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Influence of shear deformation of exterior beam-column joints on the quasi-static behavior of RC framed structures

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Abstract. In the analysis and design of reinforced concrete frames beam-column joints are sometimes assumed as rigid. This simplifying assumption can be unsafe because it is likely to affect the distributions of internal forces and moments, reduce drift and increase the overall load-carrying capacity of the frame. This study is concerned with the relevance of shear deformation of beam-column joints, in particular of exterior ones, on the quasi-static behavior of regular reinforced concrete sway frames. The included parametric studies of a simple sub-frame model reveal that the quasi-static monotonic behavior of unbraced regular reinforced concrete frames is prone to be significantly affected by the deformation of beam-column joints.

Keywords: reinforced concrete beam-column joints; joint shear deformation, quasi-static monotonic behavior; non-linear analysis; sway rc framed structures

1. Introduction

Sometimes, in the analyses of reinforced concrete (RC) frames, beam-column joints are assumed fully rigid and with an ultimate strength larger than the adjoining members. In other cases, beam-column joints are assumed perfectly pinned and with a deformation capacity large enough to accommodate the deformations required from equilibrium and compatibility conditions. In both cases, the detailing of RC beam-column joints is based in established patterns which are expected to produce an adequate solution in terms of load capacity and/or ductility. Even in the few cases where the suitability of a particular joint detailing is evaluated, usually this only comprises the ultimate load capacity of the joint, ignoring the assessment of compatibility of ultimate deformation of the beam-column joint with the adjacent members and of its effect on the overall structural performance.

Actually, the three main mechanical characteristics of a joint – stiffness, strength and ductility – can have such a strong influence on the overall behavior of RC frames, so as to render the above

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simplistic procedures unacceptable.

Paulay and Priestley (1992) show that, because shear force can be several times larger in joints than in the adjoining members, beam-column joints are sometimes the weakest link. Sharma *et al.* (2011), subjected a 3D RC frame, not designed for seismic loads, to a monotonically increasing lateral load, and concluded that in order to capture the failure modes and compute a realistic capacity curve, the joint shear behavior had to be accounted for in the analysis models. Shin and LaFave (2004), further point out that, even joints designed according to current codes or to mechanical criteria so as to remain in the elastic range of behavior, hence nearly rigid, are actually deformable, generating significant additional rotations between the ends of beams and columns, which are usually ignored in structural analysis. Lastly, the ductility of reinforced concrete beam-column joints is a fundamental aspect regarding structural robustness.

In this article attention is focused on the effect of the shear deformation of reinforced concrete beam-column (RCBC) joints, especially of exterior ones, on the overall behavior of regular unbraced frames, for the particular case of quasi-static monotonic actions. In this paper, both expressions “unbraced frames” and “sway frames” refer to frames where the resistance and stiffness to lateral loads is mainly provided by bending of frame members (beams and columns) and their connections.

The deformation of RCBC joints results essentially from the conjugation of two distinct phenomena: (i) shear deformation in the region of the joint where beams and columns intersect and (ii) anchorage slip of the longitudinal reinforcement (Biddah and Ghobarah 1999). Shear deformation is in many cases the most relevant of these components for sway frame behaviour (Altoontash 2004) and is the only one under analysis in this paper.

Because common RC structures are not, as a rule, very slender, they are not very sensitive to second order effects. Nevertheless, the construction of ever more slender RC structures is resulting from (i) the expanding use of high strength concrete and steel, to a certain extent due to a combination of relative price reduction and their superior performance with respect to durability and (ii) the urge for an enhanced architectural functionality linked to increasing span lengths and decreasing structural sections. As a result of the increasing slenderness of RC frames, second order effects turn out to be significant, particularly in unbraced frames – consequently, the deformation of beam-column joints may also become significant as, for instance, in steel sway frames.

The deformation of RCBC joints can be an important feature also in not so slender frames, because the lateral displacement caused by lateral loads is a key element in seismic performance analysis based in static nonlinear procedures (pushover analysis).

Following Gomes (2002), additional second order effects and lateral displacements caused by shear deformation of exterior beam-column joints of regular multi-storey sway frames are investigated for the particular case of quasi-static monotonic actions. The numerical analysis is performed on a representative sub-frame where, in order to circumvent the limitations of a full theoretical model, the developed beam-column joint model incorporates experimental data.

The purpose of this paper is to display the effect of RCBC joint shear deformation, explicitly modeled, upon the behavior of unbraced RC regular frames, in order to call attention to the risk of ignoring this effect. The paper does not provide explicit practical guidance to design nor a new behavior model. RCBC joint models and simplified criteria for evaluation of the effect of RCBC joints deformation on the global structural behavior are under development and will be published soon.

2. Sarsam's laboratorial results for RC beam-column joints

In this section, we first identify and validate the experimental data available in the literature and then explain how to objectively include them in the beam-column joint model – those more concerned with the structural model, numerical analysis and main results may skip it. Since RCBC joints are major sources of energy dissipation, it is obvious that during a seismic event, the stiffness degradation caused by damage of the RCBC joints has to be accounted for. However, since the focus of the present study is the quasi-static joint behavior, joint shear stiffness degradation caused by cyclic loads is not taken into account in order not to overemphasize RCBC joint shear deformation. Hence, the available results from earthquake and cycling loading tests are not of much use, which leaves us with the experimental results of Roeser (Hegger *et al*. 2003, Roeser 2002) and Sarsam (Sarsam 1983, Sarsam and Phipps 1985). This study made use only of Sarsam's experimental data.

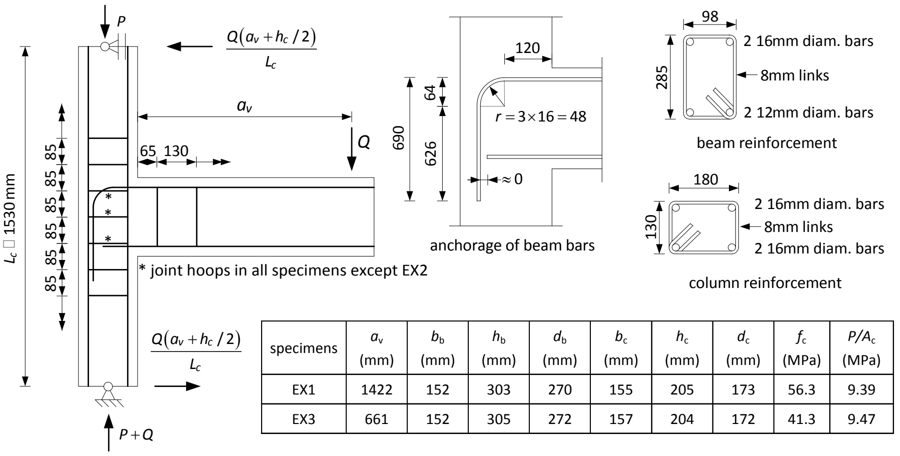
2.1 Summary of Sarsam's laboratorial results

Sarsam tested five exterior reinforced concrete beam-column joints, numbered from EX1 to EX5, subjected to quasi-static (single cycle) loads. The purpose of his investigation was to study the effect of joint reinforcement, applied moment-to-shear ratio and column axial load on the behavior of exterior RCBC joints. Fig. 1 presents the geometry of the five specimens, the load configuration, the reinforcement detailing and concrete strength values. The reinforcement bars were hot rolled deformed taken from the same supply batch and all the tested samples exhibited a yield plateau. The experimental average yield strength was about 500 MPa and the modulus of elasticity about 200 GPa. All specimens except EX2 had closed ties (hoops) in the joint. Loading was applied in two stages. Stage 1: apply the concentric load P to the column. Stage 2: while keeping the value of P fixed, apply transverse load at the beam tip and increase its magnitude until joint or beam failure.

All specimens exhibited joint diagonal cracking after testing. Up to diagonal cracking there was no noticeable difference in behavior between specimens with and without joint hoops. After diagonal cracking the influence of hoops became evident and specimen EX1, similar to EX2 but with joint hoops, revealed a much stiffer response than specimen EX2. Actually, specimen EX2 was the only one that failed due to joint shear, exhibiting afterwards a quick rise in joint deformation together with a continuous drop in joint load-carrying capacity. In the other specimens the failure occurred in the beam. After failure, specimen EX2 exhibited concrete spalling and cracks up to 2.5 mm wide on the three exposed faces of the joint. The joints of the other specimens exhibited only hairline cracks.

Specimen EX2 was excluded from the present study because (i) it has no joint hoops and hence it does not fulfill current technical specifications (see 2,3) and (ii) this study is intended to evaluate only the influence of RCBC joints deformation (not its strength), thus only full-strength joints were selected, *i.e.*, joints stronger than neighbor beams and columns; but the proposed analysis procedure could have been applicable to this specimen. From the remaining specimens, the most rigid (EX1) and the most flexible (EX3) were selected for this study, see Fig. 2, in order to offer the widest range of joint shear stiffness. These two specimens differ mainly in the concrete strength and the distance a_v from the face of the column to the location of the beam load, which equals the moment-to-shear ratio at the joint.

Fig. 2 shows the experimental moment-rotation relationships obtained by Sarsam (1983). In



specimens	a_v (mm)	b_b (mm)	h_b (mm)	d_b (mm)	b_c (mm)	h_c (mm)	d_c (mm)	f_c (MPa)	P/A_c (MPa)
EX1	1422	152	303	270	155	205	173	56.3	9.39
EX3	661	152	305	272	157	204	172	41.3	9.47

Fig. 1 Exterior RCBC joint tested by Sarsam (Sarsam 1983, Sarsam and Phipps 1985)

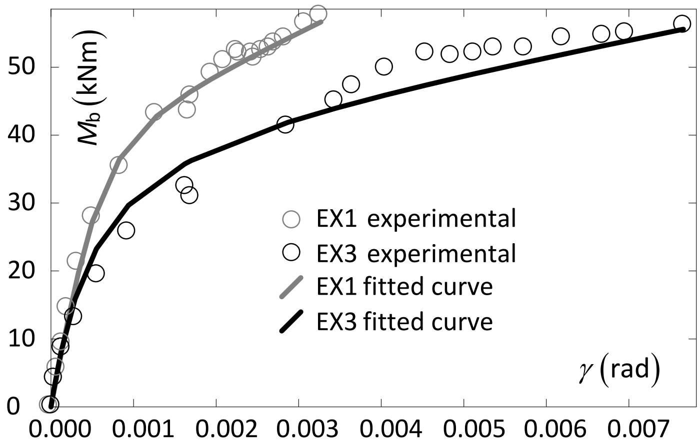


Fig. 2 Sarsam's experimental moment-rotation results for RCBC joints and fitted curves

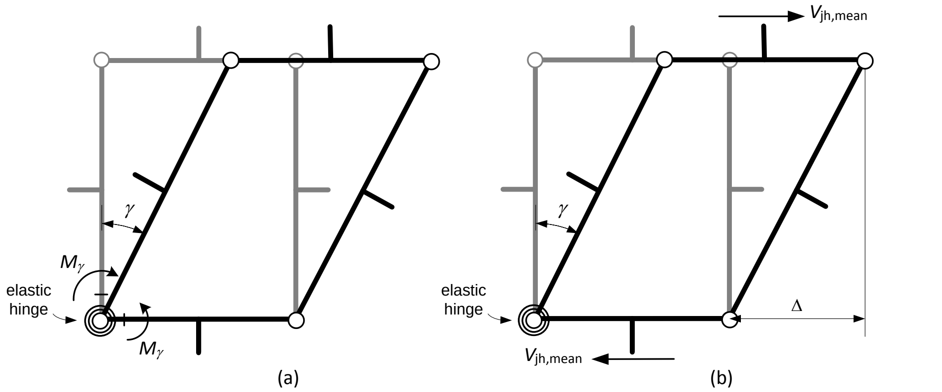


Fig. 3 Alternative conjugate pairs of joint generalized variables: (a) (M_γ, γ), (b) $(V_{\mathrm{jh,mean}}, \Delta)$

this figure, the rotation is the mean shear deformation of the joint γ (Fig. 3) while the bending moment M_{b} refers to the beam cross section at joint periphery (face of the column). Note that the peak value of the bending moment $M_{\mathrm{b,max}}$ in the experimental curves depicted in Fig. 2 does not correspond to the joint shear resistance. Since the collapse of specimens EX1 and EX3 was due to beam bending failure, these curves cannot reflect joint behavior until joint collapse. This "incompleteness" of Sarsam's records, regarding joint deformation in the neighborhood of its collapse, does not compromise the validity of our main conclusions. Actually, if experimental data for joint behavior until its collapse were available and had been employed, the practical outcome would be an even larger effect of joint deformation because usually joint flexibility increases greatly in the eminence of its collapse.

In order to remove local irregularities characteristic of sets of experimental data, we fit to Sarsam's data the smooth curve proposed by Richard and Abbott (1975), see Fig. 2.

2.2 Experimental data rescaling

When the beam-column joint deformation results predominantly from bond-slip inside the joint, the joint behavior can be reasonably modeled by an elastic hinge at the beam end. This model has been successfully employed in the simulation of seismic behavior of reinforced concrete framed structures by Anderson and Townsend (1977) and Townsend and Hanson (1973). However, whenever the joint shear behavior is relevant, such a simple characterization becomes inadequate, because the bending moment and joint shear deformation are not work-conjugate (Borkowski 1988), *i.e.*, the bending moment at the beam's end is not the unique internal force associated with joint shear deformation. Hence, Sarsam's experimental data should not be used directly for structural systems whose geometries or support/loading conditions differ from his.

Alternatively, the shear deformation of beam-column joints can be simulated with Krawinkler type models (Krawinkler 1978). With some of these models it is possible to condense in a single static variable the contribution to shear at the joint of all the internal forces at the neighbor beam and column ends. Moreover, according to Charney and Marshal (2006), these are the models whose deformation mode is closer to actual joint shear deformations. These are the reasons why the Krawinkler type model developed by Mitra and Lowes (2007) was adopted in this study.

According to Kim, LaFave and Song (2009), Hanson and Conner (1967) defined the shear force at mid-height of the joint $(V_{\mathrm{jh,max}})$ to be the best measure of the stress field in the RCBC joint; nowadays, this definition is accepted by most technical specifications.

In the following lines, Mitra and Lowes (2007) model is briefly described together with the static variable it uses to characterize joint shear behaviour $(V_{\mathrm{jh,mean}})$. The relations between (i) $V_{\mathrm{jh,mean}}$, (ii) $V_{\mathrm{jh,max}}$ and (iii) the experimental data reported by Sarsam are next investigated.

The RCBC joint model developed by Mitra and Lowes (2007) is composed of two pairs of parallel rigid straight bars hinged together and placed along the joint periphery. One hinge is elastic and characterized by the joint moment-rotation relationship, $M_{\gamma}-\gamma$, between angular distortion γ and distortional moment M_{γ}, see Fig. 3(a). In order to simulate bond-slip inside the joint and shear deformation at the beam and column ends Mitra and Lowes (2007) model also contains springs (not represented in Fig. 3) connecting the beams and columns to the rigid bars. However, since only joint shear deformations are accounted for in this study, these additional springs were assumed fully rigid.

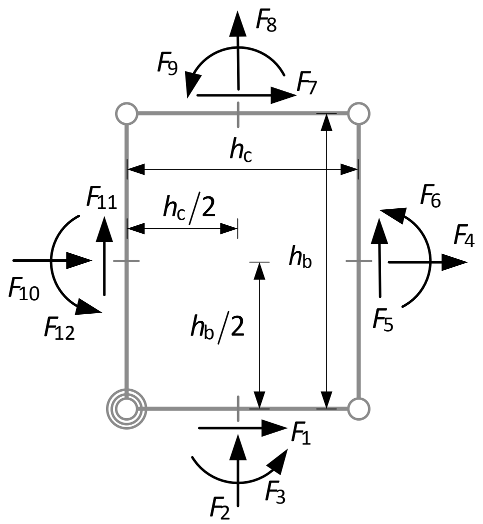


Fig. 4 Internal forces and moments at periphery of joint

Using the reference system of internal forces and moments F_i represented in Fig. 4 to express equilibrium with respect to the initial configuration, for the general case where the joint links two columns and two beams, we get the first order approximation for M_γ

$$M_\gamma^{1\text{st}} = h_c\left(\frac{F_3+F_9}{h_c}+F_5+\frac{1}{2}\left(F_2+F_8\right)\right) = h_b\left(-\frac{F_6+F_{12}}{h_b}+F_7+\frac{1}{2}\left(F_4+F_{10}\right)\right) \tag{1}$$

which, for the particular case of Sarsam´s experimental specimens (Fig. 1), gives

$$M_\gamma^{1\text{st,exp}} = -Q\,a_v\left[-1+\frac{h_b}{L_c}\left(1+\frac{h_c}{2a_v}\right)\right] = M_b\left(1-\frac{h_b/L_c}{1-h_c/L_b}\right) \tag{2}$$

where $L_b = 2a_v + h_c$. The last expression reveals that the relation between M_b and the distortional moment M_γ is not univocal. As stated before, this means that the $M_b - \gamma$ relation does not characterize the joint panel behavior properly. The conversion of the $M_b - \gamma$ relation to $M_\gamma - \gamma$ using Eq. (2) rescales Sarsam’s experimental records, providing a beam-column joint characterization independent of the structure.

The first order joint internal work associated to the pair of variables (M_γ, γ) is given by

$$W \equiv M_\gamma \cdot \gamma \tag{3}$$

This expression can be rewritten as

$$W \equiv M_\gamma \cdot \gamma = \left(\frac{M_\gamma}{h_b}\right)\cdot\left(\gamma \cdot h_b\right) = V_{\text{jh,mean}} \cdot \Delta \tag{4}$$

where Δ is the relative displacement between the top and bottom edges of the joint panel and $V_{\text{jh,mean}}$ is a measure of the joint (horizontal) shear force, Fig. 3(b), with magnitude

$$V_{\text{jh,mean}} \equiv \frac{M_\gamma}{h_\text{b}} = -\frac{F_6 + F_{12}}{h_\text{b}} + F_7 + \frac{1}{2}\left(F_4 + F_{10}\right) \tag{5}$$

Therefore, the pair $(V_{\text{jh,mean}}, \Delta)$ can be used instead of (M_γ, γ) to characterize the joint shear behavior, see Fig. 3(b), (Kato *et al*. 1988, Roeser 200).

As stated before, besides M_γ, or $V_{\text{jh,mean}}$, there is yet another static quantity which can be used to characterize the system of forces at the periphery of a beam-column joint – the horizontal shear force at mid-height of the joint, $V_{\text{jh,max}}$,which is the maximum horizontal joint shear force. In the literature, this is the most commonly used option to characterize beam-column joint strength and behavior (CEN 2004b, ACI Committee 318 2008, CEN 2005).

In order to compute $V_{\text{jh,max}}$ note that the nodal moment F_6 (and similarly F_{12}) is equivalent to a force couple of oppositely directed horizontal forces with magnitude F_6/z_b and arm z_b, where F_6/z_b is the beam longitudinal compressive force, equal to its longitudinal tension force, carried by concrete and longitudinal reinforcement at the beam's end cross section. Assuming also a uniform cross sectional stress distribution of the axial force at the beam's end, the horizontal force equilibrium condition at mid-height of the joint gives

$$V_{\text{jh,max}} = -\frac{F_6 + F_{12}}{z_\text{b}} + F_7 + \frac{1}{2}\left(F_4 + F_{10}\right) \tag{6}$$

The relation between $V_{\text{jh,max}}$ and M_γ (or $V_{\text{jh,mean}}$) is not univocal because it depends of the distribution of internal forces and moments at the periphery of the joint, *i.e.*, it is not invariant with respect to the structure overall geometry, load and support conditions. Hence, only when the distribution of internal forces and moments at the periphery of the joint is known *a priori* can the $V_{\text{jh,max}} - \gamma$ relations given in the bibliography be used in Mitra and Lowes (2007) model. However, for the particular case of Sarsam's specimens, since they are statically determined, it is straightforward to relate $V_{\text{jh,max}}$ to $V_{\text{jh,mean}}$ (or M_γ). Taking into account Sarsam's specimens geometry, Eq. (6) can be rewritten as

$$z_\text{b}\, V_{\text{jh,max}} = F_6\left(1 - \frac{z_\text{b}/z_\text{b}}{1 - h_\text{c}/L_\text{b}}\right) \tag{7}$$

Solving Eq. (2) for F_6 $(= M_\text{b})$ and introducing the result in (7) gives

$$V_{\text{jh,max}} = \frac{M_\gamma}{z_\text{b}} \frac{1-\left(h_\text{c}/L_\text{b}\right)-\left(z_\text{b}/L_\text{c}\right)}{1-\left(h_\text{c}/L_\text{b}\right)-\left(h_\text{b}/L_\text{c}\right)} = V_{\text{jh,mean}} \frac{h_\text{b}}{z_\text{b}} \frac{1-\left(h_\text{c}/L_\text{b}\right)-\left(z_\text{b}/L_\text{c}\right)}{1-\left(h_\text{c}/L_\text{b}\right)-\left(h_\text{b}/L_\text{c}\right)} \tag{8}$$

which, for Sarsam's specimens geometry, leads to $V_{\text{jh,max}} / V_{\text{jh,mean}} \approx 1.4$.

The characterization of RCBC joints by means of relationships $V_{\text{jh,max}} - \gamma$ or $V_{\text{jh,mean}} - \gamma$ is said objective because it gives a description of their shear behavior which is independent of the length of beams and columns, loading and support conditions. For instance, shear in the beam-column joint of the sub-frame increases with the column length, for constant bending moment at the beam end; this trend is satisfied by Krawinkler type models but not by Anderson and Townsend (1977) joint model.

2.3 Shear behavior and resistance of RC beam-column joint

The literature offers several shear resistance models for RCBC joints (*e.g.*, Paulay and Priestley (1992), Vollum and Newman (1999), Bakir and Boduroglu (2002), Kotsovou and Mouzakis (2012)) but only a few behavior models. The best known behavior models are based on the "Modified Compression Field Theory" (MCFT) from Vecchio and Collins (1986) and the "Softened Truss Model" (STM) from Hsu (1988). In both these models the joint is assumed to be a RC plane truss under a plain stress state produced by uniformly distributed normal and shear forces applied to the plate border. Good results from the application of MCFT and STM have been reported (Biddah and Ghobarah 1999, Lowes and Altoontash 2003) but, in fact, these models are rather complex and some simplifying assumptions are hard to justify for beam-column joints. Moreover, according to Kitayama, Otani and Aoyama (1991), the contribution of the truss mechanism is relevant to the joint shear behavior only under good bonding conditions of the longitudinal reinforcement of beam and column. Hence, these behavior models are expected to diverge from experimentally determined behavior as the magnitude of the forces in the joint increase.

More recently, Roeser (2002) and LaFave and Kim (2011) put forward simpler behavior models establishing constitutive relations $\tau_{\mathrm{jh}} - \gamma$, with $\tau_{\mathrm{jh}} = V_{\mathrm{jh,max}}/(h_{\mathrm{c}} b_{\mathrm{j}})$. Roeser employed experimental results of beam-column joints subjected to a monotonic (non-cyclic) loading, while Kim and LaFave applied a Bayesian statistical methodology to a large data base of beam-column joints subjected to cyclic loadings. Using Eqs. (2) and (8) to convert Sarsam's experimental relations $M_{\mathrm{b}} - \gamma$ to the $\tau_{\mathrm{jh}} - \gamma$ format, we can evaluate them against these behavior models, see Fig. 5 Note that Roeser's constitutive relation accounts for only two hoops (closed ties) as effective in the joint while Kim and LaFave's accounts for all hoops (three). Roeser's assumption follows Hamil (2000) who recommends that in RCBC exterior joints the transverse reinforcement in the beam compression zone should not be considered. In addition, Roeser's constitutive relation was modified in order to include the elastic shear deformation, assumed to be linear, in the uncracked phase of concrete. Fig. 5 shows that the predictions of both models agree reasonably well with the experimental data – the differences found between experimental results and these two models are close to what could be expected in the early current stage of development of behavior models for RCBC joints in shear (LaFave and Kim 2011).

Now let's evaluate Sarsam's experimental results against the shear strength of RCBC joints as given by codes of practice. Except for the reference in Annex J to strut-and-tie models of corner beam-column joints, EC2 (CEN 2004a) does not directly refer to the required resistance of beam-column joints neither to the design and detailing of their reinforcement. Therefore, ACI 318M-08 (ACI Committee 318 2008) and ACI 352R-02 (ACI-ASCE Committee 352 2002) recommendations were followed in order to assess the shear strength of joints EX1 and EX3. ACI 318M-08 recommends a minimum area of reinforcement given by

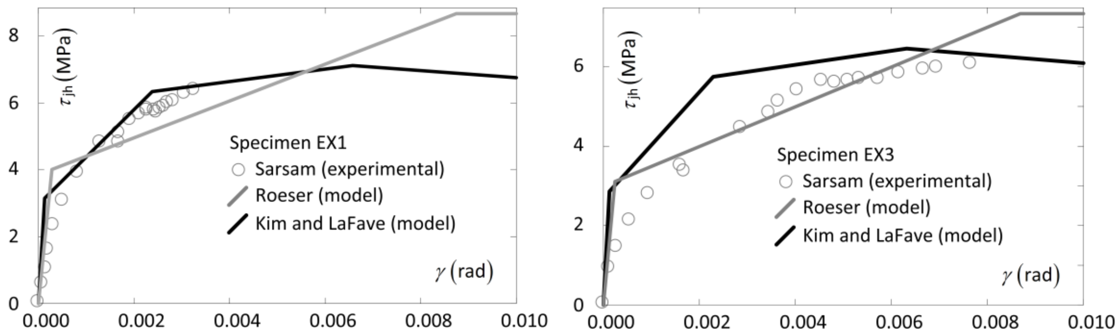


Fig. 5 Sarsam's joint shear behavior experimental results plotted against the relationships proposed by Roeser (2002) and LaFave and Kim (2011)

$$A_{v,\min} = 0.062\sqrt{f_c'(\text{MPa})}\,\frac{b_w\,s}{f_{yt}(\text{MPa})} \geq 0.035\frac{b_w\,s}{f_{yt}(\text{MPa})} \tag{9}$$

which gives $A_{v,\min}/s = 0.139\ \text{mm}^2/\text{mm}$ for specimen EX1 and $0.121\text{mm}^2/\text{mm}$ for EX3. In both cases $A_v/s = 1.188\ \text{mm}^2/\text{mm}$ and thus ACI 318M-08 conditions for a quasi-static action are satisfied. Sarsam's specimens also satisfy the maximum distance between shear reinforcement in the joint indicated by ACI 352R-02. Hence, joint shear resistance can be estimated by

$$V_n(\text{MN}) = 0.083\,C\,\sqrt{f_c'(\text{MPa})}\,b_j(\text{m})\,h_c(\text{m}) \tag{10}$$

where, for the joints tested by Sarsam (1983), $C = 15$. This expression gives $V_n = 297\,\text{kN}$ for specimen EX1 and $256\,\text{kN}$ for EX3.

According to the calculated moment-curvature relationships for the beam, the ultimate bending resistance of Sarsam's beams is $M_R = 52.1\,\text{kNm}$ for specimen EX1 and $M_R = 51.9\,\text{kNm}$ for EX3. Assuming $z_b = 0.85\,d_b$ and ignoring the benefic effect of the column shear force, gives a maximum horizontal joint shear force $V_{jh,\max} = 227\,\text{kN}$ for specimen EX1 and $V_{jh,\max} = 226\,\text{kN}$ for EX3. These results indicate that the beam-column joints in Sarsam's EX1 and EX3 specimens satisfy current technical regulations because their design guarantees that the structure collapse is initiated in the beam, *i.e.*, the joints were not under designed.

Although ACI 352R-02 recommendations for the development length l_{dh} of beam reinforcement are not met by any experimental data available for non-cyclic beam-column joint behavior (Sarsam 1983, Roeser 2002), the shear strength of the joints and the strength of reinforcement anchorage in beams and columns were enough to guarantee that the joint was stronger than the beam. Moreover, the deformation of reinforcement anchorage in beams and columns was not taken into account in the numerical analysis (only the joint shear deformation was accounted for) and the joint shear experimental behavior is close to that expected by Roeser (2002) and LaFave and Kim (2011) RCBC joints models. In view of these aspects the experimental results published by Sarsam were deemed valid and representative of common RC framed structures for the present study.

3. Numerical non-linear structural analysis

3.1 Assumptions

The behavior of multi-storey sway frames, assuming the regularity of their geometry, mechanical properties and loading, can be approximated by the behavior of a representative sub-frame (Kollár 1999). With such a sub-frame, two distinct but interconnected consequences of joint shear deformation are investigated: (i) reduction in the load-carrying capacity of the frame and (ii) magnification of the transverse deflections of the frame members for a given loading. The sub-frame model incorporates the shear behaviour of two RCBC exterior joints experimentally determined by Sarsam (1983) (Fig. 2 and Fig. 5). Even though the use of broad-spectrum analytically defined relationships for joint shear behaviour could lead to wider-ranging results, this generalization was sacrificed to what is deemed to be a more realistic description of joint shear deformation.

It must be emphasized that in RCBC joints designed according to current seismic design codes bond-slip is often the major source of deformation; hence it should also be considered in a more universal analysis. The present study aims to show that the deformation of RCBC joints is important and should not be disregarded; hence, if all sources of joint deformation had been taken into account, the relevance of joint deformation would have been even greater.

Fig. 6(b) shows the initial and deformed configurations of the sub-frame as well as the loading conditions. This sub-frame system represents part of a typical regular multi-storey sway frame, which will be called the original frame, depicted in Fig. 6(a). It includes one joint linking together one beam, whose length is half the length of the beams in the original frame, and two columns, whose length is half the length of the columns in the original frame. The sub-frame has a hinge support at the bottom end of the lower column and a simple hinge support at the beam right end. There is a pair of point loads, one horizontal, H, and the other one vertical and concentric, P, applied at the free end of the upper column. (It is clear that the axial force is zero in the beam.) The total sway deflection of the loaded section is similar to the inter-story drift of the corresponding floor of the original frame. Thus, H and P represent the contribution of all floors above the typical one under analysis.

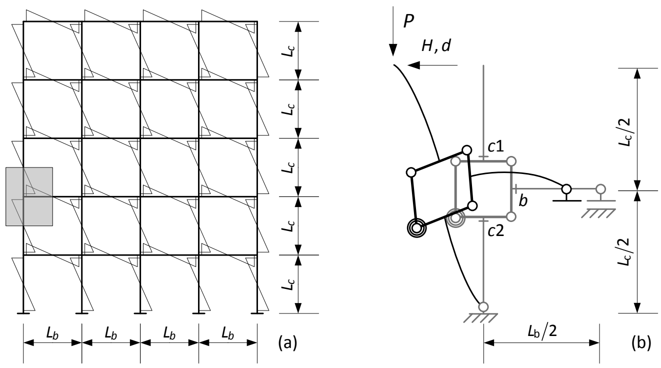


Fig. 6 (a) Bending moment diagram of regular multi-storey sway frame subjected to lateral loads, (b) initial and deformed configurations of the sub-frame implemented in the numerical analysis

If we assume that there is no relevant horizontal load transfer between exterior columns and interior columns in the original frame, *i.e.*, the axial force in the beams is negligible, then the sub-frame behavior is representative of the overall behavior of the original frame.

The beam and column cross sections are symmetric with respect to the plane defined by beam and column centerlines, and their dimensions are considered small when compared to the member length, so that the Bernoulli-Euler-Navier hypothesis is assumed to be valid. Small strains and rotations are assumed, preserving cross sectional geometry and lengths orthogonally projected to the initial configuration. Hence, the strain field in the linear elements is defined by small normal strains parallel to the element axis. The normal stresses are computed from these strains by means of (i) MC90 (MC90 (1990)) non-linear concrete stress-strain relationship (analogous to the one in section 3.1.5 of EC2 (CEN 2004a) and (ii) the reinforcement steel elastic-perfectly plastic stress-strain diagram in section 3.2.7 of EC2 (CEN 2004a).

3.2 Procedure for non-linear analysis

The numerical assessment of the effect of deformation of RCBC joints requires a geometric and material non-linear analysis. The implemented iterative procedure is based on the General Method in section 5.8.6 of EC2 (CEN 2004a), previously proposed in CEB (1974), except that creep is not taken into account. To begin with, the cross section thrust-moment-curvature relationship for the column and the moment-curvature relationship for the beam are computed using a standard fiber model analysis taking into account the geometric and material characteristics of specimens EX1 and EX3 given by Sarsam (1983). Subsequently, the concentric vertical force P is first applied and then a load control analysis is performed where the horizontal force H is progressively increased in small increments ΔH until it reaches its maximum value. For each pair (P, H), the numerical solution is calculated as follows (CEN 2004a, Westerberg 2004): (i) establish the equilibrium equations in the current deformed configuration and calculate the updated bending moment at a fixed number of closely spaced cross sections, (ii) determine the curvature using the previously computed constitutive relationships for each cross section, (iii) determine the updated deformed configuration by double integration of the curvature, assuming linear variation between these cross sections, (iv) go back to step (i) while convergence in the deformed configuration is not achieved, (v) record deformed configuration (including the displacement d) and matching loads (P, H). If, for a given increment ΔH, there is no equilibrium configuration, this iterative scheme will not converge. In this case, go back to the previous recorded solution, decrease ΔH and proceed with this new updated increment. The positive horizontal force increment ΔH can be successively decreased, till it reaches a value below a small tolerance, $\Delta H_{\min}$. When, for a given vertical force P, this happens, the last value of H for which an equilibrium solution was numerically found is defined as the maximum horizontal force $H_{\max} \equiv H_{\max}[P]$. The ultimate state of the sub-frame, associated to $H_{\max}$, may correspond to (i) a material failure, (ii) the maximum experimentally registered value of M_{γ}, or (iii) a stability failure associated with a limit point, see Section 3.3. The $H-d$ curves thus determined correspond to standard pushover curves.

The effect of the shear deformation of RCBC joints was evaluated in a second order analysis, *i.e.*, where the equilibrium equations refer to the updated deformed configuration, by comparing the solutions for the sub-frame considering deformable joints with those considering rigid joints, always including the non-linear material behavior of the beam and columns. For the non-rigid case, the constitutive relation for joint shear behavior is one of the two presented in section , based on Sarsam's experimentally obtained results.

3.3 Characterization of the collapse modes

The critical elements in the sub-frame are the joint itself and the three cross-sections at the joint periphery, where it interfaces with the columns and beam. During the iterative analysis, for each load combination, it is necessary to monitor (i) the internal bending moment at sections c1, c2 and b (see Fig. 6(b)), $M_{\mathrm{E,c1}} \equiv M_{\mathrm{E,c1}}[P,H]$, $M_{\mathrm{E,c2}} \equiv M_{\mathrm{E,c2}}[P,H]$ and $M_{\mathrm{E,b}} \equiv M_{\mathrm{E,b}}[P,H]$, respectively, which cannot exceed the matching cross-section strength, *i.e.*, $M_{\mathrm{R,c1}} \equiv M_{\mathrm{R,c1}}[P]$, $M_{\mathrm{R,c2}} \equiv M_{\mathrm{R,c2}}[P]$ and $M_{\mathrm{R,b}}$, and (ii) the distortional moment at the joint $M_{\mathrm{E},\gamma} \equiv M_{\mathrm{E},\gamma}[P,H]$ which cannot exceed the maximum experimentally recorded value, $M_{\gamma,\max}$. To evaluate the risk of material collapse or of surpassing the maximum experimentally recorded value $M_{\gamma,\max}$, the following non-negative parameters were defined

$$\mu_1 \equiv \mu_{\mathrm{c1}} \equiv \frac{|M_{\mathrm{c1}}|}{M_{\mathrm{R,c1}}}, \quad \mu_2 \equiv \mu_{\mathrm{c2}} \equiv \frac{|M_{\mathrm{E,c2}}|}{M_{\mathrm{R,c2}}}, \quad \mu_3 \equiv \mu_{\mathrm{b}} \equiv \frac{|M_{\mathrm{E,b}}|}{M_{\mathrm{R,b}}}, \quad \mu_4 \equiv \mu_{\mathrm{j}} \equiv \frac{|M_{\mathrm{E},\gamma}|}{M_{\gamma,\max}}$$

If the combination of external forces under analysis corresponds to a material failure or the attainment of $M_{\gamma,\max}$, (at least) one of the μ_e has unit value, *i.e.*, $\mu_{\max} = \max_{e=1,\cdots,4}[\mu_e] = 1$.

On the contrary, in the case of a stability failure, $\mu_{\max}$ can be, and in general is, less than one.

The stability failures identified in the present study are associated to a limit point in the column and resulted from the sudden bending stiffness reduction (Bažant and Cedolin 2003) due to beam reinforcement yielding – this is almost coincident with the attainment of the ultimate strength at the critical section where yielding takes place, and is identified by a value for $\mu_{\max}$ slightly below unity.

The value of $\mu_{\max}$ as well as the element where this maximum ratio is attained are incorporated in the graphical representation of results in Section 4.

4. Numerical analysis results

4.1 Parametric analysis

In the parametric analysis, four parameters were considered: the horizontal force H, the vertical force P and the simplified slenderness of the beam, $L_{\mathrm{b}}/h_{\mathrm{b}}$, and columns, $L_{\mathrm{c}}/h_{\mathrm{c}}$. The concentric vertical force P is defined by means of the non-dimensional axial force in the upper column, $n = P/N_{\mathrm{c}} = P/(A_{\mathrm{c}}\, f_{\mathrm{c}})$, which takes values in the range $[0.1, 0.6]$, a positive value meaning compression. The slenderness $L_{\mathrm{b}}/h_{\mathrm{b}}$ and $L_{\mathrm{c}}/h_{\mathrm{c}}$ take values in the discrete sets $\{5, 10, 15\}$ and $\{5, 7.5, 10\}$, respectively. These values cover the range which can be realistically expected for buildings in present and near future.

In order to evaluate the effect of the shear deformation of RCBC joints, we compare the second order solution of the sub-frame with deformable joints (labeled "NRig") with that of the sub-frame with rigid joints ("Rig"). For a given load level, the rigid joints solution can be thought of as an approximation to the more exact deformable joints solution, their difference giving the absolute error. The relative error α is defined by the ratio of the absolute error to a reference solution. High values of $|\alpha|$ indicate an excessive error of the rigid joints solution, meaning that this approach should not be used in common practice to replace the one with deformable joints. For the case of maximum allowable lateral load, according to ENV 1993 (CEN 1994, Gomes *et al*. 1998), a steel beam-column joint should be considered semi-rigid for $|\alpha| > 5\%$. Due to the lack of specifications for reinforced concrete structures, this limit was adopted also for RC beam-column joints. Accordingly, joint deformation is deemed relevant if the value of parameter α associated to the effect of joint shear deformation upon the maximum allowable horizontal load H is larger than 5%.

4.2 Effect of joint shear deformation on maximum allowable lateral load

This section considers the maximum value attainable by the horizontal force H on a second order analysis of the sub-frame for fixed values of the vertical load P or, more specifically, how this maximum value of the load changes when the shear deformation of the joint is taken into account, $H_{\max}^{\mathrm{NRig}}$, or not, $H_{\max}^{\mathrm{Rig}}$.

Fig. 7 depicts the maximum allowable value of the horizontal load for both cases of rigid and non-rigid joints, represents de value of $\mu_{\max}$ ($\times 100$) and identifies the corresponding critical element ("b" for beam and "j" for joint). This figure reveals that the maximum allowable horizontal load value decreases when (i) the slenderness of the column (L_c/h_c) or beams (L_b/h_b) increases, (ii) the axial compressive loading on the column increases or (iii) the joint shear deformation is incorporated, i.e., $H_{\max}^{\mathrm{NRig}} < H_{\max}^{\mathrm{Rig}}$. Moreover, when P increases, $H_{\max}$ decreases almost linearly and the absolute difference between $H_{\max}^{\mathrm{NRig}}$ and $H_{\max}^{\mathrm{Rig}}$ increases. As expected, Fig. 7 also shows that, for the same loading conditions, *i.e.*, same loads P and H, the more slender the column is, the more stressed is the joint (*i.e.*, V_{jh} increases with the ratio L_c/h_c). Hence RCBC joints are prone to be the critical elements in slender frames.

Let us define

$$\alpha_H \equiv \alpha_H\left[P\right] = \frac{H_{\max}^{\mathrm{NRig}} - H_{\max}^{\mathrm{Rig}}}{H_{\max}^{\mathrm{Rig}}} \tag{11}$$

which gives the relative reduction of the maximum lateral load due to joint shear deformation. Fig. 8 depicts the variation of $\alpha_H[P]$, confirming that this parameter is always negative, *i.e.*, joint deformation reduces $H_{\max}$. This figure also depicts the 5% critical boundary, showing that the reduction of lateral load capacity due to joint shear deformation is effectively significant. This provides clear evidence of the need (i) to include joint shear deformation in the analysis of unbraced RC regular frames, and (ii) to define criteria which identify the cases where the contribution of joint shear deformation can be neglected. In fact, many combinations of the parameters $(n, L_c/h_c, L_b/h_b)$ cause a relative decrease of the maximum allowable horizontal load greater than 5%.

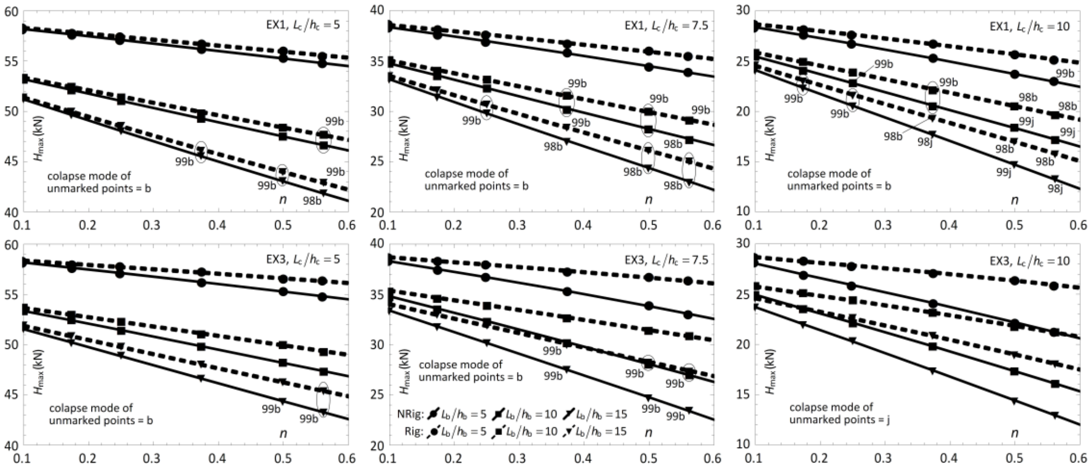


Fig. 7 Maximum allowable value of the horizontal load

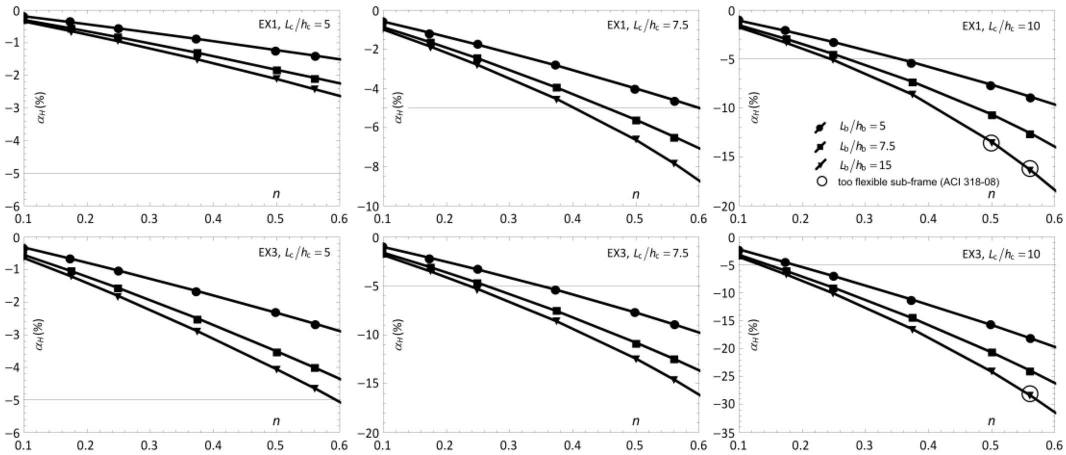


Fig. 8 Relative reduction of the maximum horizontal load due to joint shear deformation

As an alternative to the investigation of the reduction of the maximum value of H for a given value of P, we could study the reduction in the vertical load capacity for fixed values of H, but this is omitted for brevity.

Fig. 8 also presents the cases for which the structure is too flexible according to section 10.10.2.1 of ACI 318 (ACI Committee 318 (2008)) for rigid RCBC joints. Even when these cases are ignored, the deformation of RCBC joints still has a significant impact on the reduction of lateral load capacity in regular sway frames.

4.3 Effect of joint shear deformation on transverse deflections

We now consider the effect of joint shear deformation on the lateral deflection (second order analysis) of the tip loaded section of the sub-frame, d, which is equivalent to the inter-storey drift.

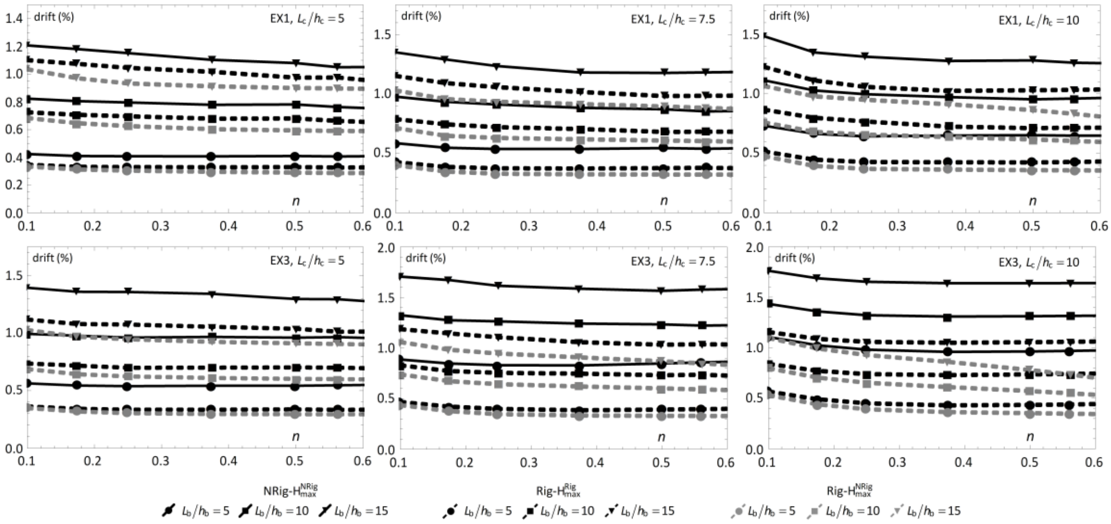


Fig. 9 Variation of column tip deflection with axial load

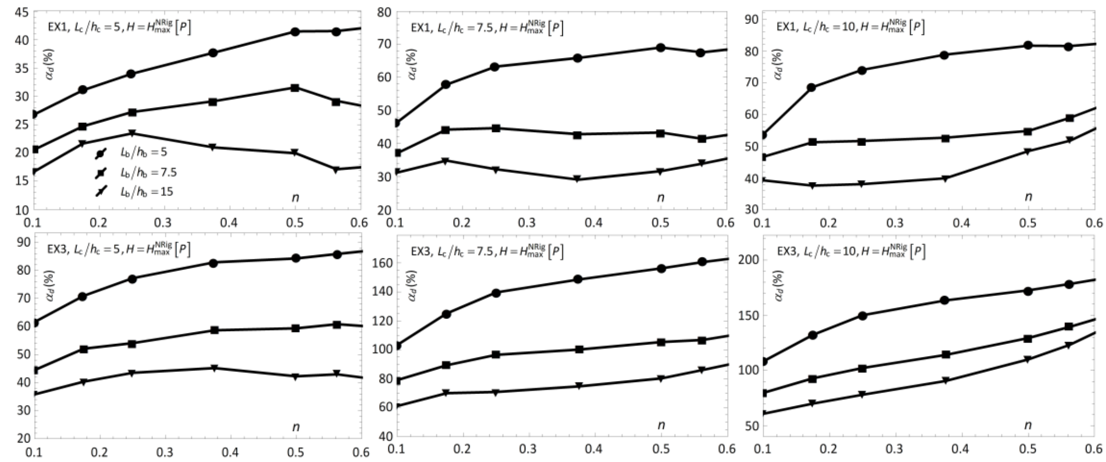


Fig. 10 Magnification of column tip deflection due to joint shear deformation

Fig. 9 depicts this deflection (i) for each pair of loads (P, H_{max}^{NRig}) for both rigid and deformable joints and (ii) for each pair of loads (P, H_{max}^{Rig}) for rigid joints only, showing that deflections increase with (i) the increase of the slenderness of the sub-frame, *i.e.*, with increasing L_c/h_c and L_b/h_b ratios, and (ii) the inclusion of the joint shear deformation. The total deflection of the sub-frame with rigid joints $d^{Rig}[H_{max}^{Rig}]$ can be decomposed into two pieces: the first one $d^{Rig}[H_{max}^{NRig}]$ is associated with H_{max}^{NRig}, and the second $d^{Rig}[H_{max}^{Rig}] - d^{Rig}[H_{max}^{NRig}]$ with the load increment $H_{max}^{Rig} - H_{max}^{NRig}$. (Total deflection of the sub-frame with deformable joints only presents the first of these components.) Fig. 9 shows that the drift hardly varies with *P*. Moreover, and perhaps rather unexpectedly, when we compare total deflection for deformable joints with total deflection for rigid joints, it shows that the increment due to joint deformation is almost independent of *P*. This justifies the almost linear variation of H_{max} with P observed in Fig. 7, because the increase of bending moments due to increasing *P* is compensated by the decrease of the maximal horizontal force $H_{\max}$. In fact, admitting that the constant bending resistance $M_{\mathrm{R,b}}$ is the conditioning parameter, the bending moments distribution at collapse does not change a lot – their magnitude increases only slightly with *P* along the columns. Therefore, the invariability of $M_{\mathrm{R,b}}$ and $M_{\gamma,\max}$ balances the effects of *P* and *H*. Accordingly, it is the difference of maximum drift between the rigid and deformable joint cases, $d^{\mathrm{NRig}}[H_{\max}^{\mathrm{NRig}}]-d^{\mathrm{Rig}}[H_{\max}^{\mathrm{Rig}}]$, which increases with the structure slenderness – not the maximum column rotation, $(d^{\mathrm{NRig}}[H_{\max}^{\mathrm{NRig}}]-d^{\mathrm{Rig}}[H_{\max}^{\mathrm{Rig}}])/L_{\mathrm{c}}$, which is approximately constant – that explains why the more slender the sub-frame, the higher the reduction of the transverse load carrying capacity defined by $H_{\max}$.

If we now consider the variation of deflection due to joint deformation for $H_{\max}^{\mathrm{NRig}}$, we get the relative transverse drift magnification

$$\alpha_d = \frac{d^{\mathrm{NRig}} - d^{\mathrm{Rig}}}{d^{\mathrm{NRig}}} \tag{12}$$

whose values are depicted in Fig 10. Note first that the magnification is surprisingly high, *i.e.*, joint deformation can almost double the transverse drift. Note also that the smaller the slenderness $L_{\mathrm{b}}/h_{\mathrm{b}}$ the larger α_d. A simple explanation for this result is that deformable joints are more effective in stiff structures than in deformable ones. This trend for the variation of the transverse drift is pointed out because it opposes the one obtained for α_H (Fig. 8) and means that, depending on the quantity being analyzed, the RCBC joint shear deformation might be equally relevant in slender and in bulky frames, *e.g.*, for a pushover analysis.

These opposite trends are not due to RC nonlinear material behavior and can also be observed in frame structures with linear material behavior. Fig. 10 also shows that the relative magnification of the transverse drift generally increases with $L_{\mathrm{c}}/h_{\mathrm{c}}$ and *P*. However, when the failure cause changes from material to instability the transverse drift diminishes with increasing *P*.

5. Conclusions

The present study shows that the shear deformation of exterior beam-column joints can have a relevant impact in the quasi-static short-term structural behavior of RC regular unbraced frames, and should thus be considered in the analysis and design of this type of structures.

Because of the large number of parameters influencing the behavior of a complete frame, the use of a simpler sub-frame is recommended as a first step for clarifying this effect. Results on RCBC joint deformation relevance using complete frames will soon be published by the authors.

The relative reduction of maximum allowable horizontal load α_H and the relative increase of horizontal displacement (drift) α_d were the parameters used in this study to evaluate the relevance of RCBC joint shear deformation.

In steel frames (CEN 1994, Gomes *et al*. 1998, Faella *et al*. 2000, Gomes 2002), the flexibility of the joint is deemed relevant to overall structural behavior when it originates a load capacity reduction above 5%. If the same criterion is applied to RC structures, our parametric study shows that RCBC joint shear deformation may have a relevant impact upon structural behavior, *i.e.*, it can reduce the load capacity of regular frames more than 5%. This reduction is higher (i) in slender frames, *i.e.*, with higher $L_{\mathrm{b}}/h_{\mathrm{b}}$ and $L_{\mathrm{c}}/h_{\mathrm{c}}$ ratios and (ii) with higher column compressive forces.

It was also shown that RCBC joint shear behaviour has a strong influence in lateral deformation of regular reinforced concrete unbraced frames. From the parametric study it was concluded that the RCBCJ shear deformation may easily increase lateral displacement in more than 50%. This increase is higher in regular frames with (i) higher L_c/h_c ratios, (ii) lower L_b/h_b ratios, and generally speaking, (iii) higher column compressive forces.

The present study covers a large range of frame slenderness values and load conditions, particularly of common unbraced sway frames which can be found nowadays or are likely to be found in the near future. However, this study is linked to a particular set of experimental data and RCBC joint shear behaviour depends on factors that were not accounted for, and which may increase or reduce joint shear stiffness, namely, reinforcement detailing, joint slenderness, joint eccentricity, presence of transversal elements, etc. Hence, the results from the parametric study should be carefully examined in order to avoid abusive generalizations.

Note finally, that our conclusions would have been even more striking, had we not ignored (i) serviceability limit states and (ii) other significant contributions to joint deformation, namely the beam and columns rebars slippage inside the joint and the consequent beam and column end additional rotations.

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