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Shear strength of full-scale steel fibre-reinforced concrete beams without stirrups

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Abstract. Although shear reinforcement in beams typically consists of steel bars bent in the form of stirrups or hoops, the addition of deformed steel fibres to the concrete has been shown to enhance shear resistance and ductility in reinforced concrete beams. This paper presents a model that can be used to predict the shear strength of fibrous concrete rectangular members without stirrups. The model is an extension of the plasticity-based crack sliding model originally developed for plain concrete beams. The crack sliding model has been improved in order to take into account several aspects: the arch effect for deep beams, the post-cracking tensile strength of steel fibre reinforced concrete and its ability to control sliding along shear cracks, and the mitigation of the shear size effect due to presence of fibres. The results obtained by the model have been validated by a large set of experimental tests taken from literature, compared with several models proposed in literature, and numerical analyses are carried out showing the influence of fibres on the beam failure mode.

Keywords: shear; steel fibre reinforced concrete; plasticity; size effect; Kani's Valley

1. Introduction

The use of discontinuous, randomly oriented fibres has long been recognized to provide post-cracking tensile resistance to concrete. As consequence, their use as shear reinforcement in reinforced concrete (RC) beams has been the focus of several investigations in the past four decades (Mansur *et al.* 1986, Li *et al.* 1992, Kwak *et al.* 2002, Cucchiara *et al.* 2004, Minelli and Vecchio 2006, Dinh *et al.* 2010, Foster 2010).

Fibre reinforcement enhances shear resistance by bridging tensile stresses across diagonal cracks and reducing diagonal crack spacing and width, which increases aggregate interlock effect. The reduction in crack spacing due to the presence of fibres indicates that the use of fibre reinforcement could potentially lead to a reduction of the shear size effect in beams without stirrups (Minelli 2005), whose shear strength is known to decrease as the overall beam depth increases (Bentz 2000). Recently, fibres have been employed in concrete to cast full-scale beams (Dinh *et al.* 2010), indicating that they can safely be used as minimum shear reinforcement in RC

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beams constructed with normal-strength concrete and within the range of member depths considered (455-685 mm). Specially in seismic areas, the optimize mix of fibres and stirrups as shear reinforcement provides outstanding results both in terms of cost and decongestion of classical reinforcement (Spinella *et al.* 2012, Colajanni *et al.* 2012).

Fibres increase the residual tensile strength of the composite, enlarging the softening branch of the stress-strain curve in compression and, hence, significantly enhance the resisting strut area, in the final arch resisting mechanism (Minelli and Vecchio 2006). Thus, the major difference in the shape of constitutive laws between plain and fibrous concrete lies in their different post-peak behaviours, especially for Steel Fibre Reinforced Concrete (SFRC) under tensile stress.

At the aim of predicting the shear strength of SFRC beams without stirrups, several empirical or semi-empirical formulations have been proposed in literature. Numerous researchers have extended known formulations, originally suggested for plain concrete beams, to fibrous concrete members, providing an additional contribution to shear strength depending on the amount of fibres (Ashour *et al.* 1992, Campione *et al.* 2006, Foster 2010, Model Code 2010 2010).

Semi-empirical models are usually obtained by a regression analysis of SFRC beam test data for a few fibre types and volume percentages (V_f) and, hence, their use is restricted to the limited range of variation of fibre parameters investigated in experimental tests.

The purpose of this paper is to present the recent upgrading of a semi-rational model for the shear strength prediction of SFRC beams without stirrups. The model is based on the upper bound principle of the theory of plasticity and limit analysis concepts (Zhang 1997, Nielsen 1999). Further, application of the upper bound principle seems to be the simplest way to extent models for plain concrete members to cover members with fibrous concrete.

Some research works (Zhang 1997, Vecchio 2000) on plain concrete shear behaviours have shown that slips along the crack can delay or prevent the development of direct strut action spanning between the loading and the support points of beams. These certainly imply that sliding displacements can occur along the crack and the failure crack can originate from a generic section between loading and support point. This failure mechanism is typical of slender beams and it is taken into account by plastic theory in an interesting extension of the original plastic solution, called Crack Sliding Model (CSM) originally proposed by Zhang (1997).

A first extension to the case of FRC beams was proposed by Voo *et al.* (2006), assuming a constant value of 0.80 for effectiveness factors in compression and tension, and calculating the maximum tensile stress by the Variable Engagement Model (VEM) suggested by the same authors (Voo and Foster 2003). Recently, the formulation of CSM for the evaluation of ultimate shear strength of RC beams without stirrups has been improved to also determine the ultimate shear capacity of short reinforced concrete members (Spinella *et al.* 2010). In this paper, an appropriate residual tensile strength law for fibrous concrete is introduced to take into account the ability of fibres to bridge tensile stress across crack. Moreover, the ability of fibres to contain shear crack slips and the width of cracks is reproduced by a modification of the efficiency factor as function of toughness of SFRC. In addition, the shear size effect is reduced by presence of fibres with respect to the case of plain concrete beams, and it is herein taking into account introducing in the CSM the dependence of efficiency factor by geometrical properties of fibres which reflects the beneficial action of fibres on the shear size effect.

The proposed formulation is corroborated by the results of a wide database of experimental tests collected in literature, and its efficacy of predicting the shear strength of specimens is compared with several models known from literature. Lastly, numerical analyses have been carried out by using the proposed model, showing the effect of steel fibres of governing the failure mode of beam.

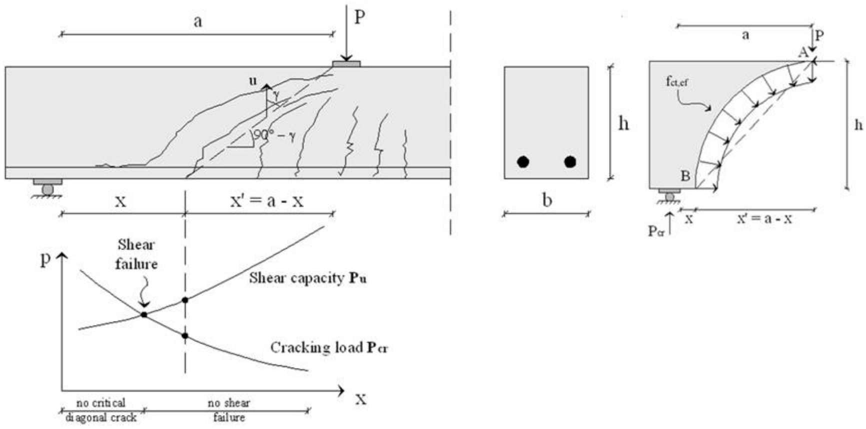
2. Review of crack sliding model formulation

In the theory of plasticity applied to concrete structural elements, reinforcement is assumed to resist forces in axial direction only, with yield stress equal to f_y. Concrete is assumed to behave as a rigid, perfectly plastic material, obeying the modified Coulomb failure criterion with the associated flow rule (Nielsen 1999).

At failure, it is necessary to account for the concrete compression strength reduction owing to cracking and softening due to tensile strains regime along the orthogonal direction to compression strain. The Modified Compression Field Theory (MCFT) accounts for the effect of cracking by operating with a compression strength influenced by a compression softening coefficient (less than one), which depends on the level of the transverse tensile strain (Vecchio and Collins 1986). For the plasticity models, a so-called effectiveness factor of concrete must be introduced. It is determined by correlating the theoretical solutions with test results. In the original plastic solution the effective compressive strength is evaluated in the following form (Nielsen 1999)

$$f_{c,ef} = \nu_c f_c = \left\{ \left(0.35/\sqrt{f_c}\right)\left[0.27\left(1+1/\sqrt{h}\right)\right]\left(0.15r+0.58\right)\left[1.0+0.17\left(a/h-2.6\right)^2\right] \right\} f_c \quad (1)$$

where ν_c= effectiveness factor for plain concrete in compression; f_c= compressive cylinder concrete strength, a= shear span, h and b height and width of beam cross section, A_s= longitudinal reinforcement area and $r= 100\ A_s/bh$ longitudinal reinforcement percentage. As concrete is not a perfectly plastic material, the dependency on f_c and h reflects compression softening and shear size effects. The dependency on r is mainly attributable to dowel action. Eq. (1) proves that ν_c is a function of shear span-height ratio a/h. This dependency has been considered unsatisfactory from a design point of view (Nielsen 1999).



(a) (b)

Fig. 1 Crack pattern in a beam without stirrups at (a) shear failure and (b) first cracking (Spinella *et al.* 2010)

Thus, for a simply supported rectangular beam loaded with two symmetrically point loads (Fig. 1(a)), the ultimate average shear stress may be evaluated using the upper bound approach of the limit analysis that is to consider the equality between internal and external works (Nielsen 1999). However, Zhang (1997) has provided some effective extensions to the original plastic theory, based on the consideration that the slippage along a crack, originated in a generic section of the shear span, may be more dangerous than slippage along the theoretical cracking line between support and load application point on the top level of beam, as is assumed in the original plastic solution. Thus, a different starting section of the critical diagonal crack must be determined, so the load level necessary to fully develop a diagonal crack in a generic section of the beam along the shear span is determined considering a semicircular crack (Fig. 1(b)) for the beam. The average cracking stress τ_{cr} is a function of the effective tensile strength of concrete $f_{ct,ef}$, which, for a beam with height h, is calculated as $f_{ct,ef}= 0.156f_c^{2/3}(h/100)^{-0.3}$ (in MPa). The distance from the support of the starting section of the critical diagonal crack x is obtained by equating the average shear stress failure τ_u and the average cracking stress τ_{cr}, that is the intersection of the shear capacity and cracking load curves in Fig. 1(a). The two curves do not always intersect, because the cracking load curve can be lower than the shear capacity curve within the x range. In these cases, the shear capacity coincides with the value of the original plastic solution ($x= 0$).

Introducing this new concept, Zhang (1997) eliminated the dependence of ν_c by shear span-height ratio a/h and the effectiveness factor for uncracked concrete only needs to take into account micro-cracking and softening effects. If sliding failure takes place in an existing crack, the effectiveness factor for plain cracked concrete in compression begins the product of two terms

$$\nu_c = \nu_s \nu_0 = \nu_s \left\{ \left(0.56 / \sqrt{f_c}\right) \left[0.27\left(1 + 1/\sqrt{h}\right)\right] \left(0.15r + 0.58\right) \right\} \tag{2}$$

where ν_s= 0.50 is the sliding reduction factor due to the reduced cohesion of cracked plain concrete when the yield line follows the diagonal crack path or crosses many cracks; ν_0 is a modified part of the empirical formula (1) obtained in the original plastic solution (Nielsen 1999). It also to be noted that the Disturbed Stress Field Model (DSFM), which updates the MCFT, has adopted an analogous coefficient to take the influence of crack sliding on compression softening into account (Vecchio 2000).

The CSM has been validated by Zhang (1997) on a large database of data collected in literature. The tests considered by the author for the model corroboration have been characterized by values of $a/h > 2$, thus most specimens collapse for diagonal tension. Because the CSM is not able to reproduce the arch action, validated numerical results fails for $a/h < 2$. In order to eliminate this drawback, the CSM has been improved retaining the correlation of the effectiveness factor for plain concrete in compression by the a/h ratio for $a/h < 2$, i.e., assuming the additional term $f_4 = [1.0+0.17(a/h-2.6)^2]$ in Eq. (2) only for $a/h \leq 2.6$ (Spinella *et al.* 2010).

3. Upgrading of crack sliding model to SFRC Large beams

In order to extend the CSM formulation to fibrous concrete beams, two important issues are: the evaluation of the tensile stress bridged across the shear crack at failure; and the capacity of model of adequately reproducing the size effect in shear.

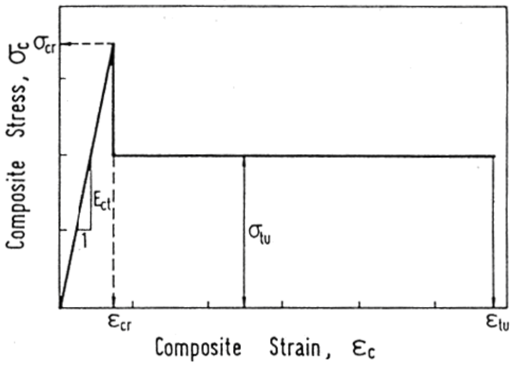


Fig. 2 Analytical constitutive tensile law proposed by Lim *et al.* (1987)

3.1 Post-cracking tensile strength of SFRC

The presence of fibres into the mixture of concrete allow to sew the cracks during the load process and allocate stress in a wide region of beams. To model this experimental evidence a first step is the use of a reliable constitutive law for fibrous concrete in tension, reproducing the toughness of mixture.

The residual tensile stress of SFRC plays a key role in the shear failure mechanism of the beam, especially for slender structural elements, and several analytical relationships have been proposed in literature (Lim *et al.* 1987, Li *et al.* 1992, Foster *et al.* 2006, Lee *et al.* 2011). In this work, the elastic-plastic model suggested by Lim *et al.* (1987) is used to evaluate the residual post-crack tensile strength of fibrous concrete.

Lim *et al.* (1987) proposed an analytical constitutive model for fibrous concrete in tension as function of strain, applying a linear technique of homogenization. The authors of the suggested constitutive law assumed a linear-elastic behaviour of composite until the first crack strain ε_{cr}, which is calculated also taking into account the presence of fibres in the mixture. Then, the ultimate tensile strength after cracking (σ_{tu}) is almost instantly reached, thus an horizontal branch is proposed by the authors to reproduce the residual tensile strength. This plateau is placed at the following stress value (Fig. 2)

$$\sigma_{tu} = 2\eta_0\eta_l V_f \frac{l_f}{d_f}\tau_f = 2\eta_0\eta_l F_\tau f_{ct} \tag{3}$$

In which l_f = fibre length; d_f = fibre diameter; V_f = volumetric percentage of fibre; η_l = length efficiency factor for fibre; η_0 = 0.405 the fibre orientation factor and τ_f the average bond strength between matrix and fibre. The $F_\tau=\beta_\tau V_f(l_f/d_f)$ is the fibre factor with $\beta_\tau= \tau_f/f_{ct}$ (Spinella *et al.* 2010), while η_l is depending of the critical length $l_c=(\sigma_{fu}d_f)/(2\tau_f)$: if l_f is less or equal to l_c then $\eta_l=0.5$, else $\eta_l=(1-l_c/2l_f)$. Therefore, besides the geometric characteristics of the fibres, the ratio between the tensile strength of fibres (σ_{fu}) and the bond fibre-matrix interface strength (τ_f) rules the fracture of fibre. The τ_f value is calculated as function of matrix tensile strength (f_{ct}), shape of fibre (hooked, plain or crimped), and type of matrix (concrete or mortar) as suggested by Voo and Foster (2003). In the typical case of concrete matrix, $\tau_f=2.5f_{ct}$ or $\tau_f=1.2f_{ct}$ for hooked and plain fibres, respectively

(Voo and Foster 2003). Furthermore, the residual tensile strength for fibrous concrete (3) depends on the tensile strength of plain concrete (f_{ct}), which can be evaluated as $0.45f_{cm}^{0.4}$ (in MPa) as suggested by Bentz (2000). In the proposed model the residual tensile strength of fibrous concrete ($f_{ct,ef}$) is assumed equal to the residual tensile strength (σ_{tu}).

3.2 Crack sliding factor for fibrous concrete

As previously introduced, Zhang (1997) suggested to use an efficiency factor to evaluate the effective compressive strength of plain concrete defined as $\nu_c = \nu_0 \nu_s$ (Eq. (2)), where ν_s is the crack sliding reduction factor. It takes into account the reduced cohesion of cracked plain concrete when the yield line follows the diagonal crack path or crosses many cracks. The original value of crack sliding factor chosen by Zhang (1997) for plain concrete beams is equal to 0.50, to adequately represents the strong influence of slips along cracks on the effective compressive strength of concrete.

The presence of fibres in the mixture limits both cracks width and slips along edges of a shear crack, thus the effective area of compression strut crossed by tensile stress is supposed larger for fibrous than plain concrete. Consequently, it is reasonable to assume that the sliding factor for fibrous concrete (ν_{sf}) has to be more than 0.50 and dependent by the amount of fibres in the mixture and its geometrical and mechanical characteristics. In a previous work (Spinella *et al.* 2010), a constant value equal to 0.82 has been proposed for the ν_{sf} parameter, taking into account the ability of fibres to contain slips along the crack edges, and obtaining satisfactory results specially for small-medium scale specimens.

Herein, a step forward is performed introducing a dependency of sliding factor for fibrous concrete by the mechanical and geometrical characteristics of fibres. The fibre factor (F_τ) is the parameter that best represents the increase in toughness of concrete due to the presence of fibres in the mixture, thus the crack sliding factor for FRC is defined as follows

$$\nu_{sf} = \nu_s \left(1 + \frac{F_\tau}{F_{\tau,\max}}\right) \tag{4}$$

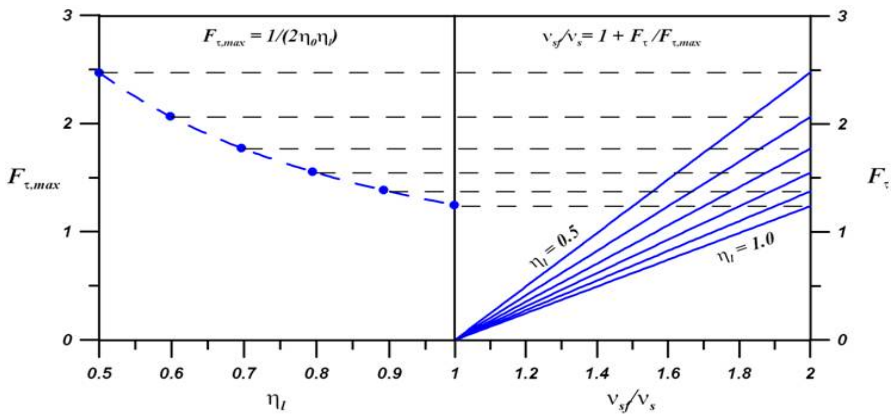


Fig. 3 Crack sliding factor as a function of fibre and concrete properties

with $F_{\tau,\max}$ the maximum admissible value of fibre factor. Imposing the condition $\sigma_{tu}/f_{ct}=1$ in the Eq. (3), the upper bound value of fibre factor is obtained as $F_{\tau,\max}=1/(2\eta_0\eta_l)$.

In Fig. 3(a), the $F_{\tau,\max}$ is plotted for various values of the length efficiency factor of fibre, η_l, which is a parameter depending by the geometrical and mechanical characteristics of fibres. Improving the bond strength between fibre and matrix, the value of η_l increases and the maximum value of fibre factor decreases. As shown in Fig. 3(b), where the curves of F_τ versus $v_{sf}/v_s = 1+(F_\tau / F_{\tau,\max})$ ratio are plotted, the cohesion of cracked plain concrete is enhanced and slips along shear crack are reduced thanks to the toughness of material (Fig. 3(b)). Since the geometrical (l_f and d_f) and mechanical properties (σ_{fu}) of fibre and concrete (f_{ct}) are known, the length efficiency factor (η_l) and the aspect ratio (l_f/d_f) are defined. The length efficiency factor allows to evaluate $F_{\tau,\max}$ in Fig. 3(a) and the corresponding curve in Fig. 3(b). Choosing the value of V_f, the fibre factor F_τ is handily calculated ($F_\tau=V_f\,\beta_\tau\,l_f/d_f$) and the v_{sf}/v_s ratio can be obtained.

3.3 Effect of fibres on size effect in shear

Several studies (Adebar *et al.* 1997, Parra-Montesinos 2006) produced a number of experimental researches on the shear resistance of SFRC beams. Even though these studies can be certainly considered a good advancement for understanding the shear behaviour of fibrous concrete beams, some of them are characterized by tests on small-medium scale specimens ($h <$ 300 mm). Recently, campaigns of tests on full scale beams ($h \geq 300$ mm) worldwide carried out (Sharma 1986, Imam *et al.* 1998, Noghabai 2000, Barragàn 2002, Rosenbusch and Teutsch 2003, Minelli 2005, Dinh *et al.* 2010, Minelli and Plizzari 2010), allow to collect a sufficiently large database to investigate the positive effect of fibres on size effect in shear.

The addition of fibres promotes a progressive evolution and stable development of several shear cracks in beams without stirrups subjected to transversal loads, and as consequence a more ductile behaviour is usually observed with evident flexural failure. The size effect is mitigated as highlighted by larger vertical deflection of specimen than those usually observed in reference plain concrete beams. In addition, fibres are able to control the development of shear crack width when the external loads increase, and allowing a spread of shear stress in a wider region of beam than that observed for plain concrete beams. This experimental behaviour is a clear evidence of size effect mitigation due to fibres, which provide a wide residual strength after the emergence of first shear crack and they allow a multi-cracking in shear with small width (0.15-0.25 mm).

Aiming at taking into account these experimental evidences, the recommendations of RILEM TC162-TDF (Vandewalle 2002) suggest a semi-empirical method to calculate the average crack spacing of fibrous concrete, based on the formulation proposed by Eurocode 2 (1993), and the beneficial effects due to fibres is considered by the geometrical term $k_s/(l_f/d_f)\leq 1$, with k_s a reference value for the fibre aspect ratio (l_f/d_f) set equal to 50. It needs to be noted, that the effectiveness factor originally proposed for plain concrete beams (2) takes into account the size effect in shear introducing in v_0 a term $f_2=0.27(1+1/h^{0.5})$ which depends by the root square of specimen's height as obtained by Nielsen (1999) on the basis of a large experimental campaign. This term contributes to take into account the compression softening effect due to slip along shear crack, which increases with the height of beam. Therefore, the beneficial effect of fibres is herein introduced in the analytical expression of effectiveness factor, modifying the f_2 term, functions of height root square, with the following formulation

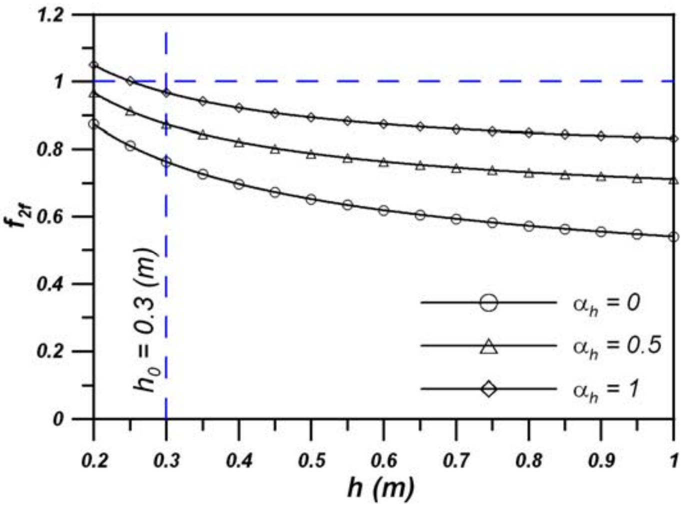


Fig. 4 Shear size effect in the effectiveness factor as function of beam depth

$$f_{2f} = 0.27\left(1 + \frac{1}{\sqrt{h/\left[1+\alpha_h\left(h/h_0\right)\right]}}\right) \tag{5}$$

being α_h=$50/(l_f/d_f) \le 1$; and h_0 a reference beam height set equal to 300 mm.

In Fig. 4, the term f_{2f} (5) versus the depth of beam is plotted for several value of α_h (0, 0.5 and 1). The curves show that when the value of α_h is increased, the detrimental effect due to size effect in shear is reduced, specially for large beams. Nevertheless, it is intuitive that the amount of fibres can affect the size effect in shear also. Therefore, the f_{2f} term should be influenced by V_f value, and an experimental campaign in this way could be provide interesting enhancements.

4. Analysis method and results

As above mentioned, many researches are available in literature regarding the shear resistance of beams without stirrups (Adebar *et al.* 1997, Parra-Montesinos 2006) and in a previous work a large database has been compiled (Spinella *et al.* 2010). For full-scale beams, however, few experimental results can be found in literature. Therefore, a literature survey is performed to find experimental series of SFRC shear tests where height is varied ($h \ge$ 300mm). From this survey, eight separate test series (Sharma 1986, Imam *et al.* 1998, Noghabai 2000, Barragàn 2002, Rosenbusch and Teutsch 2003, Minelli 2005, Dinh *et al.* 2010, Minelli and Plizzari 2010) are found consisting of a total of 45 beams (Table 1). Only tests reported by the researchers as shear failures are included and any reported flexural or bond failures are removed from the datasets.

Table 1 summarizes the data sets by author/s of testing, and the capacity of each specimen is reported as shear stress ($\tau_{u,exp}$). Note that tests are included independently of their *a/d* ratios despite the fact that shorter *a/d* ratios exhibit a different failure mode than shallow beams, but the proposed analytical model is independent by shear span-depth ratio.

Table 1 Details and observed ultimate shear stress of test specimens

Geometry			Reinforc.		Fibres			
b (mm)	h (mm)	a/d	ρ (%)	f_y (MPa)	V_f (%)	l_f/d_f	f_{cf} (MPa)	$\tau_{u,\exp}$ (MPa)
Minelli (2005); Hooked ended fibres								
200	480	2.5	1.04	512	0.38	50	24.8	1.49
200	480	2.5	1.04	512	0.64	48	61.1	2.14
Minelli and Plizzari (2010); Hooked ended fibres								
250	1000	2.9	1.05	555	0.64	63	32.1	1.16
250	1000	2.9	1.05	555	0.96	63	33.1	1.49
250	1500	3.0	0.99	518	0.64	63	32.1	1.34
250	1500	3.0	0.99	518	0.96	63	33.1	1.54
Barragan (2002); Hooked ended fibres								
250	300	3.5	2.83	500	0.50	67	32.1	2.34
250	450	3.3	3.08	500	0.50	67	32.1	1.78
250	500	3.3	2.41	500	0.50	67	32.1	1.61
250	600	3.5	2.73	500	0.50	67	32.1	1.98
Dinh *et al.* (2010); Hooked ended fibres								
152	455	3.4	1.96	496	0.75	55	44.8	2.34
152	455	3.4	2.67	448	1.50	55	31.0	1.78
152	455	3.4	2.67	448	1.50	55	44.9	1.61
152	455	3.4	2.67	448	1.00	80	49.2	1.98
152	455	3.4	1.96	496	0.75	79	43.3	2.34
205	685	3.5	2.06	455	0.75	55	50.8	2.70
205	685	3.5	2.06	455	0.75	80	28.7	2.76
205	685	3.5	1.56	455	0.75	55	42.3	2.79
205	685	3.5	1.56	455	0.75	80	29.6	1.79
205	685	3.5	2.06	455	1.50	55	44.4	3.49
205	685	3.5	2.06	455	1.50	80	44.4	3.38

Geometry			Reinforc.		Fibres			
b (mm)	h (mm)	a/d	ρ (%)	f_y (MPa)	V_f (%)	l_f/d_f	f_{cf} (MPa)	$\tau_{u,\exp}$ (MPa)
Sharma (1986); Hooked ended fibres								
150	300	2.0	1.61	400	1.00	83	48.6	3.03
Imam *et al.* (1998); Hooked ended fibres								
200	350	2.5	1.87	550	0.75	75	110.0	4.48
200	350	1.8	3.08	550	0.75	75	109.5	8.80
200	350	2.5	3.08	550	0.75	75	110.0	4.74
200	350	3.5	3.08	550	0.75	75	111.5	3.48
200	350	4.5	3.08	550	0.75	75	110.8	3.53
Noghabai (2000); Hooked ended fibres								
200	300	2.8	4.30	500	1.00	50	91.4	6.60
200	500	2.9	3.06	590	1.00	86	76.8	3.52
200	500	2.9	3.00	590	1.00	86	76.8	3.81
200	500	2.9	3.00	590	0.50	86	69.3	3.51
200	500	2.9	3.00	590	0.75	86	68.0	3.85
300	700	3.0	2.90	590	1.00	40	76.8	2.60
300	700	3.0	2.90	590	0.75	86	60.2	2.98
Rosenbusch and Teutsch (2003); Hooked ended fibres								
200	300	3.5	3.56	400	0.20	67	49.9	2.12
200	300	3.5	3.56	400	0.40	67	46.5	2.31
200	300	3.5	3.56	400	0.60	67	51.3	2.98
200	300	3.5	2.83	500	0.50	67	32.1	2.34
200	450	3.3	3.08	500	0.50	67	32.1	1.78
200	500	3.4	2.41	500	0.50	67	32.1	1.61
200	600	3.5	2.73	500	0.50	67	32.1	1.98
200	300	1.5	1.81	400	0.25	67	43.7	1.59
200	300	1.5	1.81	400	0.75	67	42.8	1.70
200	300	2.5	1.15	400	0.25	67	42.5	0.77
200	300	2.5	1.15	400	0.75	67	41.1	1.00
200	300	2.5	1.81	400	0.25	67	42.5	1.00
200	300	2.5	1.81	400	0.75	67	41.1	1.34
200	300	4.0	1.81	400	0.25	67	43.7	1.26
200	300	4.0	1.81	400	0.75	67	42.8	1.79

The studies varied with the aspect ratio of steel fibre (l_f/d_f), size of the beams tested as well as the amount of fibre in concrete. Due to this variation, the experimental data collected is divided into high ($f_{cf} \geq 50$ MPa) and normal ($f_{cf} < 50$ MPa) strength concrete (HSC and NSC) because this

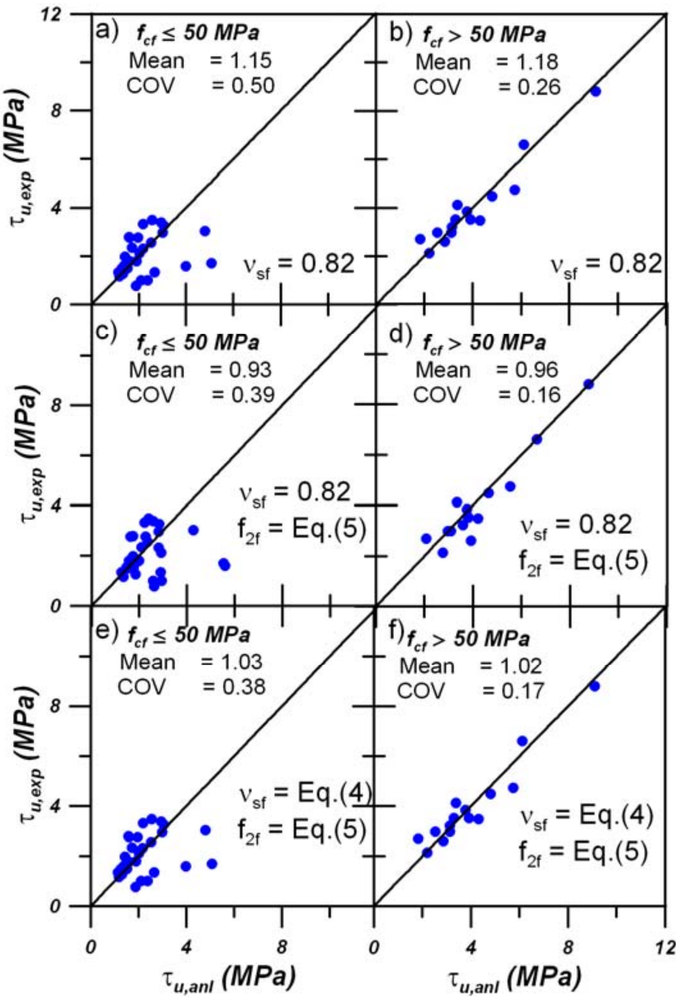


Fig. 5 Comparison between experimental and analytical results for NSC and HSC beams with different values of effectiveness factors

parameter significantly affects the shear behaviour of SFRC beams. In fact, the normal strength SFRC beams are more affected by the shape of the fibres, which is less significant in the case of HSC beams.

Thanks to the handily proposed formulation, it can analyze data using a simple spreadsheet.

In Fig. 5, the values of the ratio between experimental results given in literature and the analytical values ($\tau_{u,exp}/\tau_{u,anl}$), predicted by using the proposed model, namely CSMf, are reported together with its mean value and the Coefficient Of Variation (COV) calculated as the ratio between standard deviation and mean.

First, the constant value of crack sliding factor (ν_{sf}) previously proposed (Spinella *et al.* 2010) and equal to 0.82 is used (Figs. 5(a) and (b)). The values of mean obtained are slightly conservative both for normal and high strength fibrous concrete (1.15 and 1.18, respectively), and excessive values of COV are obtained (0.50 and 0.26, respectively) specially for $f_{cf}<50$ MPa.

These results confirm that the constant value of v_{sf} is appropriate to capture the experimental response of specimens, but the model returns a wide variation of results.

Introducing the proposed upgrade (Eq. (5)) to the term of size effect in the effectiveness factor v_0, the analytical model improves its ability to reproduce the response of SFRC beams subjected to transversal loads. As shown in Figs. 5(c) and (d), where the f_{2f} term (5) is taken into account, the mean values become a little bit unconservative, but close to one (0.93 and 0.96 for normal and high strength concrete, respectively) and the COVs are less than values previously obtained (0.39 and 0.16 for NSC and HSC, respectively). Finally, the expression of crack sliding factor (4) is used (Figs. 5(e) and (f)), and the CSMf is optimized, returning appreciable value of statistical coefficients: 1.03 and 1.02 for the mean and 0.38 and 0.17 for the COV.

The effectiveness of CSMf is proved by the reliable estimation of shear strength, independently by the cylinder strength of material in compression. Many equations have been proposed by numerous researchers to predict the shear strength of SFRC beams based on experimental investigations. The majority of the proposed formulations contain the fibre factor (*F*), which illustrates the combined effect of the fibre aspect ratio and fibre content on the shear strength of SFRC beams. It can also be noticed that the inverse of the span depth ratio (*d/a*) is used in most equations to reproduce the arch effect. In this study the models suggested in the Model Code 2010 (2010) and by Foster (2010), Kwak *et al.* (2002), Sharma (1986), Campione *et al.* (2006), Imam *et al.* (1998), Khuntia *et al.* (1999) and Narayanan and Darwish (1987) are used to perform a comparative analysis with the herein proposed model (CSMf) for predicting the shear strength of SFRC beams.

By analyzing the results (Fig. 6), accurate predictions for high strength SFRC beams are observed from the approach suggested from Imam *et al.* (1998)'s model, with a mean value of 1.03 and a COV equal to 0.15, while for normal strength SFRC beams the average of predictions (=0.98) is slightly not-conservative and the COV reaches a wide value (= 0.42). The approaches suggested in the Model Code 2010 (2010) and by Foster (2010), respectively, allow to obtain an accurate prediction of the shear strength for normal strength SFRC beams (Figs. 6(a1) and 6(b1)), while they become too conservative for the high strength SFRC specimens (Figs. 6(a2) and 6(b2)).

Prediction equations overestimating shear strength can be dangerous for designers as the amount of shear reinforcement needed to prevent shear failure contains much more uncertainty. Although a small shear strength overestimation can be tolerated as in the case of Imam *et al.* (1998)'s model, larger shear strength overestimations as in the case of some of the previously proposed models (Sharma 1986, Narayanan and Darwish 1987, Kwak *et al.* 2002) for NSC beams cannot be used in practice unless a proper safety factor or reduction factor is incorporated with the equations.

The model proposed by Sharma (1986) is inaccurate compared to those of the other models for NSC beams (Fig. 5(d1)). This is because of its simplicity, which does not even include the fibre aspect ratio, fibre volume or the reinforcement ratio, whereas numerous studies clearly indicate that these have substantial effect on the shear strength of SFRC beams (Narayanan and Darwish 1987). For most of the considered cases the equation proposed by Sharma (1986), which is currently being used by ACI, predicts the shear strength with a larger amount of scatter than many of the other proposed equations. Sharma (1986)'s equation is also simple and only contains the concrete tensile strength and the shear-span to depth ratio. Therefore, it could not handle larger variations in database, such as those found with SFRC beams considered in this study.

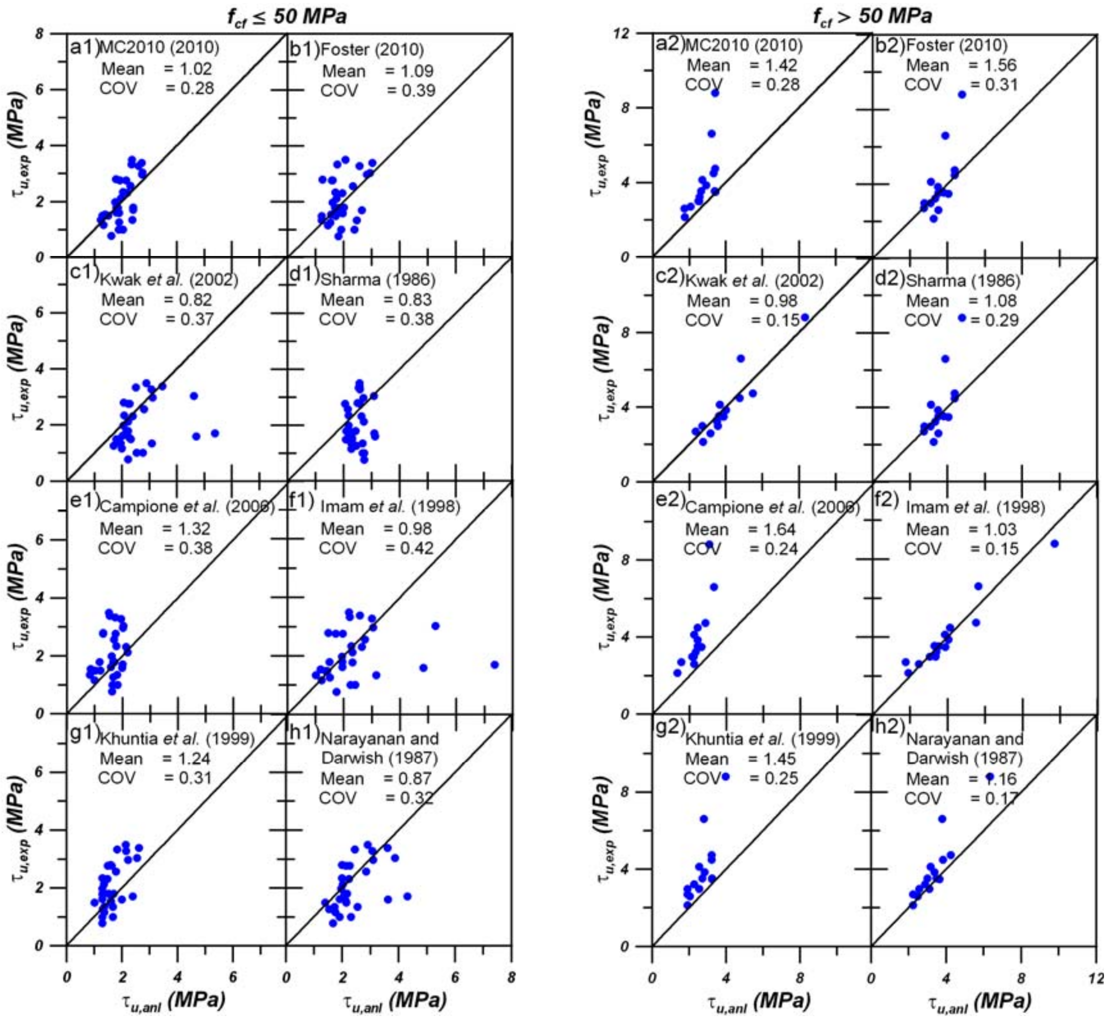


Fig. 6 Comparison between experimental and analytical results for NSC and HSC beams performed by several models

5. Numerical analysis

The analyzed results of experimental tests on fibrous concrete beams without stirrups subjected to shear load show that fibres are highly effective in reducing the chance to obtain a brittle shear failure. As known in literature, this experimental behaviour is analytically represented by the Kani Valley (Kani 1967). It is a 3D surface which represents the relative shear capacity (V_u/V_{fl}) or the ratio between the shear at failure and the shear at the theoretical flexure failure, as function of the *a/d* ratio and the geometrical percentage of flexural reinforcement (ρ).

A numerical analysis carried out with the proposed model is performed to reproduce this experimental evidence. To this aim, the valleys of diagonal shear failure are drawn.

The investigation is performed by assuming three different beam depths: 300, 600 and 900 mm; four different values of volumetric percentage of fibres and fibre factor: V_f = 0, 0.33, 0.67 and 1%;

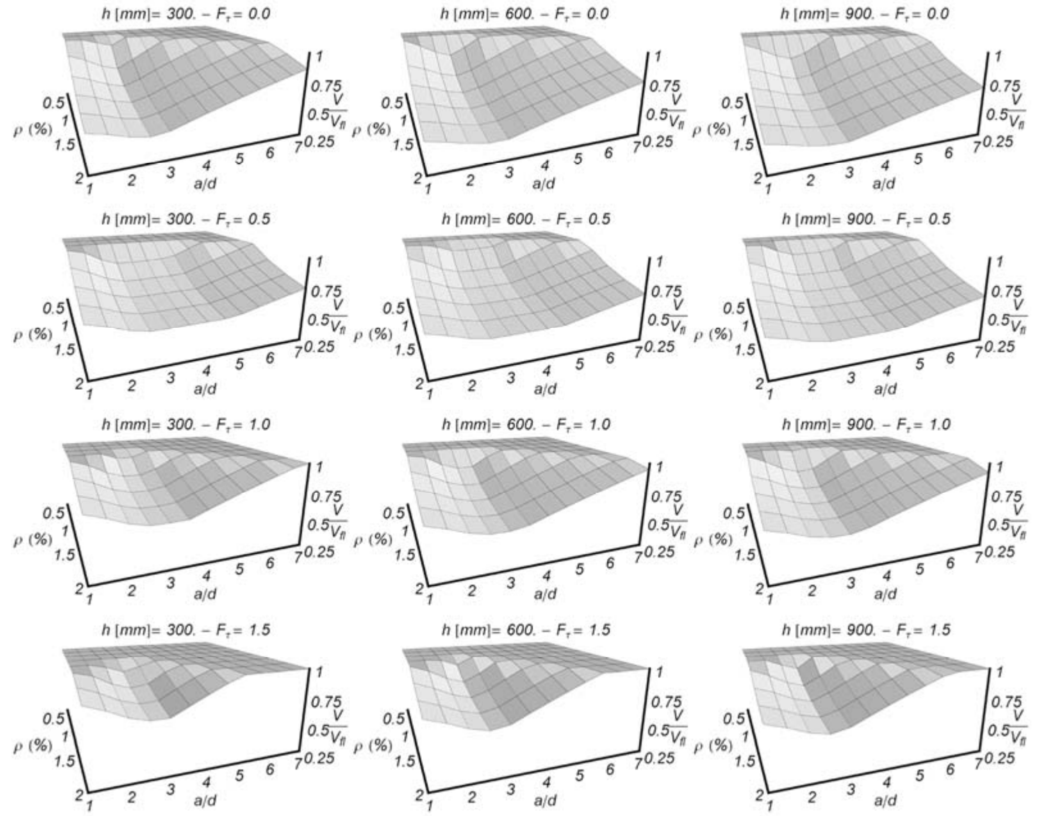


Fig. 7 Valleys of diagonal failure for NSC and HSC and for different fibre factor and beam depth values

F_τ = 0, 0.5, 1 and 1.5. The variation of longitudinal reinforcement percentage is limited to a range of 0.5% to 2% to reflect practical arrangements. Finally, hooked ended fibres, with a length of 30 mm, a diameter of 0.5 mm (l_f/d_f = 60) and a yield strength of 1000 MPa are considered. The cylinder strength of concrete (f_{cf}) and the yield strength of steel (f_y) are assumed equal to 40 MPa and 400 MPa, respectively.

To evaluate the relative flexural capacity, the bending moment corresponding to flexural failure, M_{fl}, is calculated according to the formulation proposed by Imam *et al.* (1998) for fibrous concrete

$$M_{fl} = \frac{1}{2}\rho f_y bd^2\left(2-\eta\right) + 0.83Fbd^2\left(0.75-\eta\right)\left(2.15+\eta\right) \tag{6}$$

where F is the fibre factor distinguished by F_τ for the definition of the β coefficient; and η a parameter which depends by F, ρ, f_y and f_{cf}.

Fig. 7 shows that shear failure domains are extended by using low fibre factor values and high longitudinal reinforcement percentage. This trend is different for each value of the beam depth, being greater for small scale beam.

Increasing the amount of fibre in the mixture (F_τ= 0.5, 1.0) the shear failure valley tend to disappear (Fig. 7). However it is still wide for deep beams, while in case of small-medium beams the domain of shear failure is extended to *a/d* ratios between 1.0 and 3.5.

This trend is confirmed in Fig. 6(d) where F_τ= 1.5 is assumed. The valley of diagonal shear failure is narrower, while shear collapse is reported just for a few *a/d* ratios and high longitudinal reinforcement ratios. This behaviour is emphasized for small beams, where the fibres strongly help to tighten the shear failure valley.

As seen in Fig. 7, despite the CSM has been updated for predicting the behaviour of short beams by introducing an additional term (Spinella *et al.* 2010), depending on *a/h* ratio, and determined by tests on plain concrete members, it still predicts a shear failure for beams with $a/d \approx 1$. In these conditions, experimental tests show a shear capacity higher than flexural ones, which the former that depends by the compressive strength of concrete. Such deep beams tend to be substantially stronger than shallow beams as they resist load with a direct compression strut to the support and are more appropriately modelled by strut-and-tie analysis rather than the shear equations.

4. Conclusions

A simple mechanical model is proposed for shear capacity prediction of fibrous concrete beams without stirrups under transversal loads.

The model has been derived on the basis of plastic theory and limit analysis and takes into account the fibre concrete contribution to shear strength, including the high residual post cracking tensile strength of SFRC. To this aim, the constitutive plastic law suggested by Lim *et al.* (1987) has been used.

The effectiveness factor of fibrous concrete in compression has been modified for deep beams, by introducing an additional term depending on the shear span-depth ratio (Spinella *et al.* 2010). The reduction slide factor for fibre concrete, v_{sf}, has been increased for fibrous concrete respect to plain concrete and as function of fibre toughness, taking into account the ability of fibres in reducing slips along shear cracks.

The ability of fibre of mitigating the shear size effect has been taken into account by an appropriate analytical term in the efficiency factor. It is function of the geometrical characteristic of fibres.

The comparison between experimental and analytical values of ultimate shear stress shows the ability of the CSMf model as modified to determine both the collapse strength of fibrous concrete beams and the section where the critical diagonal crack starts from the bottom face of the beam.

The formulation proposed in this study for several configurations of SFRC beams is able to produce more precise results than those of the other models proposed in literature since this study considers a semi-rational model and reproduces an experimental behaviour by an analytical scheme, whereas some of the previous equations proposed by different researchers which are developed on the basis of regression analyses and are often unable to capture the physical mechanism of shear failure.

The numerical analyses indicate that the addition of steel fibres enhanced ultimate loads of NSC and HSC beams. This enhancement is evident in the Kani Valleys plotted for several values of fibre factor and beam height. The use of fibre factor greater than 0.5 allows the disparition of the shear failure for small beams, while for large beams a wide amount of fibres (i.e., toughness) is needed to obtain a ductile behaviour.

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Appendix

As an illustration of the calculation involved, consider beam B18-3b tested by Dinh *et al.* (2010). The geometrical and material properties of the beam are as follows: b= 152 mm, h= 455 mm, a/d= 3.44, r= 2.67%, V_f= 1.50%, l_f/d_f= 30.0/0.55= 54.5, f_{cf}= 31.0 MPa, and experimental average shear stress at failure $\tau_{u,exp}$= 2.56 MPa. In the following, steps of calculation of shear strength are illustrated:

1. Being β_τ = 2.50 (hooked end fibres), then the fibre factor F_τ = 2.05 and $F_{\tau,\max}$ = 2.47. Therefore, the residual tensile strength (Eq. (3)) $f_{ct,ef} = \sigma_{tu}$ = 1.52 MPa and the crack sliding factor for FRC (Eq. (4)) v_{sf} = 0.91.
2. For the effectiveness factor in compression, first estimate α_h = 0.92, second use the Eq. (5) to calculate the term f_{2f} = 0.89, third use Eq. (2) to obtain v_0 = 0.82 and, consequently, the effective compressive strength is $v_{0f} v_{sf} f_{cf}$ = 0.82×0.91×31 = 23.2 MPa.
3. Iteratively, calculate the starting section of the critical diagonal crack x_c as the intersection (if it exists) between the average shear stress failure $\tau_u(x)$ and the average cracking stress $\tau_{cr}(x)$ (Fig. 1(a)): x_c/h = 0.23.
4. Finally, find the analytical average shear stress at failure calculating the τ_u in correspondence of the critical diagonal crack section (x_c): $\tau_{u,anl}$= $\tau_u(x_c)$ = 2.53 MPa.

Note that all calculations involved are simple in nature and just few iterations (usually less than five) are needed.

Notations

a	Shear span
A_s	Longitudinal reinforcement area
b, d, h	Width, effective depth and depth of beam cross section
h_0	Reference value for beam depth
d_f, l_f	Diameter and length of fibre
f_c, $f_{c,ef}$	Cylinder and effective compressive strength of concrete
f_{cf}, f_{ctf}	Cylinder compressive strength and direct tensile strength of fibrous concrete
f_{ct}, $f_{ct,ef}$	Direct and effective tensile strength of concrete
f_{cft}, $f_{cft,ef}$	Direct and effective tensile strength of fibrous concrete
f_y	Yielding stress of rebar reinforcement
f_2, f_{2f}	Term responsible of size effect in the effectiveness factor for plain and fibrous concrete
F, F_τ	Fibre factors
$F_{\tau,\max}$	Maximum value of fibre factor
l_c	Critical length of fibre
k_s	Reference value for fibre aspect ratio
M_u, M_{fl}	Ultimate and nominal flexural capacity
r	Geometrical percentage of longitudinal reinforcement
V_f	Fibre volume percentage
x	Distance between the support and critical crack
α_h	$= 50\, l_f/d_f \le 1$

β, β_τ	Fibre bond factors
γ	Diagonal crack angle
η	Parameter in the formulation of flexural capacity (Imam *et al.* 1998)
η_0, η_l	Parameter in the formulation of Lim *et al.* (1987)'s model
ν_0	Effectiveness factors for concrete in compression
ν_s, ν_{sf}	Crack sliding factors for plain and fibrous concrete
ρ	Geometrical percentage of longitudinal reinforcement
σ_c, σ_f , σ_{cf}	Tensile tension of concrete, fibres and fibrous concrete
σ_{fu}	Yielding stress of steel fibre
σ_{tu}	Residual tensile strength of SFRC
τ_f	Mean shear stress between fibre and matrix
$\tau_{u,exp}$, $\tau_{u,anl}$	Experimental and analytical ultimate shear stress
τ_u, τ_{cr}	Ultimate and cracking shear stress