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Reliability of column capacity design in shear

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Abstract. The capacity design of shear forces is one of the special demands of EC8 by which the ductile behavior of structures is implemented. The aim of capacity design is the formation of plastic hinges without shear failure of the elements. This is achieved by deriving the design shear forces from equilibrium conditions, assuming that plastic hinges, with their possible over-strengths, have been formed in the adjacent joints of the elements. In this equilibrium situation, the parameters (dimensions, material properties, axial forces etc) are random variables. Therefore, the capacity design of shear forces is associated with a probability of non-compliance (probability of failure). In the present study the probability of non-compliance of the shear capacity design in columns is calculated by assuming the basic variables as random variables. Parameters affecting this probability are examined and a modification of the capacity design is proposed, in order to achieve uniformity of the safety level.

Keywords: reliability; shear; capacity design; columns.

1. Introduction

The capacity design rule for shear forces of EC8 (2004) aims to prevent the brittle shear failure of building elements (beams, columns and walls). Avoiding shear failure provides structures the ability to dissipate energy through the formation of plastic hinges. According to the capacity design rule, the formation of plastic hinges can be ensured if the shear resistance of the elements is larger than the shear force corresponding to the development of plastic hinges at both ends of the element. The design values of shear forces are equal to the shear forces that must be developed in order for the equilibrium condition to be satisfied in the seismic design situation. In this equilibrium condition the moments at the ends of the elements are considered equal to the resisting moments multiplied by partial safety factors which give the desirable overstrength to the shear resistance.

The success of the capacity design rule to fulfil its scope depends on the effectiveness of the shear strength and flexural strength models that are used to predict the corresponding resistances. Several studies have dealt with the assessment of these models using specimen results (e.g. Yoshimura 2008 examined the behaviour of half-scale model specimens of columns designed to fail in shear or in flexural yielding and in NCHRP 2005 the large experimental database, consisting of 878 RC and 481 prestressed concrete members was used to compare the shear strength estimated according to different design codes with the test results). Specially for the shear strength, the studies have revealed that the shear models used in codes fail to predict the strength with satisfactory small values of coefficients of variation (Mwafy and Elnashai 2008) and thus new formulas for calculating the

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shear strength of reinforced concrete elements are proposed by many authors (e.g. Ghiassi and Soltani 2010, Bentz *et al.* 2006, Collins *et al.* 2008).

In the present study the capacity design of shear forces of EC8 is examined under a probabilistic point of view in order to take into account all these model uncertainties with all other material uncertainties. The success of the capacity design rule of beam-column joints of EC8 has also been examined by the authors (Thomos and Trezos 2011). The variables that affect the capacity design (concrete strength, steel strength, dimensions etc.) are considered as random variables and the probability of shear failure to appear before the formation of plastic hinges is calculated for columns for which the shear capacity design has been implemented. This corresponds to the non-compliance (or failure) of the shear capacity design.

More precisely, failure of the capacity design for shear forces of columns is considered as the event in which "the shear force induced from the formation of plastic hinges at the ends of the columns (or at the beams connected to the joints into which the columns ends frame, if the plastic hinges form there first) is greater than the shear resistance of the considered member".

The aim of this study is to investigate the safety level of the capacity design of shear forces of columns. The safety level is quantified by the probability of non-compliance (called also "probability of failure"). Parameters affecting this probability are determined and a modification of the partial safety factor of the capacity design of shear forces of columns is proposed, in order to achieve uniform safety level.

2. Methodology

2.1 Capacity design rule for shear forces of columns

According to EC8, the design values of shear forces of columns shall be determined in accordance to the capacity design rule. For columns with fixed ends the design value of shear force is

$$V_{d,c} = (M_{1d} + M_{2d})/l_c \tag{1}$$

Where,

l_c is the length of the column and

$M_{i,d}$ (with $i = 1,2$ denoting the end sections of the column) are the end moments (calculated from the relation (2)) that correspond to plastic hinge formation for positive and negative directions of seismic loading. Plastic hinges should be formed either at the end of the column or at the end of the beam connected to the joint into which the column frames

$$M_{i,d} = \gamma_{Rd} M_{Rd,c,i} \min\left(1, \frac{\Sigma M_{Rd,b}}{\Sigma M_{Rd,c}}\right) \tag{2}$$

Where,

γ_{Rd} is the safety factor accounting for overstrength due to steel strain hardening and confinement of the concrete of the compression zone of the section, taken as being equal to 1.1 for DC M (medium ductility) columns and 1.3 for DC H (high ductility) columns.

$M_{Rdc,i}$ is the design value of the column resisting moment at the end i in the direction of the seismic moment under the seismic action considered.

$\Sigma M_{Rd,c}$ is the sum of the design values of the resisting moments of the columns framing in the joint and

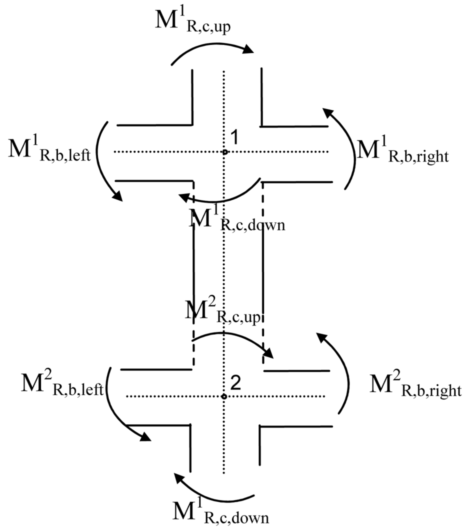


Fig. 1 Directions of the resisting moments for calculating relation 3

$\Sigma M_{Rd,b}$ is the sum of the design values of the resisting moments of the beams framing in the joint.

The shear strength (V_{Rd}) and the resisting moments of relation (2) are affected by many parameters such as model uncertainties, concrete strength, steel strength and dimensions etc. that are random variables. Therefore, there is a probability ("probability of failure") of the shear strength being less than the shear force developed during plastic hinges formation (although relation (1) has been used for calculating the shear force design value). This probability of failure, p_f, can be written for a column (Fig. 1) as

$$P_f = P\left(V_R < \frac{M^1_{R,c,down} \cdot \min\left(1, \frac{M^1_{R,b,left} + M^1_{R,b,rightt}}{M^1_{R,c,down} + M^1_{R,c,up}}\right) + M^2_{R,c,up} \cdot \min\left(1, \frac{M^2_{R,b,left} + M^2_{R,b,rightt}}{M^2_{R,c,down} + M^2_{R,c,up}}\right)}{l_c} \right) \tag{3}$$

Where

V_R is a random variable that represents the shear strength of the column

$M^i_{R,b,left}$ $M^i_{R,b,right}$, $M^i_{R,c,up}$ and $M^i_{R,c,down}$ are random variables that represent the resisting moments of the members with the directions that are shown in Fig. 1.

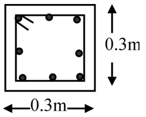
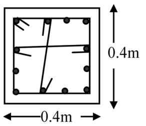
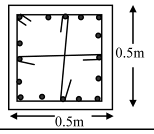
The Monte Carlo simulation can be used for calculating relation (3). Assuming that the probability function of the resisting moments and the shear resistances may be approximated by the normal distribution, the probability of failure can be directly related to the safety index β

$$P_f = P(r<0) = Erf\left(-\frac{m_r}{\sigma_r}\right) = PHI(-\beta) \tag{4}$$

with

$$r = \min(V_R)- \tag{4a}$$

Table 1 Examined cases

Columns				Beams				
Section/ dimensions	Longitudinal reinforcement ratio ρ_{tot} (%)	Height of the column	Axial force (ν)	Dimensions	b_{eff} (m)	Reinforcement ratio ρ_t=ρ_c (%)	γ_{Rd}	Number of cases
 	1.0, 2.0, 3.0, 4.0	1.5 2.0 3.0 4.5	0.00 0.20 0.40 0.60	0.25/0.50	2.00 (with thickness of the plate 0.16 m)	0.18 0.45 0.74 1.22	1.1 1.3 1.7	768
 	1.0, 2.0, 3.0, 4.0	1.5 2.0 3.0	0.00 0.20 0.40 0.60	0.25/0.50	2.00 (with thickness of the plate 0.16 m)	0.18 0.45 0.74 1.22	1.1 1.3 1.7	768
 	1.0, 2.0, 3.0, 4.0	1.5 2.0 3.0	0.00 0.20 0.40 0.60	0.25/0.50	2.00 (with thickness of the plate 0.16 m)	0.18 0.45 0.74 1.22	1.1 1.3 1.7	768
							Total:	2304

Where,

PHI( ) is the cumulative distribution function of the standard normal distribution.

In the following, the safety index β is used instead of the probability of failure p_f, for alleviating the presentation (large values of β correspond to small probability of failure).

In order to examine the influence of basic variables to the probability of failure of the capacity design of shear forces of columns, the cases shown in Table 1 have been examined. The beams of the upper joint and the beams of the lower joint (see Fig. 1) have been considered to have the same dimensions and reinforcement. Moreover, the two columns framing in a joint have been considered to have the same cross section and the same reinforcement.

The shear capacity design is applied to each column of Table 1: The design value of shear force ($V_{d,c}$) is calculated using the relation (1). The shear reinforcement is calculated according to EC2 (2004). The design method used in EC2 is known as the variable strut inclination method and is based on a truss model (see Fig. 6.5 of EC2). For columns with vertical reinforcement the shear resistance V_{Rd} is the smaller value of

$$V_{Rd,s} = A_{sw}/s \cdot 0.9 \cdot d \cdot f_{ywd} \cdot \cot\theta, \text{ [(6.8) of EC2]} \tag{5}$$

and

$$V_{Rd,\max} = b_w \cdot 0.9 \cdot d \cdot 0.6 \cdot f_{cd}/(\cot\theta + \tan\theta) \text{ [(6.9) of EC2]} \tag{6}$$

Where

A_{sw} is the cross-sectional area of the shear reinforcement

s is the spacing of the stirrups

d, b_w are the effective depth and the width of the column

f_{ywd} is the design yield strength of the shear reinforcement

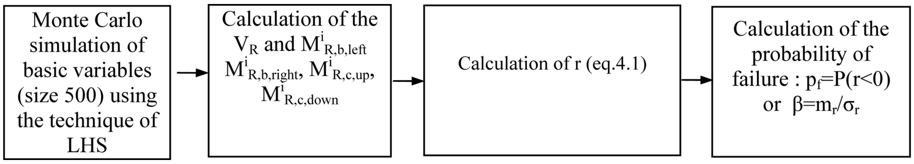


Fig. 2 Methodology for calculating the probability of failure of the capacity design of shear forces of columns

θ is the angle between the concrete compression strut and the column axis perpendicular to the shear force.

For cotθ, the recommended limits of EC2 have been used ($1 \leq \cot\theta \leq 2.5$) in the present study. For a given design shear force the required amount of shear reinforcement is dependent upon cotθ. The largest possible value of cotθ should be used in order to minimize the amount of stirrups required. This value can be calculated by equating the design shear force to the maximum shear resistance. This consideration leads to the below procedure which describes how the shear reinforcement is calculated in order the column to resist the design value of shear force ($V_{d,c}$).

Step 1: Calculate ω: $\omega(=\cot\theta+\tan\theta) = b_w \cdot 0.9 \cdot d \cdot 0.6 \cdot f_{cd} / V_{d,c}$

Step 2: if $\omega > 2.9$ then $\cot\theta = 2.5$

if $2.0 \leq \omega \leq 2.9$ then $\cot\theta = \dfrac{\omega + \sqrt{\omega^2 - 4}}{2}$

if $\omega < 2.0$ the dimensions of the column must be changed.

Step 3: Calculate A_{sw}/s: $A_{sw}/s = V_{d,c}/(0.9 \cdot d \cdot f_{ywd} \cdot \cot\theta)$, where cot$\theta$ the calculated value of step 2.

Once the columns have been designed and the shear reinforcement has been calculated, the probability of failure is calculated using the procedure shown in Fig. 2. The random variable simulation is implemented using the technique of Latin Hypercube Sampling (LHS) (Ayyub and Lai 1989, Iman and Conover 1980, McKay *et al.* 1979, Nowak and Collins 2000). It was found, by examining different sizes simulation results, that a sample size of 500 offers an adequate accuracy level for the present analysis problem, as the results for the index β do not change from simulation to simulation.

3. Random variables

Many probabilistic models for random variables are given in the international literature (Ditlevsen and Madsen 1996, Melcher 1999, Joint Committee on Structural Safety 2001, Gardoni e*t al.* 2002, Epaarachchi and Stewart 2004, Lu *et al.* 2005). In the present study consideration of random variables is based on probabilistic models that have been thoroughly investigated by Thomos and Trezos (2006). In the following, the assumed distributions are presented.

3.1 Materials

The stress-strain diagrams of the materials are shown in Fig. 3. For the conventional design, the σ-ε diagrams in the left column of Fig. 3 were used, while for the simulation, the diagrams in the right column of Fig. 3.

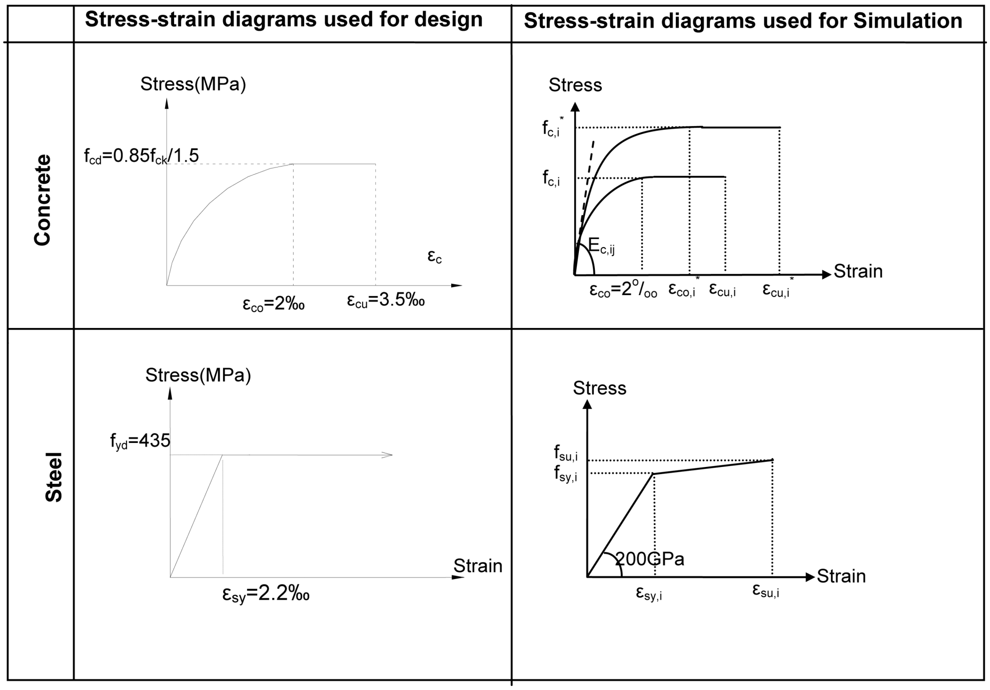


Fig. 3 Stress-strain diagrams of concrete and steel

3.1.1 Unconfined concrete

The models of concrete properties for a particular element *i* in a particular floor, are (see Fig. 3)

$$\text{Compressive strength: } f_{c,i} = f_{co,i} \cdot Y_1 \tag{7}$$

$$\text{Modulus of elasticity: } E_{c,i} = 10.5 \cdot f_{c,i}^{1/3} \cdot Y_2 \tag{8}$$

$$\text{Ultimate strain: } \varepsilon_{cu,i} = 6 \cdot 10^{-3} \cdot f_{c,i}^{-1/6} \cdot Y_3 \tag{9}$$

Where,

$f_{co,i}$ a normal random variable with mean value related to the 5% characteristic value of the compressive strength $f_{co,i,k}$: $E[f_{co,i}] = f_{co,i,k}/(1-1.64 \cdot Cov[f_{co,i}])$ and coefficient of variation: $Cov[f_{co,i}] = 0.15$ (Joint Committee on Structural Safety 2001)

Y_1,: log-normal variables reflecting floor to floor variation of casting conditions with mean value 1 and coefficient of variation 0.06 (Joint Committee on Structural Safety 2001).

Y_2, Y_3 lognormal variables reflecting factors not well accounted for by concrete compressive strength (e.g. gravel type and size, chemical composition of cement and other ingredients, climatic conditions) with mean value 1 and coefficient of variation 0.15 (Joint Committee on Structural Safety 2001).

3.1.2 Confined concrete

The deterministic model for the confinement, proposed Tassios and Lefas (1984) adopted in CEB-FIP Model Code 1990 (1993), has been used for simulating the confined concrete. This model has been converted to a probabilistic model by introducing random variables $Y_{conf,1}$, $Y_{conf,2}$, $Y_{conf,3}$, taking into account the uncertainties of the model (Eqs. (10)-(12)).

$$f^*_{c,i} = Y_{conf,1} \cdot f_{c,i} \cdot \begin{cases} 1 + 2.5 \cdot \alpha \cdot \omega_w,\ for \ldots \omega_w \le 0.1/\alpha \\ 1.125 + 1.25 \cdot \alpha \cdot \omega_w,\ for \ldots \omega_w > 0.1/\alpha \end{cases} \tag{10}$$

$$\varepsilon^*_{co,i} = Y_{conf,2} \cdot \varepsilon_{co,i} \cdot (f^*_{c,i}/f_{c,i})^2 \tag{11}$$

$$\varepsilon^*_{cu,i} = Y_{conf,3} \cdot (\varepsilon_{cu,i} + 0.1 \cdot \alpha \cdot \omega_w) \tag{12}$$

Where:

$\varepsilon_{co,i}$: 0.002, deterministic value corresponding to the maximum stress (strength) of the unconfined concrete

α : confinement effectiveness factor

ω_w: mechanical volumetric ratio of the transverse reinforcement

Y_{conf1}, Y_{conf2}, Y_{conf3}: lognormal random variables representing model uncertainties with a mean value of 1 and coefficients of variation 0.15, 0.10 and 0.50 respectively (Thomos and Trezos 2006).

3.1.3 Steel properties

Yield stress $f_{sy,i}$ (see Fig. 3):normal random variable with mean value related to the 5% characteristic value of the yield stress: $E[f_{sy,i}] = f_{sy,i,k}/(1-1.64 \cdot Cov[f_{sy,i}])$ and $Cov[f_{sy,i}] = 0.05$

Tensile strength $f_{su,i}$: perfectly correlated to the yield stress, $f_{su,i} = 1.15 f_{sy,i}$

Ultimate strain: normal variable with mean value $E[\varepsilon_{su,i}] = 0.05$ and a coefficient of variation of $Cov[\varepsilon_{su,i}] = 0.1$ (Joint Committee on Structural Safety 2001).

3.2 Dimensions

The cross-sectional dimensions are modeled as random variables that follow a normal distribution with mean values equal to the nominal values $E[X_i] = X_{i,nom}$ and standard deviations $\sigma_{Xi} = 4$ mm$+ 0.006 \cdot X_{i,nom}$.

Areas of re-bars are assumed to be independent normal random variables with mean values equal to the nominal values $E[A_{s,i}] = A_{si,nom}$ and coefficient of variation $Cov[A_{s,i}] = 0.02$ (Joint Committee on Structural Safety 2001).

3.3 Shear capacity

The shear strength of a column is calculated by choosing the value of $\cot\theta$ in such a way to maximize the shear strength

$$V_{Rs,\max} = \max\{\min[V_{Rs},\ V_{R,\max}]\} = \max\{\min[A_{sw,i}/s_i \cdot 0.9 \cdot d_i \cdot f_{y,wd,i} \cdot \cot\theta,\ b_{w,i} \cdot 0.9 d_i \cdot 0.6 f_{c,i}/(\cot\theta + \tan\theta)]\},\ \text{with } 1 \le \cot\theta \le 2.5 \tag{13}$$

Eq. (13) leads to the following relations depending on the magnitude of V_{Rs} and $V_{R,\max}$ (see Fig. 4)

$$V_R' = A_{sw,i}/s_i \cdot 0.9 \cdot d_i \cdot f_{y,wd,i} \cdot 2.5, \text{ for case A} \tag{14a}$$

$$V_R' = b_{w,i} \cdot 0.9 d_i \cdot 0.6 \cdot f_{c,i}/2, \text{ for case B} \tag{14b}$$

$$V_R' = A_{sw,i}/s_i \cdot 0.9 \cdot d_i \cdot f_{y,wd,i} \cdot \cot\theta, \text{ where}$$

$$\cot\theta = [(b_{w,i} \cdot 0.9 d_i \cdot 0.6 \cdot f_{c,i})/(A_{sw,i}/s_i \cdot 0.9 \cdot d_i f_{y,wd,i}) - 1]^{0.5} \text{ for case } C \tag{14c}$$

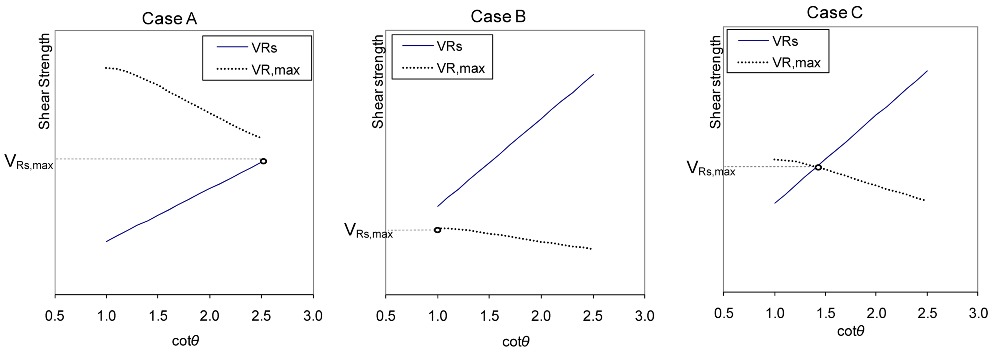


Fig. 4 Calculating of shear strength according to EC2

The model uncertainties of the shear capacity are taken into account by inserting a random variable (Y_{VR}) into the shear strength model of EC2. Y_{VR} is supposed to be lognormal random variable with a mean value of 1 and coefficient of variation equal to 0.35 (Mwafy and Elnashai 2008). The shear strength is calculated as

$$V_R = Y_{VR} \cdot V_{Rs,\max} \tag{15}$$

4. Results

In Fig. 5 the safety index β for the 1728 cases of Table 1 is shown, as a function of the partial safety factor γ_{Rd}. For the code values of $\gamma_{Rd} = 1.1$ and $\gamma_{Rd} = 1.3$ the variation of the safety index β is significant. The values of β vary from -1.16 to 0.76 for $\gamma_{Rd} = 1.1$ and from -0.5 to 1.44 for $\gamma_{Rd} = 1.3$ corresponding to probabilities of failure 88%, 22% and 69%, 7% respectively. So it is clear that the probability of “the shear force induced from the formation of the plastic hinges to be greater than

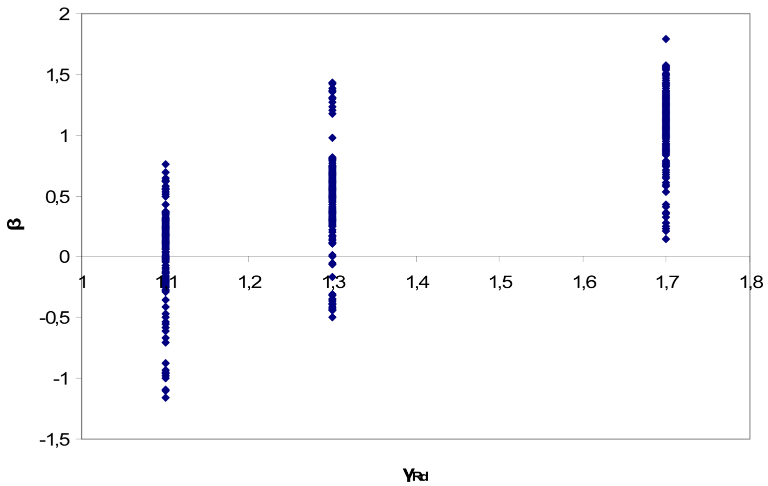


Fig. 5 Safety index β as a function of γ_{Rd}

the shear resistance" is quite large for many of the examined columns although the shear capacity design (as it is proposed by EC8 with γ_{Rd} = 1.1 or 1.3) has been satisfied.

In order to investigate the parameters affecting the probability of failure of the capacity design, the diagrams 6, 7 and 8 have been plotted. In these diagrams the safety index β is presented as a function of the ratio α_c and the reduced axial force ν for γ_{Rd} = 1.1, γ_{Rd} = 1.3 and γ_{Rd} = 1.7 respectively. The ratio α_c is defined as

$$\alpha_c = \frac{\Sigma M_{Rd,b}}{\Sigma M_{Rd,c}} \tag{16}$$

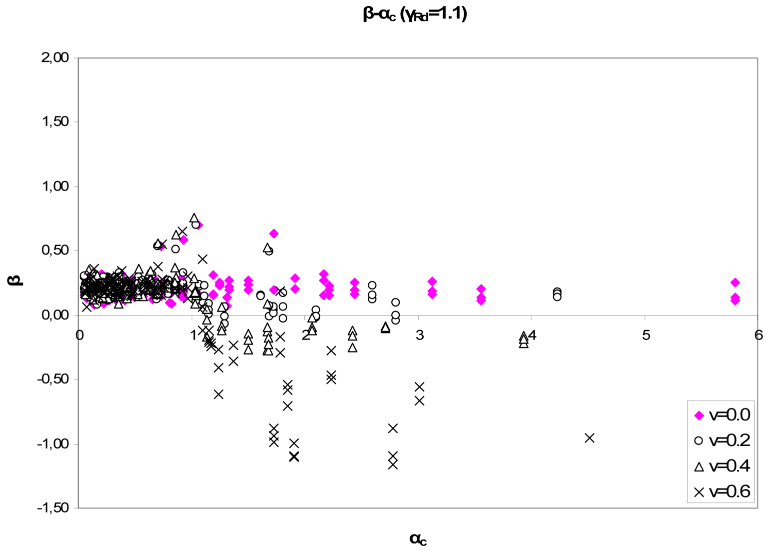


Fig. 6 Safety index β as a function of α_c and v, for γ_{Rd} = 1.1

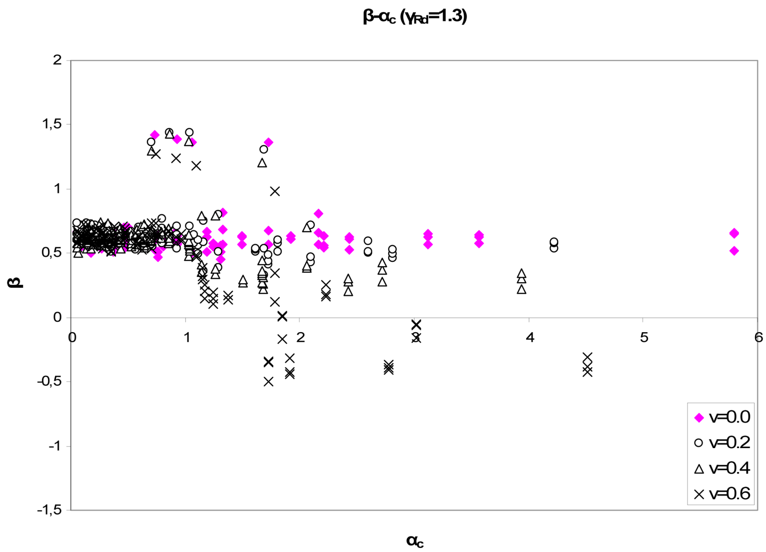


Fig. 7 Safety index β as a function of α_c and v, for γ_{Rd} = 1.3

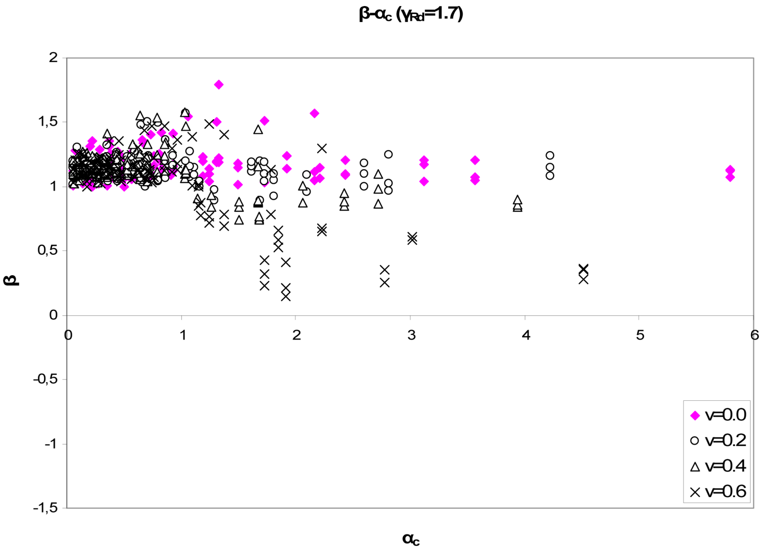


Fig. 8 Safety index β as a function of α_c and v, for $\gamma_{Rd} = 1.7$

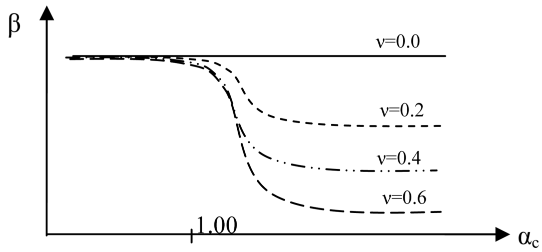


Fig. 9 Schematically presentation of β as a function of ν and α_c. β is also affected from γ_{Rd} and cotθ, β takes larger values when γ_{Rd} increases and when cotθ decreases

For the examined cases (of Table 1), the values of α_c are the same for the upper and the lower joint of the column, having common nominal cross-section and nominal reinforcement. If α_c is larger than 0.77 (=1/1.3) the joint corresponds to cases where the capacity design of joints (relation (4.29) with safety factor 1.3 of EC8) has not been satisfied. If a_c is smaller than 0.77 the capacity design of joints has been fulfilled.

Comparing Figs. 6, 7 and 8 it can be seen that values of β increase when γ_{Rd} increases. From Figs. 6, 7 and 8 it also can be seen that the values of β are affected from the axial force ν and the factor a_c. Fig. 9 describes this behavior.

From Fig. 9, it can be seen that when α_c is significantly larger than 1, the safety factor β decreases for increased values of ν (this is due to the fact that the ratio of mean value to design value of the resisting moment ($M_{R,m}/M_{Rd}$) increases when v increases (Trezos 2000)). When α_c is smaller than 1, the probability of failure depends only on the resisting moments of beams as the plastic hinges are expected to be formed at the ends of the beams connected to the joints into which the column frames. This can be seen from relation 4.1: using common values for $M^i_{R,b,left}$, $M^i_{R,b,right}$, $M^i_{R,c,up}$ and $M^i_{R,c,down}$ for the joints 1 and 2 and assuming that $\Sigma M_{R,b}<\Sigma M_{R,c}$ the relation becomes independent of the resisting moments of columns. So, for these cases, the axial force does not affect

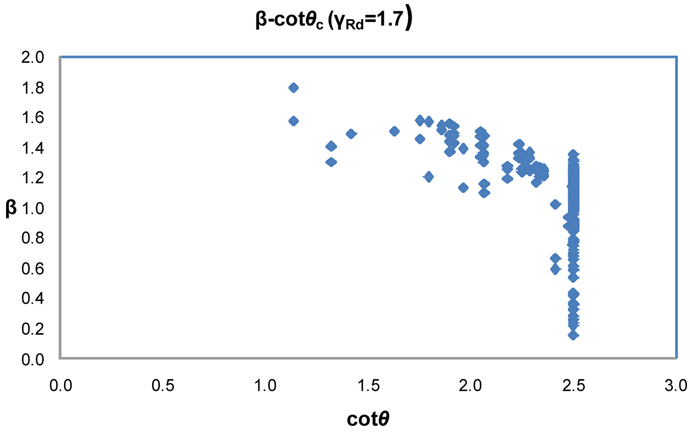


Fig. 10 Values of β as a function of cotθ for γ_{Rd}=1.7

the safety index β because the resisting moments of columns are not taken into account in the calculation of β.

Values of β are also affected from the value of cotθ that has been used for calculating the stirrups of the columns. The shear strength of the columns for which the value of cot$\theta = 2.5$ has been used for calculating the shear reinforcement is, probably, calculated through the relation 14.1 while the shear strength of the columns designed with cot$\theta < 2.5$ is calculated through relation 14.3 (Relation 14.2 refers to extremely rare cases. For deterministic variables, relation 14.2 refers to not allowed cases because of the relation 6.12 of EC2 which gives the maximum shear reinforcement). The use of these two different relations affects the probabilistic characteristics of the results of the simulations in such a way that values of β to increases when cotθ decreases. The increase of the safety index β when cotθ decreases, can be observed in Fig. 10 in which the safety index β is presented as a function of cotθ for the cases of columns that have been designed with $\gamma_{Rd} = 1.7$.

Relation 17 has been inspired from Fig. 9. Several functions have been examined by applying nonlinear regression analysis to the data of Table 1 and the results of the simulations. Relation 17 turned up as the most suitable. It gives the estimated value of the safety index β as a function of the partial safety factor γ_{Rd}, the axial force ν and the beam to column resisting moment ratio a_c.

$$\beta = 2.19\cdot\ln\gamma_{Rd} + 2.22\cdot\nu^2\cdot\left[\exp\left(-\exp\left(\frac{a_c - 1.28}{0.14}\right)\right) - 1\right] + (1.76 - 0.70\cdot\cot\theta) \qquad (17)$$

(with $R^2 = 0.96$, standard error = 0.09)

where

ν the average axial force of the column [$\nu = 0.5(\nu_{up} + \nu_{down})$, positive for compression]

cotθ the value that has been used for calculating the shear reinforcement

In the diagram of Fig. 11, the values of β calculated from the simulation are compared to the values of β derived from Eq. (17).

Relation (17) takes into account the parameters that affect mainly the safety index β. There are other parameters such as the dimensions, the confinement $a\omega_w$, ρ_{tot} etc. that also affect the safety index β. These parameters are not taken into account in relation (17) because they affect insignificantly and with a non systematical way the safety index β.

Relation (17) could be used for modifying the shear capacity design in such a way that the

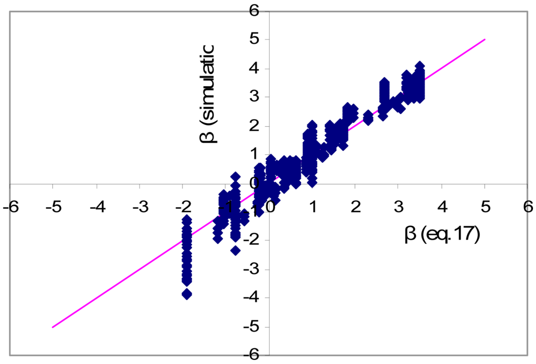


Fig. 11 Values of β calculated from Eq. (17) and simulation

probability to be satisfactory small and uniform for all types of columns. Using a value of β as the desirable value, the necessary value of the safety factor γ_{Rd} can be calculated with relation (17). The use of the calculated value of γ_{Rd} in the shear capacity design would lead to cases of columns with β equal to the desirable value.

In the design practice, the calculation of cotθ depends on the value of $V_{d,c}$ which is calculated from relation (1). Relation (1) contains the safety factor γ_{Rd} which depends, as is it is shown from relation (17), from cotθ. Therefore, although relation (17) provides the means for predicting the safety level of the shear capacity design, it is not easy to be implemented in the design practice as it contains the parameter cotθ, which is the result of the shear capacity design. To bypass this difficulty, relation (17) is changed to (18) in which the parameter ($1.76–0.70\cdot\cot\theta$) has been neglected. This relationship is less accurate than relationship (17), as the safety index β is actually influenced by cotθ, but it is more useful as it does not include unknown parameters. Besides, relation (18) gives larger values from relation (17) as the value of cotθ that corresponds to $1.76–0.70\cdot\cot\theta = 0$ is $\cot\theta = 1.76/0.7 = 2.5$ (the maximum permitted value). So, it is a more conservative relation.

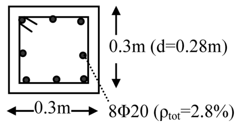
$$\beta = 2.19\cdot\ln\gamma_{Rd} + 2.22\cdot\nu^2\cdot\left[\exp\left(-\exp\left(\frac{a_c - 1.28}{0.14}\right)\right) - 1\right] \tag{18}$$

Using relation (18) a modification of the shear capacity design of columns can be proposed. The scope of the modified design is for all cases of columns to yield a common value of the safety index β. The common value of β is considered equal to 0.24 for DC M (medium ductility) columns and 0.57 for DC H (high ductility) columns. These values correspond to the values of β that are given from relation (17) if the values $\gamma_{Rd} = 1.1$, $\nu = 0$ and $\gamma_{Rd} = 1.3$, $\nu = 0$ are used respectively. The

Table 2 γ_{Rd} values for achieving a uniform safety level for the shear capacity design of columns ($\beta = 0.21$ for DC M columns and $\beta = 0.57$ for DC H columns)

	DC M (medium ductility) columns			DC H (high ductility) columns		
	$0 \le \nu < 0.2$	$0.2 \le \nu < 0.4$	$0.4 \le \nu \le 0.6$	$0 \le \nu < 0.2$	$0.2 \le \nu < 0.4$	$0.4 \le \nu \le 0.6$
$\alpha_c < 0.7$	1.10	1.10	1.11	1.30	1.30	1.31
$0.7 \le \alpha_c < 1.4$	1.14	1.27	1.53	1.35	1.51	1.81
$1.4 \le \alpha_c$	1.15	1.29	1.58	1.36	1.53	1.87

Table 3 Example of implementing the proposed shear capacity design

<table>
<tr><th colspan="3">Columns</th><th colspan="4">Beams</th></tr>
<tr><th>Section/dimensions/ reinforcement</th><th>Height of the column</th><th>Axial force (ν)</th><th>Dimensions</th><th>b_{eff} (m)</th><th>Reinforcement ratio ρ_t=ρ_c (%)</th><th>a_c</th></tr>
<tr><td>
</td><td>3 m</td><td>ν = 0.6</td><td>25/50</td><td>2.0</td><td>6Φ14 (ρ_t=ρ_c=0.7%)</td><td>$\Sigma M_{Rd,b}/\Sigma M_{Rd,c}$= 366.7/249.9 =1.47</td></tr>
<tr><td></td><td colspan="3">Shear capacity design as proposed by EC8</td><td colspan="3">Modified shear capacity design</td></tr>
<tr><td>DC M (medium ductility) columns</td><td colspan="3">Shear capacity design with γ_{Rd}=1.1: $V_{d,c}$ =1.1·249.9/3.0=91.63 kN, ω=0.3·0.9·0.28·0.6·20000/1.5/ 91.63=6.6>2.9, so A_{sw}/s=91.63/ (0.9·28·50/1.15·2.5)=0.033 cm^2/cm, needed stirrups Φ8/30 (ρ_w=0.0011)</td><td colspan="3">ν=0.6, αc=1.47→γ_{Rd}=1.58: $V_{d,c}$=1.58·249.9/3.0=134.61 kN, ω=0.3·0.9·0.28·0.6·20000/1.5/134.61=4.5>2.9, so A_{sw}/s=134.61/(0.9·28·50/1.15·2.5)=0.049 cm^2/cm, needed stirrups Φ8/20 (ρ_w=0.0016)</td></tr>
<tr><td>DC H (high ductility) columns</td><td colspan="3">Shear capacity design with γ_{Rd}=1.3: $V_{d,c}$ =1.3·249.9/3.0=108.29 kN, ω=0.3·0.9·0.28·0.6·20000/1.5/108.29 =5.6>2.9, so A_{sw}/s= 108.29/ (0.9·28·50/1.15·2.5)=0.0395 cm^2/cm, needed stirrups Φ8/25 (ρ_w=0.0013)</td><td colspan="3">ν=0.6, αc=1.47→ γ_{Rd}=1.87: $V_{d,c}$=1.87·249.9/3.0=155.77 kN, ω=0.3·0.9·0.28·0.6·20000/1.5/157.77=3.8>2.9, so A_{sw}/s=155.77/(0.9·28·50/1.15·2.5)=0.057 cm^2/cm, needed stirrups Φ8/17.6 (ρ_w=0.0019)</td></tr>
</table>

values of the safety factor γ_{Rd} that must be used for any case of column are presented in Table 2, in order for the safety level of the shear capacity design to be equal to 0.63 for DC M columns and 1.74 for DC H columns. These values of γ_{Rd} have been calculated using the relation (18).

Each value of Table 2 is the value of γ_{Rd} that corresponds to the most critical case of each region. For example the value of 1.27 is the value of γ_{Rd} that corresponds to a column with ν = 0.4, α_c = 1.4. If the value 1.27 is used for a different column of this region, larger values of β would result.

In Table 3, an example of how the proposed modification of the shear capacity design can be implemented is presented. The transverse reinforcement is calculated for DC M and DC H columns with the relation (1) of EC8 and with the proposed modified values for γ_{Rd} of Table 2.

The results show that the shear reinforcement required in the example according to the proposed modification is almost 50% higher than the shear reinforcement calculated with the relation of EC8 (45% increase for DC M and 46% increase for DC H columns). This is a logical result that would be expected for any kind of column, as the proposed values of γ_{Rd} of Table 2 are larger than the values of 1.1 and 1.3 of EC8.

5. Conclusions

In the present study, the safety level of the shear capacity design of columns has been examined. The case of “shear failure to appear before the formation of the plastic hinges at the ends of the elements” is considered as failure (or non-compliance) of the capacity design. A methodology based on simulation techniques (Monte Carlo, LHS) has been developed for calculating this probability. The methodology has been used for calculating the probability of failure for 2304 columns (for which the shear capacity design, as proposed by EC8, had been implemented).

The results showed that the shear capacity design of columns does not offer a uniform level of safety as the values of β vary from -1.16 to 0.76 for the code value of γ_{Rd} = 1.1 and from -0.50 to 1.44 for the code value of γ_{Rd} = 1.3. So a modification of the shear capacity design is proposed in order to make uniform the safety level. Appling regression analysis to the data and to the results of the examined columns a relation that gives the safety level of the shear capacity design as a function of the partial safety factor γ_{Rd} and the parameters of the columns was found. This relation is used for calculating the proper values of partial safety factors for each case of column in order for the safety level to be uniform (Table 2).

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