Computers and Concrete, *Vol. 14, No. 5 (2014) 503-526*
DOI: http://dx.doi.org/10.12989/cac.2014.14.5.503

Failure criteria of concrete- A review

Muthukumar G*[1] and Manoj Kumar[2a]

[1]*Civil Engineering Department, BITS, Pilani – 333 031, India*
[2]*Civil Engineering Department, BITS, Pilani – 333 031, India*

(*Received June 24, 2012, Revised September 20, 2014, Accepted October 1, 2014*)

Abstract. Concrete is a versatile construction material used in many engineering structures. The design of concrete structures requires a thorough understanding of their material properties under various loading conditions. Several experimental investigations have been carried out to examine the behavior of concrete. This paper is an attempt to summarize the behavioral aspects of concrete under different loading conditions. Failure models developed out of these experimental investigations are reported in this paper with their merits and demerits.

Keywords: failure criteria; concrete; uniaxial; biaxial; triaxial; unified

Nonmenclature

σ, σ_i – Principal normal stress, principal normal stress in direction i.

τ – Shear stress

I_1 – First invariant of stress tensor

J_2 – Second invariant of deviatoric stress tensor

$$= \frac{1}{6}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2\right]$$

J_3- Third invariant of deviatoric stress tensor

σ_{oct}- Octahedral normal stress = $I_1/3$

τ_{oct} – Octahedral shear stress = $\left[\frac{2}{3}J_2\right]^{1/2}$

*Corresponding author, Lecturer, E-mail: muthug@bits-pilani.ac.in
[a]Associate Professor & Head

http://www.techno-press.org/?journal=cac&subpage=8 ISSN: 1598-8198 (Print), 1598-818X (Online)

1. Introduction

Concrete is a versatile material used for the construction of civil engineering structures. The inevitable use of concrete has sparked many investigations on its behavior for various nature (tension, compression) and types of loading (uniaxial, biaxial, triaxial) conditions. To understand the behavior of concrete completely, there is a need to investigate its behavior in all ranges i.e. compression, tension, combinations of tension and compression. In order to predict the strength of concrete in compression and tension, several experimental investigations have been reported so far. These tests have been done on concrete subjected to uniaxial, biaxial and triaxial loadings. The behavior of concrete changes drastically when the nature of loading changes from compressive to tensile. When subjected to tension, the response of concrete is same irrespective of uniaxial or biaxial loads. Nevertheless, the concrete shows different behavior for uniaxial and biaxial loads under compression. Thus, the nature and type of loading plays a major role on the behavior of concrete, thus highlighting the importance of the current study.

Several uniaxial, biaxial, triaxial tests have been reported in literature (Kupfer *et al*. (1969); Kotsovos and Newman (1977); Mills and Zimmerman (1970); Lee *et al*. (2004); Sinha *et al*. (1964), Ren *et al*. (2008), Fehking *et al*. (2011)). In order to simulate the actual behavior of concrete in compression and tension, several analytical models have been developed. The analytical models are formulated based on microscopic, mesoscopic and macroscopic behavior of concrete. The models based on macroscopic behavior are invariably used in practice especially in construction field. The deformation patterns and stress-strain curves are the two important indicators of the behavior of concrete. In order to define completely the deformational and stress-strain behavior of concrete, one must analyze the structure till failure. The deformation of the structure is linearly elastic till the yield limit, and beyond this point plastic deformation (irreversible) takes place. A model should be capable of producing the above mentioned behavior till failure. One such way of obtaining a model is based on plasticity theory (Chen (1982)).

The plasticity theory defines the yield limit as the limit below which the material property remains elastic and any further loading beyond this yield limit results in plastic flow. In the case of elastic-perfectly plastic, the initial yield surface becomes a failure/bounding surface, reflecting the increase in strain without further change in stress. Nevertheless, for concrete having elasto-plastic behavior, strain hardening and strain softening behavior are to be included. Strain hardening is the region between the yield and peak stress reflecting the hardening nature of concrete with the increase in stress value. Once the concrete hardens and attains the peak stress, further loading results in the decrease in stress with the increase in strain, thus enabling the softening behavior. Thus initial loading surface/yield surface is allowed to expand on the application of load resulting in strain-hardening behavior of concrete, defining the subsequent loading surface. Hardening may be isotropic, kinematic and mixed hardening. Isotropic hardening assumes that the expansion of initial loading surface takes place uniformly thus completely neglecting Bauschinger effect. It essentially means that direction of strain and stress are assumed to progress in the same fashion. Thus, the isotropic hardening is only valid for monotonic loading without any load reversals. On the other hand, the kinematic hardening assumes that the loading surface translates as a rigid body in stress space thus accounting for Bauschinger effect. The typical isotropic hardening rule for the monotonic loading condition is shown in Fig. 1. At times, mixed hardening rules can be used which combines the isotropic and kinematic hardening.

In general, plasticity theory is based on either stress-based formulation or strain-based formulation. Many failure theories have evolved based on stress-based formulation. However, in

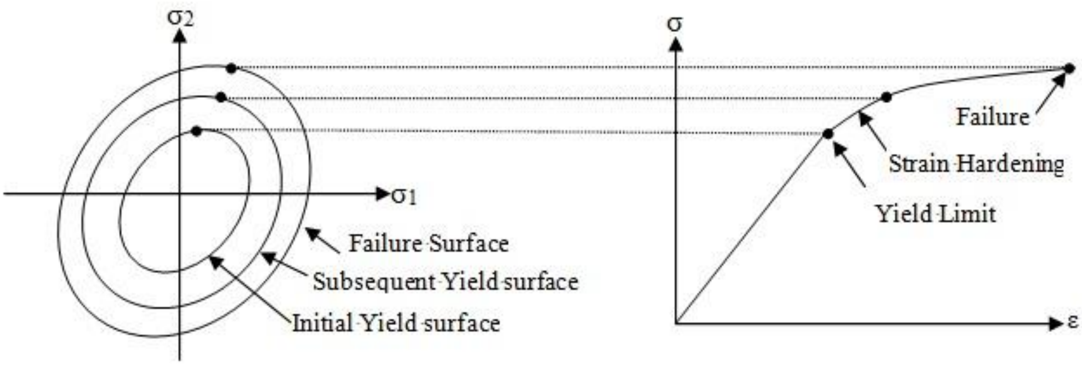

Fig. 1 Isotropic Hardening with expanding yield surfaces and the corresponding uni-axial

defining the loading/unloading criteria, there is a little bit of ambiguity in expressing the loading function in terms of stresses. This ambiguity is avoided in strain-based formulation. While stress-based formulation is based on Drucker's postulate, strain-based formulation is based on II'yushin's postulate (Chen (1982)). Keeping in view the fact that the not many failure criteria are developed in strain-based formulation, the stress-based formulation is adopted in practice.

To this end, the behavior of concrete under monotonic and cyclic loading under various combinations of tension & compression is studied. In order to adopt a mathematical model to simulate completely the experimental behavior, it is necessary to investigate the various yield/failure models. To keep abreast with the latest developments in material modeling, a detailed review has been made in identifying the various failure models adopted for the concrete.

2. Experimental behaviour of concrete

The behavior of plain concrete has been found to be complex due to inherent characteristics of the material. Several experimental works have been performed to examine mechanisms that lead to the propagation of failure from initial stage to ultimate collapse. On the other hand, the presence of reinforcement in the concrete makes it more complex. It has been reported that the presence of reinforcement alters the behavior of concrete considerably. Nevertheless, Brestler and Pister (1958) have suggested that the conditions responsible for local failure are essentially the same for both plain and reinforced concrete, thus highlighting the importance of examining the behavior for plain concrete. Keeping in view the above mentioned facts, a brief summary has been made on the various experimental investigations carried out on predicting the behavior of plain concrete under uni-axial, biaxial and triaxial loadings for different combinations of tension and compression.

In order to plot stress-strain curve under compressive loading in uni-direction, strength tests on cylindrical or cubical concrete specimens are obtained at an age of 28 days. The various stages of stress-strain curve which characterizes different behavior of concrete are mentioned in the literature (Hognestad *et al*. (1955); Kwak and Filippou (1990); Fardis *et al*. (1983), Tsai (1988)).

Carreira (1985) has reported that the four parameters, namely uni-axial compressive strength, strain corresponding to uniaxial compressive strength, initial tangent modulus and ultimate strain at failure can be regarded as the characteristic values for the stress-strain curve of concrete under uni-axial compression. The direct tensile tests by various researchers (Hughes and Chapman (1966); Ansari (1987), Gopalarathnam and Shah (1985)) show that the load-elongation curve presents a peak followed by a softening branch. Even though uniaxial loading conditions characterize the different stages of deformation it does not simulate the actual behavior of structure which are generally subjected to multiaxial loading conditions. Therefore, it is essential to predict the behavior of concrete under multi-axial stress state to obtain the more generalized response (Kupfer (1973); Darwin and Pecknold (1977); Cedolin and Mulas (1984); Hussein and Marzouk (2000); Bellamy (1961); Tasuji *et al*. (1978); Buyukozturk and Nilson (1971). Brestler and Pister have performed experimental tests on 65 tubular specimens of plain concrete subjected to combined stresses to predict the failure of the concrete specimens (Brestler and Pister (1958)). Based on experimental results they suggested that strength of the concrete is a function of the state of stress and cannot be predicted without considering the interaction of stresses.

Kupfer (1969, 1973) has conducted the test on plate-type specimens (20 × 20 × 5 cms) under proportional monotonically increasing biaxial loading. Nevertheless, concrete strength under biaxial tension was found to be the approximately equal to the uni-axial tensile strength (Kupfer *et al*. (1969); Kwak and Filippou (1990)). Tasuji *et al*. (1978) have conducted experimental investigations on thin plain concrete plates subjected to biaxial loading which includes all combinations of compressive and tensile loadings. It has been reported that the concrete possesses higher compressive strength when subjected to biaxial compression as compared to uniaxial compression (Tasuji *et al*. (1978); Liu *et al*. (1972)). On the other hand, when the concrete is subjected to combined compression and tension, the compressive strength has been reported to decrease linearly as the tensile stress increases (Tasuji *et al*. (1978); Kupfer *et al*. (1969); Kwak and Filippou (1990)).

On the basis of above mentioned uni-axial and bi-axial tests on concrete, it is emphasized that the inclusion of the three basic parameters in any analytical model namely uni-axial compressive strength, biaxial compressive strength and uni-axial tensile strength is indispensable.

To understand the behavior of concrete subjected to tri-axial loading, several experimental investigations have been done (Mills and Zimmerman (1970), Imran and Pantazopoulou (1991)). Mills and Zimmerman (1970) have reported that majority of investigations have been performed on cylindrical specimens where two principal stresses out of three retain the same value. They conducted the test on cubical specimens and concluded that cubical specimens provide a realistic estimation of failure strength as it incorporates the effect of intermediate stress component.

It has been reported by Gardner (1969) that all mechanical properties can be improved with increase in the confinement. Gardner (1969) and Zhi *et al*. (1987) carried out the biaxial and triaxial experiments to investigate the influence of confinement on compressive strength of concrete and observed that confinement significantly enhances the compressive strength. Linhua *et al*. (1991) found that strength of concrete under triaxial compressive-compressive-tensile loading is higher compared to strength of concrete subjected to biaxial Compression-tension loading.

On the other hand, cyclic loading occurs when there is a load reversal with several loading, unloading and reloading cycles. Loading, unloading & reloading constitutes a hysteresis loop. In order to understand the behavior of concrete under dynamic effects, it is essential to know the behavior of concrete under compressive and tensile cyclic loadings. The concrete, subjected to compressive loading of high amplitude and low cycle is predominantly significant from earthquake point of view. When the number of cycles is large, the continuous growth of micro cracks can lead to the reduction in the strength of concrete (Sinha *et al*. (1964); Karsan and Jisra

(1969); Reinhardt (1986)). Vecchio (1990) has discussed the importance of cyclic load modeling of reinforced concrete and analyzed the shear wall using nonlinear elasticity model. Several experimental investigations have been done on concrete subjected to uniaxial cyclic loading (Lam (1980); Karsan and Jirsa (1969); Sinha *et al*. (1964)).

Mlakar *et al*. (1985) examined the bi-axial tensile-compressive behavior of concrete under dynamic loading and concluded that the tensile stress at failure decreases while the compressive stress gets increased. This conclusion was similar to one observed under monotonic loading. It has been observed that strength envelope for cyclic and monotonic loading has not been found to show significant difference (Buyukozturk and Tseng (1984); Lan and Guo (1999)).

Based on the above experimental works, it has been concluded that confinement of a concrete has a very strong influence on compressive strength and hence should be sufficiently incorporated in any analytical model. Moreover, since condition responsible for local failure is essentially the same regardless of randomness of loading, information available on the monotonic loading can be used effectively in developing failure model capable of capturing the responses under monotonic and cyclic loading. In order to describe the response of structures under monotonic loadings, various failure models have been developed (Willam and Warnke (1975), Chen (1982), Bagheripour (2011), Karam and Tabbara (2012)). These models are categorized from one-parameter through five-parameter models depending upon the number of parameters appearing in the expression of the failure surface. The next section briefly summarizes various characteristic features of failure models.

3. Basic features of failure criteria

The capacity of the material at any state of stress is defined by criterion known as failure criterion/yield criterion. The yield criterion is the limiting point at the yield of a material representing the threshold of elastic and plastic deformations; whereas failure criterion is the limiting point at failure stage. In general yield criterion and failure criterion may be interchangeably used as both represent a certain state of material. Nevertheless, a single yield criterion may not be suitable to capture characteristics of all materials. Hence, it is inevitable to develop several yield criteria for different materials according to the requirement. Therefore, the choice of yield criteria considerably affects the estimation of strength characteristics of the material (Chen (1982), Yu (2002b, 2004)). Yield limit for uniaxial case is represented by a point whereas for biaxial and triaxial cases, it is represented by a curve and surface respectively.

The general form of the failure surface can be described in terms of principal stresses and strain hardening parameters $F(\sigma_1,\sigma_2,\sigma_3,\kappa_1,\kappa_2,....)=0$. In the above failure surface, $\sigma_1,\sigma_2,\sigma_3$ are principal stresses and κ_1,κ_2 are material constants to be determined experimentally. At every point inside the stressed body, there exist at least three planes called principal planes. The directions normal to these planes are called principal directions and the stresses along these directions are called principal stresses. The failure surface can at best be represented in terms of three stress invariants, I_1, J_2 and J_3 as

$$\text{(Chen 1982)} \quad F(I_1, J_2, J_3) = 0 \tag{1}$$

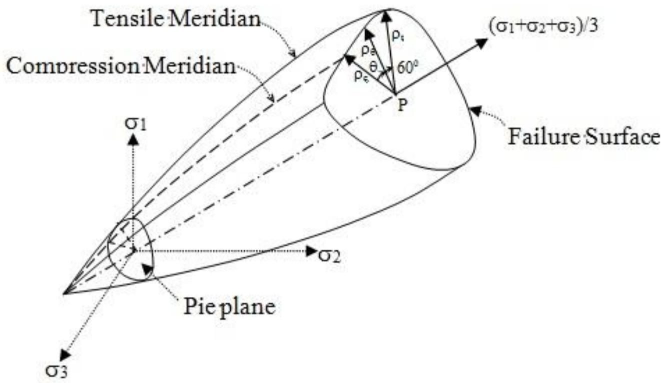

Fig. 2 Yield surface in 3-dimensional principal stress space

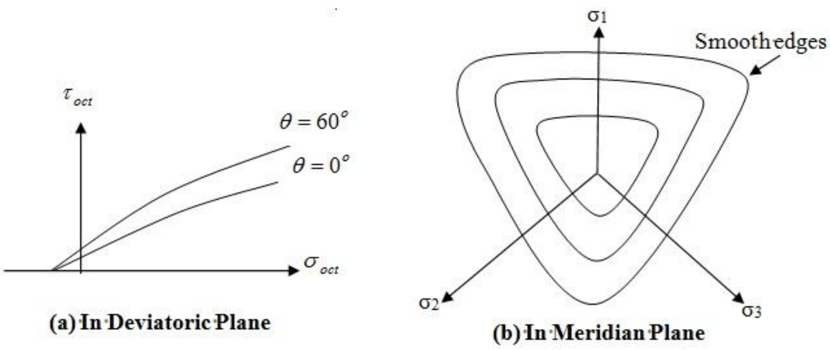

Fig. 3 Characteristics of failure surface

where $J_2 = \frac{1}{3}\left(I_1^{\ 2} - 3I_1\right)$; $J_3 = \frac{1}{27}\left(2I_1^{\ 3} - 9I_1I_2 + 27I_3\right)$

$I_1 = \sigma_1 + \sigma_2 + \sigma_3$; $I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$; $I_3 = \sigma_1\sigma_2\sigma_3$

The typical yield surface in 3-dimensional principal stress space is shown in Fig. 2. The diagonal which has equal distances from three principal axes is called the hydrostatic axis ($\sigma_1 = \sigma_2 = \sigma_3$). Deviatoric plane is a plane perpendicular to the hydrostatic axis. Pie plane is the plane perpendicular to the hydrostatic axis and passes through the origin and is considered as a special type of deviatoric plane. The meridians of the failure surface are the intersection curves between the failure surface and the plane containing the hydrostatic axis with constant Θ. The meridian planes corresponding to $\Theta = 0^\circ$ and $\Theta = 60^\circ$ are called tensile and compressive meridians respectively. The meridian plane corresponding to $\Theta = 30^\circ$ is known as shear meridian. The deviatoric plane and meridian planes are pictorially represented in Fig. 3.

Nevertheless, for better geometric representation, the failure criterion may be expressed in Haigh-Westergaard coordinate system (Chen and Han (1987)) which is defined in terms of three parameters namely hydrostatic stress invariant ξ, deviatoric stress invariant ρ and deviatoric polar angle θ. $F(\xi, \rho, \theta) = 0$ In the Haigh-Westergaard coordinate system, the cross-section of the failure surface is represented in deviatoric plane and the meridians are described in the meridian plane. At any point within the failure surface, the principal stresses may be expressed in terms of Haigh-Westergaard coordinates as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} \xi \\ \xi \\ \xi \end{bmatrix} + \sqrt{\frac{2}{3}}\rho \begin{bmatrix} \cos\theta \\ \cos\left(\theta - \frac{2\pi}{3}\right) \\ \cos\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} \tag{2}$$

where $\xi = \frac{1}{\sqrt{3}} I_1$; $\rho = \sqrt{2J_2}$ and the angle θ is the lode angle. The lode angle is expressed in terms of second and third deviatoric stress invariant (Nayak and Zeinkiewicz (1972)) as $\cos 3\theta = \frac{3\sqrt{3}}{2} \frac{J_3}{\sqrt{J_2^3}}$

On the basis of the various experimental results, the following characteristics of the have been considered to be desirable and the same has to be reflected in any failure model.

(i) Failure surface should be convex and should contain smooth edges

(ii) Cross-section in the deviatoric plane changes from triangular to circular with the increase in confinement.

(iii) Compressive and tensile meridians in the meridian plane should be parabolic in nature.

Based on the knowledge on the shape of failure surface for concrete materials, a variety of failure criteria models were proposed. Failure models have been developed in the past based on (i) the empirical approach, (ii) physical approach, (iii) phenomenological approach. In the empirical approach, the tensile and compressive meridians in the meridian plane were derived through curve fitting over a cluster of experimental data points obtained by various researchers. To represent the failure surface in the deviatoric plane, the smooth interpolation between the two meridians were obtained to get the shape functions. It has also been reported by Fan and Wang (2002) that the formulations based on empirical approach lacks theoretical background on its hypothetical smooth interpolation between the meridians in the deviatoric plane although it has been found to give satisfactory results. On the other hand, failure models have also been developed based on the material structure which has been claimed as the only true property of a material. The above approach captures the pressure sensitivity of the material. Failure models are also based on phenomenological approach where criterion is based on experimental observations of the global shape of the failure contour.

In the early days, multiple equations were used to describe the failure criteria (Kupfer and Gerstle (1973); Willam and Warnke (1975); Kotsovos (1979) for different zones such as tension, compression, tension-compression, tension-tension, compression etc. In order to avoid complexity of using multiple functions, models based on a single unified function have been used to describe the entire range of failure surface (Ottosen (1977); Lade (1982); and Kang and William (1999)). Nevertheless, the use of single function has been found to be plagued by unsatisfactory results (Kang and William (1999)). In general, failure criteria may be developed based on conventional method or by a unified method. A recently developed unified method encompasses several other criteria as special cases by changing the parameter. In this paper, a detailed review has been made on the various conventional failure criteria.

4. Failure models

Failure surface is well described in terms of principal stresses (Chen 1982). Since the tensile strength of concrete is much lower than its compressive strength, the failure criterion for concrete for uniaxial case is defined in terms of two parameters namely tensile strength (f_t) and Compressive strength (f_c). Under biaxial tension, the strength of the concrete is almost the same as that under uni-axial tension. However, for the tensile-compressive stress state with low tensile/compressive stress ratio, a substantial reduction in tensile strength of concrete arises. Under biaxial compression, the maximum compressive strength of concrete increases significantly. Therefore, at least one additional parameter that is biaxial compressive becomes an important parameter to be considered in the failure criterion. When concrete is subjected to triaxial loading, the failure surface similar to the one shown in Fig. 3 has been obtained. Failure models are classified from 1-parameter model through 5-parameter model depending on the number of material constants appearing in the expressions.

4.1 One-parameter model

When the state of stress is a function of either uni-axial tension or uni-axial compression acting normal to the cross-section of the material, the yield condition is represented by $\sigma = \pm\sigma_y$. Since, this yield model depends only on the yield stress in tension or compression, it is known as one parameter model. In order to develop a unified failure criterion applicable in compression as well as in tension, the one-parameter tension model can be coupled with other compression failure criterion to define a fracture/cut-off region and referred as tension cut-off criterion. Sometimes concrete may fail in shear and, therefore, there is a need to define failure criterion in terms of shear stress also. Tresca considered yield stress at shear plane as the parameter to characterize the shear failure of concrete. Mathematically the failure form of Tresca criterion is given by

$$F(\sigma_1, \sigma_3) = \frac{\sigma_1 - \sigma_3}{2} - k = 0 \tag{3}$$

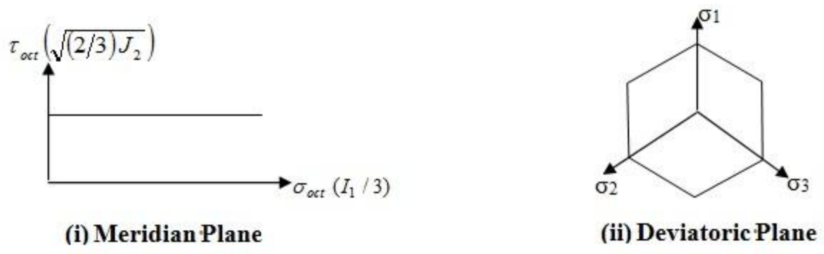

Fig. 4 Tresca criterion

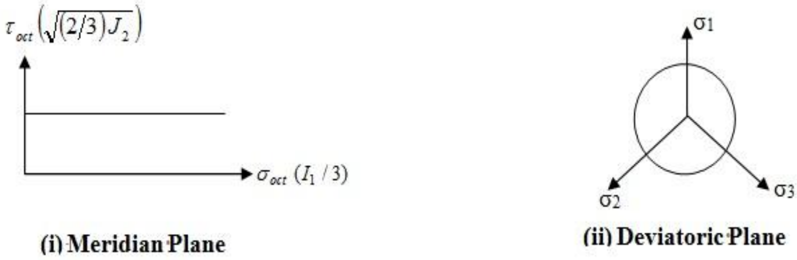

Fig. 5 Von-mises criterion

where σ_1 and σ_3 are the maximum and minimum principal stresses respectively and k is the strength of concrete in shear which is equal to $(\sigma_y/2)$. Now the above failure criterion may be represented as

$$F(\sigma_1,\sigma_3)=\frac{\sigma_1-\sigma_3}{2}-\frac{\sigma_y}{2}=0 \quad\Rightarrow\quad \sigma_1-\sigma_3=\sigma_y \tag{4}$$

In the above equation, σ_y is the yield stress in simple tension.

The above failure surface can be represented invariant form as

$$F(J_2,J_3)=4J_2^3-27J_3^2-36k^2J_3^2-36k^2J_2^2+96k^4J_2-64k^6=0 \quad \text{(Invariant Form)} \tag{5}$$

In the above equation, it is essential to note that the there is no influence of I_1(First invariant stress tensor). Octahedral normal stress is given by $\sigma_{oct}=I_1/3$. Thus, the relationship between octahedral shear stress and stress is represented by a parallel line. It is interesting to note that the intermediate principal stress value has nothing to do with Tresca criterion. Moreover, the Tresca criterion assumes that strength of concrete same in compression as well as in tension. Hence Tresca criterion cannot be used for concrete since concrete has different strengths in compression and tension.

In order to incorporate the effect of intermediate principal stress, von-Mises proposed another one–parameter model in terms of octahedral shear stress as

$$F(\tau_{oct})=\tau_{oct}-\sqrt{\frac{2}{3}}\,k=0 \tag{6}$$

where $\tau_{oct}=\frac{1}{3}\sqrt{(\sigma_1-\sigma_3)^2+(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2}$ and k is the strength of concrete in pure shear $k=f_y/\sqrt{3}$ at octahedral plane. von-Mises criterion can be expressed in the stress invariant form as

$$F(J_2)=J_2-k^2=0 \tag{7}$$

From the above equation it may be observed that yielding of material begins when the second deviatoric stress invariant (J_2) reaches a critical value and hence this criterion is also referred as J_2 Plasticity theory or J_2 flow theory. Since the von-Mises criterion includes all three principal stress states, it is also known as three-stress criterion.

Since the above one-parameter models are independent of hydrostatic pressure as evident by absent of I_1 term in the failure equations, they cannot be used as failure criteria for concrete.

4.2 Two-parameter models

Failure models have also been developed based on the material structure which has been claimed as the only true property of a material. The criterion for strength is expressed in terms of the cohesive strength of the cement paste and the frictional adhesion of aggregate interaction. The models based on this category have the pressure sensitivity and tensile strength criteria. The failure model proposed by Mohr-Coulomb & Drucker-Prager is based on this category. The Drucker-Prager and Mohr-Coulomb failure models are probably the simplest types of hydrostatic pressure dependent failure models, where the pure shear or octahedral shear stress depends linearly on the hydrostatic stress or octahedral normal stress respectively. The above failure criteria depend on two parameters.

In Mohr's criterion, maximum shear stress is the only critical measure of looming failure.

The general form of Mohr-Coulomb criterion is expressed as $F(\sigma_1, \sigma_3) = 0$;

$$F(\sigma_1, \sigma_3) = \sigma_1 - \alpha\sigma_3 - f_t = 0 \tag{8}$$

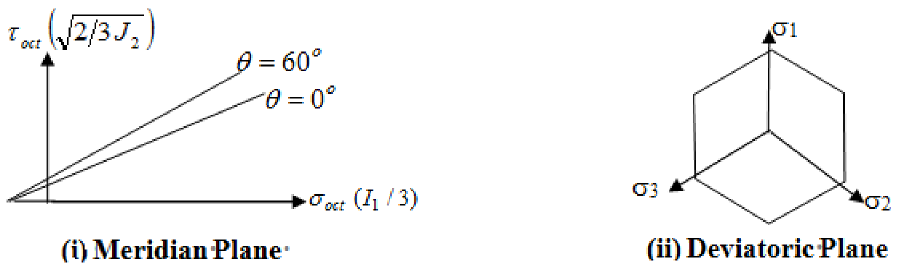

Fig. 6 Mohr-coulomb criterion

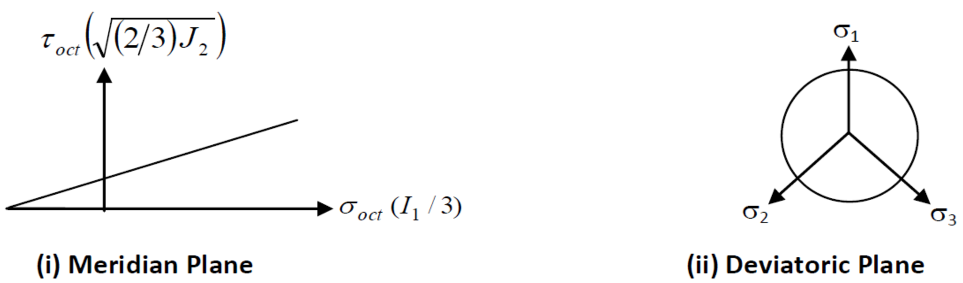

Fig. 7 Drucker-prager criterion

where $\alpha = \frac{f_t}{f_c}$; $f_t = \frac{2c\cos\phi}{1+\sin\phi}$; and $f_c = \frac{2c\cos\phi}{1-\sin\phi}$

Where f_c and f_t are the strength of concrete in uniaxial compression and tension respectively and may be obtained from uni-axial tests on concrete. It is interesting to note that, the constant shear stress Tresca criterion is a special case of Mohr's criterion and may be obtained by substituting $\alpha = 1$ in Mohr's criterion

The Mohr-Coulomb failure criterion can be expressed in terms of invariant form as

$$F(I_1, J_2, \theta) = \frac{1}{3} I_1 \sin\phi + \sqrt{J_2}\, \sin\left(\theta + \frac{\pi}{3}\right) + \sqrt{\frac{J_2}{3}} \cos\left(\theta + \frac{\pi}{3}\right) \sin\phi - c\cos\phi = 0 \tag{9}$$

The above equation involves the hydrostatic pressure term and thus gives the linear relationship between the octahedral shear stress and normal stress.

Smoothness of the failure surface in the deviatoric sections has been considered as desirable property. Nevertheless, the Mohr-Coulomb criterion has been found to possess certain problems in numerical computations because of corners observed in the deviatoric section, thus highlighting the shortcomings associated with Mohr-Coulomb failure criteria (Chen and Han (2008)). In addition to that, Mohr-Coulomb criterion is not influenced by intermediate principal stress. In order to overcome the problem possessed by Mohr-Coulomb, Drucker-Prager proposed a yield criterion similar to the von-Mises criterion by including one additional parameter. This additional parameter takes into the account the strength difference of concrete between tension and compression.

The failure surface for the Drucker-Prager criterion is given by

$$f(I_1, J_2) = \beta I_1 + \sqrt{J_2} - \kappa = 0;\ \mathrm{k} = f_y / \sqrt{3} \tag{10}$$

when $\beta = 0$; the above equation leads to von-Mises criterion.

The failure surface obtained using the Drucker-Prager criterion is a right-circular cone. It overcomes the problem posed by Mohr-Coulomb criteria where failure surface in hexagonal. The Drucker-Prager criterion, as a smooth approximation to the Mohr-Coulomb criterion, can be made to match the Mohr-Coulomb by adjusting the material parameters of both criteria. Nevertheless, in the Drucker-Prager criterion the relation between octahedral shear stress and octahedral normal stress is linear, and failure surface in the Pie (π) plane is a circle (Fig. 7). This is contradictory to the experimental results which confirm that the failure surface is not a circle and the linear relationship is not valid between octahedral shear and normal stresses (Chen and Han (2008), Chen (1982)). It is concluded that the above two parameter models fail to comply experimental results. Hence, there is a need to develop refined models with more parameters in order to replicate the actual behavior.

4.3 Three-Parameter models

In a step to overcome the problems posed by Drucker-Prager criterion, Brestler and Pister (1958) proposed a yield criterion consisting of three-parameters, which is basically an extension of the Drucker-Prager yield criterion. The failure form in terms of principal stresses is expressed as

$$F(\sigma_1,\sigma_2,\sigma_3)=\frac{1}{\sqrt{6}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2} \\ -c_0-c_1(\sigma_1+\sigma_2+\sigma_3)-c_2(\sigma_1+\sigma_2+\sigma_3)=0; \tag{11}$$

where, $c_0=\frac{\sigma_c}{\sqrt{3}}+c_1\sigma_c+c_2\sigma_c^{\ 2}$; $c_1=\left(\frac{\sigma_t-\sigma_c}{\sqrt{3}(\sigma_t+\sigma_c)}\right)\left(\frac{4\sigma_b^{\ 2}-\sigma_b(\sigma_c+\sigma_t)+\sigma_c\sigma_t}{4\sigma_b^{\ 2}+2\sigma_b(\sigma_t-\sigma_c)-\sigma_c\sigma_t}\right)$;

$c_2=\left(\frac{1}{\sqrt{3}(\sigma_t+\sigma_c)}\right)\left(\frac{\sigma_b(3\sigma_t-\sigma_c)-2\sigma_c\sigma_t}{4\sigma_b^{\ 2}+2\sigma_b(\sigma_t-\sigma_c)-\sigma_c\sigma_t}\right)$

where σ_c , σ_t and σ_b are the uniaxial compressive strength, uniaxial tensile and biaxial compressive strength of concrete respectively. In the invariant form the Brestler and Pister three parameter model may be represented as

$$F(I_1,J_2)=A+BI_1+CI_1^2-\sqrt{J_2}=0 \tag{12}$$

From the above equation, it is clear that there is a parabolic dependence between I_1 and J_2, which is reflected in Fig. 8. The material constants A, B and C are determined from the experimental tests.

Willam and Warnke (1975) have proposed a three parameter failure criterion in order to characterize the characteristics of concrete. In terms of hydrostatic stress σ_m and deviatoric stress τ_m, the above failure model is represented as

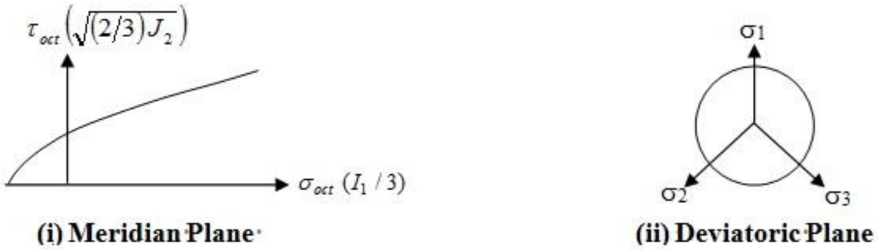

Fig. 8 Brestler –Pister criterion

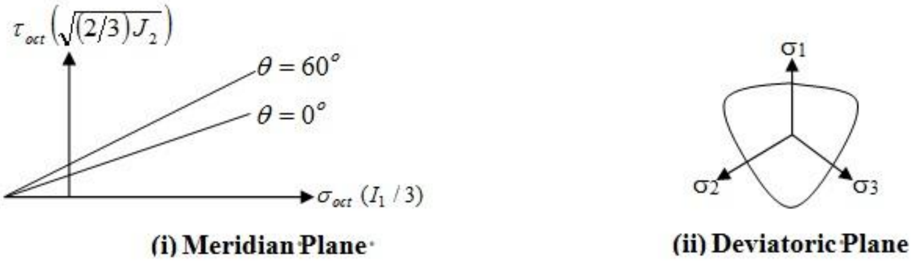

Fig. 9 Willam Warnke critetion

$$F(\sigma_m, \tau_m, \theta) = \frac{1}{\alpha}\frac{\sigma_m}{f_c} + \frac{1}{\rho(\theta)}\frac{\tau_m}{f_c} - 1 = 0 \tag{13}$$

$$\text{Where } \sigma_m = (I_1/3); \quad \tau_m = \sqrt{(2/5)J_2} \tag{14}$$

The radius of the elliptical curve can be described by the following equation as

$$\rho(\sigma_m, \theta) = \frac{2\rho_c(\rho_c^2 - \rho_t^2)\cos\theta + \rho_c(2\rho_t - \rho_c)[4(\rho_c^2 - \rho_t^2)\cos^2\theta + 5\rho_t^2 - 4\rho_t\rho_c]^{1/2}}{4(\rho_c^2 - \rho_t^2)\cos^2\theta + (\rho_c - 2\rho_t)^2} \tag{15}$$

where ρ_c and ρ_t are the radiuses corresponding to compressive and tensile meridians respectively. The three parameters ρ_c, ρ_t and α are determined from uniaxial compressive strength, uniaxial tensile strength and equal biaxial compressive strength. Nevertheless, Willam Warnke model (Fig. 9) has been found inadequate to representing the concrete compressive failure where the ratio (ρ_t/ρ_c) approaches unity. This is due to the linear dependence of octahedral normal and octahedral shear stresses. It is also reported that this criterion estimates lower value of concrete tensile cracking (Chen (1982)).

It has reported in the literature that for uni-axial loading, the criteria for cracking and crushing depend on tension and compression. Nevertheless, for combined state of stress like tension-compression, it may be difficult to obtain a simple model. Li and Harmon (1990) have proposed a three-parameter model in which the failure surface depends on the deviation in the volumetric strain. They assumed that the volumetric strain is non-positive for crushing region as the crushing should essentially happen due to compression. The volumetric strain components are related to first strain invariant by

$$I_1' = \varepsilon_x + \varepsilon_y + \varepsilon_z < 0 \tag{16}$$

$$\sigma_x + \sigma_y + \sigma_z < 0 \tag{17}$$

On the basis of above equation, they have proposed to use different failure equations in stress space for three different regions namely tension-tension, compression-compression and tension-compression instead of unified equations. The failure functions for different regions are mentioned below:

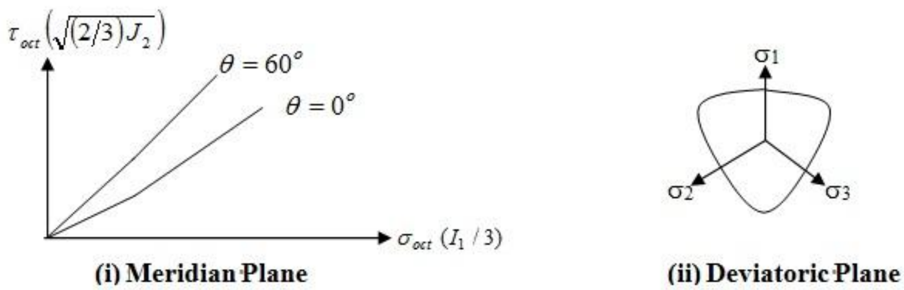

Fig. 10 Li & Harmon Failure Critetion

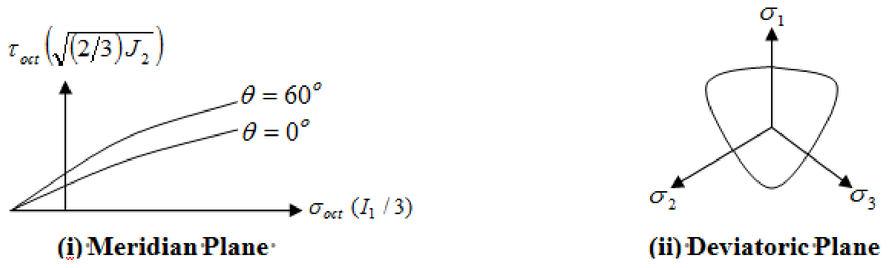

Fig. 11 Menetrey and Willam criterion

$$F_1(\sigma_1,\sigma_2,\sigma_3)=(\sigma_1-f_t)(\sigma_2-f_t)(\sigma_3-f_t)=0\quad [Cracking\ region\ I_1<0] \tag{18}$$

$$F_2(I_1,I_2,I_3)=a_1I_1^2+a_2I_1-I_2+a_3I_3+a_4=0\quad [Crushing\ region\ 1\ (-f_c\le I_1\le 0] \tag{19}$$

$$F_3(I_1,I_2,I_3)=b_1I_1^2+b_2I_1-I_2+b_3I_3+b_4=0\quad [Crushing\ region\ 2\ (I_1\le -f_c) \tag{20}$$

Eqs. (18), (19) and (20) represent the failure surface in the region of tension, compression and compression-compression respectively.
where a_1, a_2, a_3 a_4 and b_1, b_2, b_3 b_4 are the constants determined from the following conditions.
The C^0 continuous conditions between separation functions

$$F_1 = F_2 \quad for \quad I_1 = 0 \tag{21}$$

$$F_2 = F_3 \quad for \quad I_1 = -f_c \tag{22}$$

The uniaxial and equal biaxial compressive failure conditions

$$F_2 = 0 \quad for \quad \sigma_1 = \sigma_2 = 0; \quad \sigma_3 = -f_c \tag{23}$$

$$F_3 = 0 \quad for \quad \sigma_1 = 0; \quad \sigma_2 = \sigma_3 = -f_b \tag{24}$$

The C^1 continuous condition is expressed by

$$\frac{\partial F_1}{\partial I_1} = \frac{\partial F_2}{\partial I_1} \quad for \quad I_1 = 0 \tag{25}$$

$$F_3 = b_2 I_1 + J_2 + b_4 \tag{26}$$

With the help of equations (18-23), the constants can be determined based on uniaxial compressive strength, uniaxial tensile strength and equal biaxial compressive strength of concrete. The above failure models capture the behavior of concrete adequately and have been found to be simple to implement as different equations are used for different stages. It is also observed from the Fig. 10 that the Li and Harmon model is more close to experimental results than Chen and Chen model for the cracking region. However, Li and Harmon model adopted the same criterion for compression as proposed by chen and chen (Chen and Chen (1975)). Nevertheless, for this criterion the significant deviation of results between the experimental and analytical results has been observed in the high compressive regime (Li and Harmon (1990)).

Menetrey and Willam (1995) have presented another three parameter failure criterion which satisfies the basic characteristics of a failure surface in meridian plane and deviatoric plane as shown in Fig. 11. The failure form of Menetrey-Willam criterion is represented as

$$F(\sigma_1,\sigma_3)=\left[\frac{\sigma_1-\sigma_3}{f_c'}\right]^2+m\frac{\sigma_1}{f_c'}-c=0 \tag{27}$$

In terms of stress invariants, the above failure criterion is expressed as

$$F(I_1,J_2,\theta)=\left[\left(\frac{\sqrt{1.5}\times\sqrt{2J_2}}{f_c'}\right)^2+m\left(\frac{\sqrt{2J_2}}{\sqrt{6}f_c'}\rho(\theta,e)+\frac{I_1}{3f_c'}\right)\right]-c=0$$

$$\rho(\theta,e)=\frac{4(1-e^2)\cos^2\theta+(2e-1)^2}{2(1-e^2)\cos\theta+(2e-1)\left[4(1-e^2)\cos^2\theta+5e^2-4e\right]^{1/2}}$$

In the above equations, m and c are frictional strength and cohesive strength of concrete respectively. The cohesion and frictional strength can be calibrated using the uniaxial compression and uniaxial tension test results. In order to obtain the smooth trace of the curve in the deviatoric plane, the eccentricity should be greater than 0.6. The Menetrey-Willam three parameter model possesses parabolic dependence between octahedral shear and normal stresses apart from satisfying the criteria in deviatoric sections. The Menetrey-Willam three-parameter model has been verified with biaxial and triaxial strength data on plain concrete. The failure criterion includes Huber-Mises, Drucker-Prager, Rankine, Mohr-Coulomb as special cases. This criterion has been considered to be simplification of Willam Warnke five parameter model.

4.4 Four-Parameter models

In order to meet the geometric requirements of failure model, namely parabolic dependence between octahedral shear and normal stresses and also angle of similarity dependence, the more refined models have been found to be developed based on phenomenological approach where criterion is based on experimental observations of the global shape of the failure contour. Ottosen (1977) developed a four-parameter criterion based on phenomenological approach for short-time loading of concrete. The meridians and deviatoric plane for the Ottosen model are shown in Fig. 12. It involves all the essential features of tri-axial stress state and shows good agreement with the experimental results. The four parameters in the Ottosen model depend heavily on the ratio of uni-axial tensile to compressive strength. The Ottosen failure criterion is expressed as

$$F(I_1,J_2,\theta)=A\frac{J_2}{f_c^2}+\mu\frac{\sqrt{J_2}}{f_c}+B\frac{I_1}{f_c}-1=0 \tag{28}$$

where A and B are material constants and μ is a function of angle θ

$$\mu = \left\{ C \cos\left[\frac{1}{3}\cos^{-1}\left(D \cos 3\theta\right)\right] \right. \qquad for \;\; \cos 3\theta \le 0 \tag{29}$$

$$\mu = \left\{ C \cos\left[\frac{\pi}{3} - \frac{1}{3}\cos^{-1}\left(D \cos 3\theta\right)\right] \right. \quad for \;\; \cos 3\theta \ge 0 \tag{30}$$

In the above equation, C and D are material parameters

The four parameters of the Ottosen criterion namely A, B, C and D are determined using the (i) uniaxial compressive strength, (ii) Uniaxial tensile strength, (iii) Biaxial compressive strength, and (iv) a point on compressive meridian (σ_{oct}, τ_{oct}). This criterion also suffers from the drawback that error in predicting the strength characteristics of concrete increases with the increase in compressive strength. Thus, for high compressive stresses, Ottosen criterion may not yield accurate results.

The Hseih-Ting-Chen presented the Ottosen four parameter model in a simplified form as (Dede (2010))

$$F(I_1, J_2, \sigma_1) = C_1 \frac{J_2}{f_c^2} + C_2 \frac{\sqrt{J_2}}{f_c} + C_3 \frac{\sigma_1}{f_c} + C_4 \frac{I_1}{f_c} - 1 = 0 \tag{31}$$

In the above equation, C_1, C_2, C_3 and C_4 the material constants similar to A, B, C and D in

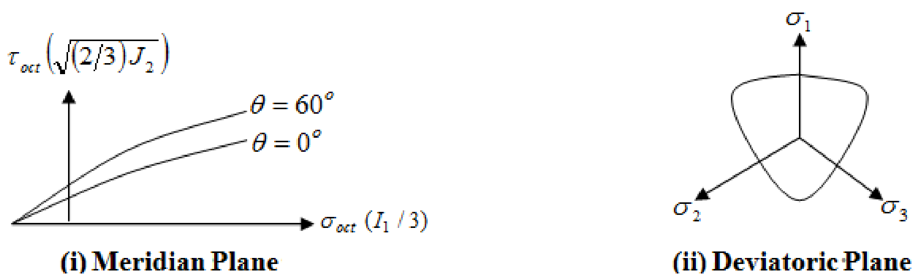

Fig. 12 Ottosen criterion

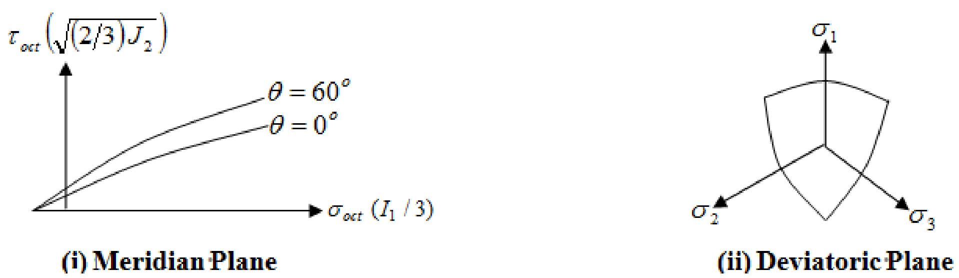

Fig. 13 Hseih-Ting-Chen criterion

Ottosen model and are determined in similar way. The above mentioned functional form of the Hseih-Ting-Chen model has been found to be a linear combination of three failure criteria, namely von-Mises criterion, Drucker-Prager criterion, Rankine criterion. The Hseih-Ting-Chen model satisfies almost all the necessary conditions. Nevertheless, the Hseih-Ting-Chen model still has corners along the compressive meridians as shown in Fig. 13, thus smoothness has not been achieved which is one of the fundamental characteristics of a failure surface. Other failure criteria have been developed in the past such as Reimann-Janda criterion and Cedolin, Crutzen and Dei Poli criterion using four parameters. Nevertheless, these two criteria suffer from the drawback of having corners in deviatoric cross-section (Ottosen (1977)).

4.5 Five-Parameter models

Willam and Warnke (1975) modified their three parameter model by including two more degrees of freedom which results in the effectiveness of the criterion in high compressive state of stress.

The failure surface of five-parameter model has been expressed conveniently by hydrostatic and deviatoric Sections (Fig. 14). The failure surface of the five-parameter model can be described by the following equation

$$F(\sigma_m, \tau_m, \theta) = \frac{\tau_m}{\rho(\sigma_m, \theta) f_c'} - 1 = 0\text{;}\quad \tau_m = \sqrt{2/5 J_2} \tag{32}$$

The elliptical trace is expressed by polar coordinates as

$$\rho(\sigma_m, \theta) = \frac{2\rho_c(\rho_c^2 - \rho_t^2)\cos\theta + \rho_c(2\rho_t - \rho_c)[4(\rho_c^2 - \rho_t^2)\cos^2\theta + 5\rho_t^2 - 4\rho_t\rho_c]^{1/2}}{4(\rho_c^2 - \rho_t^2)\cos^2\theta + (\rho_c - 2\rho_t)^2} \tag{33}$$

and the angle of similarity is defined as

$$\cos\theta = \frac{\sigma_1 + \sigma_2 - 2\sigma_3}{\sqrt{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}} \tag{34}$$

The tensile and compressive meridians in the five parameter model are expressed by the two following equations using second order parabolic form as

$$\frac{\rho_t}{\sqrt{5} f_c'} = a_0 + a_1\left(\frac{\sigma_m}{f_c'}\right) + a_2\left(\frac{\sigma_m}{f_c'}\right)^2 \qquad at\ \theta = 0^0 \tag{35}$$

$$\frac{\rho_c}{\sqrt{5} f_c'} = b_0 + b_1\left(\frac{\sigma_m}{f_c'}\right) + b_2\left(\frac{\sigma_m}{f_c'}\right)^2 \qquad at\ \theta = 60^0 \tag{36}$$

The above equations may be written in terms of stress-invariants as

$$\frac{2J_2}{5\sqrt{f_c^{'}}} = a_o + a_1\left(\frac{I_1}{3f_c^{'}}\right) + a_2\left(\frac{I_1}{3f_c^{'}}\right)^2 \quad (for\ \ tension) \tag{37}$$

$$\frac{2J_2}{5\sqrt{f_c^{'}}} = b_o + b_1\left(\frac{I_1}{3f_c^{'}}\right) + b_2\left(\frac{I_1}{3f_c^{'}}\right)^2 \quad (for\ \ compression) \tag{38}$$

In above equation six parameters namely a_0, a_1, a_2, b_0, b_1, b_3 are required to define the compressive and tension meridians. Since, the two meridians intersect at the hydrostatic axes at a common apex of equisectrix, the above six parameters are reduced to five parameters namely a_0, a_1, a_2, b_1 and b_2. These five parameters are determined from experimental results which are uniaxial compressive strength, biaxial compressive strength, uniaxial tensile strength and triaxial compressive strength at two different stages. The five – parameter model has been used extensively to model the concrete behavior (Yan and Pantelides (2006)) and found to give satisfactory results. Bhargava *et al*. (2006) performed the three-dimensional finite element modeling of confined High-Strength concrete columns in which the non-linear behavior of concrete material has been idealized by Willam-Warnke five-parameter model. It has also been reported in the paper that Willam-Warnke model is widely accepted and sophisticated criterion to predict the nonlinear material behavior of confined concrete. Ribeiro and Oliveira (1998) conducted elasto-plastic analysis of RC plates using the Reissner's model and used Willam-Warnke five parameter criterion to model the concrete behavior in compression. Mansour (2010) in his study on theoretical analysis of tunnel lining used Willam-Warnke five parameter model as a yield and failure criteria for concrete.

It has been a general consensus that the failure criterion should be different for reinforced concrete. The above statement is augmented by Seow and Swaddiwudhipong (2005)) that the failure surface of Willam and Warnke five parameter model intersects the negative hydrostatic axis under very high compressive stresses which is contrary to the real behavior of steel fibre reinforced concrete (Seow and Swaddiwudhipong (2005)). They have discussed the failure surface for concrete under multi-axial load and developed a unified approach using five-parameters. This new five-parameter model works well for different strengths of concrete and also for steel

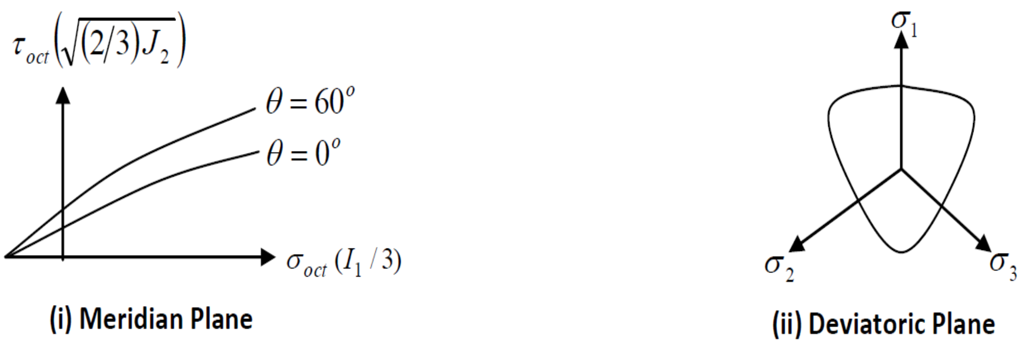

Fig. 14 Willam –warnke five-parameter model

Table 1 Summary of various failure models

Type of failure criteria	Type of failure model	Failure model
Hydrostatic pressure independent	One parameter model	Rankine criterion
		Tresca criterion
		von-Mises criterion
Hydrostatic pressure dependent	Two parameter model	Mohr Coulomb criterion
		Drucker-Prager criterion
		Leon criterion
		Vallabhan-Mehta criterion
		Mills-Zimmerman criterion
	Three parameter model	Brestler & Pister criterion
		Willam & Warnke criterion
		Li & Harmon criterion
		Hoek & Brown criterion
		Menetrey & Willam criterion
		Chen & Chen criterion
		Lade criterion
	Four parameter model	Ottosen criterion
		Hseigh Ting Chen
		De-Boer Dresenkamp criterion
		Reimann-Janda criterion
	Five parameter model	Willam Warnke five parameter criterion
		Podgorski criterion
		Modified five parameter criterion
	Seven-Parameter model	Boswell-Chen criterion

fiber reinforced concrete under various loading conditions. Many failure models have been developed in the recent past based on the unified approach so that the same failure criteria can be applied to different materials as well (Hinchberger (2009), Seow and Swaddiwudhipong (2005). The failure surface of the five parameter model proposed by Seow and Swaddiwudhipong (2005) is represented by

$$\frac{\xi}{f_{cu}} = a_2\left(\frac{\kappa\rho_t}{f_{cu}}\right)^2 + a_1\left(\frac{\rho_t}{f_{cu}}\right) + a_0 \qquad \kappa \le 1 \tag{39}$$

$$\frac{\xi}{f_{cu}} = b_2\left(\frac{\rho_c}{f_{cu}}\right)^2 + b_1\left(\frac{\rho_c}{f_{cu}}\right) + b_0, \tag{40}$$

In Eq. (39) κ is a parameter used to modify the failure surface to incorporate the effect of steel fiber reinforcement. In the case of plain concrete, the value of κ equals unity;

$$\rho(\theta) = \frac{2\rho_c\left(\rho_c^{\,2} - \rho_t^{\,2}\right)\cos\theta + \rho_c\left(2\rho_t - \rho_c\right)\left[4\left(\rho_c^{\,2} - \rho_t^{\,2}\right)\cos^2\theta + 5\rho_t^{\,2} - 4\rho_t\rho_c\right]^{1/2}}{4\left(\rho_c^{\,2} - \rho_t^{\,2}\right)\cos^2\theta + \left(\rho_c - 2\rho_t\right)^2} \tag{41}$$

$$\cos\theta = \left[\frac{3\left(\sigma_3 - I_1/3\right)}{\sqrt{6}\sqrt{\sigma_1^{\,2} + \sigma_2^{\,2} - 3\left(I_1/3\right)^2}}\right] \qquad for\ \sigma_3 \geq \sigma_2 \geq \sigma_1 \tag{42}$$

Finite element analysis was carried out to show the wide range of applicability of new five-parameter model especially in steel fibre reinforced concrete (Seow and Swaddiwudhipong (2005)).

4.6 3 Recent developments

It has been recently established that the performance based failure criterion be adopted for normal and high strength concretes in order to define the maximum strength of concrete. This falls under the category of unified strength criterion as like Seow and Swaddiwudhipong (2005). Folino *et al*. (2009) reported that the compressive strength and mortar qualities are the two most important parameters to define the failure surface of concrete. However, it has been reported that no parameter has been established to measure the quality of mortar in the failure criteria. They have developed the failure model in which the form of maximum strength surface varies depending on the quality of material, thus incorporating the effect the material quality in the failure criterion. It has also been reported that this strength criterion leads to better results than reported by Seow and Swaddiwudhipong (2005). The four material parameters are required to define this criterion.

In the recent times, the use of nonlinear strength criteria based on unified approach is being used as failure criteria of concrete. Du *et al*. (2010) recently proposed four-parameter failure criterion based on unified approach and concluded that the criterion works satisfactorily in predicting the failure criterion of concrete. The use of such models in finite element analysis is relatively easy as it encompasses several other criteria as special cases.

Some failure models have not been discussed in order to avoid complexity. The various failure models are summarized & reported in Table 1.

5. Concluding remarks

Based on the detailed review made on the behavior of concrete and failure criteria, the following salient observations have been made.

The experimental behavior of concrete subjected to uniaxial, biaxial, triaxial loading performed by various investigators has been reviewed. Since the von-Mises, Tresca and Rankine failure criteria depend on only one parameter and not possessing the smooth deviatoric plane, it has been reported in the literature that these failure models fail to capture the behavior of concrete adequately. Addition of Rankine criterion to von-Mises and Tresca to include tension-cutoff condition, makes these criteria two-parameter models. Nevertheless, these failure models are insensitive to hydrostatic pressure and the shape of the deviatoric plane does not comply with the characteristics of failure surface of concrete. Hence the above one parameter failure models are adopted as a failure criterion for concrete. The Mohr-Colomb criterion widely used as a failure criterion for concrete because of its hydrostatic pressure dependence, but has straight edges and sharp corners in the failure surface in the deivatoric plane. The straight edges essentially mean that the linear interpolation has been used between the tensile and compressive meridian. The Drucker-Prager failure criterion gives better results than Mohr-Coulomb criterion because of the smoothness of the failure surface in the deviatoric section. The Mohr-Coulomb and Drucker-Prager criteria results to Tresca and von-Mises criteria respectively as special cases. Furthermore, the Rankine's criterion can be added to either Mohr-Coulomb or Drucker-Prager criteria to extrapolate these two-parameter models into three-parameter model. Several investigations have been conducted on the behavior of concrete subjected to biaxial stress state of concrete and concluded that the biaxial compressive strength of concrete is higher than uniaxial compressive strength of concrete. Nevertheless, most of the researchers have suggested that not much variation was observed between the biaxial tensile and uniaxial tensile strength of concrete. Thus, it has been reported that minimum three parameters, namely (i) uniaxial compressive strength (ii) uniaxial tensile strength and (iii) biaxial compressive strength are required to predict the behavior of concrete. Thus, the use of Mohr-Coulomb, Drucker-Prager criteria have been found having limitations in predicting the response of concrete. On the experimental investigations, various three-parameter models have been proposed. The Menetrey-Willam three parameter model has been considered the simplification of Willam-Warnke five parameter model and possesses the desired characteristics of a failure surface. Brestler and Pister have also proposed a three parameter model. While Menetrey-Willam failure model has the parabolic dependence between octahedral shear and normal stress, Brestler and Pister has the circular section in the deviatoric plane. The extensive investigations lead to the representation of the failure surface with the help of four parameters. Ottosen and Hseih-Ting-Chen model falls in this category. Hseih-Ting Chen model is considered a computationally simple than Ottosen, the failure model has edges in the failure surface in the deviatoric plane. The Willam-Warnke five parameter model (1975) has been the best pick over all the above mentioned failure criteria because of its versatility in satisfying all the characteristics of the failure surface and also its performance in predicting the behavior of concrete (Ribeiro and Oliveira 1998; Fanning 2001; Mansour 2010). The above mentioned failure criteria including five-parameter Willam-Warnke model are based on monotonic loading condition and is applicable only for plain concrete.

The new five parameter model proposed by Seow and Swaddiwudhipong (2005) is based on unified approach and its applicability has been found to be better than five parameter model especially for steel fibre-reinforced concrete (SFRC). Experimental investigations by Seow and Swaddiwudhipong (2005) confirm that the new unified 5-parameter failure surface is suitable for concrete strengths ranging from 20 MPa to 165 MPa and also for SRFC. The comparison of Willam-Warnke five parameter failure model with the unified failure model was mentioned in Seow and Swaddiwudhipong (2005). The advantage of unified approach is that it is not restricted to only one type of materials. Other failure models have been found to be developed based on this unified approach. Recently, performance based failure criterion has also been developed by incorporating the quality of material as performance index parameter. The performance based criterion has been found to be in use for different range of materials ranging from normal strength concrete to high strength concrete. Nevertheless, five parameter Willam-Warnke model (1975) and the Menetrey Willam three parameter model (1995) have all the credentials to be used as a failure criterion for plain concrete.

References

Ansari, F. (1987), “Stress-strain response of microcracked concrete in direct tension”, *ACI Mater. J.*, **84**(6), 481-490.

Bagheripour, M.H., Rahgozar, R., Pashnesaz, H. and Malekinejad, M. (2011), “A complement to hoek-brown failure criterion for strength prediction in anisotropic rock”, *Geomech. Eng.*, **3**(1), 61-81.

Balmer (1949), *Shearing Strength of Concrete under High Triaxial Stress—Computation of Mohr's Envelope as a Curve*, Structural Research Laboratory Report, No SP-23, United States Department of the Interior, Bureau of Reclamation, Washington, DC.

Bhargava, P., Bhowmick, R., Sharma, U. and Kaushik, S.K. (2006), “Three-dimensional finite element modeling of confined high-strength concrete columns”, *Special Publication, ACI* , **238**, 249-266.

Bortolotti, L., Carta, S. and Cireddu, D. (2005), “Unified yield criterion for masonry and concrete in multiaxial stress states”, *ASCE, J. Mater. Civil Eng.*, **17**(1), 54-62.

Brestler, B. and Pister. (1958), “Behavior of concrete under triaxial compressive-compressive-tensile stresses”, *ACI Mater. J.*, **8**(2), 181-185.

Brestler, B. and Pister. (1958), “Strength of concrete under combined stresses”, *Proceedings of ACI Journal*, **55**(3), 321-346.

Buyukozturk, O. and Shareef. S.S. (1985), “Constitutive modeling of concrete in finite element analysis”, *Comput. Struct.*, **21**(3), 581-610.

Carreira, D.J. and Chu, K.H. (1986), “Stress-strain relationship for plain concrete in compression”, *Proceedings of ACI Journal*, **82**(6), 797-804.

Cedolin, L. and Mulas, M.G. (1984), “Biaxial stress-strain relation for concrete”, *ASCE, J. Eng. Mech. Div.*, **110**(2), 187-206.

Cedolin, L., Crutzen, Y.R.J. and Dei Poli, S. (1977), “Triaxial stress-strain relationship for concrete”, *ASCE, J. Eng. Mech. Div.*, **103**(3), 423-439.

Chen, A.C.T. and Chen, W.F. (1975), “Constitutive relations for concrete”, *J. Eng. Mech. Div.* **101**(4), 465-481.

Chen, W. F. (1982), “Plasticity in reinforced concrete”, McGraw-Hill, New York.

Chen, W.F. and Han, D.J. (2008), “Plasticity for structural engineers”, *J. Ross Publishing*, India.

Darwin, D. and Pecknold, D.A. (1977), “Nonlinear biaxial stress-strain law for concrete”, *ASCE, J. Eng. Mech. Div.*, **103**(2), 229-241.

Dede, T. and Ayvaz, Y. (2010), “Comprative study of plasticity models for concrete material by using different criteria including Hseih-Ting-Chen criterion”, *Mater. Des.*, **31**(3), 1482-1489.

Du, X.L., Lu D.C., Gong, Q.M. and Zhao, M. (2010), “Nonlinear unified strength criterion for concrete under three-dimensional stress states”, *ASCE, J. Eng. Mech. Div.*, **131**(1), 51-59.

Fanning, P. (2001), “Nonlinear models of Reinforced and post-tensioned concrete beams”, *Elect. J. Struct.*, **2**, 111-119.

Fan, S.C. and Wang, F. (2002), “A new strength criterion for concrete”, *ACI Struct. J.*, **99**(3), 317-326.

Fardis, M.N., Alibe, B. and Tasoulas, J.L. (1983), “Monotonic and cyclic constitutive law for concrete”, *ASCE, J. Eng. Mech. Div.*, **109**(2), 516-536.

Fehling, E., leutbecher, T. and Roeder, F.K. (2011), “Compression-Tension strength of Reinforced and Fiber-Reinforced concrete”, *ACI Struct. J.*, **108**(3), 350-359.

Folino, P., Etse, G. and Will, A. (2009), "Performance dependent failure criterion for normal-and high-strength concretes", *ASCE, J. Eng.Mech. Div.*, **35**(12), 1393-1409.

Gardner, N.J. (1989), "Triaxial behavior of concrete", *Proceedings of ACI Journal*, **66**(2), 136-146.

Gopalarathnam, V.S. and Shah, S.P. 1985), "Softening response of plain concrete in direct tension", *ACI Struct. J.*, **82**(2), 310-323.

Han, D.J. and Chen, W.F. 1987), "Constitutive modelling in analysis of concrete structures", *ASCE, J. Eng. Mech. Div.*, **113**(4), 577-593.

Hinchberger, S.D. (2009), "Simple single-surface failure criterion for concrete", Technical Notes. *ASCE, J. Eng. Mech.*, **135**(7), 729-732.

Hognestad, E., Hansen, N.W. and McHenry, D. (1955), "Concrete stress distribution in ultimate strength design", *ACI Journal*, **52**(4), 455-480.

Hughes, B.P. and Chapman, G.P. (1966), "The complete stress-strain curve for concrete in direct tension", *RILEM Bulletin*, **30**, 95-97.

Hussein, A. and Marzouk, H. (2000), "Behavior of high-strength concrete under biaxial stresses", *ACI Mater. J.*, **97**(1), pp. 27-36.

Imran, I. and Pantazopoulou, S.J. (2001), "Plasticity model for concrete under triaxial compression", *ASCE, J. Eng. Mech. Div.*, **127**(3), 281-290.

Imran, I. and Pantazopoulou, S.J. (1991), "Experimental study of plain concrete under triaxial stresses", *ACI Mater. J.*, **93**(6), 589-601.

Karam, G. and Tabbara, M. (2012), "Hoek-brown strength criterion for actively confined concrete", *J. Mater. Civil Eng.*, **21**(3), 110-118.

Karsan, I.D. and Jirsa, J.O. (1969), "Behavior of concrete under compressive loadings", *ASCE, J. Struct. Div.*, **95**(12), 2543-2563.

Kotsovos, M.D. and Newman J.B. (1977), "Behavior of concrete under multiaxial stress", *Proceedings of ACI J.*, **74**(9), 443-446.

Kupfer, H. and Gerstle. (1973), "Behavior of concrete under biaxial stresses", *ASCE, J. Eng. Mech. Div.*, **99**(4), 853-866.

Kupfer, H., Hilsdorf, H.K. and Rusch, H. (1969), "Behavior of concrete under biaxial stresses", *Proceedings of ACI J.*, **66**(8), 656-666.

Kwak, H.G. and Filippou, F.P. (1990), *Finite Element Analysis of Reinforced Concrete Structures under Monotonic Loads*, Report No. UCB/SEMM-90/14, *Earthquake Engineering Research Centre*, University of California, Berkeley.

Li, L.Y. and Harmon T.G. (1990), "Three-parameter failure criterion for concrete", *ASCE, J. Mater. Civil Eng.*, **2**(4), 215-222.

Lan, S. and Guo, Z. (1999), "Biaxial compression behavior of concrete under repeated loading", *ASCE, J. Mater. Civil Eng.*, **105**(11), 105-115.

Lee, S.K., Song, Y.C. and Han, S.H. (2004), "Biaxial behavior of plain concrete of nuclear containment building", *Nuclear Eng. Des.*, **227**(2), 143-153.

Linhua, J., Dahai, H. and Nianxiang, X. (1991), "Behavior of concrete under triaxial compressive-compressive tensile stresses", *ACI Mater. J.*, **88**(2), 181-185.

Liu, T.C.Y., Nilson, A.H. and Slate, S.F.O. (1972), "Stress-strain response and fracture of a concrete in uniaxial and biaxial compression", *Proceedings of ACI Journal*, **69**(5), 291-295.

Menetrey, P. and Willam, K.J. (1995), "Triaxial failure criterion for concrete and its generalization", *Proceedings of ACI Journal*, **92**(3), 311-318.

Mills, L.L. and Zimmerman, R.M. (1970), "Compressive Strength of Plain Concrete Under Multiaxial Loading Conditions", *Proceedings of ACI J.*, **67**(10), 802-807.

Mlakar, P.F., Vitaya-Udom, K.P. and Cole, R.A. (1985), "Dynamic tensile-compressive behavior of concrete", *ACI Journal*, **86**(5), 484-491.

Mansour, M.H. (2010), "Theoretical analysis of tunnel lining subjected to fire", *J. Eng. Sci.*, **38**(3), 619-640.

Nayak, G.C. and Zeinkiewicz, O.C. (1972), "Convenient forms of stress invariants for plasticity", *ASCE, J. Struct. Eng.*, **98**(4), 949-954.

Ottosen N.S. (1993), “A failure criterion for concrete”, *ASCE, J. Eng. Mech. Div.*, **103**(4), 527-535.
Ren, X., Yang, W., Zhou, Y. and Li, J. (2008), “Behavior of high performance concrete under uniaxial and biaxial loading”, *ACI Mater. J.*, **105**(6), 548-557.
Ribeiro, G. de O. and Oliveira, A.L. (1998), “Elastoplastic analysis of RC plates using the Reissner's model and the boundary element method”, *Proceedings of Computational Mechanics, New Trends and Applications*, Barcelona, Spain.
Seow, P.E.C. (2005), “A unified failure criterion for normal, high-strength and steel fibre-reinforced concrete”, PhD Thesis, Department of Civil Engineering, National University of Singapore, Singapore.
Seow, P.E.C. and Swaddiwudhipong S. (2005), “Failure surface for concrete under multiaxial load - a unified approach”, *ASCE, J. Mater.Civil Eng.*, **17**(2), 219-228.
Sinha, B.P., Gerstle, K.H. and Tulin, L.G.(1964), “Stress-strain relations for concrete under cyclic loading”, *Proceedings of ACI Journal*, **61**(2), 195-212.
Tasuji, M.E., Slate, F.O. and Nilson, A.H. (1978), “Stress-strain response and fracture of concrete in biaxial loading”, *Proceedings of ACI Journal*, **75**(7), 306-312.
Tsai, W.T. (1988), “Uniaxial compressional stress-strain relation of concrete”, *ASCE, J. Struct. Div., ASCE*, **114**(9), 2133-2166.
Vecchio (1998), “Lessons from the analysis of a 3-D concrete shear wall”, *Struct. Eng. Mech.*, **6**(4), 439-455.
Willam, K. and Warnke, E. (1975), “Constitutive model for triaxial behaviour of concrete”, *Proceedings of the International Association for Bridge and Structural Engineering*, Zurich, Switzerland, 1-30.
Yan, Z. and Pantelides, C.P. (2006), “Fiber-Reinforced polymer jacketed and shape - modified compression members: II – Model”, *ACI Struct. J.*, **103**(6), 894-903.
Yin, W.S., Su, E.C.M., Mansur, M.A. and Hsu, T.T.C. (1989), “Biaxial tests of plain and fibre concrete”, *ACI Mater. J.*, **86**(3), 236-243.
Yu, M.H. (2002b), “Advances in strength theories for materials under complex stress state in the 20th century”, *Appl. Mech. Rev. ASME*, **55**(3), 169-218.
Yu, M.H. (2004), “Unified strength theory and its applications”, *Springer, Berlin*.
Zhi, W.C., Hai, G.Z. and Qin, Z.X. (1987), “Experimental investigation of biaxial and triaxial compressive concrete strength”, *ACI Mater. J.*, **84**(2), 92-100.

CC