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Reliability analysis of reinforced concrete haunched beams shear capacity based on stochastic nonlinear FE analysis

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Abstract. The lack of experimental studies on the mechanical behavior of reinforced concrete (RC) haunched beams leads to difficulties in statistical and reliability analyses. This study performs stochastic and reliability analyses of the ultimate shear capacity of RC haunched beams based on nonlinear finite element analysis. The main aim of this study is to investigate the influence of uncertainty in material properties and geometry parameters on the mechanical performance and shear capacity of RC haunched beams. Firstly, 65 experimentally tested RC haunched beams and prismatic beams are analyzed via deterministic nonlinear finite element method by a special program (ATENA) to verify the efficiency of utilized numerical models, the shear capacity and the crack pattern. The accuracy of nonlinear finite element analyses is verified by comparing the results of nonlinear finite element and experiments and both results are found to be in a good agreement. Afterwards, stochastic analyses are performed for each beam where the RC material properties and geometry parameters are assigned to take probabilistic values using an advanced simulating procedure. As a result of stochastic analysis, statistical parameters are determined. The statistical parameters are obtained for resistance bias factor and the coefficient of variation which were found to be equal to 1.053 and 0.137 respectively. Finally, reliability analyses are accomplished using the limit state functions of ACI-318 and ASCE-7 depending on the calculated statistical parameters. The results show that the RC haunched beams have higher sensitivity and riskiness than the RC prismatic beams.

Keywords: haunched beams; reinforced concrete; nonlinear finite element analysis; stochastic analysis; reliability analysis

1. Introduction

Although reinforced concrete (RC) haunched beams are widely used as bridges or portal frames and precast roof girders, there is a lack of studies in the literature investigating this topic (Nilson *et al*. 2011). Scarce in experimental studies is the main impediment to include this topic in details by

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international building practice codes. As a result of various experimental studies, it can be concluded that the behavior and failure of the RC haunched beams differs as compared to prismatic section RC beams (Debaiky and El-Niema 1982, Stefanou 1983, Tena*et al*. 2008, Nghiep 2010).

The researchers proved that the depth variance along the beam has a clear influence on the shear behaviour as well as shear capacity. Although the researchers suggested some formulas to estimate the shear capacity, the mechanical model that can explain the inclination effect is not available (Neghiep 2010). Therefore, the nonlinear finite element analysis may give answers about this problem. This article discusses the stochastic and reliability analyses of ultimate load capacity for RC haunched beams. Analysis of beams is based on the nonlinear finite element analysis of available tested beams in the literature. The stochastic and the reliability analyses are useful tools for code calibration purposes aiming to bridge the gap in this field due to experimental knowledge indigence.

The design of engineering structures after 1970 has turned into a new direction regarding reliability analysis and its concept developed by Ang and Cornell (1974). The fundamental concept states that the uncertainties of material and geometry parameters should be considered in the design where reliability analysis is applied to RC structures together with the ultimate limit state design. Furthermore, evaluation of structural safety associated with the design procedure was studied by Ellingwood and Ang (1974). Recently, reliability analysis has been applied successfully to structural members (Eamon and Jensen 2012, 2013, Kim *et al*. 2013).

In this study, firstly, the load capacities of tested beams collected from the literature are verified by nonlinear finite element analysis using the 2D ATENA (Červenka 2012) program. Secondly, the verified beams are analyzed with random values of material properties and geometry parameters. An advanced Monte Carlo sampling technique is used to generate the samples of the material properties depending on statistical parameters of the materials. The final step includes the calculation of the statistical parameters regarding the results of stochastic analysis and uses these parameters to predict the reliability index of the limit state functions. The procedure followed in this study includes several steps given in the flowchart shown in Fig. 1.

2. Experimental studies

The first experimental study which investigated the RC haunched beams shear behavior in details is achieved by Debaiky and Elniema (1982), where 33 simply supported slender beams are prepared and tested with varying geometries and concrete properties (see Fig. 2). The authors proved that the nominal shear contribution of the concrete was affected by the haunch's inclination, and proposed an expression influenced by the inclination angle α based on the equation of shear strength in ACI-318 code.

Stefanou (1983) conducted an experimental study of shear failure of the RC haunched beams and the mode of shear failure. Furthermore, he compared RC haunched beams with RC prismatic beams. This work includes beams with and without stirrups with different inclination cases, as shown in Fig. 3. All beams were simply supported and tested under concentrated load. The author discussed the performance of international building codes compared to experimental results and recommended modifications for codes' equations.

Tena *et al*. (2008) tested two RC prismatic beams and eight RC haunched beams with support depth higher than mid-span depth reinforced, where half of beams are reinforced with shear stirrups and others without shear reinforcement. Fig. 4 shows the tested beams with different

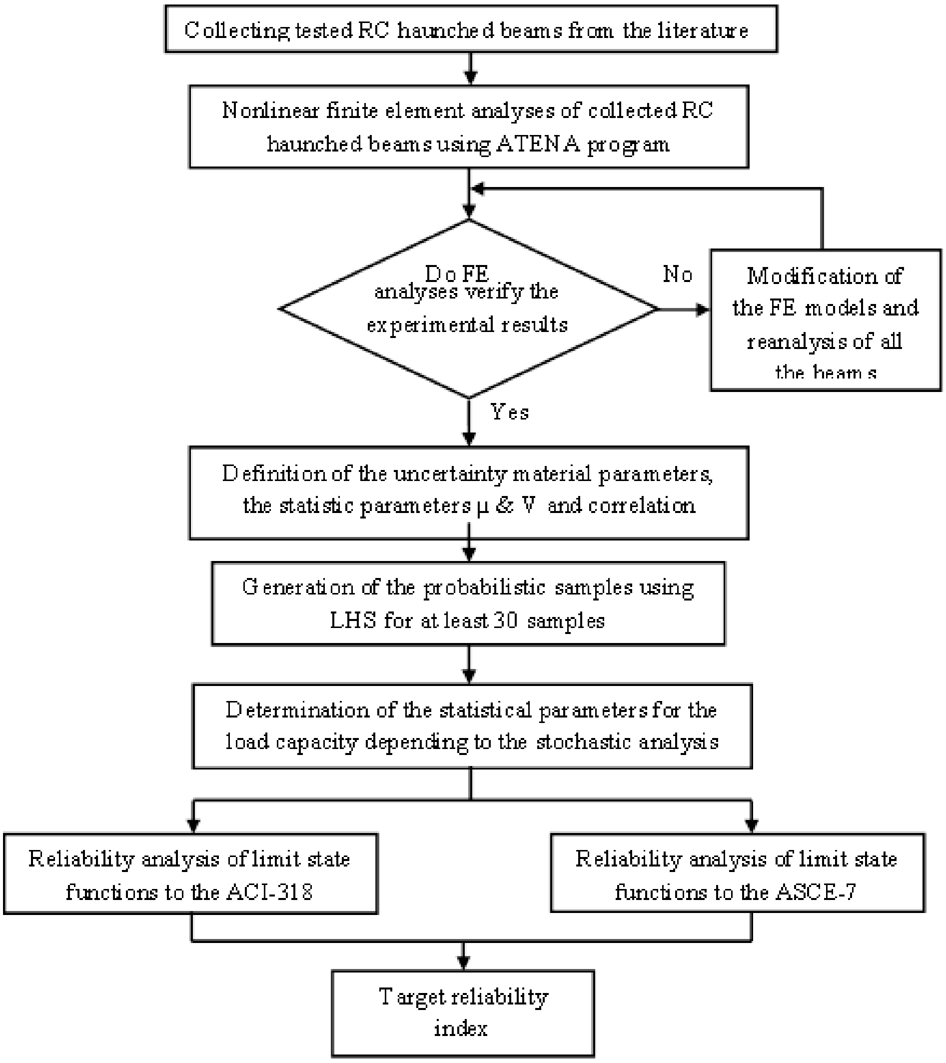


Fig. 1 Analysis flow chart

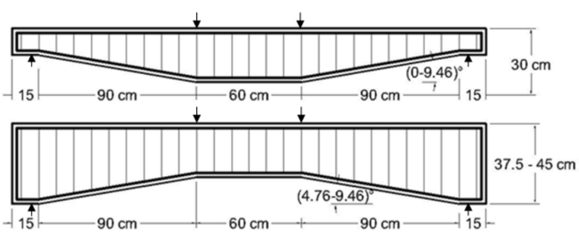


Fig. 2 Experimental tested beams byDebaiky and Elniema(1982)

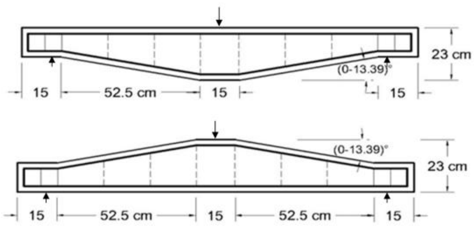


Fig. 3 Experimental tested beams by Stefanou (1983)

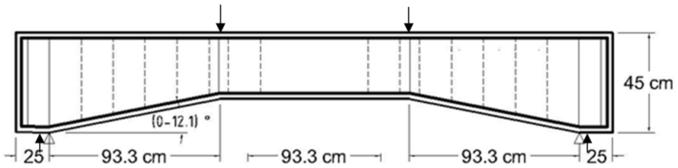


Fig. 4 Experimental tested beams by Arturo *et al*. (2008)

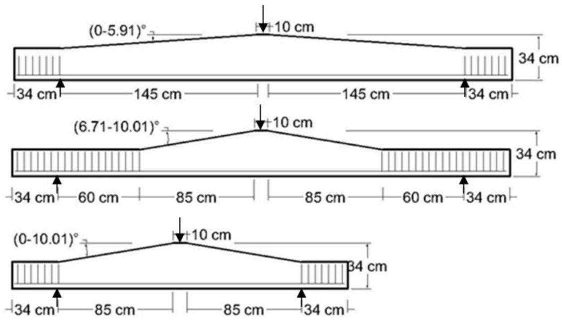


Fig. 5 Experimental tested beams by Nghiep (2010)

slopes in bottom surface. The study upgraded previous design equations depending on the experimental test results conducted by various authors. Nghiep (2010) performed experimental tests for simply supported RC haunched beams without shear reinforcement; the beams were inclined in upper surface and tested under concentrated load at mid-span (see Fig. 5). The research proposed new practice shear design models valid for prismatic and haunched beams without shear reinforcement.

3. Nonlinear finite element analysis

3.1 Concepts of models

In this study, 2D ATENA program which is developed by Červenka consulting (Červenka 2012), is used to simulate and verify the real behavior of reinforced concrete structures including the concrete cracking, crushing and reinforcement yielding.

The plain concrete as a material is modeled using SBETA material model available in the ATENA material library that employs nonlinear fracture mechanics and energy based concept of concrete fracture and includes all concrete material parameters where reinforcement bars are modeled as discrete approach. The constitutive models in the ATENA material library which simulate concrete behavior include:

- KUPFER model is used to represent the biaxial failure criterion; the principal stresses are based on the equivalent uniaxial stress-strain relationship.
- The tension behavior without cracks is assumed linear elastic. After cracking, the stress-strain relation is modeled based on the crack opening law and fracture energy. Fig. 6 shows the utilized exponential curve-softening model.

- The compression behavior for the concrete before the peak strength value is represented by CEB-FIP Model. After the peak stress, the softening law in compression decreases linearly as shown in Fig. 7.
- The shear strength of the concrete is related to the smeared crack model. The principle axis is assumed to be fixed in the principle direction at moment of crack initiation as shown in Fig. 8 (Cervenka 1985). The shear modulus reduces with growing normal strain to the crack. Fig. 9 shows the shear stiffness reduction due to the crack opening.

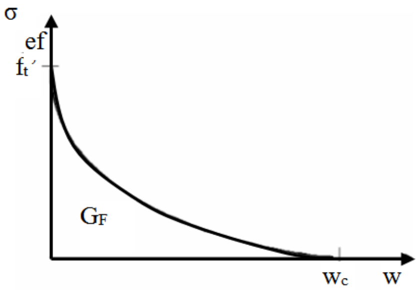


Fig. 6 Exponential tension softening, Červenka *et al*. (2012)

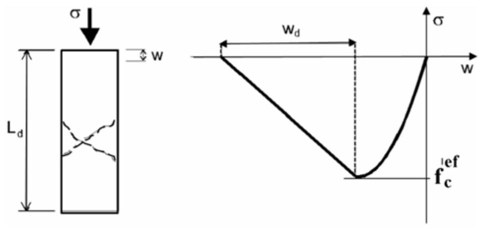


Fig. 7 Concrete cracked compression model, Červenka *et al*. (2012)

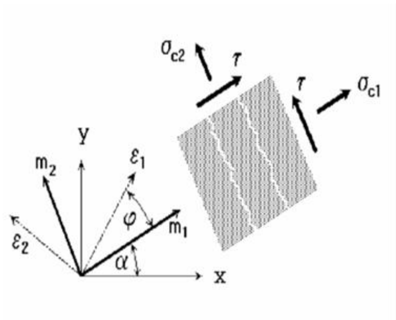


Fig. 8 Fixed crack model, Cervenka (1985)

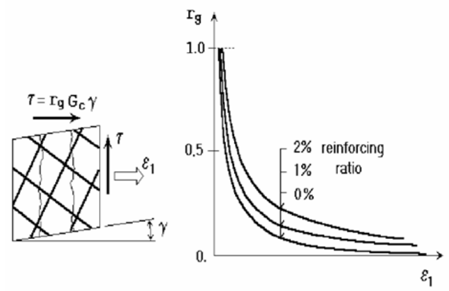


Fig. 9 Shear retention factor, Cervenka (1985)

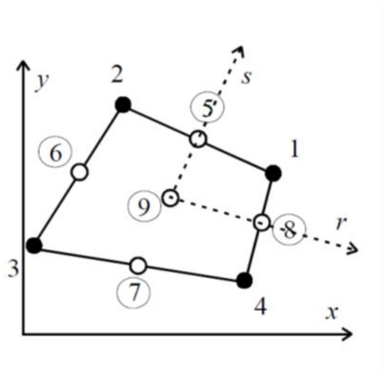


Fig. 10 Quadrilateral element, Červenka *et al*. (2012)

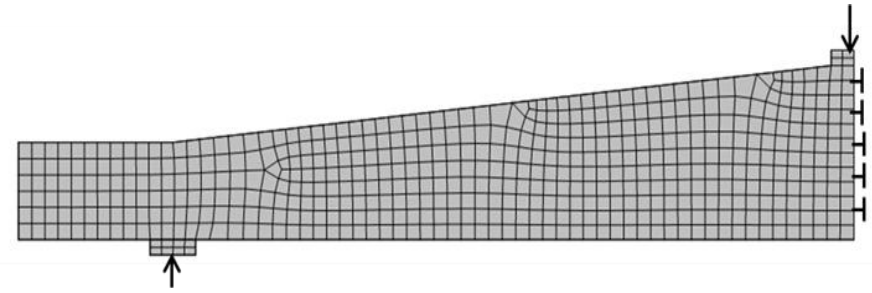


Fig. 11 Finite element modeling

The element that has been adopted in the modeling is a plane quadrilateral element. This element is anisoparametric element integrated by Gauss integration from 4 to 9 integration points for the case of bilinear or bi-quadratic interpolation. This element is suitable for plane 2D axisymmetric problems. Geometry, interpolation functions and integration points of the elements are given in Fig. 10. Also, Modified Newton-Raphson Method is adopted to solve the nonlinearity.

3.2 Verification of RC haunched beams

This article investigates 65 (prismatic 16 and haunched 49) RC beams gathered from the experimental works by (Debaiky and El-niema 1982, Stefanou 1983, Tena *et al*. 2008, Nghiep 2010). Available experimental studies are very limited and investigate the following parameters: concrete compressive strength, slenderness ratio, reinforcement bar strength, longitudinal reinforcement, shear reinforcement and beam geometries. All beams are simply supported, tested under point load and slender beams have a shear span ratio a/d greater or equal to 2.5.

To verify the numerical analysis models, the beams are analyzed using the 2D-ATENA program. To reduce the time consumption for analysis, the beams are modeled symmetrical and only one half size of the beam is considered as shown in Fig. 11.All beams are analyzed under load increment control with breakdown instability conditions. The beams are modeled using fixed smeared crack plain concrete elements which include discrete reinforcement rebar elements. Concrete is modeled using the SBETA element as mentioned previously. The mesh size of the finite element models for the RC haunched beams is unified along the inclined part. The results of the analysis are recorded using monitor points in specified locations of the member to compare with the experimental results. On the other hand, the program has capabilities to show the results of principle stresses and crack pattern development that reflect the beam behavior.

3.3 Finite element analysis results discussion

In this step, the tested beams are analyzed using the deterministic nonlinear finite element approach. The material properties and geometries are taken to be the same values with those experiments. The analysis results for the same authors are separated into two tables: haunched beams in Table 1(a) and prismatic beams in Table 1(b).

Tables 1(a)-(b) shows FE analysis of ultimate load capacity of the beams and corresponding modeling bias compared with experimental ultimate load capacity values. The results show a very

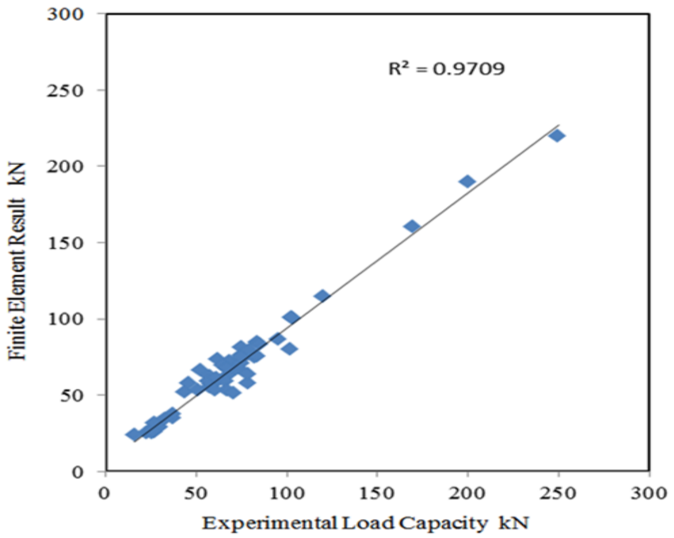
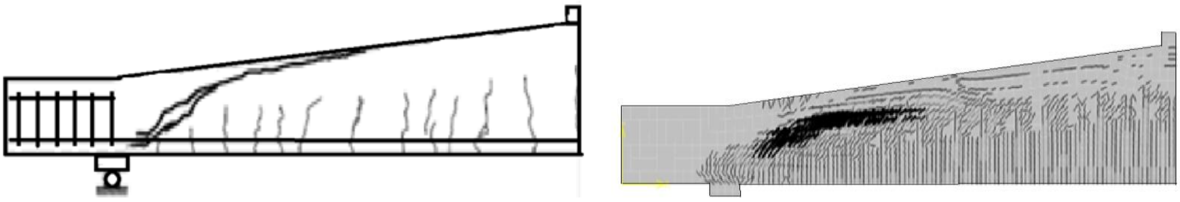
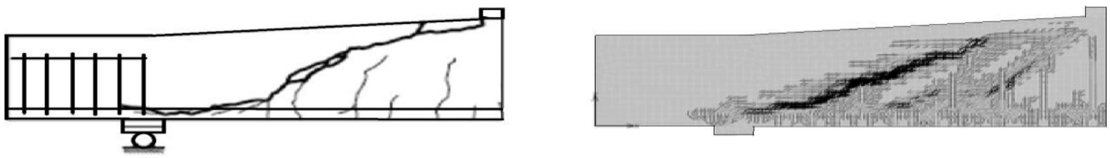


Fig. 12 Nonlinear Finite Element Analysis Results vs. the Experimental



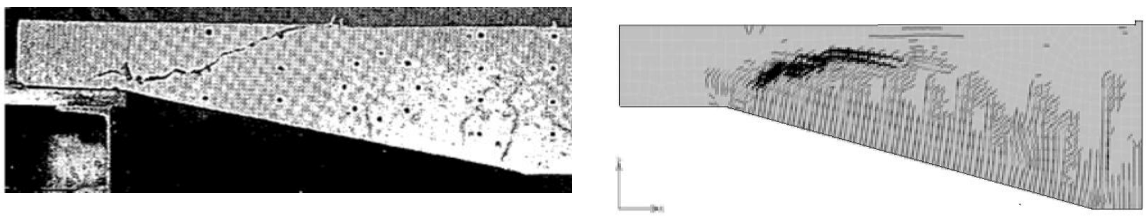
Experimental FE analysis

Fig. 13 Crack pattern of beam L3 (Nghiep 2010)



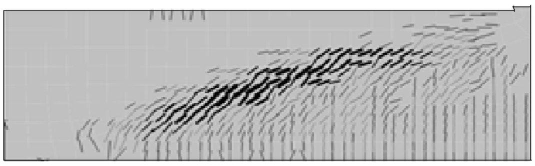
Experimental FE analysis

Fig. 14 Crack pattern of beam K2 (Nghiep 2010)



Experimental recorded FE analysis

Fig. 15 Crack pattern of beam B1 (Stefanou 1983)

Experimental recorded FE analysis

Fig. 16 Crack pattern of beam B5 (Stefanou 1983)

good correlation with the experimental results; where the modeling bias for the haunched beams equal to 1.025 and for the prismatic beams the modeling bias equal to 1.003. Fig.12 shows the correlation between experimental and FE results with a correlation coefficient ofR^2=0. 97.

The crack pattern of analyzed beams is displayed and matches very well with those of experiments as shown in Figs. 13-16. The similarity of crack pattern and the correlation of the results of FE analysis and experimental studies verify the effectiveness and accuracy of the proposed FE model.

4. Stochastic analysis

4.1 Basic assumptions

As properties of most construction materials exhibit complex random variation, it is generally difficult to model the real behavior by deterministic analysis. Probabilistic models are needed to quantify the uncertainties of these properties to develop realistic representations of the output and failure state of these systems and to obtain a rational and safe design. The stochastic response program FReET (Novak *et al*. 2014) has been used for the sampling of the models with randomly separated parameters, which enables to visualize the sensitivity and uncertainty of parameters' randomness. The program uses Latin hypercube sampling (LHS) to simulate the uncertainty of parameters. LHS simulation technique is a special type of Monte Carlo simulation method that uses stratification of the theoretical probability distribution function of input random variables (Nowak and Collins 2000). The sampling by LHS stratifies the probability range into N equivalent intervals, and then the simulation process takes one value from each interval as shown in Fig. 17. The mean value of each interval can be used in order to capture the means and variances. (Huntington and Lyrintzis 1998, Vorechovsky and Novak 2009):

$$x_{i,k} = N.\int_{y_{i,k-1}}^{y_i,k} x.fi(x)dx \tag{1}$$

where f_i is the probability density function and the limits of integration are given by

$$y_{i,k} = \phi_i^{-1}\left(\frac{k}{N}\right) \tag{2}$$

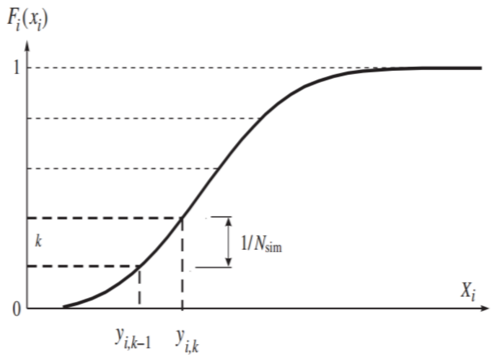


Fig. 17 Variables domain intervals(Novak *et al*. 2014)

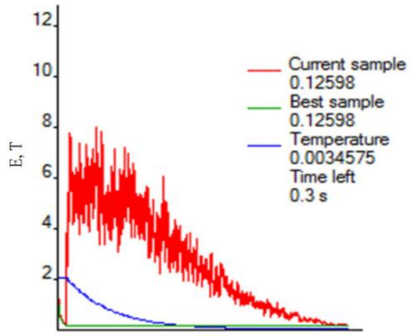


Fig. 18 Correlation error in the simulated annealing procedure(Novak *et al*. 2014)

where Φ^{-1} is the inverse cumulative probability distribution function CPDF for variable X_i.

The statistical correlations between the random variables are calculated using a robust technique based on the simulated annealing stochastic optimization method (Vorechovsky and Novak 2009). The optimization imposed that the difference between the prescribed sample and the generated sample correlation matrices should be as small as possible, Fig. 18.

4.2 Uncertainties of material properties and geometry

In this study, the uncertainties of the parameters are modeled as random variables described by the probability distribution functions (PDF). Then, a statistical assessment of such experimental data should be done, resulting in selection the most appropriate PDF (e.g. Gaussian, lognormal, Gumbel, Weibull). Also, it is possible to work directly with measured histograms (raw data) without mathematical model.

The material parameters that are taken to investigate the behavior of the RC haunched beams are the elastic modulus E_c, the compressive strength f_c, the tensile strength f_t and the fracture energy G_f for concrete. Also, the elastic modulus E_s and the yielding strength f_y of steel are considered as uncertain parameters. On the other hand, the geometry factors considered are the beam width, effective depth, longitudinal reinforcement area and shear reinforcement area. The values representing random parameters are generated according to probabilistic distribution methods.

Table2 summarizes the statistical parameters for the materials and geometries uncertainty parameters as random variables by the coefficient of variation. The coefficient of variation of each parameter is taken from previous studies available in the literature (Choi *et al*. 2004 and Strauss *et al*.2006). Table 3 shows the statistical correlation between the individual basic variables as a correlation matrix based on intuitive judgment and experimental results (Matos *et al*. 2010).

4.3 Stochastic analysis results

In the stochastic analysis step, each model is analyzed at least 50 times with random values of the parameters that are mentioned in Table 2. The stochastic analysis results in Tables1(a)-(b) are the outcome of more than 3200 independent analyses, which are presented as minimum load limit,

Table 2 Statistical parameters of the material and the geometry parameters

		Parameter description	Distribution	COV
Materials	Concrete	Modulus of elasticity	Normal	0.119
		Compressive strength	Normal	0.176
		Tensile strength	Normal	0.218
		Fracture energy	Weibull	0.17
	Steel	Modulus of elasticity	Normal	0.03
		Yield strength	Normal	0.05
Geometry		Beam width	Normal	0.045
		Effective depth	Normal	0.045
		Longitudinal area of steel	Normal	0.024
		Shear area of steel	Normal	0.024

Table 3 Correlation factors between the material parameters

	E_c	f_c	f_t	G_f
E_c	1	0.9	0.7	0.5
f_c		1	0.8	0.6
f_t			1	0.9
G_f				1

maximum load limit, mean value, standard deviation and coefficient of variation. Fig. 19 shows a sample of PDF and CDF histograms for the ultimate load capacity of a beam after stochastic analysis. The shape of the histogram indicates that the probabilistic parameters affect the ultimate load response significantly.

The stochastic analysis results represented by the bias and coefficient of variation; the statistical factors reflect the effect of material uncertainty, geometries (fabrication) uncertainty and modeling (professionalism) uncertainty. Material and geometry factors are obtained together directly from SARA and ATENA programs as a result of the material and geometry variation. The modeling factoris computed from the variance between the experimental recorded values and the deterministic finite element analysis.

The deterministic and stochastic finite element analysis results are summarized in Table 1(a) for RC haunched beams and Table 1(b) for the RC prismatic beams. The results show different coefficient of variation values between the RC haunched beams and RC prismatic beams. Coefficient of variations of shear capacity due to material properties and geometry parameters variance are equal to 0.067 for RC prismatic beams and 0.078 for RC haunched beam.

The values of coefficient of variation for shear capacity due to modeling are equal to 0.082 for RC prismatic beams and 0.113 for RC haunched beams. The difference of coefficient of variation between the two types of beams demonstrated that the uncertainty of modeling, material properties and geometries substantially influences the ultimate shear capacity of RC haunched beams more than the prismatic beams. Table 4, summarized the statistical factors for both of RC prismatic and RC haunched beams.

Table 4 Statistical factors

	Statistic factors	Material and geometry	Modeling
Prismatic beams	Coefficient of variation	0.067	0.082
	Bias	1.024	1.003
Haunched beam	Coefficient of variation	0.078	0.113
	Bias	1.025	1.028

5. Statistical parameters of RC haunched beams

The most important parameters that reflect the uncertainty of resistance are bias factor (λ) and coefficient of variation (V). The values of λ and $V$are determined by the following equations:

$$\lambda_R = \lambda_M \times \lambda_P \times \lambda_F \tag{3}$$

$$V_R = \sqrt{V_M^2 + V_P^2 + V_F^2} \tag{4}$$

where R indicates the resistance, M indicates the material, P indicates the professionalism (modeling) and F indicates the fabrication (geometry).

Due to the lack of experimental data for RC haunched beams, the statistic factors are predicated from the deterministic and stochastic analysis results and presented in Table 4. The values of statistical resistance factors calculated according to Eq. 3 and 4. For the RC prismatic beam, the value of λ_R and V_R are equal to 1.026 and 0.106 respectively, and for the RC haunched beams the resulted values of λ_R and V_R are 1.053 and 1.37 respectively. The determined values of resistance statistic parameters show that the risk is higher for RC haunched beams as compared to RC prismatic beams regarding material and geometry parameters uncertainty.

6. Reliability analysis

6.1 Concepts

Structural reliability is the ability of a structure or a structural member to fulfill the specified requirements for which it has been designed (EN 1990) i.e. the element fails if the applied load (Q)exceeds the resistance of the member (R). The corresponding limit state function can be simplified as follows:

$$\begin{aligned} g &= R - Q \\ R &= \phi . R_n .(X_1, X_2, ..., X_n) \\ Q &= \lambda_D . D + \lambda_L . L \end{aligned} \tag{5}$$

wherex_i represents the random parameters, D dead load, L live load, ϕ reduction factor and λ bias factor.

The performance of the structureis assessed by the failure probability of limit state function which is given explicitly as follows (see Fig 20):

$$P_f = P(g < 0) \tag{6}$$

6.2 Reliability Index

The reliability index is the shortest distance from the origin to the failure surface, line $g(Z_R,Z_Q)=0$ as shown in Fig. 21, where Z_R is the reduced variable for resistance and Z_Q is the reduced variable for the load. This definition was introduced by (Hasofer and Lind 1974). The reliability index β is related to the probability of failure, P_f, by:

$$\beta = -\phi^{-1}(P_f) \tag{7}$$

whereϕ^{-1} is the inverse of the probabilistic distribution function, P_f is the failure probability and β is the reliability index. The expression of the reliability index is expressed in Eq. 8:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{8}$$

In this part of the study, reliability indexes of RC haunched and prismatic beams are calculated using Eq. (8). Two ultimate limit state load cases are considered for RC haunched beams and prismatic beams. The load case in Eq. (9) is specified by ASCE-7 and ACI-318 (2011) whereas;Eq. (10) is adopted from ACI-318 (1999). This equation (Eq. (10)) is no longer valid in the newer versions of ACI-318 code and is useful for comparison purpose only.

$$1.4.D < \phi R \; L=0$$

$$1.2.D + 1.6.L < \phi R \tag{9}$$

$$1.4.D + 1.7.L = \phi.R \tag{10}$$

Where D is the dead load, L is the live load and ϕ is the resistance reduction factor.

The statistical load factors λ_D, λ_L, V_D and V_L are shown in Table (5) are taken as suggested in the literature (Ellingwood *et al*. 1980 and Nowak and Szerszen 2003). For the RC haunched beams, the values of the statistical resistance factors were determined previously as λ_R =1.053 and V_R=0.136 and for RC prismatic beams as λ_R=1. 026 and V_R=0.106.Three levels of reduction values (ϕ= 0.8, 0.85and 0.9) are considered for each limit state function. The reliability indexes are calculated for variousratios of $D/(D+L)$ratio ranging from (0.3 to 1.0). Figs. 22 and 23 show the reliability indexes of both limit state functions.

The target reliability index of RC beam depends on the consequence of failure, cost and feasibility of structural use. The load ratio $D/(D+L)$ for RC beams usually varies from 0.3 to 0.7. Table 6 shows the calculated average value of the reliability index for a typical $D/D+L$ ratio of 0.5 and resistance reduction factor 0.85. The values of the reliability indexes for RC haunched beams and RC prismatic beams are found to be different from each other. The target reliability indices for

Table 5 Statistical factors for loads

Load component	Bias factor	COV
Dead load	1.05	0.1
Live load	1.0	0.18

Table 6 Reliability Index (β) at ($D/D+L$)=0.5

Reduction factor ϕ	ASCE7		ACI 318	
	Haunched beams	Prismatic beams	Haunched beams	Prismatic beams
0.8	2.82	3.24	3.25	3.77
0.85	2.56	2.9	3.00	3.45
0.9	2.31	2.58	2.77	3.14

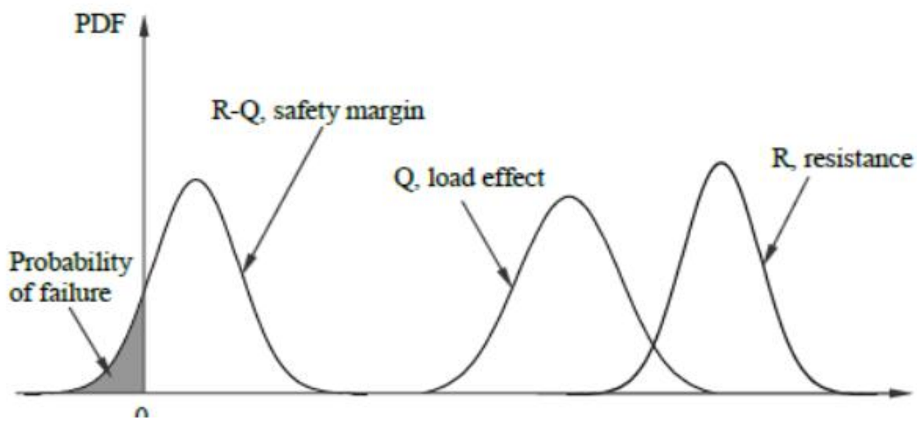


Fig. 20 Probability density function of load, resistance and safety margin(Nowak and Collins 2000)

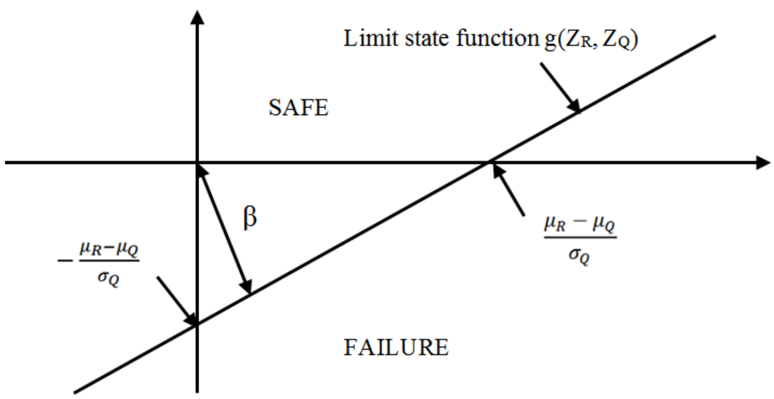
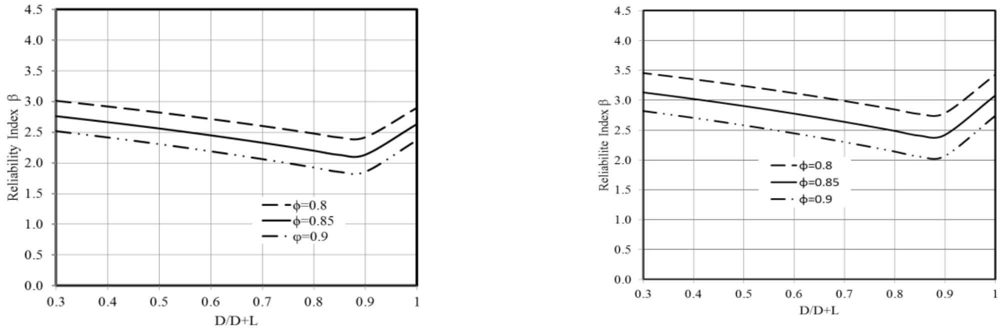
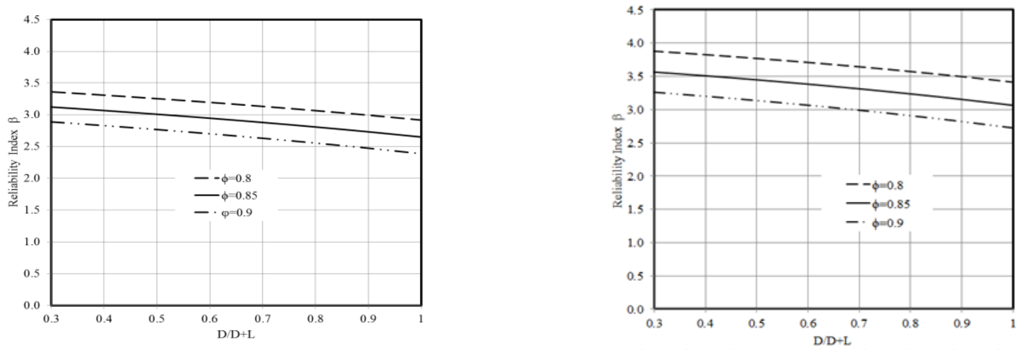


Fig. 21 Reliability Index definition (Nowak and Collins 2000)



a. Haunched beam b. Prismatic beam

Fig. 22 Reliability index for ASCE-7 and ACI-318 (2011) limit state functions



a. Haunched beam b. Prismatic beam

Fig. 23 Reliability index for AC1-318 (1999) limit state function

RC haunched beams are computed equal to 2.56 for (ASCE-7 and ACI-318(2011)) and 2.9 for ACI-318(1999).

On the other hand, for RC prismatic beams the values of the target reliability indices are found equal to 3.0 for (ASCE-7 and ACI-318(2011)) and 3.45 for ACI-318(1999). As a result of these findings, it can be concluded that the risk of failure for RC haunched beams is higher than that of the RC prismatic beams. The target reliability indices for prismatic beams in this work were found to be different than previous work by (Szerszen and Nowak 2003). The reason is due to different deterministic analysis models as well as different material statistical parameters.

7. Conclusions

This study is the first one in literature investigating the behavior of RC haunched beams using deterministic and stochastic nonlinear finite element analysis. The motivation to discuss this topic is the lack of studies investigating the behavior of this type of RC members. Available data on experimentally tested RC haunched beams are collected from the literature. At first, the beams are analyzed by using a specialized FE element program (2D-ATENA)considering deterministic values of the material properties. The second step of the analysis is the stochastic FE analysis accomplished by using a module of 2D-ATENA known as SARA, which is combined with the stochastic package called FReET. In this step the material properties and geometry parameters are taken as probabilistic values for the stochastic analysis. At least 50 samples are generated for each beam using the LHS technique by FReET program.

The deterministic analysis results show a perfect agreement with the ultimate load capacity of experimental tested beams with a coefficient of correlation of R^2=0.97. The modeling bias was found to be1.003 for RC prismatic beams and 1.028 for RC haunched beams. Additionally, a perfect match in crack pattern is observed between the experimental tested beams and FE analysis results. These results verified the accuracy and efficiency of ATENA program and its material library for the use of probabilistic material properties.

The parameters that are considered as uncertain parameters include the properties of concrete and reinforcement in addition to the geometry parameters. The results of stochastic analysis demonstrate that the uncertainty of material properties and geometry parameters substantially influence the ultimate load capacity of RC haunched beams.

The statistical parameters for ultimate shear strength capacity, bias factor λ_R and coefficient of variation V_R are calculated from three uncertainty values:*M*, materialproperty factor, *F*, geometry (fabrication) factor and *P*, professional (modeling) factor. Computed statistical parameters of resistance are found to beλ_R=1.053 and $V_{R.}$=1.136 for RC haunched beams whereas the values for RC prismatic beams areλ_R=1.026 and V_R=0.106.

The last part of this study is based on the reliability analysis of both types of beams using the ultimate state functions of (ASCE-7 and ACI-318(2011)) and ACI-318(1999). The results of the analyses have shown that the safety margin due to failure in ACI-318(1999) is higher than (ASCE-7and ACI-318(2011)).Moreover, reliability indexes of RC haunched beams and RC prismatic beams are also computed to compare the risk of failure where RC haunched beams show a higher risk than RC prismatic beams.

Appendix

See the Tables 1(a)-(b)

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Appendix

Table 1(a) Results of deterministic and stochastic FE analyses for RC haunched beams.

Beam ID	Experimental Data								Deterministic Analysis		Stochastic analysis					V_{FEM}/V_{Mean}
	b cm	d_s cm	α^o	ρ_s^*	ρ_v^*	f_c MPa	f_y MPa	V_{EXP} kN	V_{FEM} kN	Model bias	V_{min} kN	V_{max} kN	V_{Mean} kN	St.D	C.O.V.	Bias
Debaiky and El-niema (1982)																
A2	12	11	9.46	0.00198	0.03	20	461	58	55.5	1.045	39.5	60	52.46	4.2	0.08	1.058
B3	12	11	9.46	0.00198	0.03	18.6	461	65.5	59	1.110	40	65	55.56	5.63	0.101	1.062
B4	12	11	9.46	0.00198	0.03	21	461	101.5	80	1.269	57.5	84	74.81	4.89	0.065	1.069
C2	12	11	9.46	0.00198	0.03	28.2	461	72	67	1.075	58	75	64.97	9.06	0.139	1.031
D3	12	11	9.46	0.00396	0.03	29.6	461	69	72	0.958	66	86	69.07	5.5	0.079	1.042
D4	12	11	9.46	0.00419	0.03	27.5	461	58.5	57	1.026	51	59	54.38	1.93	0.035	1.048
F3	12	11	9.46	0.00235	0.019	21.5	461	44	52.5	0.838	32.5	57.5	49.5	5.31	0.107	1.061
F4	12	11	9.46	0.00235	0.024	21	461	45.5	58.5	0.778	38.5	63.5	55.06	5.59	0.102	1.062
A3	12	18.5	4.76	0.00198	0.03	17.8	461	78.5	58	1.353	45	65	55.49	5.15	0.093	1.045
C3	12	18.5	4.76	0.00198	0.03	27.8	461	52	66	0.788	52	77	67.313	4.87	0.073	0.980
A4	12	33.5	-4.76	0.00198	0.03	22	461	51.3	53	0.968	37.5	62	51.48	5.61	0.109	1.030
C5	12	33.5	-4.76	0.00198	0.03	31.4	461	57.5	62	0.927	51	61	55.8	2.64	0.047	1.111
E2	12	33.5	-4.76	0.00314	0.032	33.5	461	75	74	1.014	58	76	69.12	4.46	0.065	1.071
A5	12	41	-9.46	0.00198	0.03	22.5	461	57	59	0.966	39.5	60	52.575	4.14	0.079	1.122
B5	12	41	-9.46	0.00198	0.03	20.6	461	78.5	64	1.227	57	80	69.2	6.47	0.094	0.925
C4	12	41	-9.46	0.00198	0.03	31.1	461	61	61	1.000	53	69	61.9	3.55	0.058	0.985
D5	12	41	-9.46	0.00396	0.03	28.9	461	65	70	0.929	47.5	79	65.85	5.76	0.088	1.063
D6	12	41	-9.46	0.00419	0.03	32.2	461	75	71	1.056	53	80	67.72	6.32	0.093	1.048
E1	12	41	-9.46	0.00314	0.032	34.8	461	95	87	1.092	66	93	81.4	6.82	0.084	1.069
F1	12	41	-9.46	0.00235	0.019	21.1	461	67	53.5	1.252	37.5	59.5	50.58	4.23	0.084	1.058
F2	12	41	-9.46	0.00235	0.024	20.8	461	70.5	51.5	1.369	36.5	55.5	48.82	3.88	0.08	1.055
Tena *et al*. (2008)																
TASC1-0	22	41	-3.07	-	0.0263	32.1	412	67.5	67	1.007	55	77.5	67.8	5.09	0.075	0.988
TASC2-0	22	41	-6.12	-	0.0308	29.5	412	60	53	1.132	47.5	62.5	53.6	3.38	0.063	0.989
TASC3-0	22	41	-9.13	-	0.0372	23.6	412	37.5	37.5	1.000	27.5	40	35.2	3.34	0.095	1.065
TASC4-0	22	41	-12.1	-	0.047	28.1	412	30	29	1.034	23	35	29.66	2.1	0.071	0.978
TASC1-1	22	41	-3.07	0.0025	0.0263	26.9	412	200	190	1.053	165	210	187.8	8.7	0.046	1.012
TASC2-1	22	41	-6.12	0.0025	0.0308	29.2	412	170	160	1.063	130	170	154.4	8.95	0.058	1.036
TASC3-1	22	41	-9.13	0.0025	0.0372	28.8	412	120	115	1.043	100	130	116.1	6.17	0.053	0.991
TASC4-1	22	41	-12.1	0.0025	0.047	21.1	412	80	79	1.013	64	84	73.6	4.96	0.067	1.073

Table 1 Continued

Beam ID	Experimental data								Deterministic analysis		Stochastic analysis					V_{FEM}/ V_{Mean}
	b mm	d_s mm	α^o	ρ_s^*	ρ_v^*	f_c MPa	f_y MPa	V_{EXP} kN	V_{FEM} kN	Model bias	V_{min} kN	V_{max} kN	V_{Mean} kN	St.D	C.O.V.	Bias
Beams with shear reinforcement																
Stefanou (1983)																
B1-Ib	10	10	13.39	-	0.02	19.9	361	25	26	0.962	21	29	25.6	1..97	0.077	1.016
B2-Ia	10	15	8.13	-	0.013	19.9	361	26.5	27	0.981	16	30	25.02	2.78	0.111	1.079
B2-Ib	10	15	8.13	-	0.02	19.9	361	30	32	0.938	24	34	30.92	2.19	0.071	1.035
B3-Ia	10	10	13.39	-	0.013	15.7	361	27.5	26	1.058	21	28	24.51	1.64	0.067	1.061
B3-Ib	10	10	13.39	-	0.02	15.7	361	25	25	1.000	24	31	27.4	1.48	0.054	0.912
B4-Ia	10	15	8.13	-	0.013	15.7	361	26.5	28	0.946	22	31	27.46	2.0	0.073	1.020
B4-Ib	10	15	8.13	-	0.02	15.7	361	32.5	34	0.956	26	35	30.8	2.43	0.079	1.104
B5- Ias	10	10	13.39	0.0032	0.013	19.9	361	22.5	25	0.900	19	29	25.18	1.71	0.068	0.993
B5-Ibs	10	10	13.39	0.0032	0.02	19.9	361	27	28	0.964	20	33	28.46	2.65	0.093	0.984
B6- Ias	10	15	8.13	0.0032	0.013	19.9	361	29	28	1.036	24	30	27.4	1.3	0.048	1.022
B6-Ibs	10	15	8.13	0.0032	0.02	19.9	361	37.75	35	1.079	26	41	35.96	2.73	0.076	0.973
B7- Ias	10	10	13.39	0.0032	0.013	15.7	361	29	28	1.036	19	28	24.63	1.75	0.071	1.137
B8- Ias	10	15	8.13	0.0032	0.013	15.7	361	27.5	27	1.019	23	31	28.1	1.71	0.061	0.961
Nghiep (2010)																
2L	20	20	3.95	-	0.0157	49.4	550	75	76.5	0.980	67.5	87.5	75.75	3.57	0.047	1.010
3L	20	15	5.91	-	0.0157	50.2	550	66.5	68	0.978	45	82.5	67.65	12.56	0.093	1.005
2K	20	24.3	3.95	-	0.0157	54	550	83.5	75.5	1.106	67.5	105	84.75	7.65	0.091	0.891
3K	20	20	6.71	-	0.0157	54	550	79.5	79.4	1.001	80	210	93.3	12.02	0.064	0.851
4K	20	15	10.01	-	0.0157	54	550	85	84	1.012	50	100	84.4	20.24	0.12	0.995
Average										1.028					0.078	1.025
St.D										0.116						

*The reinforcement's ratio in middle section

Table 1(b) Results of deterministic and stochastic FE analyses for RC prismatic beams

Beam ID	Experimental Data								Deterministic Analysis		Stochastic analysis					V_{FEM}/V_{Mean}
	b mm	d_s mm	α^o	ρ_s^*	ρ_v^*	f_c MPa	f_y MPa	V_{EXP} kN	V_{FEM} kN	Model bias	V_{min} kN	V_{max} kN	V_{Mean} kN	St.D	C.O.V.	Bias
Beams with shear reinforcement																
Debaiky and El-niema (1982)																
A1	12	26	0	0.00198	0.03	25	461	73.5	68	1.081	54	75	68.31	4.9	0.072	0.995
B1	12	26	0	0.00198	0.03	25	461	68.8	70	0.983	54	74	67.34	4.1	0.061	1.040
B2	12	26	0	0.00198	0.03	17.6	461	82.5	75	1.100	69	93	82.12	4.64	0.057	0.913
C1	12	26	0	0.00198	0.03	28.6	461	72.5	71.5	1.014	56	75	68.36	4.2	0.061	1.046
D1	12	26	0	0.00396	0.03	30.4	461	83.5	84.5	0.988	60	89.5	78.04	6.16	0.079	1.083
D2	12	26	0	0.00419	0.03	31.2	461	75	75	1.000	59	90	74.94	5.94	0.079	1.001
E3	12	26	0	0.00314	0.03	32	461	62.5	74	0.845	52	80	70.58	5.26	0.074	1.048
F5	12	26	0	0.00235	0.024	20.6	461	67.5	63	1.071	45	64	58.2	4.29	0.073	1.082
F6	12	26	0	0.00235	0.019	20.9	461	62.5	59	1.059	45	61	54.22	4.04	0.074	1.088

Table 1(b) Continued

Beam ID	Experimental Data								Deterministic Analysis		Stochastic analysis					V_{FEM}/V_{Mean}
	b mm	d_s mm	α^o	ρ_s^*	ρ_v^*	f_c MPa	f_y MPa	V_{EXP} kN	V_{FEM} kN	Model bias	V_{min} kN	V_{max} kN	V_{Mean} kN	St.D	C.O.V.	Bias
Tena et al. (2008)																
TASC0-1	22	41	0	0.0025	0.0229	31.5	412	250	220	1.136	190	225	217.8	13.94	0.064	1.010
TASC0-0	22	41	0	-	0.0229	33.4	412	75	75	1.000	67.5	92.5	80.25	5.38	0.067	0.935
Nghiep (2010)																
1L	20	30	0	-	0.0157	48	550	75.5	81.5	0.926	67.5	92.5	81.8	9.72	0.059	0.996
1K	20	30	0	-	0.0157	54	550	75.5	75	1.007	60	82.5	74.13	9.3	0.063	1.012
Stefanou (1983)																
B9-Ia	10		0	-	0.013	18.6	361	26.5	27	0.981	22	29	25.78	1.44	0.056	1.047
B9-Ib	10		0	-	0.02	18.6	361	27.3	32	0.853	24	33	30.22	1.93	0.064	1.059
Average										1.003					0.067	1.024
St.D										0.082						

*The reinforcement's ratio in middle section