b) M/bh^2 plotted against ρ (f_c' = 60 MPa and $P/(A_g f_c')$ = 0.3)

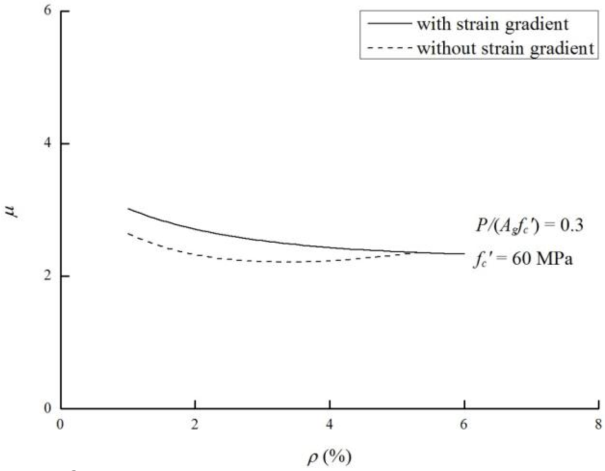


(c) M/bh^2 plotted against $P/(A_g f_c')$ (f_c' = 60 MPa and ρ = 3%)

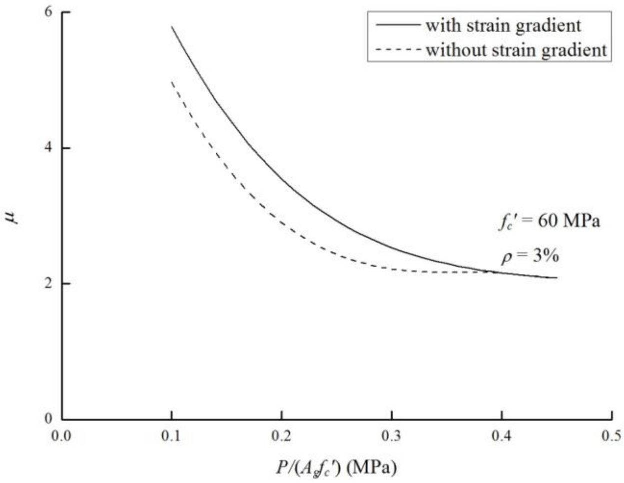
Fig. 6 Flexural strength of RC columns with and without strain gradient effect considered



(a) M/bh^2 plotted against f_c' ($P/(A_g f_c') = 0.3$ and $\rho = 3\%$)

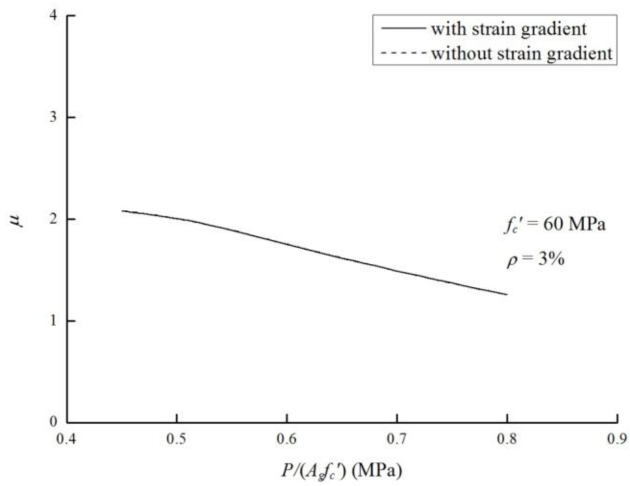


(b) M/bh^2 plotted against ρ ($f_c' = 60$ MPa and $P/(A_g f_c') = 0.3$)



(c) μ plotted against $P/(A_g f_c')$ in tension failure ($f_c' = 60$ MPa and $\rho = 3\%$)

Fig. 7 Flexural ductility of RC columns with and without strain gradient effect considered



(d) μ plotted against $P/(A_g f_c')$ in compression failure (f_c' = 60 MPa and ρ = 3%)

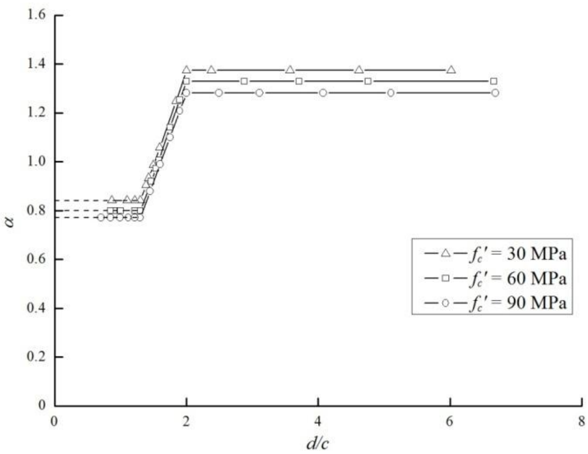
Fig. 7 Flexural ductility of RC columns with and without strain gradient effect considered

The effect of strain gradient on ductility is different for columns failing by tension steel yielding (i.e. at low axial load level) and by concrete crushing (i.e. at high axial load level). Figs. 7(c) and 7(d) show the strain gradient effect on columns ductility under tension and compression failure respectively. From Fig. 7(c), it is seen that strain gradient improves the ductility of RC columns. Nevertheless, the extent of improvement decreases as the axial load level increases which increases the neutral axis depth and decreases strain gradient. From Fig. 7(d), it is evident that strain gradient will not improve the ductility because the neutral axis depth is so deep that the strain gradient become negligible in this type of columns.

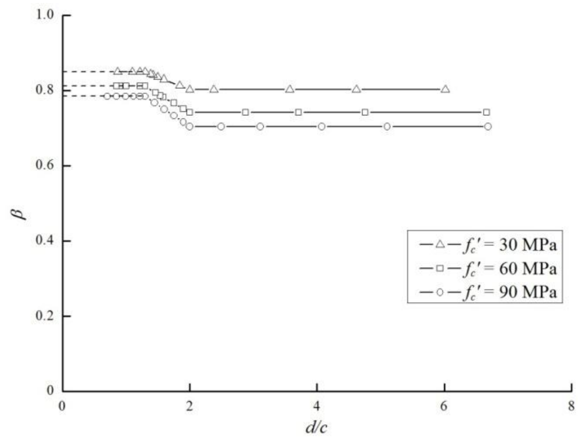
4. Practical design formulas and charts

4.1 Flexural strength design formulas of RC Columns

In practical flexural strength design of RC columns, an equivalent rectangular stress block for concrete, which is defined by two parameters α and β, is normally adopted to replace the non-linear concrete stress distribution in the compression zone for the convenience of calculation as shown in Fig. 1. By equating the force and moment obtained from nonlinear concrete distribution and the simplified concrete stress block, the values of α and β for concrete strength from 30 to 100 MPa and a wide range of strain gradients are derived. The values of these two stress block parameters have been plotted against d/c in Figs 8(a) and 8(b) respectively. Unlike those stress block parameters stipulated in most of the current RC design codes that depend solely on the concrete strength, it is apparent from the above results that they should instead vary with both concrete strength and strain gradient. To enable a more accurate prediction of flexural strength, a new set of equivalent rectangular concrete stress block parameters incorporating the combined effects of concrete strength and strain gradient is developed by empirical formulas that fit the data well in Figs. 8(a) and 8(b):

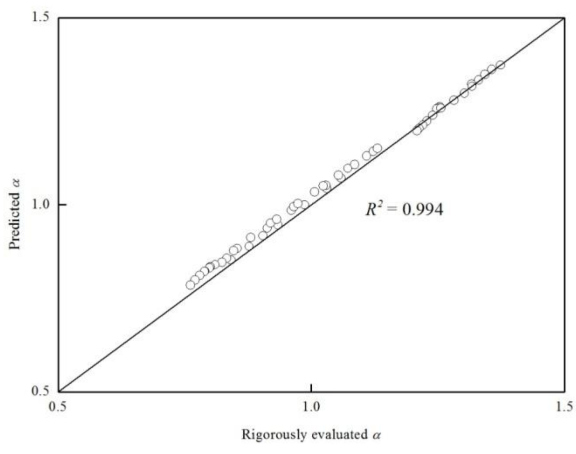


(a) α plotted against *d/c*



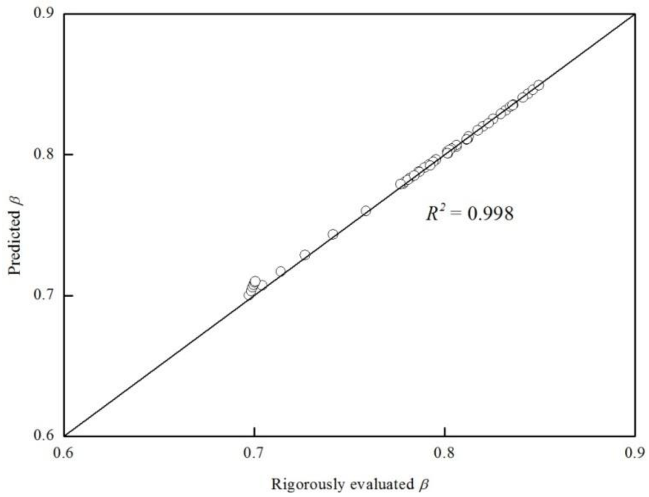
(b) β plotted against *d/c*

Fig. 8 Graphs of α and β plotted against strain gradient *d/c*



(a) α plotted against *d/c*

Fig. 9 Predicted equivalent stress block parameters plotted against rigorously evaluated values



(b) Predicted ß against rigorously evaluated values

Fig. 9 Predicted equivalent stress block parameters plotted against rigorously evaluated values

$$\alpha = \begin{cases} \alpha_1 & \text{for } 0 \le d/c \le 1.3 \\ (\alpha_2 - \alpha_1)\left(\dfrac{d/c - 1.3}{0.7}\right) + \alpha_1 & \text{for } 1.3 < d/c < 2.0 \\ \alpha_2 & \text{for } 2.0 \le d/c \end{cases} \tag{6a}$$

$$\alpha_1 = -0.07(f_c'/100)^2 - 0.005(f_c'/100) + 0.86 \tag{6b}$$

$$\alpha_2 = -0.076(f_c'/100)^2 - 0.066(f_c'/100) + 1.4 \tag{6c}$$

$$\beta = \begin{cases} \beta_1 & \text{for } 0 \le d/c \le 1.3 \\ (\beta_2 - \beta_1)\left(\dfrac{d/c - 1.3}{0.7}\right) + \beta_1 & \text{for } 1.3 < d/c < 2.0 \\ \beta_2 & \text{for } 2.0 \le d/c \end{cases} \tag{7a}$$

$$\beta_1 = 0.069(f_c'/100)^2 - 0.19(f_c'/100) + 0.9 \tag{7b}$$

$$\beta_2 = 0.12(f_c'/100)^2 - 0.3(f_c'/100) + 0.88 \tag{7c}$$

The comparison between rigorously evaluated and the predicted values of α and β is depicted in Figs. 9(a) and 9(b) respectively. Within the range of parameters covered in this study ($30 \le f_c' \le 100$ MPa), it is verified that Eqs. (6) and (7) are accurate (R^2 = 0.994 and 0.998) for predicting the flexural strength of RC columns considering the effects of both concrete strength and strain gradient. Further, Eqs. (6) and (7) should be adopted together with appropriate ultimate concrete strain ε_{cu}, which can be taken as 0.0032 for concrete strength ranging from 30 to 100 MPa with strain gradient effect considered (Chen and Ho 2014).

For RC columns subjected to pure axial compression, the axial capacity of the column section can be evaluated by axial force equilibrium as shown in Eq. (8).

$$P = \alpha_1 f_c' A_c + f_y A_s \tag{8}$$

where the value of α_1 is proposed in Eq. (6b), A_c is the area of concrete, f_y and A_s are the yield strength and area of longitudinal steel respectively. For RC columns subjected to combined axial load and flexure, the corresponding flexural strength can be calculated by conditions of axial force and moment equilibrium.

$$P = \alpha\beta f_c' bc + \sum_{i=1}^{n} \sigma_{si} A_{si} \tag{9a}$$

$$M = \alpha\beta f_c' bc\left(\frac{h}{2} - \frac{\beta}{2}c\right) + \sum_{i=1}^{n} \sigma_{si} A_{si}\left(\frac{h}{2} - d_i\right) \tag{9b}$$

where α and β follow Eqs. (6) and (7), σ_{si} and A_{si} is the stress and area of the i^{th} longitudinal steel reinforcement respectively, d_i is the distance of the i^{th} steel bar from the extreme concrete compressive fibre, c is the neutral axis depth of the section.

4. 2 Verification of the proposed design formulas

Validation of the proposed stress block parameters as well as the design value of ultimate concrete strain in flexural design of RC column is carried out by comparing the moment capacities M_t of 275 RC column specimens tested by other researchers with those predicted by the proposed stress block parameters M_p, as well as with the theoretical strengths calculated using various RC design codes (i.e. M_{NZ} based on NZS 3101 (Standards New Zealand 2006), M_{ACI} based on ACI 318M-08 (ACI Committee 318 2008), M_{EC} based on Eurocode 2 (European Committee for Standardization 2004). It is worth noting that f_c' ($\leq$ 50 MPa) = 0.8f_{cu} and f_c' (> 50 MPa) = f_{cu} – 11 MPa (Carrasquilio and Nilson 1981) are used for the conversion between concrete cylinder strengths f_c' and cube strengths f_{cu} for normal- and high-strength concrete respectively.

In the comparison, the selected 275 RC column specimens have been divided into three categories according to the axial load level:

(1) Columns with low axial load level ($0 < P/A_g f_c' \leq 0.2$) (Saatcioglu and Ozcebe 1989; Watson and Park 1994; Basappa Setty and Rangan 1996; Lloyd and Rangan 1996; Xiao and Martirossyan 1998; Mo and Wang 2000; Ho and Pam 2002, 2003; Marefat *et al*. 2005; Woods *et al*. 2007);

(2) Columns with medium axial load level ($0.2 < P/A_g f_c' \leq 0.5$) (Park *et al*. 1982; Sheikh and Yeh 1990; Sheikh and Khoury 1993; Azizinamini *et al*. 1994; Watson and Park 1994; Basappa Setty and Rangan 1996; Lloyd and Rangan 1996; Foster and Attard 1997; Bayrak and Sheikh 1998; Claeson and Gylltoft 1998; Ahn *et al*. 2000; Lee and Son 2000; Mo and Wang 2000; Ho and Pam 2002; Lam *et al*. 2003; Marefat *et al*. 2005; Tan and Nguyen 2005; Tao and Yu 2008; Ho 2012);

(3) Columns with high and ultra-high axial load level ($0.5 < P/A_g f_c' \leq 0.7$ and $P/A_g f_c' > 0.7$) (Park *et al*. 1982; Sheikh and Yeh 1990; Sheikh and Khoury 1993; Sheikh *et al*. 1994; Watson and Park 1994; Basappa Setty and Rangan 1996; Ibrahim and MacGregor 1996; Lloyd and Rangan 1996; Foster and Attard 1997; Claeson and Gylltoft 1998; Lee and Son 2000; Ho and Pam 2002; Lam *et al*. 2003; Němeček *et al*. 2005; Tan and Nguyen 2005; Kim 2007; Hadi and Widiarsa 2012);

Tables 3 to 5 compare the proposed flexural strength M_p calculated by Eqs. (6) and (7) with their respective measured strengths M_t and theoretical strengths calculated as per various design codes (in absolute values and ratio to measured strengths). It can be concluded that:

(1) For columns subjected to low axial load level, the average ratio and standard deviation of the predicted M_p to experimentally measured M_t flexural strengths is 1.01 and 0.069 respectively, whilst the average ratios of the codified theoretical strengths (M_{NZ}, M_{ACI} and M_{EC}) to M_t are 0.90, 0.92 and 0.90 respectively. The corresponding standard deviations are 0.052, 0.056 and 0.053 respectively. It is apparent that the proposed method that includes the combined effects of strain gradient and concrete strength can improve the flexural strength prediction of RC columns subjected to low axial load level by 10% on average.

(2) For columns subjected to medium axial load level, the average ratio and standard deviation of the predicted M_p to experimentally measured M_t flexural strengths is 1.04 and 0.145 respectively, whilst the average ratios of the codified theoretical strengths (M_{NZ}, M_{ACI} and M_{EC}) to M_t are 0.84, 0.88 and 0.82 respectively. The corresponding standard deviations are 0.118, 0.132 and 0.124 respectively. It should be noted that the consideration of the combined effects of concrete strength and strain gradient can improve the flexural strength prediction of RC columns subjected to medium axial load level by up to 19% on average. Apparently, the underestimation of codified strength is no longer insignificant in this type of columns, and the need of more accurate flexural strength assessment taking into account concrete strength and strain gradient is justified.

(3) For columns subjected to high/ultra-high axial load levels, the average ratio and standard deviation of the predicted M_p to experimentally measured M_t flexural strengths is 0.91 and 0.182 respectively, whilst the average ratios of the codified theoretical strengths (M_{NZ}, M_{ACI} and M_{EC}) to M_t are 0.84, 0.94 and 0.82 respectively. The corresponding standard deviations are 0.152, 0.220 and 0.159 respectively. Because of the small strain gradient present in this type of column, the proposed flexural strength is very close to the codified flexural strength, in which the difference is about 4% only. Under this circumstance, both proposed method and the current design code can provide a satisfactory estimate of the flexural strength.

Table 3 Comparison of proposed theoretical flexural strengths of RC columns subjected to low axial load level with other researchers' results

Specimen code	f_c' (MPa)	P/A_gf_c'	Moment (kNm)					$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
			M_p (1)	M_{NZ} (2)	M_{ACI} (3)	M_{EC} (4)	M_t (5)				
Saatcioglu and Ozcebe (1989)											
U1	43.6	0.00	240.4	232.0	232.0	232.1	275.0	0.87	0.84	0.84	0.84
U2	30.2	0.16	314.2	275.7	275.4	284.0	270.0	1.16	1.02	1.02	1.05
U3	34.8	0.14	308.7	278.3	278.0	287.4	268.0	1.15	1.04	1.04	1.07
U4	32.0	0.15	309.5	275.2	274.9	283.8	326.0	0.95	0.84	0.84	0.87
U6	37.3	0.13	314.3	284.5	284.3	294.3	343.0	0.92	0.83	0.83	0.86
U7	39.0	0.13	315.8	287.3	287.1	297.7	342.0	0.92	0.84	0.84	0.87

Table 3 Continued

Specimen code	f_c' (MPa)	$P/A_g f_c'$	Moment (kNm)					$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
			M_p (1)	M_{NZ} (2)	M_{ACI} (3)	M_{EC} (4)	M_t (5)				
Watson and Park (1994)											
1	47.0	0.10	319.9	302.0	302.0	306.9	335.2	0.95	0.90	0.90	0.92
Basappa Setty and Rangan (1996)											
8	79.1	0.18	24.3	20.5	21.5	20.2	22.6	1.08	0.91	0.95	0.89
9	79.1	0.15	22.0	18.9	19.6	18.8	22.7	0.97	0.83	0.86	0.83
10	79.1	0.18	26.7	22.1	23.6	21.7	23.9	1.12	0.92	0.99	0.91
12	79.1	0.18	24.2	20.4	21.4	20.2	23.3	1.04	0.88	0.92	0.86
Lloyd and Rangan (1996)											
IIC	58.0	0.20	22.8	20.3	20.4	20.1	22.3	1.02	0.91	0.91	0.90
IVC	58.0	0.14	17.1	15.3	15.3	15.2	17.9	0.95	0.85	0.86	0.85
VIB	92.0	0.16	28.3	24.9	25.7	–	27.6	1.03	0.90	0.93	–
VIC	92.0	0.15	27.1	23.9	24.6	–	27.4	0.99	0.87	0.90	–
VIIIC	92.0	0.11	20.9	18.5	19.1	–	19.6	1.07	0.95	0.97	–
XB	97.2	0.13	23.8	21.1	21.7	–	25.3	0.94	0.84	0.86	–
XC	97.2	0.11	21.8	19.4	19.9	–	20.7	1.06	0.94	0.96	–
XIIB	97.2	0.17	27.9	24.4	25.3	–	26.8	1.04	0.91	0.95	–
XIIC	97.2	0.10	21.1	18.8	19.3	–	19.4	1.09	0.97	0.99	–
Xiao and Martirossyan (1998)											
HC4-8L19-T10-0.1P	76.0	0.10	169.8	151.4	155.6	149.0	166.6	1.02	0.91	0.93	0.89
HC4-8L19-T10-0.2P	76.0	0.20	207.7	175.2	180.4	171.6	196.6	1.06	0.89	0.92	0.87
HC4-8L16-T10-0.1P	86.0	0.10	142.8	131.5	136.2	127.8	136.7	1.05	0.96	1.00	0.94
HC4-8L16-T10-0.2P	86.0	0.19	189.9	160.3	166.5	155.3	166.1	1.14	0.96	1.00	0.93
Mo and Wang (2000)											
C1-1	24.9	0.11	328.0	300.4	300.4	305.2	351.4	0.93	0.85	0.85	0.87
C2-1	25.3	0.11	328.9	301.2	301.2	305.9	347.3	0.95	0.87	0.87	0.88
C3-1	26.4	0.11	331.2	303.3	303.3	307.8	353.4	0.94	0.86	0.86	0.87
C1-2	26.7	0.16	355.9	322.5	322.5	329.3	374.6	0.95	0.86	0.86	0.88
C2-2	27.1	0.16	356.9	324.2	324.2	330.3	399.9	0.89	0.81	0.81	0.83
C3-2	27.5	0.15	357.8	325.5	325.5	331.0	395.5	0.90	0.82	0.82	0.84

Table 3 Continued

Specimen code	f_c' (MPa)	$P/A_g f_c'$	Moment (kNm)					$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
			M_p	M_{NZ}	M_{ACI}	M_{EC}	M_t				
			(1)	(2)	(3)	(4)	(5)				
Ho and Pam (2002)											
BS-80-01-09	75.0	0.16	249.1	220.8	227.0	218.2	256.5	0.97	0.86	0.89	0.85
NEW-80-01-09	77.8	0.15	236.5	211.6	218.4	209.3	243.5	0.97	0.87	0.90	0.86
Ho and Pam (2003)											
BS-80-01-09-R6	72.6	0.17	248.1	219.5	225.0	217.1	256.5	0.97	0.86	0.88	0.85
BS-80-01-09-R8	74.6	0.15	239.4	214.0	219.8	211.8	237.0	1.01	0.90	0.93	0.89
BS-80-01-09-R10	72.4	0.16	240.1	214.0	219.1	211.8	237.8	1.01	0.90	0.92	0.89
NEW-80-01-09-R12	77.8	0.15	238.5	214.2	221.0	211.8	243.5	0.98	0.88	0.91	0.87
Marefat *et al*. (2005)											
STCM-9	24.0	0.19	25.4	22.5	22.5	22.8	23.3	1.09	0.97	0.97	0.98
SBCC-7	27.0	0.16	48.0	43.6	43.6	43.6	45.1	1.06	0.97	0.97	0.97
Woods *et al*. (2007)											
S3.2-76	69.0	0.16	71.6	64.1	65.4	63.5	69.8	1.03	0.92	0.94	0.91
S4.8-76	69.0	0.16	71.6	64.1	65.4	63.5	67.8	1.06	0.95	0.97	0.94
S6.4-76	69.0	0.16	71.6	64.1	65.4	63.5	69.7	1.03	0.92	0.94	0.91
S8.0-76	69.0	0.16	71.6	64.1	65.4	63.5	69.3	1.03	0.93	0.94	0.92
V5.5-66	69.0	0.16	71.6	64.1	65.4	63.5	71.8	1.00	0.89	0.91	0.88
V6.4-86	69.0	0.16	71.6	64.1	65.4	63.5	68.5	1.05	0.94	0.96	0.93
V8.0-135	69.0	0.16	71.6	64.1	65.4	63.5	66.9	1.07	0.96	0.98	0.95
Average								**1.01**	**0.90**	**0.92**	**0.90**
Standard Deviation								**0.069**	**0.052**	**0.056**	**0.053**

Note:
–: Concrete strength is beyond the limit specified in Eurocode 2, no result can be reported in Table 3 to 5.

Table 4 Comparison of proposed theoretical flexural strengths of RC columns subjected to medium axial load level with other researchers' results

Specimen code	f_c' (MPa)	$\frac{P/A_g}{f_c'}$	Moment (kNm)					$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
			M_p (1)	M_{NZ} (2)	M_{ACI} (3)	M_{EC} (4)	M_t (5)				
Park *et al*. (1982)											
1	23.1	0.26	769.2	667.7	667.7	667.7	864.0	0.89	0.77	0.77	0.77
2	41.4	0.21	1006.0	893.5	893.3	894.9	1010.0	1.00	0.88	0.88	0.89
3	21.4	0.42	846.4	658.8	658.8	664.0	843.0	1.00	0.78	0.78	0.79
Sheikh and Yeh (1990)											
D-5	31.2	0.46	228.7	170.6	169.9	172.5	204.5	1.12	0.83	0.83	0.84
Sheikh and Khoury (1993)											
AS-19	32.3	0.47	232.7	170.9	170.3	181.2	202.2	1.15	0.85	0.84	0.90
Azizinamini *et al*. (1994)											
D60-7-4-2 $\frac{5}{8}$ -0.2P	53.7	0.21	251.1	215.8	215.9	219.0	248.0	1.01	0.87	0.87	0.88
D60-7-3C-1 $\frac{5}{8}$ -0.2P	50.8	0.21	242.6	207.5	207.6	212.9	237.7	1.02	0.87	0.87	0.90
D60-4-3C-2 $\frac{5}{8}$ -0.2P	26.3	0.25	181.2	156.7	156.7	160.8	173.2	1.05	0.90	0.90	0.93
D60-4-3C-2 $\frac{5}{8}$ -0.4P	27.0	0.50	192.7	152.6	152.6	154.3	168.2	1.15	0.91	0.91	0.92
D60-15-3C-1 $\frac{5}{8}$ -0.3P	103.8	0.28	422.4	329.7	358.9	–	304.0	1.39	1.08	1.18	–
Watson and Park (1994)											
2	44.0	0.30	482.8	405.2	405.1	409.4	486.0	0.99	0.83	0.83	0.84
3	44.0	0.30	483.5	406.0	405.9	410.2	479.1	1.01	0.85	0.85	0.86
4	40.0	0.30	454.7	383.2	382.9	386.1	448.1	1.01	0.86	0.85	0.86
5	41.0	0.50	519.2	377.3	376.8	387.1	525.8	0.99	0.72	0.72	0.74
6	40.0	0.50	523.5	373.4	372.8	382.5	526.4	0.99	0.71	0.71	0.73
Basappa Setty and Rangan (1996)											
5	79.1	0.26	29.6	22.3	24.1	21.9	31.1	0.95	0.72	0.78	0.71
6	79.1	0.27	32.8	23.6	25.7	23.2	30.0	1.09	0.79	0.85	0.77
7	79.1	0.23	30.3	23.0	24.9	22.6	29.8	1.02	0.77	0.83	0.76
11	79.1	0.21	29.3	22.7	24.6	22.3	27.9	1.05	0.81	0.88	0.80

Table 4 Continued

Specimen code	f_c' (MPa)	$\frac{P/A_g}{f_c'}$	Moment (kNm)					$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
Lloyd and Rangan (1996)											
IB	58.0	0.46	53.9	43.7	44.3	44.7	52.8	1.02	0.83	0.84	0.85
IC	58.0	0.37	59.5	45.4	45.9	47.2	52.2	1.14	0.87	0.88	0.90
IIB	58.0	0.25	25.5	21.0	21.2	21.7	23.8	1.07	0.88	0.89	0.91
IIIB	58.0	0.41	56.0	40.4	40.9	41.2	46.0	1.22	0.88	0.89	0.90
IIIC	58.0	0.28	46.6	40.7	41.1	40.7	40.5	1.15	1.00	1.01	1.00
IVB	58.0	0.24	22.6	19.1	19.2	19.5	20.7	1.10	0.92	0.93	0.94
VB	92.0	0.36	81.7	58.1	64.4	–	60.5	1.35	0.96	1.07	–
VC	92.0	0.28	72.1	58.7	64.2	–	61.5	1.17	0.95	1.04	–
VIA	92.0	0.44	31.7	29.1	32.7	–	33.7	0.94	0.86	0.97	–
VIIB	92.0	0.32	71.1	53.8	59.1	–	55.6	1.28	0.97	1.06	–
VIIC	92.0	0.23	58.9	51.7	53.7	–	55.0	1.07	0.94	0.98	–
VIIIA	92.0	0.38	30.4	28.1	31.2	–	24.3	1.25	1.16	1.28	–
IXB	97.2	0.33	76.0	56.4	62.2	–	60.5	1.26	0.93	1.03	–
IXC	97.2	0.25	64.1	55.1	58.0	–	59.9	1.07	0.92	0.97	–
XIB	97.2	0.32	74.7	56.4	62.0	–	58.9	1.27	0.96	1.05	–
XIC	97.2	0.25	64.3	55.2	58.1	–	59.2	1.09	0.93	0.98	–
Foster and Attard (1997)											
2L50-30	40.0	0.49	25.4	21.5	21.4	22.6	26.0	0.98	0.83	0.83	0.87
2L50-60	43.0	0.49	27.7	22.9	22.9	24.1	27.6	1.00	0.83	0.83	0.87
2L50-120	40.0	0.49	23.8	20.9	20.9	22.2	26.0	0.92	0.81	0.81	0.86
2M50-30	74.0	0.38	41.0	29.5	31.8	28.6	37.5	1.09	0.79	0.85	0.76
2M50-60	74.0	0.45	32.4	29.4	32.1	28.1	45.9	0.71	0.64	0.70	0.61
2M50-120	74.0	0.39	41.5	30.0	32.3	29.0	40.1	1.04	0.75	0.81	0.72
4M50-30	74.0	0.39	37.5	30.3	32.6	29.4	39.0	0.96	0.78	0.83	0.75
4M50-60	75.0	0.41	41.9	32.4	34.9	31.2	40.8	1.03	0.79	0.86	0.76
4M50-120	74.0	0.41	41.3	32.1	34.5	30.9	40.3	1.03	0.80	0.86	0.77
2H50-30	92.0	0.36	48.8	34.7	38.3	–	44.7	1.09	0.78	0.86	–
2H50-60	92.0	0.33	47.9	34.3	37.6	–	41.1	1.17	0.84	0.92	–
2H50-120	92.0	0.41	40.4	34.5	38.5	–	49.6	0.81	0.70	0.78	–
4H50-30	88.0	0.39	47.9	36.4	40.1	33.2	47.2	1.01	0.77	0.85	0.70
4H50-60	88.0	0.40	47.3	36.4	40.1	33.1	47.0	1.01	0.77	0.85	0.70
4H50-120	92.0	0.40	48.6	37.6	41.5	–	48.7	1.00	0.77	0.85	–
2M50-60R	67.0	0.44	34.4	28.6	30.1	27.8	39.1	0.88	0.73	0.77	0.71
2M50-120R	73.0	0.41	43.4	30.8	33.2	29.7	42.5	1.02	0.73	0.78	0.70
4M50-60R	73.0	0.49	38.3	33.2	35.8	31.5	46.8	0.82	0.71	0.76	0.67
4M50-120R	70.0	0.40	40.6	31.3	33.1	30.5	37.7	1.08	0.83	0.88	0.81

Table 4 Continued

Specimen code	f_c' (MPa)	$\frac{P/A_g}{f_c'}$	Moment (kNm)					$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
Bayrak and Sheikh (1998)											
ES-1HT	72.1	0.50	273.6	249.1	272.3	242.1	309.0	0.89	0.81	0.88	0.78
AS-2HT	71.7	0.36	368.7	266.4	284.0	262.6	323.0	1.14	0.82	0.88	0.81
AS-3HT	71.8	0.50	274.9	250.3	273.1	244.2	320.0	0.86	0.78	0.85	0.76
AS-4HT	71.9	0.50	273.2	248.9	271.7	242.0	324.0	0.84	0.77	0.84	0.75
AS-5HT	101.8	0.45	353.1	323.4	367.2	–	402.0	0.88	0.80	0.91	–
AS-6HT	101.9	0.46	345.6	319.9	364.7	–	396.0	0.87	0.81	0.92	–
AS-7HT	102.0	0.45	352.7	323.9	367.8	–	359.0	0.98	0.90	1.02	–
ES-8HT	102.2	0.47	343.6	318.2	364.4	–	377.0	0.91	0.84	0.97	–
Claeson and Gylltoft (1998)											
24	43.0	0.45	13.7	12.1	12.1	13.1	12.3	1.11	0.98	0.98	1.06
25	86.0	0.30	25.6	17.5	19.0	16.4	20.7	1.24	0.84	0.92	0.79
26	86.0	0.27	25.8	17.4	18.9	16.3	22.1	1.17	0.79	0.86	0.74
33	93.0	0.41	109.3	85.5	95.5	–	90.3	1.21	0.95	1.06	–
34	93.0	0.42	105.2	85.3	95.4	–	95.2	1.11	0.90	1.00	–
Ahn *et al*. (2000)											
L2-30-3N	35.0	0.30	97.7	82.2	82.2	83.3	106.8	0.92	0.77	0.77	0.78
L2-30-5N	35.0	0.50	108.3	77.8	77.6	79.0	119.4	0.91	0.65	0.65	0.66
H3-20-3N	52.0	0.30	124.9	102.1	102.3	105.5	170.8	0.73	0.60	0.60	0.62
H3-37-3N	52.0	0.30	124.9	102.1	102.3	105.5	153.7	0.81	0.66	0.67	0.69
H3-37-5N	52.0	0.50	127.5	98.1	98.2	101.0	195.0	0.65	0.50	0.50	0.52
U3-20-3N	59.0	0.30	136.0	108.7	110.2	111.6	170.8	0.80	0.64	0.64	0.65
U3-37-3N	59.0	0.30	136.0	108.7	110.2	111.6	175.7	0.77	0.62	0.63	0.64
U3-37-5N	70.0	0.50	130.8	113.2	122.9	108.2	209.3	0.62	0.54	0.59	0.52
Lee and Son (2000)											
LS-2	41.8	0.35	20.4	16.9	16.8	16.9	17.5	1.16	0.96	0.96	0.97
LM-2	41.8	0.34	20.0	16.8	16.8	16.8	20.7	0.97	0.81	0.81	0.81
LL-1	34.9	0.47	15.2	14.2	14.1	14.3	15.5	0.98	0.92	0.91	0.92
HS-2	70.4	0.33	19.4	15.4	16.4	15.5	15.9	1.22	0.97	1.03	0.97
HM-2	70.4	0.30	18.6	15.5	16.3	15.5	17.1	1.09	0.91	0.95	0.91
HL-2	70.4	0.20	14.8	13.6	13.8	13.5	13.0	1.14	1.04	1.06	1.03
HS-3A	70.4	0.34	26.7	21.3	22.6	21.7	23.1	1.16	0.92	0.98	0.94
HM-3A	70.4	0.27	24.4	22.3	22.7	22.1	20.6	1.18	1.08	1.10	1.07
HL-1A	70.4	0.48	21.9	19.1	20.5	19.3	18.8	1.17	1.02	1.09	1.03
HL-3A	70.4	0.21	22.3	20.9	21.2	20.6	19.1	1.17	1.10	1.11	1.08
VS-1	93.2	0.49	19.2	17.2	20.1	–	18.1	1.06	0.95	1.11	–
VS-2	93.2	0.31	24.4	18.9	20.9	–	19.8	1.23	0.95	1.05	–
VM-1	93.2	0.48	19.4	17.5	20.2	–	18.0	1.08	0.97	1.12	–
VM-2	93.2	0.24	21.2	18.7	19.4	–	19.0	1.12	0.99	1.02	–
VS-2A	93.2	0.40	30.0	22.8	25.4	–	25.2	1.19	0.90	1.01	–
VM-2A	93.2	0.35	31.8	23.4	26.0	–	27.1	1.17	0.87	0.96	–

Table 4 Continued

Specimen code	f_c' (MPa)	$\frac{P/A_g}{f_c'}$			Moment (kNm)			$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
Mo and Wang (2000)											
C1-3	26.1	0.22	377.5	330.8	330.8	341.1	427.7	0.88	0.77	0.77	0.80
C2-3	26.8	0.21	379.2	333.4	333.4	343.6	427.2	0.89	0.78	0.78	0.80
C3-3	26.9	0.21	379.5	333.9	333.9	344.1	423.8	0.90	0.79	0.79	0.81
Ho and Pam (2002)											
BS-100-03-24	82.8	0.35	494.7	350.1	384.5	330.7	408.6	1.21	0.86	0.94	0.81
NEW-100-03-24	83.3	0.36	497.1	345.9	380.5	325.6	426.4	1.17	0.81	0.89	0.76
Lam *et al*. (2003)											
X-6	31.9	0.45	36.9	29.8	29.6	30.2	40.3	0.92	0.74	0.74	0.75
X-7	35.7	0.45	42.0	32.9	32.7	33.7	39.3	1.07	0.84	0.83	0.86
Tan and Nguyen (2005)											
S40-B-E60/1	49.0	0.49	62.3	53.5	53.5	56.8	62.1	1.00	0.86	0.86	0.91
S40-B-E60/2	49.0	0.49	63.9	53.1	53.1	56.4	61.2	1.04	0.87	0.87	0.92
S70-B-E60	76.1	0.34	100.1	70.2	75.9	68.1	68.7	1.46	1.02	1.11	0.99
Marefat *et al*. (2005)											
NTCM-14	20.1	0.31	18.6	16.0	16.0	16.1	16.8	1.11	0.95	0.95	0.96
NBCC-12	25.2	0.23	25.3	22.0	22.0	22.4	21.7	1.17	1.01	1.01	1.03
NBCM-11	24.5	0.25	45.0	38.6	38.6	38.5	44.6	1.01	0.87	0.87	0.86
SBCM-8	28.0	0.22	51.9	46.0	46.0	46.0	58.7	0.88	0.78	0.78	0.78
Tao and Yu (2008)											
US-2U	49.2	0.22	23.9	21.9	21.9	22.0	21.1	1.14	1.04	1.04	1.04
BS-2U	49.8	0.23	23.8	21.7	21.7	21.8	25.1	0.95	0.87	0.87	0.87
Ho (2012)											
NEW-100-03-24-S	83.3	0.36	497.1	345.9	380.5	325.6	492.8	1.01	0.70	0.77	0.66
NEW-100-03-24-C	96.4	0.36	550.7	387.0	426.1	–	545.6	1.01	0.71	0.78	–
NEW-80-03-24-C	80.6	0.36	484.9	337.5	371.0	322.7	479.3	1.01	0.70	0.77	0.67
NEW-100-03-61-C	94.7	0.35	727.1	514.7	559.0	–	802.7	0.91	0.64	0.70	–
BS-100-03-24-S	82.8	0.35	494.7	350.1	384.5	330.7	480.5	1.03	0.73	0.80	0.69
BS-100-03-24-C	87.5	0.38	531.9	363.2	401.3	331.6	537.6	0.99	0.68	0.75	0.62
Hadi and Widiarsa (2012)											
0C50	79.5	0.42	82.1	67.2	75.5	64.0	70.3	1.17	0.95	1.07	0.91
Average								**1.04**	**0.84**	**0.88**	**0.82**
Standard Deviation								**0.145**	**0.118**	**0.132**	**0.124**

Table 5 Comparison of proposed theoretical flexural strengths of RC columns subjected to high and ultra-high axial load levels with other researchers' results

Specimen code	f_c' (MPa)	$P/A_g f_c'$	Moment (kNm)					$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
			M_p (1)	M_{NZ} (2)	M_{ACI} (3)	M_{EC} (4)	M_t (5)				
Park *et al*. (1982)											
4	23.5	0.60	619.1	609.0	609.0	613.0	911.0	0.68	0.67	0.67	0.67
Sheikh and Yeh (1990)											
E-2	31.4	0.61	169.2	163.3	162.8	164.0	169.3	1.00	0.96	0.96	0.97
A-3	31.8	0.61	172.1	165.4	164.8	166.2	197.8	0.87	0.84	0.83	0.84
F-4	32.2	0.60	178.7	167.6	167.0	168.6	198.3	0.90	0.85	0.84	0.85
F-6	27.2	0.75	136.2	134.6	134.6	135.2	145.3	0.94	0.93	0.93	0.93
D-7	26.2	0.78	121.4	119.5	119.5	121.3	133.2	0.91	0.90	0.90	0.91
E-8	25.9	0.78	130.6	128.9	128.9	129.7	129.2	1.01	1.00	1.00	1.00
F-9	26.5	0.77	132.9	131.3	131.3	132.0	152.0	0.87	0.86	0.86	0.87
E-10	26.3	0.77	132.6	131.0	131.0	131.7	132.7	1.00	0.99	0.99	0.99
A-11	27.9	0.74	141.2	139.6	139.6	140.2	135.1	1.04	1.03	1.03	1.04
F-12	33.4	0.60	183.0	171.7	171.2	173.3	161.0	1.14	1.07	1.06	1.08
E-13	27.2	0.74	137.7	136.2	136.2	136.7	127.9	1.08	1.06	1.06	1.07
D-14	26.9	0.75	127.2	125.3	125.3	127.0	116.5	1.09	1.08	1.08	1.09
D-15	26.2	0.75	124.9	123.0	123.0	124.7	134.5	0.93	0.91	0.91	0.93
A-16	33.9	0.60	182.9	172.7	172.2	174.4	157.4	1.16	1.10	1.09	1.11
Sheikh and Khoury (1993)											
AS-3	33.2	0.60	169.4	159.5	159.2	167.6	192.9	0.88	0.83	0.83	0.87
FS-9	32.4	0.76	139.1	133.3	133.2	139.8	174.2	0.80	0.76	0.76	0.80
ES-13	32.5	0.76	137.9	132.2	132.2	139.2	163.3	0.84	0.81	0.81	0.85
AS-17	31.3	0.77	135.7	129.8	129.7	136.2	180.1	0.75	0.72	0.72	0.76
AS-18	32.8	0.77	136.3	130.6	130.6	137.5	204.0	0.67	0.64	0.64	0.67
Sheikh *et al*. (1994)											
AS-3H	54.1	0.62	209.0	203.8	204.0	206.4	252.8	0.83	0.81	0.81	0.82
AS-18H	54.7	0.64	203.6	199.8	200.0	199.6	269.0	0.76	0.74	0.74	0.74
AS-120H	53.6	0.64	201.6	197.5	197.7	200.5	297.9	0.68	0.66	0.66	0.67
A-17H	59.1	0.65	210.5	203.0	208.9	194.3	261.1	0.81	0.78	0.80	0.74
Watson and Park (1994)											
7	42.0	0.70	298.3	298.2	298.1	303.3	516.8	0.58	0.58	0.58	0.59
8	39.0	0.70	289.7	288.3	288.1	293.1	524.5	0.55	0.55	0.55	0.56
9	40.0	0.70	291.1	290.2	290.0	294.9	599.0	0.49	0.48	0.48	0.49
Basappa Setty and Rangan (1996)											
1	79.1	0.58	24.7	21.8	26.2	19.5	24.6	1.01	0.89	1.07	0.79
2	79.1	0.51	25.3	22.7	26.5	21.1	20.4	1.24	1.12	1.30	1.04
3	79.1	0.58	23.3	20.3	25.1	18.1	23.9	0.98	0.85	1.05	0.75
4	79.1	0.62	23.6	20.6	25.4	17.9	26.8	0.88	0.77	0.95	0.67

Table 5 Continued

Specimen code	f_c' (MPa)	$P/A_g f_c'$	Moment (kNm)					$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
			M_p (1)	M_{NZ} (2)	M_{ACI} (3)	M_{EC} (4)	M_t (5)				
Ibrahim and MacGregor (1996)											
V2	82.8	0.65	129.1	101.1	144.9	81.3	159.9	0.81	0.63	0.91	0.51
V7	84.7	0.62	143.8	118.2	158.7	96.1	139.3	1.03	0.85	1.14	0.69
V13	72.5	0.66	119.7	102.8	128.8	93.0	103.8	1.15	0.99	1.24	0.90
V16	59.3	0.77	67.3	67.7	74.0	58.2	110.4	0.61	0.61	0.67	0.53
Lloyd and Rangan (1996)											
IA	58.0	0.82	21.9	22.4	23.7	19.9	35.1	0.62	0.64	0.67	0.57
IIA	58.0	0.68	18.4	18.4	18.8	17.6	26.6	0.69	0.69	0.71	0.66
IIIA	58.0	0.64	33.3	32.9	33.8	31.7	27.9	1.19	1.18	1.21	1.14
IVA	58.0	0.52	20.4	20.1	20.4	19.6	21.9	0.93	0.92	0.93	0.89
VA	92.0	0.60	51.0	44.4	56.1	–	36.5	1.40	1.22	1.54	–
VIIA	92.0	0.61	44.6	37.9	50.7	–	40.4	1.11	0.94	1.26	–
IXA	97.2	0.66	39.4	32.7	48.3	–	42.8	0.92	0.77	1.13	–
XA	97.2	0.54	28.7	26.0	31.9	–	36.8	0.78	0.71	0.87	–
XIA	97.2	0.65	41.1	34.6	49.6	–	39.7	1.04	0.87	1.25	–
XIIA	97.2	0.56	27.7	24.9	31.2	–	38.0	0.73	0.66	0.82	–
Foster and Attard (1997)											
4L50-30	40.0	0.57	23.8	22.9	22.8	24.7	35.4	0.67	0.65	0.64	0.70
4L50-60	40.0	0.61	27.0	25.6	25.6	26.7	31.9	0.85	0.80	0.80	0.84
4L50-120	40.0	0.58	25.7	24.3	24.3	26.0	30.5	0.85	0.80	0.80	0.85
2M20-30	74.0	0.70	23.7	20.1	25.6	17.3	30.2	0.78	0.67	0.85	0.57
2M20-120	74.0	0.64	26.7	23.5	28.1	20.8	26.7	1.00	0.88	1.05	0.78
4M20-30	75.0	0.62	31.4	28.4	32.1	25.7	25.2	1.24	1.13	1.27	1.02
4M20-60	75.0	0.59	30.6	28.0	31.4	25.6	25.1	1.22	1.12	1.25	1.02
2H20-30	92.0	0.58	32.8	29.0	35.8	–	32.0	1.03	0.91	1.12	–
2H20-60	92.0	0.60	31.4	27.5	34.6	–	31.5	1.00	0.87	1.10	–
4H20-30	88.0	0.68	32.9	28.7	35.8	21.8	36.5	0.90	0.79	0.98	0.60
4H20-60	88.0	0.69	33.1	28.8	36.0	22.0	37.3	0.89	0.77	0.96	0.59
4H20-120	92.0	0.66	34.3	30.4	37.6	–	37.1	0.93	0.82	1.01	–
4L50-30R	40.0	0.61	24.6	23.6	23.6	25.3	32.8	0.75	0.72	0.72	0.77
4M20-120R	73.0	0.67	30.6	27.7	31.3	24.5	30.1	1.02	0.92	1.04	0.82
2L8-60	43.0	0.89	10.7	11.0	11.0	11.0	12.0	0.89	0.92	0.92	0.92
2L8-120	43.0	0.94	7.2	7.7	7.7	7.7	12.8	0.57	0.60	0.60	0.60
2L20-30	40.0	0.83	13.5	13.6	13.6	13.6	18.6	0.72	0.73	0.73	0.73
2L20-60	43.0	0.72	17.8	17.8	17.8	18.0	18.3	0.97	0.97	0.97	0.98
2L20-120	43.0	0.81	14.3	14.5	14.5	14.5	19.9	0.72	0.73	0.73	0.73
4L8-120	43.0	1.01	13.5	13.8	13.8	14.1	13.4	1.01	1.03	1.04	1.06
4L20-120	40.0	1.00	14.7	14.9	14.9	15.3	21.6	0.68	0.69	0.69	0.71
2M8-120	75.0	0.73	20.3	16.4	22.8	12.6	14.9	1.36	1.10	1.53	0.85
4M8-60	75.0	0.83	20.5	17.1	23.0	11.6	16.8	1.21	1.01	1.36	0.69
4M8-120	74.0	0.84	20.5	17.3	23.2	12.2	16.1	1.27	1.07	1.44	0.76

Table 5 Continued

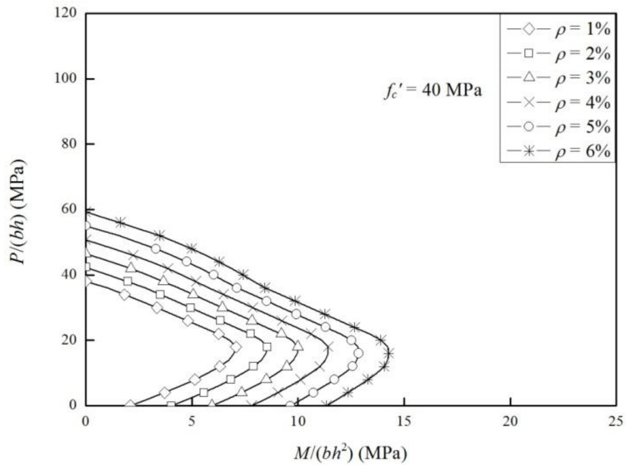
Specimen code	f_c' (MPa)	$P/A_g f_c'$	Moment (kNm)					$\frac{(1)}{(5)}$	$\frac{(2)}{(5)}$	$\frac{(3)}{(5)}$	$\frac{(4)}{(5)}$
			M_p (1)	M_{NZ} (2)	M_{ACI} (3)	M_{EC} (4)	M_t (5)				
Foster and Attard (1997)											
4M20-120	75.0	0.73	28.3	25.1	30.0	21.2	30.7	0.92	0.82	0.98	0.69
2H8-30	93.0	0.75	18.5	13.3	24.5	–	18.1	1.02	0.73	1.35	–
4H8-30	91.0	0.78	25.0	20.4	29.8	–	20.5	1.22	1.00	1.46	–
4H8-60	92.0	0.82	21.2	16.3	26.9	–	23.0	0.92	0.71	1.17	–
4H8-120	92.0	0.80	22.9	18.4	28.3	–	20.2	1.14	0.91	1.40	–
2L8-120R	56.0	0.87	10.8	11.7	12.0	10.0	13.7	0.79	0.86	0.88	0.73
2L20-120R	56.0	0.71	20.2	20.4	20.6	19.6	22.4	0.90	0.91	0.92	0.87
4L8-120R	56.0	0.99	12.9	14.1	14.4	12.0	15.0	0.86	0.94	0.96	0.80
4L20-120R	53.0	0.79	23.2	23.4	23.4	23.2	24.6	0.94	0.95	0.95	0.94
2M20-60R	73.0	0.73	20.9	17.5	23.0	14.3	32.5	0.64	0.54	0.71	0.44
4M20-60R	68.0	0.78	24.6	22.9	26.0	19.5	29.2	0.84	0.78	0.89	0.67
Claeson and Gylltoft (1998)											
23	43.0	0.52	12.3	11.8	11.8	12.7	14.7	0.84	0.80	0.80	0.86
27	33.0	0.75	39.2	37.6	37.6	41.2	41.6	0.94	0.90	0.90	0.99
28	33.0	0.75	39.2	37.6	37.6	41.2	40.6	0.97	0.93	0.93	1.02
29	91.0	0.63	73.3	62.5	80.6	–	99.3	0.74	0.63	0.81	–
30	92.0	0.64	73.0	62.2	80.8	–	94.0	0.78	0.66	0.86	–
31	37.0	0.61	48.3	46.6	46.5	50.1	54.0	0.89	0.86	0.86	0.93
32	37.0	0.62	47.7	46.0	46.0	49.5	51.5	0.93	0.89	0.89	0.96
Lee and Son (2000)											
LS-1	41.8	0.70	11.4	11.5	11.5	11.5	15.6	0.73	0.73	0.73	0.74
LM-1	41.8	0.62	13.7	13.7	13.6	13.7	17.6	0.78	0.77	0.77	0.78
HS-1	70.4	0.52	15.0	13.6	15.0	13.3	13.7	1.09	0.99	1.10	0.97
HM-1	70.4	0.50	15.3	13.9	15.3	13.6	14.3	1.07	0.97	1.07	0.96
HL-1	70.4	0.52	15.1	13.7	15.1	13.4	20.8	0.73	0.66	0.73	0.64
HS-1A	70.4	0.66	17.6	15.8	17.5	15.5	17.6	1.00	0.90	0.99	0.88
HM-1A	70.4	0.62	18.5	16.6	18.2	16.5	16.7	1.11	0.99	1.09	0.99
VS-1A	93.2	0.62	20.2	17.7	21.4	–	22.7	0.89	0.78	0.94	–
VM-1A	93.2	0.59	21.0	18.5	22.0	–	24.3	0.87	0.76	0.91	–
Ho and Pam (2002)											
BS-60-06-61	51.1	0.67	408.5	384.9	385.4	410.2	417.7	0.98	0.92	0.92	0.98
NEW-60-06-61	50.0	0.66	400.0	368.2	368.5	402.6	466.4	0.86	0.79	0.79	0.86
Lam *et al*. (2003)											
X-4	31.9	0.65	25.5	25.2	25.1	25.4	38.1	0.67	0.66	0.66	0.67
Tan and Nguyen (2005)											
S40-B-E20/2	49.0	0.87	22.0	22.8	22.7	24.9	37.1	0.59	0.61	0.61	0.67
S40-B-E40/1	49.0	0.71	40.9	40.9	40.9	43.3	59.5	0.69	0.69	0.69	0.73
S40-B-E40/2	49.0	0.71	41.3	41.2	41.2	43.7	59.7	0.69	0.69	0.69	0.73
S70-B-E20	76.1	0.68	53.6	44.0	58.4	35.8	47.3	1.13	0.93	1.23	0.76
S70-B-E40	76.1	0.51	72.1	64.7	73.5	60.3	67.4	1.07	0.96	1.09	0.90

Table 5 Continued

Němeček *et al*. (2005)											
N50	30.0	0.91	9.8	9.1	9.1	10.8	11.6	0.84	0.78	0.79	0.93
N100	30.0	0.90	10.2	9.5	9.6	11.2	11.4	0.90	0.84	0.84	0.98
N150	30.0	0.89	10.4	9.8	9.8	11.4	11.1	0.94	0.88	0.88	1.02
N50	30.0	0.94	9.0	8.2	8.3	10.0	12.0	0.75	0.69	0.69	0.83
N100	30.0	0.92	9.7	9.0	9.1	10.7	11.6	0.84	0.78	0.78	0.92
N150	30.0	0.92	9.8	9.1	9.1	10.8	11.4	0.86	0.79	0.80	0.94
H50	67.2	0.73	20.3	18.6	21.6	15.6	19.8	1.03	0.94	1.09	0.79
H100	67.2	0.72	20.8	19.1	22.1	16.2	19.2	1.08	0.99	1.15	0.84
H150	67.2	0.70	21.6	19.9	22.8	17.2	18.5	1.17	1.08	1.24	0.93
Kim (2007)											
10E2	54.5	0.68	167.1	164.1	164.4	164.6	162.5	1.03	1.01	1.01	1.01
A10E1	75.2	0.87	107.5	81.7	127.3	56.6	122.5	0.88	0.67	1.04	0.46
Hadi and Widiarsa (2012)											
0C25	79.5	0.61	60.6	49.8	65.5	42.8	52.4	1.16	0.95	1.25	0.82
Average								**0.91**	**0.84**	**0.94**	**0.82**
Standard Deviation								**0.182**	**0.152**	**0.220**	**0.159**

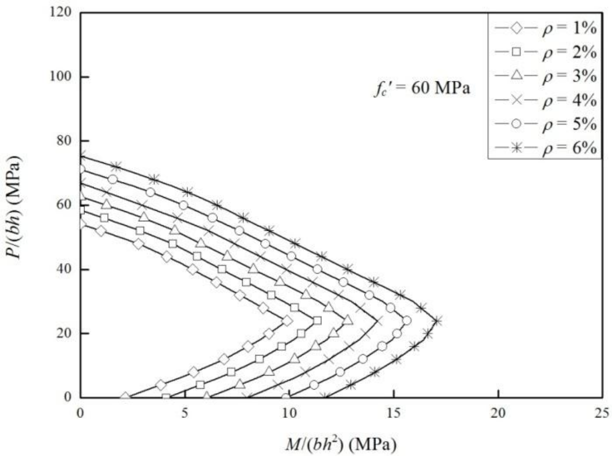
4.3 Column interaction diagrams

To faciliate the process of practical flexural strength design of RC columns, Eqs. (6) to (9) are converted into a series of column interaction diagrams with strain gradient effect considered. To eliminate the size effect of column section, the column interaction diagrams plot $P/(bh)$ against $M/(bh^2)$ as shown in Fig. 10. It covers concrete strength from 40 to 100 MPa, longitudinal ratios from 1% to 6% and yield strength of 460 MPa.

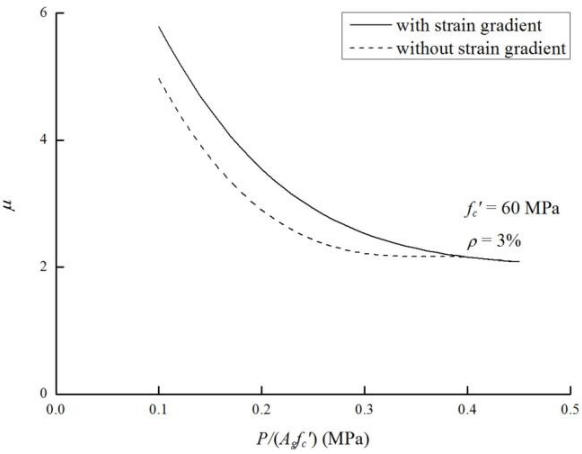


(a) f_c' = 40 MPa

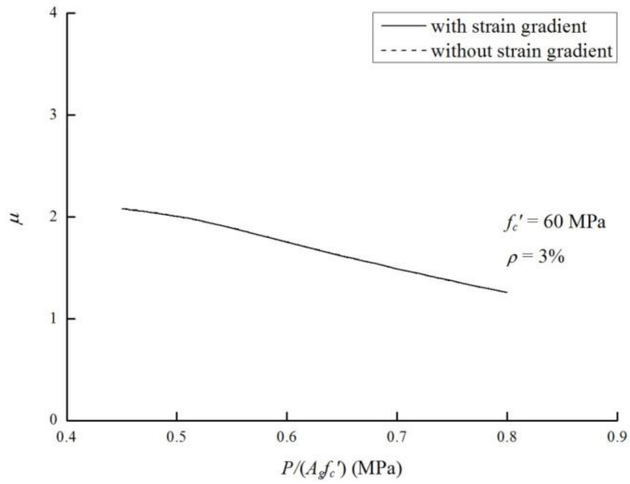
Fig. 10 Flexural ductility of RC columns with and without strain gradient effect considered



(b) f_c' = 60 MPa



(c) f_c' = 80 MPa



(d) f_c' = 100 MPa

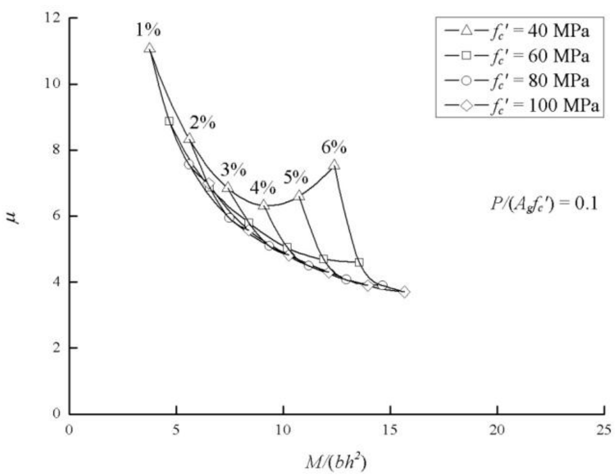
Fig. 10 Continued

For a given pair of concrete strength and steel ratio, the flexural strength of the specified section under certain axial load can be determined from the appropriate diagrams in Fig. 10(a) to 10(d). The diagrams in Fig. 10 serves as practical flexural strength design charts for designing concrete strength and longitudinal steel ratios of RC columns subjected to various axial load levels. To use the charts, it is suggested that the design could start with a low strength concrete say $f_c' = 40$ MPa in Fig. 10(a) and look for an appropriate longitudinal ratio that can provide adequate flexural strength. Alternatively, a higher concrete strength than 40 MPa can be used coupling with smaller ρ ratio. In the case where the prescribed flexural strength cannot be achieved when $\rho = 6\%$ is used, f_c' should be gradually increased until 100 MPa. If the flexural strength capacity requirement cannot be satisfied even by using $f_c' = 100$ MPa, the section is recommended to be enlarged.

4.4 Design charts for flexural ductility design of RC columns

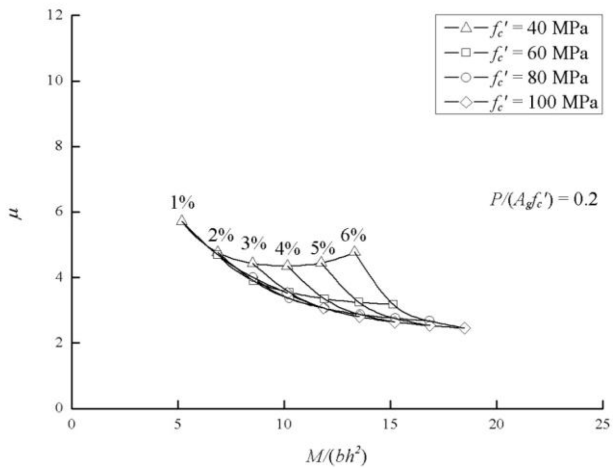
To enable a one-step direct design for flexural ductility of RC column without conducting nonlinear moment-curvature analysis, the authors propose to use a series of design charts which plot the ductility against flexural strength of columns of various concrete strength and longitudinal steel ratios incorporating strain gradient effect. These charts are shown in Fig. 11 for columns failed in tension, and in Fig. 12 for columns failed in compression. In these graphs, the lines plotted represent the maximum flexural strength and ductility that can be simultaneously achieved by a column section with certain concrete strength f_c' at a given axial load level $P/A_g f_c'$. The intermediate lines crossing the curves of constant concrete strength represent the section having the same longitudinal steel ratio.

For a given pair of prescribed flexural strength and ductility requirement, the required longitudinal steel ratio at a given axial load level can be determined from the respective charts. If the use of high-strength concrete is allowed, the corresponding required steel ratio can be determined from the graph using $f_c' = 100$ MPa in Fig. 11 or 12. This is because the use of high-strength concrete can decrease the size of column, save space, which is more durable and

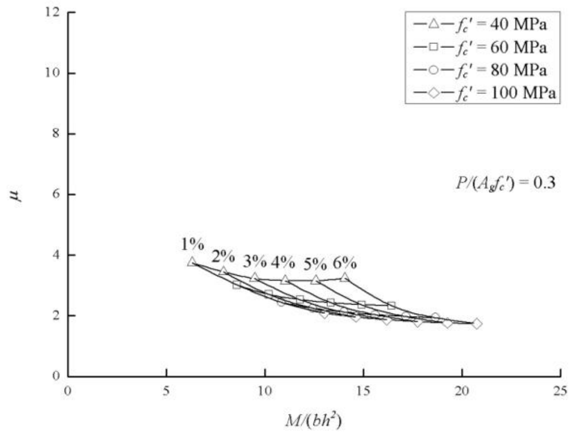


(a)

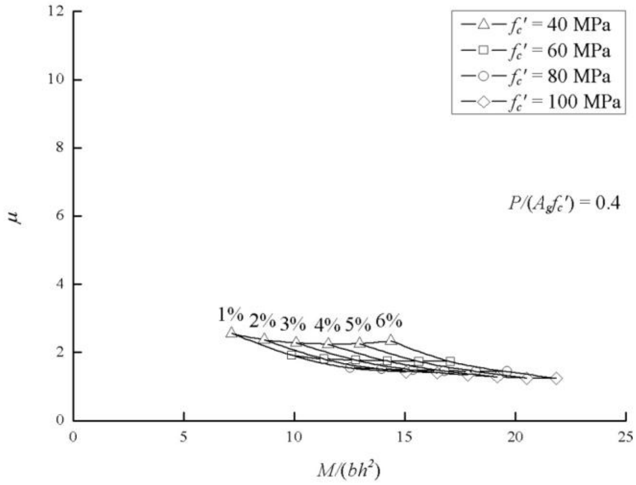
Fig. 11 Flexural ductility of RC columns with and without strain gradient effect considered



(b)

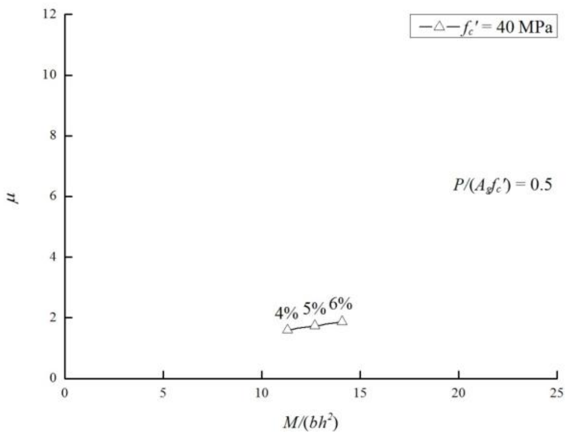


(c)



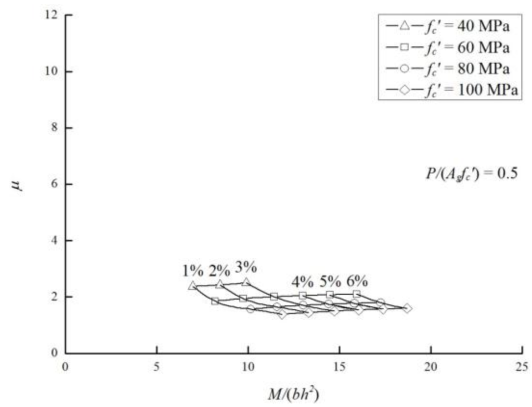
(d)

Fig. 10 Continued

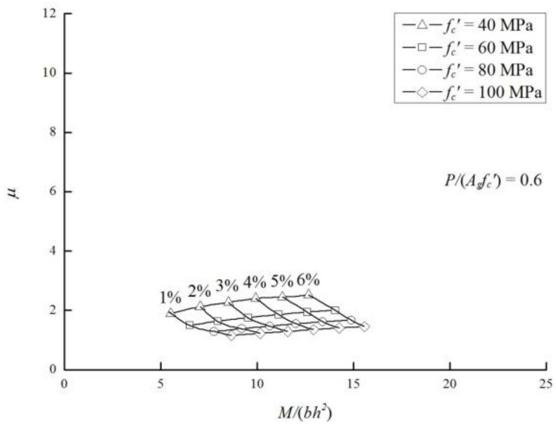


(e)

Fig. 11 Flexural Strength-Ductility design charts for columns in tension failure incorporating strain gradient effect



(a) $P/(A_g f_c') = 0.5$



(b) $P/(A_g f_c') = 0.6$

Fig. 12 Flexural Strength-Ductility design charts for columns in compression failure incorporating strain gradient effect environmentally friendly [Wong and Kwan 2008] than lower strength concrete. In the event that the prescribed strength and ductility requirement cannot be achieved, the concrete strength can be successively lowered to 80, 60 and finally 40 MPa. If the use of f_c' = 40 MPa still cannot achieve the required strength and ductility, the section should be enlarged or some confinement should be added.

5. Conclusions

A strain-gradient-dependent stress-strain curve of concrete previously developed by the authors has been adopted in this study to model the flexural stress-strain behavior of concrete with strain gradient effect considered. Using the proposed stress-strain curve, the combined effects of strain gradient and concrete strength on flexural strength and ductility design of RC columns up to f_c' = 100 MPa were investigated by nonlinear moment-curvature analysis. A series of comprehensive parametric study was then conducted on concrete strength from 40 to 100 MPa, axial load level from 0.1 to 0.6 and longitudinal steel ratio from 1% to 6% to study the effect of strain gradient on column's flexural strength and ductility.

Based on the results obtained, two equations were proposed for the equivalent rectangular stress block parameters α and β for flexural strength design of RC columns with the combined effects of strain gradient and concrete strength considered. The validity of the proposed parameters was verified by comparing the proposed theoretical strength with that of 275 RC columns measured experimentally by other researchers. It is evident from the comparison that the proposed equations can predict more accurately the flexural strength of RC columns than the current RC design codes. For RC columns subjected to low and medium axial load levels, the proposed flexural strength is about 10% and 19% closer to the measured strength on average respectively. It therefore indicates that both effects of concrete strength and strain gradient should be considered in the flexural strength design of RC columns.

With respect to the flexural ductility design of RC columns, it is seen that the ductility improves as a consequence of the strain gradient effect for column failing under tension. For columns failing under compression, the ductility is irrespective of the strain gradient because the neutral axis depth is very large and the strain gradient becomes insignificant. Lastly for practical design purpose, column interaction diagrams and concurrent flexural strength-and-ductility design charts have been produced incorporating the combined effects of strain gradient and concrete strength. For a given flexural strength and/or ductility requirement, the respective required amount of longitudinal steel taken into account the combined effects of strain gradient and concrete strength can be determined directly from the proposed design charts of column interaction and/or concurrent strength-and-ductility diagram.

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List of notations

A_c Area of concrete
A_g Gross area of the column section
A_s Area of longitudinal steel reinforcement
A_{si} Area of i^{th} longitudinal steel reinforcement
b Breadth of rectangular section
c Neutral axis depth in the section
d Depth to centroid of bottom reinforcement (effective depth)
d_i Distance of the i^{th} steel bar from the extreme concrete compressive fibre
E_c Initial Young's modulus of concrete
E_s Young's modulus of steel reinforcement
f_c' Uni-axial concrete compressive strength represented by cylinder strength
f_{co} Concrete stress developed under flexure
f_{cu} Uni-axial concrete compressive strength represented by cube strength
f_o Maximum compressive stress of concrete
f_r Confining stress of concrete
f_y Yield strength of steel reinforcement
h Total height of rectangular section
k_o Ratio of concrete strains corresponding to maximum concrete stress developed in flexure a nd under uni-axial load
k_3 Ratio of maximum concrete stress developed in flexure to uni-axial strength
M Moment capacity (flexural strength)
M_{ACI} Moment calculated based on ACI318M-08
M_{EC} Moment calculated based on Eurocode 2
M_{NZS} Moment calculated based on NZS 3101
M_p Moment calculated based on the proposed of equivalent rectangular concrete stress block parameters
M_t Measured moment capacity
P Prescribed compressive axial load in the section
RC Reinforced concrete
α Ratio of equivalent concrete compressive stress developed under flexure to concrete cylind er strength
β Ratio of depth of equivalent rectangular concrete compressive stress block to neutral axis depth
ε Strain in concrete
ε_{co} Strain of concrete at maximum uni-axial stress
ε_{cu} Ultimate strain of concrete
ε_o Strain of concrete at maximum stress under flexure
ε_p Residual plastic strain in the tension steel reinforcement
ε_s Strain in the steel reinforcement
ϕ_u Ultimate curvature
ϕ_y Yield curvature
μ Curvature ductility factor
ρ Longitudinal steel ratio ($=A_s/bh$)
σ Stress in concrete
σ_s Stress in the steel reinforcement
σ_{si} Stress in the i^{th} longitudinal steel reinforcement

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