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Numerical and statistical analysis about displacements in reinforced concrete beams using damage mechanics

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Abstract. This work intends to contribute for the improvement of the procedure suggested by Brazilian Technical Code that takes into account the cracked concrete stiffness in the estimative of the displacement of reinforced concrete beams submitted to service loads. A damage constitutive model accounting for induced anisotropy, plastic deformations and bimodular elastic response is used in order to simulate the concrete behaviour, while an elastoplastic behaviour is admitted for the reinforcement. The constitutive models were implemented in a program for bars structures analysis with layered finite elements. Initially, the damage model is briefly presented as well as the parametric identification of the materials that have been used in the reinforced concrete beams. After that, beams with different geometries and reinforcement area are analyzed and a statistical method (ANOVA) is employed in order to identify the main variables in the problem. Soon after, the same procedure is used with another resistance of concrete, where the compression strength is changed. The numerical responses are compared with the ones obtained by Brazilian Technical Code and experimental tests in order to validate the use of the damage model. Finally, some remarks are discussed based on responses presented in this work.

Keywords: damage mechanics; reinforced concrete; technical code; non-linear analysis.

1. Introduction

In the usual estimative of displacements in structures using Solid Mechanics, it can be appealed, for instance, to Principle of Virtual Work. However, in the case of reinforced concrete structures, that estimative is complex because this kind of material is composed by concrete and steel with different elasticity moduli leading to a different bending behaviour. Besides, some parts of the structure will probably present different behaviours even when the structure is submitted to small value loads, i.e., areas where the tensioned concrete present cracks while other ones don't present any damage. The existence of cracks induces to the inertia decreasing processes, where theoretically just the reinforcement resists to the traction stresses. In this moment, the bending moment at the cross section of the structure is called cracking moment, M_r.

With the increase of the knowledge of the materials behaviour and the numerical techniques, besides the development of computers more and more efficient, it has been possible the consideration of mechanisms of RC structures behaviour through mathematical models more realistic. These developments can be noted in the Brazilian Technical Code (NBR 6118:2003 2007), where a procedure to estimate the displacements in bar elements considering the non-linear behaviour of the concrete is

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suggested, where the cracking is the main phenomenon that induces the decrease of the material stiffness.

The approach attempt between mathematical model and reality of the structural behaviour has the advantage of obtain numerical responses more accurate. However, the amount of parameters is excessive and its parametric identification is more complex with the need of experimental test more sophisticated. Note yet, the computational and theoretical formulation costs are forbidding for the use of the model in practical situations of structural engineering. On the other hand, the search for mathematical models that try to balance the simplicity with robustness of results is desirable. In this context, this present work intends to show the potentialities of a constitutive model for the concrete when it is used in practical situations. This damage model is based on the Continuum Damage Mechanics (CDM) following the formalism proposed in Pituba (2006a). In some works are reported the use of the damage model in complex loading situations (Proença and Pituba 2003, Pituba 2006b, 2008, 2009, 2010, Pituba and Fernandes 2011). Now, it intends to use a more simplified version of the model, however efficient, in the analysis of RC structures submitted to usual loading.

The CDM is a tool for simulating the material deterioration in equivalent continuous media due exclusively to microcracking process (Zhu *et al.* 2009, Kucerova *et al.* 2009, Ibrahimbegovic *et al.* 2008, Brancherie and Ibrahimbegovic 2009). A certain material can be assumed as a continuous medium being the internal changes caused by the microcracks taken into account by scalar or tensor damage variables which decrease the initial stiffness of the equivalent medium.

The damage model describes the stiffness loss process through the reduction of the elasticity modulus of the material in a given point of the structure. On the other hand, the model suggested by NBR 6118:2003 (2007) reduces the inertia moment in a studied section and, besides this new inertia moment is representative for throughout beam as if the beam was cracking in a homogeneous way. These questions together the reliability of the numerical responses presented by the damage model and low cost of the numerical analyses, when compared to the experimental ones, motivated the discussion that follows.

Finally, the work contributes to a discussion about the consideration of the non-linear behaviour of the concrete in the estimative of displacements in RC beams suggested in the NBR Procedure. For that, the NBR Procedure and damage model are used in analyses of RC beams submitted to gravitational and accidental loads. Besides, the span, cross section, reinforcement arrangement and concrete strength of the beams are changed. The aim is to verify the differences among the numerical responses by the damage model, analytical responses by NBR Procedure and experimental ones. For a better discussion, a statistical method (Variance Analysis – ANOVA) is employed in order to verify the main variables involved in the problem taking into account the numerical and analytical results obtained in this work. Obviously, it is necessary to investigate more cases in future works in order to propose an alternative procedure to estimate displacements in RC beams. This is the goal of this research.

2. Damage model

This model was proposed by Pituba (2006a) in order to simulate the concrete behaviour. The material is assumed as an initially isotropic material that starts to present transverse isotropy and bimodular responses induced by the damage. To take into account the bimodularity conveniently, two damage tensors governing the rigidity in tension or compression regimes are introduced.

Moreover, the model tries to respect the principle of energy equivalence between damaged real medium and equivalent continuous medium established in the Continuum Damage Mechanics, Lemaitre and Chaboche (1990). Here in after, the damage model is briefly described. So, for the tension dominant states, the following damage tensor is adopted

$$\boldsymbol{D_T} = f_1(D_1, D_4, D_5)(\boldsymbol{A}\otimes\boldsymbol{A}) + 2 f_2(D_4, D_5)[(\boldsymbol{A}\,\underline{\overline{\otimes}}\,\boldsymbol{I}+\boldsymbol{I}\,\underline{\overline{\otimes}}\,\boldsymbol{A})-(\boldsymbol{A}\otimes\boldsymbol{A})] \quad (1)$$

where $f_1(D_1, D_4, D_5) = D_1 - 2f_2(D_4, D_5)$ and $f_2(D_4, D_5) = 1 - (1 - D_4)(1 - D_5)$.

The variable D_1 represents the damage in the orthogonal direction to the transverse isotropy local plane of the material, while D_4 is representative of the damage generated by the sliding movement between the crack faces. The third damage variable, D_5, is only activated if a previous compression state accompanied by damage has occurred.

In the Eq. (1), the tensor $\boldsymbol{I}$ is the second-order identity tensor and the tensor $\boldsymbol{A}$, by definition Pituba (2006a) and Curnier *et al.* (1995), is formed by the dyadic product of the unit vector perpendicular to the transverse isotropy plane for itself.

The same observations given for damage tensor to the dominant tension states are valid for dominant compression states. Therefore the damage tensor is given by

$$\boldsymbol{D_C} = f_1^*(D_2, D_4, D_5)(\boldsymbol{A}\otimes\boldsymbol{A}) + f_2(D_3)[(\boldsymbol{I}\,\overline{\otimes}\,\boldsymbol{I})-(\boldsymbol{A}\otimes\boldsymbol{A})] + 2f_3(D_4, D_5)[(\boldsymbol{A}\,\underline{\overline{\otimes}}\,\boldsymbol{I}+\boldsymbol{I}\,\underline{\overline{\otimes}}\,\boldsymbol{A})-(\boldsymbol{A}\otimes\boldsymbol{A})] \quad (2)$$

where $f_1^*(D_2, D_4, D_5) = D_2 - 2f_3(D_4, D_5)$, $f_2(D_3) = D_3$ and $f_3(D_4, D_5) = 1 - (1-D_4)(1-D_5)$.

Note that in the compression damage tensor expression two additional scalar variables are introduced: D_2 and D_3. The variable D_2 (damage perpendicular to the transverse isotropy local plane of the material) reduces the Young's modulus in that direction. On the other hand, the variable D_2 together with D_3 (that represents the damage in the transverse isotropy plane) degrades the Poisson's ratio on the perpendicular planes to the one of transverse isotropy.

Finally, the resultant constitutive tensors $\boldsymbol{E_T}$ and $\boldsymbol{E_C}$ may be described as follow

$$\begin{aligned} E_T = {} & \lambda_{11}[I\otimes I]+2\mu_1(I\,\underline{\overline{\otimes}}\,I)-\lambda_{22}^{+}(D_1,D_4,D_5)[A\otimes A] \\ & -\lambda_{12}^{+}(D_1)[A\otimes I+I\otimes A]-\mu_2(D_4,D_5)[A\,\underline{\overline{\otimes}}\,I+I\,\underline{\overline{\otimes}}\,A] \end{aligned} \quad (3)$$

$$\begin{aligned} E_C = {} & \lambda_{11}[I\otimes I]+2\mu_1[I\,\underline{\overline{\otimes}}\,I]-\lambda_{22}^{-}(D_2,D_3,D_4,D_5)[A\otimes A]-\lambda_{12}^{-}(D_2,D_3)[A\otimes I+I\otimes A] \\ & -\lambda_{11}^{-}(D_3)[I\otimes I]\frac{(1-2\nu_0)}{\nu_0}\lambda_{11}^{-}(D_3)-\mu_2(D_4,D_5)[A\,\underline{\overline{\otimes}}\,I+I\,\underline{\overline{\otimes}}\,A] \end{aligned} \quad (4)$$

where $\lambda_{11} = \lambda_0$; $\mu_1 = \mu_0$. The remaining parameters will only exist for no-null damage, evidencing in that way the anisotropy and bimodularity induced by damage. Those parameters are given by

$$\begin{aligned} & \lambda_{22}^{+}(D_1,D_4,D_5) = (\lambda_0+2\mu_0)(2D_1-D_1^2)-2\lambda_{12}^{+}(D_1)-2\mu_2(D_4,D_5); \\ & \lambda_{12}^{+}(D_1) = \lambda_0 D_1;\ \mu_2(D_4,D_5) = 2\mu_0[1-(1-D_4)^2(1-D_5)^2]; \\ & \lambda_{22}^{-}(D_2,D_3,D_4,D_5) = (\lambda_0+2\mu_0)(2D_2-D_2^2)-2\lambda_{12}^{-}(D_2,D_3)+\frac{(\nu_0-1)}{\nu_0}\lambda_{11}^{-}(D_3)-2\mu_2(D_4,D_5); \\ & \lambda_{12}^{-}(D_2,D_3) = \lambda_0[(1-D_3)^2-(1-D_2)(1-D_3)];\ \lambda_{11}^{-}(D_3) = \lambda_0(2D_3-D_3^2) \end{aligned} \quad (5)$$

In Curnier *et al.* (1995), for the identification of the bimodularity constitutive response, a hypersurface is defined either in the stress or strain space to be used. In this work, a particular form is adopted for the hypersurface in the strain space: a hyperplane $g(\varepsilon)$ defined by the unit normal $\boldsymbol{N}$ ($||\boldsymbol{N}|| = 1$) and characterized by its dependence of the strain and damage states. Thus, the criterion presented by Curnier *et al.* (1995) is here extended so that the actual damage state can influence the hyperplane definition. Therefore, the following relation is proposed

$$g(\varepsilon, \boldsymbol{D_T}, \boldsymbol{D_C}) = \boldsymbol{N}(\boldsymbol{D_T}, \boldsymbol{D_C}) \,.\, \varepsilon^e = \gamma_1(D_1, D_2)\,\varepsilon_V^e + \gamma_2(D_1, D_2)\,\varepsilon_{11}^e \tag{6}$$

where $\gamma_1(D_1, D_2) = \{1+H(D_2)[H(D_1)\text{-}1]\}\,\eta(D_1)+\{1+H(D_1)[H(D_2)-1]\}\,\eta(D_2)$ and $\gamma_2(D_1, D_2) = D_1+D_2$.

The Heaviside functions employed above are given by

$$H(D_i) = 1 \text{ for } D_i > 0;\ H(D_i) = 0 \text{ for } D_i = 0\ (i = 1, 2) \tag{7}$$

The $\eta(D_1)$ *e* $\eta(D_2)$ functions are defined, respectively, for the tension and compression cases, assuming for the first one that there was no previous damage in compression affecting the present tension damage variable D_1. Analogously, for the second one it is assumed that has not had previous damage in tension affecting variable D_2.

$$\eta(D_1) = \frac{-D_1+\sqrt{3-2D_1^2}}{3};\ \eta(D_2) = \frac{-D_2+\sqrt{3-2D_2^2}}{3} \tag{8}$$

As it has already been pointed out, in the model formulation the damage induces anisotropy in the concrete. Therefore, it is convenient to separate the damage criterion into two criteria: the first one is used only to indicate damage incipience when the material is no longer isotropic and the second one is used for loading and unloading when the material is already considered as transverse isotropic. The second criterion identifies if there is or not evolution of the damage variables. That division is justified by the difference between the complementary elastic strain energies of an isotropic and a transverse isotropic material. For more details see Pituba and Fernandes (2011).

In the loading case, i.e., when $\dot{\boldsymbol{D}}_T \neq \boldsymbol{0}$ or $\dot{\boldsymbol{D}}_c \neq \boldsymbol{0}$, one needs to update the values of the scalar damage variables that appear in the $\boldsymbol{D_T}$ and $\boldsymbol{D_C}$ tensors, considering their evolution laws. In this work, the evolution laws of the damage variables are written in terms of conjugate forces, i.e., associated variables (Lemaitre and Chaboche 1990). In a general way, the expression that defines the associated variables may be represented by

$$Y_{T,C} = F(\sigma, E_0, D_{T,C}) \tag{9}$$

Taking into account an implicit representation, the damage evolution laws may be given by

$$\dot{D}_{T,C} = F^*(Y_{T,C}, b_{T,C}) \tag{10}$$

where $b_{T,C}$ are groups of parameters incorporated in the evolution laws of $\boldsymbol{D_T}$ or $\boldsymbol{D_C}$. Observe that in case of monotonic loading, the Eq. (10) can be integrated directly. However, the set of relations formed by $\boldsymbol{Y_{T,C}}$ and $\boldsymbol{D_{T,C}}$ leads to an implicit system whose solution can be obtained by an iterative procedure. In the numerical applications presented in this work, the monotonic loading is considered. The evolution laws for the scalar damage variables have been proposed according to the experimental results. Moreover, these laws present similar characteristics to the ones described in the works of Mazars (1986) and La Borderie (1991). Thus, the general form proposed is

$$D_i = 1 - \frac{1+A_i}{A_i+\exp[B_i(Y_i - Y_{0i})]} \qquad i = 1,5 \tag{11}$$

where A_i, B_i and Y_{0i} are parameters that must be identified. The parameters Y_{0i} are understood as initial limits for the damage activation. The parametric identification of the model is accomplished through uniaxial tension tests in order to obtain A_1, B_1 and $Y_{01} = Y_{0T}$, through uniaxial compression tests for the identification of the parameters A_2, B_2 and Y_{02}, and finally through biaxial compression tests to obtain A_3, B_3 and $Y_{03} = Y_{02} = Y_{0C}$. On the other hand, the identification of the parameters for the evolution laws corresponding to the damage variables D_4 and D_5 needs a direct shear test. However, for the one-dimensional version of the model used in this work, it is necessary the parametric identification related to uniaxial stress tests.

When the damage process is activated, the formulation starts to involve the tensor $\boldsymbol{A}$ that depends on the knowledge of the normal to the transverse isotropy plane. Therefore, it is necessary to establish some rules to identify its location for an actual strain state.

Initially, a general criterion is established for the existence of the transverse isotropy plane. It is proposed that the transverse isotropy due to damage only arises if positive strain rates exist at least in one of the principal directions. Moreover, some rules to identify its location must be defined. First of all, considering a strain state in which one of the strain rates is no-null or has sign contrary to the others, the following rule is applied:

"In the principal strain space, if two of the three strain rates are extension, shortening or null, the plane defined by them will be the transverse isotropy local plane of the material."

For this work is interesting observe that the uniaxial tension is an example of the case above where the transverse isotropy plane is perpendicular to the tension stress direction. The same observation is valid for uniaxial compression case.

3. NBR 6118:2003 Procedure

The models of displacement evaluation in RC structures consider the behaviour of a structural element submitted to the bending moment in the Stage I (section without crack, when is considered the contribution of the concrete in the tensioned area) and Stage II (cracked section, when is not considered the contribution of the tensioned concrete for the cross section equilibrium).

The NBR Procedure presents a criterion for the estimative of the excessive displacement in RC deformed elements based on a contribution of the inertias in the Stages I (I_1) and II (I_2) what results an equivalent inertia, I_{eq}. That equivalent inertia is calculated by Eq. (12) obtained through the proposed model by Branson (1968). Such procedure is valid for the active moment in the critical section, M_a, larger than the cracking moment M_r.

$$I_{eq} = \left(\frac{M_r}{M_a}\right)^3 \cdot I_c + \left[1 - \left(\frac{M_r}{M_a}\right)^3\right] \cdot I_2 \leq I_c \tag{12}$$

In Eq. (12), I_c is the inertia of the undamaged section without the contribution of the reinforcement bars in the cross section of the beam. The cracking moment M_r is given by Eq. (13). Note that in Eq. (13), the Brazilian Technical Code does not take into account the favourable effect of the reinforcement bars. Therefore, there is a decrease of the M_r value.

$$M_r = \frac{\alpha \cdot f_t \cdot I_c}{y_t} \tag{13}$$

The value of a is 1.2 for T or double T cross sections and 1.5 for rectangular cross sections. The tension strength of the concrete (f_t) is given by Eq. (14) and y_t is the distance of the gravity centre of the section to the more tensioned fiber of the cross section.

$$f_t = 0.3 \cdot f_c^{2/3} \tag{14}$$

where f_c is the characteristic compression strength of the concrete.

The moment in the critical section, M_a, is obtained through a load combination named quasi-permanent. This load combination is given by Eq. (15) and reduces the intensity of the accidental loading through a statistical coefficient ψ_{2j} whose value can be 0.3, 0.4 or 0.6, depending for what the structure is used.

$$F_{d,ser} = \sum_{i=1}^{n} F_{gi,k} + \sum_{i=1}^{m} \psi_{2j} \cdot F_{qj,k} \tag{15}$$

In Eq. (15), F_g represents the permanent load values like as gravitational loads and F_q represents the accidental load values.

4. ANOVA method

The variance analysis is a statistical test very knew among the analysts, and it intends to verify if there is a significant difference among the averages and if the factors influence some dependent variable.

The proposed factors can be of qualitative or quantitative origin, but the dependent variable should be continuous. The main application of ANOVA is the comparison of averages obtained from different groups. There are two problem kinds that ANOVA is used as a tool to resolve them: fixed or random factors. The fixed factors are present in the majority of the cases.

The ANOVA is thoroughly used in several areas, for instance: in the industry, for the production line optimizing; in medicine sciences, for identify which factors are important in the treatment of certain pathology. In civil engineering, the use of ANOVA is still restricted, however, there are some works that has been used this technique, for instance, Ramadoss and Nagamani (2012), Delalibera and Giongo (2008) and Lima Júnior and Giongo (2004).

The ANOVA developed in this work uses fixed factors (Montgomery 1996). It has been chosen four variables for the analyses: the cross section of the beams; the effective span length; the compression strength of concrete and the rate of longitudinal reinforcement bars. For the chosen variables it has been obtained fifty four cases of combinations.

4.1 ANOVA formulation

Consider A, B and C the fixed main factors of the variance analysis and a, b and c the variations of those factors and n the number of replicas. In general way, there are $a\ b\ c...n$ possible combinations. If all the experiment factors were fixed, it can formulate the problem, obtaining results that indicate which of the analyzed factors are important as well as their combinations.

To verify the relevance of a certain fixed main factor or combinations among the main factors, it is made the relationship between the squares average of each main factor or combination of the main factors for the squares average of the errors. The division between the squares average of each main factor or combination of the main factors for the errors average is called F_0.

Table 1 ANOVA: general formulation

Factors	Squares sum	Freedom degrees	Squares average	F_0
A	SS_A	$a-1$	$MS_A = SS_A$ / (a –1)	$F_0 = \dfrac{MS_A}{MS_E}$
B	SS_B	$b-1$	$MS_B = SS_b / (b-1)$	$F_0 = \dfrac{MS_B}{MS_E}$
C	SS_C	$c-1$	$MS_C = SS_c / (c-1)$	$F_0 = \dfrac{MS_C}{MS_E}$
$A \times B$	SS_{AB}	$(a-1)(b-1)$	$MS_{AB} = SS_{AB} / [(a-1)(b-1)]$	$F_0 = \dfrac{MS_{AB}}{MS_E}$
$A \times C$	SS_{AC}	$(a-1)(c-1)$	$MS_{AC} = SS_{AC} / [(a-1)(c-1)]$	$F_0 = \dfrac{MS_{AC}}{MS_E}$
$B \times C$	SS_{BC}	$(b-1)(c-1)$	$MS_{BC} = SS_{BC} / [(b-1)(c-1)]$	$F_0 = \dfrac{MS_{BC}}{MS_E}$
$A \times B \times C$	SS_{ABC}	$(a-1)(b-1)(c-1)$	$MS_{ABC} = SS_{ABC} / [(a-1)(b-1)(c-1)]$	$F_0 = \dfrac{MS_{ABC}}{MS_E}$
Erro	SS_E	$abc(n-1)$	$MS_E = SS_E / [abc(n-1)]$	
Total	SS_T	$abcn-1$		

The number of freedom degrees of each main factor is equal to the number of variations of each less factor the unit. The number of freedom degrees of the combined main factors is the product among the main factors that were combined. For instance, Table 1 shows a variance analysis with three factors.

The total sum of squares is given by

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n} y_{ijkl}^2 - \frac{y_{...}^2}{abcn} \tag{16}$$

The sum of the squares of each main factor is defined by $A(y_{i.})$, $B(y_{.j})$ and $C(y_{..k.})$ factors.

$$SS_A = \sum_{i=1}^{a} \frac{y_{i...}^2}{bcn} - \frac{y_{...}^2}{abcn} \tag{17}$$

$$SS_B = \sum_{i=1}^{a} \frac{y_{.j..}^2}{acn} - \frac{y_{...}^2}{abcn} \tag{18}$$

$$SS_C = \sum_{i=1}^{a} \frac{y_{..k.}^2}{abn} - \frac{y_{...}^2}{abcn} \tag{19}$$

The squares sum of the combinations $A \times B$, $A \times C$ and $B \times C$ is expressed by Eqs. (20) to (22). The Eq. (23) defines the squares sum of the combination of all the factors.

$$SS_{AB} = \sum_{i=1}^{a}\sum_{j=1}^{b}\frac{y_{ij..}^2}{cn} - \frac{y_{...}^2}{abcn} - SS_A - SS_B \tag{20}$$

$$SS_{AC} = \sum_{i=1}^{a}\sum_{k=1}^{c}\frac{y_{i.k.}^2}{bn} - \frac{y_{...}^2}{abcn} - SS_A - SS_C \tag{21}$$

$$SS_{BC} = \sum_{j=1}^{b}\sum_{k=1}^{c}\frac{y_{.jk.}^2}{an} - \frac{y_{...}^2}{abcn} - SS_B - SS_C \tag{22}$$

$$SS_{ABC} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\frac{y_{.ijk.}^2}{n} - \frac{y_{...}^2}{abcn} - SS_A - SS_B - SS_C - SS_{AB} - SS_{AC} - SS_{BC} \tag{23}$$

Finally, the squares sum of the error is defined by

$$SS_E = SS_T - \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\frac{y_{.ijk.}^2}{n} - \frac{y_{...}^2}{abcn} \tag{24}$$

The test F is applied in order to verify the relevance of a certain fixed main variable or combined. There are tables in Montgomery (1996) that contain values for $F_{critical}$. The calculated value of F_0 is compared with the value of $F_{critical}$. If the value F_0 is larger than the fixed value of $F_{critical}$, this means that the factor studied is relevant, otherwise, it implicates that the factor s not important. The values of $F_{critical}$ are function of the number of freedom degrees of each variable and of the number of total freedom degrees.

5. Numerical test

5.1 Finite element models

The one-dimensional version of the damage model was implemented in a program for bars structures analysis with finite layered elements. In the layered elements it is assumed as hypotheses that the distortions strains are negligible. The assumed to govern the concrete layers behaviour are the ones reported above and for the longitudinal reinforcement bars, elastoplastic behaviour is admitted. In the transversal section, a certain layer can contain steel and concrete. By assuming a perfect adherence between the materials, it is defined, for each layer, an elastic modulus and an inelastic strain equivalent, by using homogenization rule.

Now, the finite element models of the beams used in order to verify the influence of some parameters in the estimative of displacements are described. Those models have been used in the numerical analyses as well as in the analytical analyses by NBR Procedure.

In this work, some parameters involved in the problem were changed, such as: effective span length, height of the cross section, reinforcement distribution and compression strength of the concrete. The finite element models are named according to the properties contained in the Table 2 and their geometries are described in the Fig. 1.

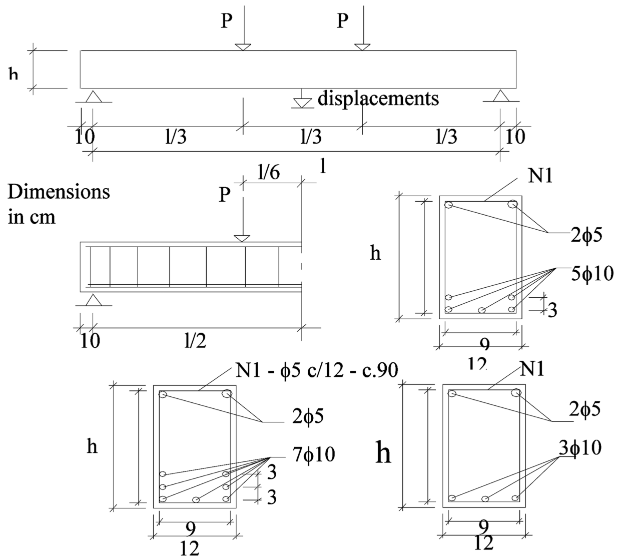

Fig. 1 Geometry properties of the finite element models

Table 2 Properties of the finite element models

Beam	f_c (MPa)	Span (m)	*As* (cm^2)	Beam	f_c (MPa)	Span (m)	*As* (cm^2)	Beam	f_c (MPa)	Span (m)	*As* (cm^2)
*V*31-12×30	30.8	2.4	2.36	*V*32-12×40	30.8	2.4	2.36	*V*33-12×50	30.8	2.4	2.36
*V*51-12×30	30.8	2.4	3.93	*V*52-12×40	30.8	2.4	3.93	*V*53-12×50	30.8	2.4	3.93
*V*71-12×30	30.8	2.4	5.5	*V*72-12×40	30.8	2.4	5.5	*V*73-12×50	30.8	2.4	5.5
*V*34-12×30	30.8	3	2.36	*V*35-12×40	30.8	3	2.36	*V*36-12×50	30.8	3	2.36
*V*54-12×30	30.8	3	3.93	*V*55-12×40	30.8	3	3.93	*V*56-12×50	30.8	3	3.93
*V*74-12×30	30.8	3	5.5	*V*75-12×40	30.8	3	5.5	*V*76-12×50	30.8	3	5.5
*V*37-12×30	30.8	4	2.36	*V*38-12×40	30.8	4	2.36	*V*39-12×50	30.8	4	2.36
*V*57-12×30	30.8	4	3.93	*V*58-12×40	30.8	4	3.93	*V*59-12×50	30.8	4	3.93
*V*77-12×30	30.8	4	5.5	*V*78-12×40	30.8	4	5.5	*V*79-12×50	30.8	4	5.5
*V*311-12×30	30	2.4	2.36	*V*322-12×40	30	2.4	2.36	*V*333-12×50	30	2.4	2.36
*V*511-12×30	30	2.4	3.93	*V*522-12×40	30	2.4	3.93	*V*533-12×50	30	2.4	3.93
*V*711-12×30	30	2.4	5.5	*V*722-12×40	30	2.4	5.5	*V*733-12×50	30	2.4	5.5
*V*344-12×30	30	3	2.36	*V*355-12×40	30	3	2.36	*V*366-12×50	30	3	2.36
*V*544-12×30	30	3	3.93	*V*555-12×40	30	3	3.93	*V*566-12×50	30	3	3.93
*V*744-12×30	30	3	5.5	*V*755-12×40	30	3	5.5	*V*766-12×50	30	3	5.5
*V*377-12×30	30	4	2.36	*V*388-12×40	30	4	2.36	*V*399-12×50	30	4	2.36
*V*577-12×30	30	4	3.93	*V*588-12×40	30	4	3.93	*V*599-12×50	30	4	3.93
*V*777-12×30	30	4	5.5	*V*788-12×40	30	4	5.5	*V799*-12×50	30	4	5.5

In order to check the vertical displacement obtained by numerical analyses presented in this work, it has been calculated analytically the vertical displacements of the RC beams submitted to the action of bending moment, using the criteria suggested by NBR Procedure, where it has been considered as permanent loads the weight of the beams and as accidental variable loads, the force values of the F_r and $3F_r$, applied to the $l/3$ distances and $2l/3$ from the support of the left of the beam (see Fig. 1). The force Fr has been obtained by Eq. (25) and its value depends on the cracking moment value (Eq. 13).

$$F_r = \left(M_r - \frac{g \cdot l^2}{8}\right) \cdot \frac{3}{l} \tag{25}$$

5.2 Parametric identification of the damage model

A perfect elasto-plastic model has been used in the mechanical behavior simulation of the reinforcement. The damage model proposed by Pituba (2006a) has been used in the mechanical behavior simulation of the concrete. For the parametric identification of the damage model, a process based on error minimization has been used. For details, see Pituba and Fernandes (2011).

The stress-strain curves of the concretes used in this work are presented in Figs. 2 and 3. Note that the experimental tests on the concrete with f_c = 30.8 MPa was performed by Álvares (1993) and for the concrete with f_c = 30 MPa was performed by Vecchio and Emara (1992). The parameter values obtained by parametric identification for each concrete are illustrated in Table 3.

According with experimental data reported in Álvares (1993), the first concrete has tension strength of 2.25 MPa and elasticity module of 29,200 MPa. According with Vecchio and Emara (1992), the second concrete has 30,400 MPa for the elasticity modulus. The strength in tension was estimated taking into account the numerical response presented by Mazars' model Pituba (2009). The steel used in reinforcement has Es = 196,000 MPa, yielding stress of 500 MPa.

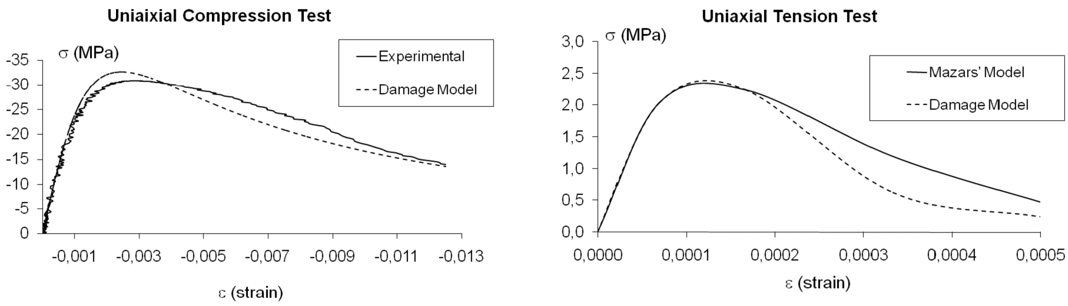

Fig. 2 Parametric identification for the concrete f_c = 30.8 MPa

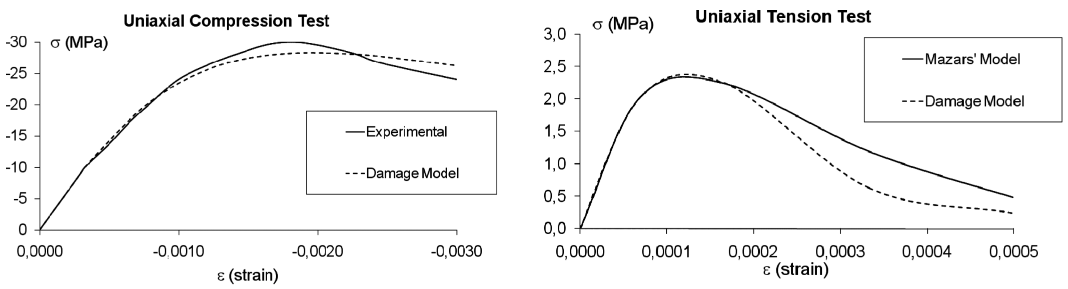

Fig. 3 Parametric identification for the concrete f_c = 30.0 MPa

Table 3 Parameters of the damage model for the concretes used in this work

Concrete f_c = 30.8 MPa		Concrete f_c = 30.0 MPa	
Tension parameters	Compression parameters	Tension parameters	Compression parameters
Y_{01} = 0.72×10^{-4} MPa A_1 = 50 B_1 = 6700 MPa^{-1}	Y_{02} = 0.5×10^{-3} MPa A_2 = -0.9 B_2 = 0.4 MPa^{-1}	Y_{01} = 0.72×10^{-4} MPa A_1 = 50 B_1 = 6700 MPa^{-1}	Y_{02} = 1.7×10^{-3} MPa A_2 = -0.8 B_2 = 1.1 MPa^{-1}

5.3 Numerical results

In order to show the accuracy of the numerical responses obtained by the damage model in the RC structures analyses, it is presented here some of those responses.

The *V*31, *V*51 and *V*71 finite element models (Fig. 1) was analyzed using the damage model and their numerical responses were compared with experimental ones reported in Álvares (1993). In the analyses were used the geometry symmetries and, therefore, half beam was just analyzed and dicretized into 20 finite elements. The cross sections were divided in 15 layers, being one layer of steel and concrete in the beam with 3#10,0 mm, two in the beam with 5#10,0 mm and three in the beam with 7#10,0 mm. The numerical and experimental responses are displayed in Figs. 4, 3 and 5.

On the other hand, the second numerical application is about RC frame (Vecchio and Emara 1992) whose geometry and reinforcement distribution are illustrated in Fig. 7. In the experimental test, it was initially applied an axial load of 700 kN for each column, which was maintained

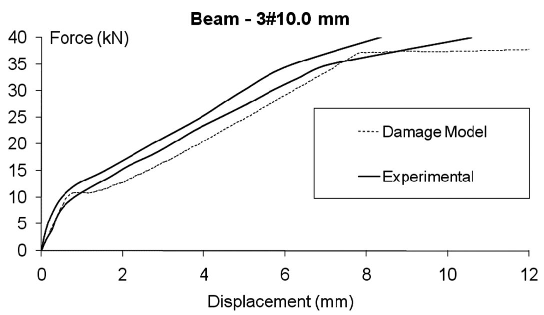

Fig. 4 Load-displacement of the Beam – 3#10.0 mm

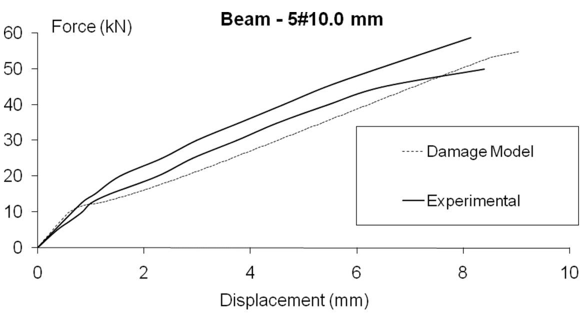

Fig. 5 Load-displacement of the Beam – 5#10.0 mm

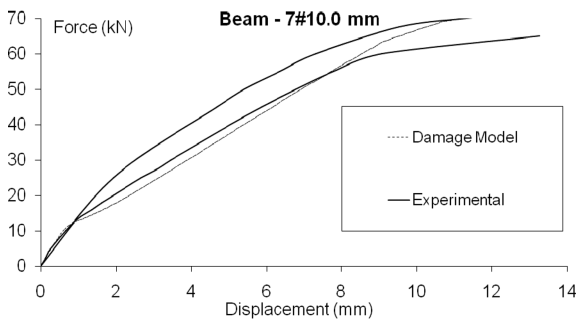

Fig. 6 Load-displacement of the Beam – 7#10.0 mm

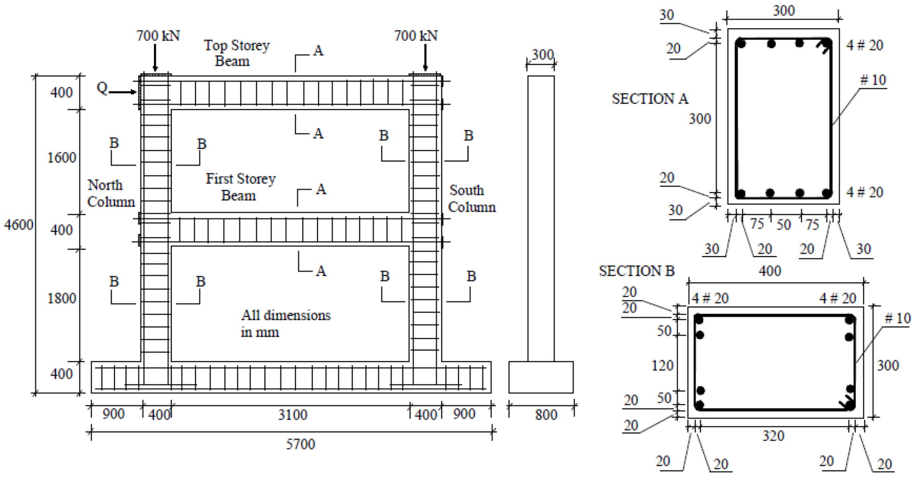

Fig. 7 Geometry and reinforcement details

constant during all the lateral load application. This force was applied in increments up to the frame ultimate load.

In the numerical analysis, displacements increments were enforced in the application point of the horizontal force. The frame was discretized into 30 finite elements, 10 of which were used in the discretization of each column and 5 in each beam. The transversal sections were divided into 10 layers. The numerical and experimental responses are displayed in Fig. 8, where the graphs represent the applied horizontal force *x* horizontal displacement relationship computed at the superior floor of the frame.

These analyses are illustrated below and the load-displacement responses obtained by the damage confirm the good recovery of the global experimental responses of the RC structures.

Here and after are presented the numerical and analytical analyses in order to show the difference between the damage model responses and NBR Procedure ones.

The tables below describe the results obtained for displacements in the middle span of each finite element model when it is employed the NBR Procedure, as well as those ones obtained in the numerical analyses. The displacement values have been considered for $P = F_r$ and $P = 3F_r$ in order to investigate the NBR Procedure related to the damage evolution of the beams.

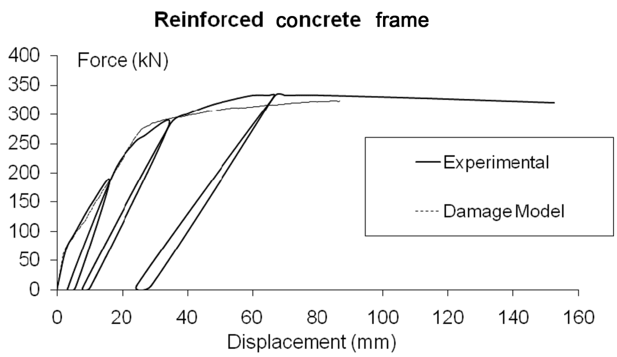

Fig. 8 Load-displacement curve of the RC frame

Table 4 Cracking force values and displacement values for $P = F_r$ and $f_c = 30.8$ MPa

Beam	P = $F_r - f_c = 30.8$ MPa				
	F_r NBR (kN)	F_r Num. (kN)	Displ. NBR (cm)	Displ. Num. (cm)	Difference (%)
*V*31-12×30	9.14	6.28	0.07	0.04	75.00
*V*51-12×30	9.14	6.44	0.07	0.04	75.00
*V*71-12×30	9.14	6.51	0.07	0.04	75.00
*V*34-12×30	6.95	4.89	0.11	0.06	83.33
*V*54-12×30	6.95	4.91	0.11	0.06	83.33
*V*74-12×30	6.95	4.97	0.11	0.06	83.33
*V*37-12×30	4.62	3.39	0.19	0.10	90.00
*V*57-12×30	4.62	3.43	0.19	0.10	90.00
*V*77-12×30	4.62	3.69	0.19	0.10	90.00
*V*32-12×40	16.61	10.39	0.05	0.03	66.67
*V*52-12×40	16.61	10.66	0.05	0.03	66.67
*V*72-12×40	16.61	10.82	0.05	0.03	66.67
*V*35-12×40	12.8	7.85	0.08	0.04	100.00
*V*55-12×40	12.8	8.05	0.08	0.04	100.00
*V*75-12×40	12.8	8.17	0.08	0.04	100.00
*V*38-12×40	8.81	6.23	0.14	0.08	75.00
*V*58-12×40	8.81	6.09	0.14	0.08	75.00
*V*78-12×40	8.81	6.18	0.14	0.08	75.00
*V*33-12×50	26.29	20.35	0.04	0.03	33.33
*V*53-12×50	26.29	20.89	0.04	0.03	33.33
*V*73-12×50	26.29	21.26	0.04	0.03	33.33
*V*36-12×50	20.42	15.39	0.06	0.05	20.00
*V*56-12×50	20.42	15.79	0.06	0.05	20.00
*V*76-12×50	20.42	16.07	0.06	0.05	20.00
*V*39-12×50	14.33	8.77	0.11	0.06	83.33
*V*59-12×50	14.33	8.99	0.11	0.06	83.33
*V*79-12×50	14.33	10.13	0.11	0.07	57.14

The difference of the displacement values has been estimated taking into account as reference the values obtained by NBR Procedure.

It is noted that the NBR Procedure is very conservatism when compared with the numerical responses, see Tables 4 to 7. In general way, it is observed the decrease of the difference among analytical and numerical displacement values when the applied load P increases. The model adopted by NBR is based on Branson (1968) and it estimates the beam stiffness related to whole beam leading to high displacement values. Note that the damage model reduces the material stiffness in agreement with the considered direction, i.e., the damage process occurs in a selectable way giving a panorama more realist of the damage in the beam. Thus, the damage model presents smaller displacements than ones obtained with the analytical model of NBR Procedure.

In the Table 8 the cracking moment values obtained by analytical and numerical analyses are

Table 5 Displacement values for $P = 3F_r$ and $f_c = 30.8$ MPa

Beam	$P = 3\ F_r - f_c = 30.8$ MPa		
	Displ. NBR (cm)	Displ. Num. (cm)	Difference (%)
*V*31-12×30	0.57	0.35	62.86
*V*51-12×30	0.44	0.26	69.23
*V*71-12×30	0.40	0.23	73.91
*V*34-12×30	0.85	0.53	60.38
*V*54-12×30	0.66	0.38	73.68
*V*74-12×30	0.60	0.33	81.82
*V*37-12×30	1.36	0.84	61.90
*V*57-12×30	1.07	0.61	75.41
*V*77-12×30	0.98	0.58	68.97
*V*32-12×40	0.5	0.31	61.29
*V*52-12×40	0.37	0.21	76.19
*V*72-12×40	0.32	0.18	77.78
*V*35-12×40	0.75	0.43	74.42
*V*55-12×40	0.56	0.31	80.65
*V*75-12×40	0.49	0.26	88.46
*V*38-12×40	1.24	0.83	49.40
*V*58-12×40	0.94	0.55	70.91
*V*78-12×40	0.81	0.45	80.00
*V*33-12×50	0.45	0.38	18.42
*V*53-12×50	0.33	0.26	26.92
*V*73-12×50	0.28	0.22	27.27
*V*36-12×50	0.69	0.55	25.45
*V*56-12×50	0.51	0.38	34.21
*V*76-12×50	0.43	0.31	38.71
*V*39-12×50	1.16	0.68	70.59
*V*59-12×50	0.85	0.47	80.85
*V*79-12×50	0.72	0.44	63.64

described. It is observed that NBR Procedure supplies an only one value of M_r that it is independent of the reinforcement distribution in the beam. This fact does not happen in the numerical analyses whose M_r values change according with reinforcement distribution. However, for practical applications the values of M_r can be adopted the same in a beam with same cross section and different reinforcement arrangements. However, the beam behaviour history taking into account the beginning of the cracking process to its collapse, obviously it will be influenced by the reinforcement distribution and other factors.

In Fig. 9 is illustrated a cracking moment (M_r) versus inertia (I_{eq}) relationship. Note that to the NBR Procedure, the cracking moment is not affected by compression strength of the concrete (f_c) and reinforcement distribution when it is dealing with low values for I_{eq}. However, in the numerical analyses there are more disperse results evidencing that the reinforcement distribution is a parameter that it contributes to different values of the M_r. Besides, the influence of the compression strength of

Table 6 Cracking force values and displacement values for $P = F_r$ and $f_c = 30.0$ MPa

Beam	$P = F_r - f_c = 30.0$ MPa				
	F_r NBR (kN)	F_r Num. (kN)	Displ. NBR (cm)	Displ. Num. (cm)	Difference (%)
*V*311-12×30	8.97	6.52	0.07	0.04	75.00
*V*511-12×30	8.97	6.68	0.07	0.04	75.00
*V*711-12×30	8.97	6.74	0.07	0.04	75.00
*V*344-12×30	6.81	4.46	0.11	0.05	120.00
*V*544-12×30	6.81	5.09	0.11	0.06	83.33
*V*744-12×30	6.81	5.14	0.11	0.06	83.33
*V*377-12×30	4.52	3.52	0.19	0.10	90.00
*V*577-12×30	4.52	3.56	0.19	0.10	90.00
*V*777-12×30	4.52	3.59	0.19	0.10	90.00
*V*322-12×40	16.30	10.78	0.05	0.03	66.67
*V*522-12×40	16.30	11.05	0.05	0.03	66.67
*V*722-12×40	16.30	11.21	0.05	0.03	66.67
*V*355-12×40	12.55	8.14	0.08	0.04	100.00
*V*555-12×40	12.55	8.35	0.08	0.04	100.00
*V*755-12×40	12.55	8.47	0.08	0.04	100.00
*V*388-12×40	8.63	5.85	0.14	0.07	100.00
*V*588-12×40	8.63	6.31	0.14	0.08	75.00
*V*788-12×40	8.63	6.41	0.14	0.08	75.00
*V*333-12×50	25.8	21.10	0.04	0.03	33.33
*V*533-12×50	25.8	21.64	0.04	0.03	33.33
*V*733-12×50	25.8	22.01	0.04	0.03	33.33
*V*366-12×50	20.04	15.96	0.06	0.05	20.00
*V*566-12×50	20.04	16.36	0.06	0.05	20.00
*V*766-12×50	20.04	16.63	0.06	0.05	20.00
*V*399-12×50	14.04	9.11	0.11	0.06	83.33
*V*599-12×50	14.04	9.32	0.11	0.06	83.33
*V*799-12×50	14.04	9.47	0.11	0.06	83.33

the concrete increases when the inertia has high values. However, the values of cracking moment in numerical analyses are lower than analytical ones. Nevertheless the beam stiffness in all loading process is more appropriated simulated by the damage model in sense that the model takes into account the concrete contribution to resist tension stress (tension stiffening).

5.4 Statistical results

In the Tables 9, 10, 11 and 12 the ANOVA method results are presented for the concrete $f_c = 30.8$ MPa. In particular, the Tables 9 and 10 show the analytical and numerical results for $F = F_r$, where it can be observed that the cross section A_c is the most important factor in the estimative of displacements. The effective span length l is the second relevant factor and the coupling between A_c

Table 7 Displacement values for P=3F_r and f_c = 30.0 MPa

Beam	P = 3 F_r– f_c = 30.0 MPa		
	Displ. NBR (cm)	Displ. Num. (cm)	Difference (%)
*V*311-12×30	0.56	0.37	51.35
*V*511-12×30	0.43	0.27	59.26
*V*711-12×30	0.39	0.24	62.50
*V*344-12×30	0.83	0.47	76.60
*V*544-12×30	0.65	0.40	62.50
*V*744-12×30	0.59	0.34	73.53
*V*377-12×30	1.34	0.89	50.56
*V*577-12×30	1.06	0.63	68.25
*V*777-12×30	0.96	0.54	77.78
*V*322-12×40	0.49	0.31	58.06
*V*522-12×40	0.37	0.22	68.18
*V*722-12×40	0.32	0.18	77.78
*V*355-12×40	0.74	0.45	64.44
*V*555-12×40	0.56	0.31	80.65
*V*755-12×40	0.48	0.26	84.62
*V*388-12×40	1.22	0.75	62.67
*V*588-12×40	0.92	0.56	64.29
*V*788-12×40	0.80	0.47	70.21
*V*333-12×50	0.45	0.39	15.38
*V*533-12×50	0.33	0.28	17.86
*V*733-12×50	0.28	0.22	27.27
*V*366-12×50	0.68	0.57	19.30
*V*566-12×50	0.50	0.40	25.00
*V*766-12×50	0.42	0.33	27.27
*V*399-12×50	1.14	0.73	56.16
*V*599-12×50	0.84	0.49	71.43
*V*799-12×50	0.71	0.40	77.50

and l is the third one. In spite of the reinforcement area A_s be relevant, its importance is much smaller than the three previous factors. On the other hand, the only difference between the analyses is that the reinforcement area does not present relevant importance in the numerical analyses.

The Tables 11 and 12 show the analytical and numerical results for $F = 3F_r$, where it can be observed that the cross section A_c remains the most important factor in the estimative of displacements. Now, reinforcement area A_s starts to present an important contribution in the beam behaviour as expected because in this stage the concrete is very damaged. The effective span length l is now the third relevant factor.

Besides, it is important to observe that the coupling between A_s and l is the fourth relevant factor in the analytical analyses while the coupling between A_c and l is the fourth one in the numerical analyses. This observation shows that the damage model considers that the concrete is not so damaged as considered by the NBR Procedure what it is confirmed in the item 5.3.

Table 8 Cracking moment values for f_c = 30.8 MPa and f_c = 30.0 MPa

f_c = 30.8 MPa			f_c = 30.0 MPa		
Beam	M_r Num. (KN.m)	M_r NBR (KN.m)	Beam	M_r Num. (KN.m)	M_r NBR (KN.m)
*V*31-12×30	5.03	7.96	*V*311-12×30	5.21	7.82
*V*51-12×30	5.15	7.96	*V*511-12×30	5.34	7.82
*V*71-12×30	5.21	7.96	*V*711-12×30	5.40	7.82
*V*34-12×30	4.89	7.96	*V*344-12×30	4.46	7.82
*V*54-12×30	4.91	7.96	*V*544-12×30	5.09	7.82
*V*74-12×30	4.97	7.96	*V*744-12×30	5.14	7.82
*V*37-12×30	4.51	7.96	*V*377-12×30	4.68	7.82
*V*57-12×30	4.57	7.96	*V*577-12×30	4.73	7.82
*V*77-12×30	4.9	7.96	*V*777-12×30	4.78	7.82
*V*32-12×40	8.31	14.15	*V*322-12×40	8.62	13.90
*V*52-12×40	8.52	14.15	*V*522-12×40	8.84	13.90
*V*72-12×40	8.66	14.15	*V*722-12×40	8.97	13.90
*V*35-12×40	7.85	14.15	*V*355-12×40	8.14	13.90
*V*55-12×40	8.05	14.15	*V*555-12×40	8.35	13.90
*V*75-12×40	8.17	14.15	*V*755-12×40	8.47	13.90
*V*38-12×40	8.28	14.15	*V*388-12×40	7.78	13.90
*V*58-12×40	8.1	14.15	*V*588-12×40	8.40	13.90
*V*78-12×40	8.22	14.15	*V*788-12×40	8.52	13.90
*V*33-12×50	16.28	22.11	*V*333-12×50	16.88	21.72
*V*53-12×50	16.71	22.11	*V*533-12×50	17.31	21.72
*V*73-12×50	17.01	22.11	*V*733-12×50	17.61	21.72
*V*36-12×50	15.39	22.11	*V*366-12×50	15.96	21.72
*V*56-12×50	15.79	22.11	*V*566-12×50	16.36	21.72
*V*76-12×50	16.07	22.11	*V*766-12×50	16.63	21.72
*V*39-12×50	11.67	22.11	*V*399-12×50	12.12	21.72
*V*59-12×50	11.95	22.11	*V*599-12×50	12.40	21.72
*V*79-12×50	13.47	22.11	*V*799-12×50	12.49	21.72

In the Tables 13, 14, 15 and 16 the ANOVA method results are presented for the concrete f_c = 30.0 MPa. In particular, the Tables 13 and 14 show the analytical and numerical results for $F = F_r$ and the Tables 15 and 16 show the analytical and numerical results for $F = 3F_r$.

The same observations about the concrete f_c = 30.8 MPa is valid for the concrete f_c = 30.0 MPa. However, in the concrete f_c = 30.0 MPa, it is noted that the A_s factor presents a contribution more relevant because of the decrease of the

compression strength of the concrete.

Based on results from Tables 8, 9 and 13, it was done a second degree polynomial regression in order to obtain an equation that represents the cracking moment value related to initial inertia of the reinforced concrete section. The choice of the polynomial function is due to the importance of the cross section inertia for the estimative of the cracking moment (see Table 9, $F_0 = 5.89 \cdot 10^{15}$). The

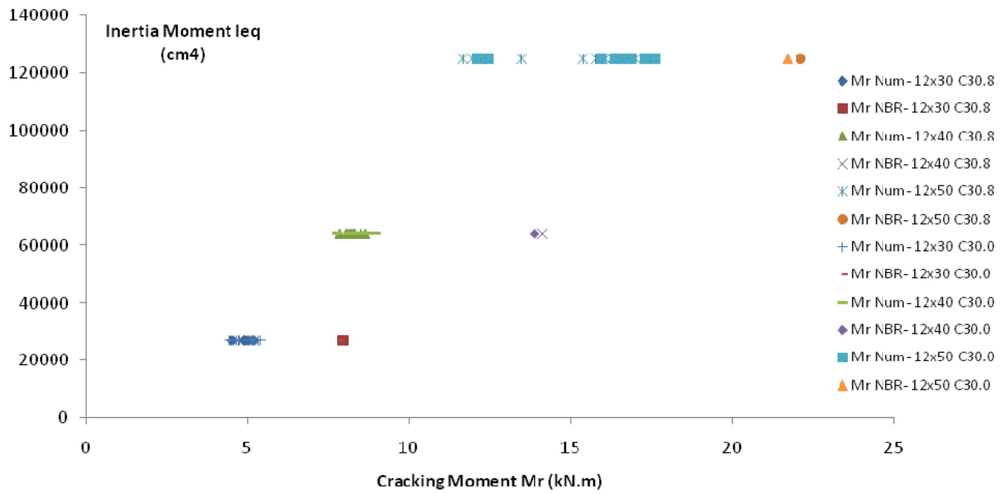

Fig. 9 Investigation about cracking moment

Table 9 ANOVA method results for analytical analyses with concrete f_c = 30.8 MPa and $F = F_r$

Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical,\ 0,05}$ $N = 26$
A_c	0.041	2	0.02	$5.89{\cdot}10^{15}$	3.37
l	0.013	2	0.0065	$1.883{\cdot}10^{15}$	3.37
A_s	0	2	0	12	3.37
$A_c \times l$	0.0019	4	0.00049	$1.393{\cdot}10^{14}$	2.74
$A_c \times A_s$	0	4	0	-6	2.74
$l \times A_s$	0	4	0	-10	2.74
Erro	0	8	0	-	-
Total	0.056	26	-	-	-

Table 10 ANOVA method results for numerical analyses with concrete f_c = 30.8 MPa and $F = F_r$

Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical,\ 0,05}$ $N = 26$
A_c	0.011	2	0.0053	$1.429{\cdot}10^{3}$	3.37
l	0.0019	2	0.00096	259	3.37
A_s	0.0000075	2	0.0000037	1	3.37
$A_c \times l$	0.0000904	4	0.000226	61	2.74
$A_c \times A_s$	0.0000148	4	0.0000037	1	2.74
$l \times A_s$	0.0000148	4	0.0000037	1	2.74
Erro	0.0000294	8	0.0000037	-	-
Total	0.013	26	-	-	-

effective span length as well as the cross section of the reinforcement bars was considered in the estimative of the beam displacements and they were considered in an indirect way in the estimative of the cracking moment.

Table 11 ANOVA method results for analytical analyses with concrete f_c=30.8 MPa and F=3F_r

Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical,\ 0{,}05}$ $N = 26$
A_c	1.716	2	0.858	$1.144 \cdot 10^4$	3.37
l	0.129	2	0.065	836.259	3.37
A_s	0.382	2	0.191	$2.55 \cdot 10^3$	3.37
$A_c \times l$	0.00936	4	0.0024	31.185	2.74
$A_c \times A_s$	0.049	4	0.012	2.296	2.74
$l \times A_s$	0.00069	4	0.00017	162.074	2.74
Erro	0.0006	8	0.000075	-	-
Total	2.288	26	-	-	-

Table 12 ANOVA method results for analytical analyses with concrete f_c = 30.8 MPa and F = 3F_r

Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical,\ 0{,}05}$ $N = 26$
A_c	0.524	2	0.262	255.64	3.37
l	0.021	2	0.011	10.276	3.37
A_s	0.224	2	0.112	109.301	3.37
$A_c \times l$	0.029	4	0.00727	7.089	2.74
$A_c \times A_s$	0.024	4	0.00592	0.325	2.74
$l \times A_s$	0.00133	4	0.00033	5.772	2.74
Erro	0.0082	8	0.001025	-	-
Total	0.831	26	-	-	-

Table 13 ANOVA method results for analytical analyses with concrete f_c=30.0 MPa and F=F_r

Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical,\ 0{,}05}$ $N = 26$
A_c	0.041	2	0.02	$5.89 \cdot 10^{15}$	3.37
l	0.013	2	0.0065	$1.883 \cdot 10^{15}$	3.37
A_s	0	2	0	12	3.37
$A_c \times l$	0.0019	4	0.00049	$1.393 \cdot 10^{14}$	2.74
$A_c \times A_s$	0	4	0	-6	2.74
$l \times A_s$	0	4	0	-10	2.74
Erro	0	8	0	-	-
Total	0.056	26	-	-	-

Therefore, the equations below can be used in the estimative of the cracking moment for the concretes analyzed in this work.

$$M_r = 0.000181 \cdot (246760 \cdot I_c + 7.571 \cdot 10^9)^{\frac{1}{2}} - 13.81 \tag{26}$$

$$M_r = 0.00188 \cdot (2383 \cdot I_c + 7.294 \cdot 10^7)^{\frac{1}{2}} - 14.03 \tag{27}$$

Table 14 ANOVA method results for numerical analyses with concrete f_c=30.0 MPa and F=F_r

Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical,\ 0{,}05}$ $N = 26$
A_c	0.0095	2	0.00483	521.12	3.37
l	0.002	2	0.00096	103.6	3.37
A_s	0.000029	2	0.000014	1.6	3.37
$A_c \times l$	0.00113	4	0.0028	30.4	2.74
$A_c \times A_s$	0.0000148	4	0.0000037	0.4	2.74
$l \times A_s$	0.0000148	4	0.0000037	0.4	2.74
Erro	0.000074	8	0.0000093	-	-
Total	0.013	26	-	-	-

Table 15 ANOVA method results for analytical analyses with concrete f_c = 30.0 MPa and $F = 3F_r$

Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical,\ 0{,}05}$ $N = 26$
A_c	1.656	2	0.828	$1.754 \cdot 10^4$	3.37
l	0.121	2	0.06	$1.279 \cdot 10^3$	3.37
A_s	0.369	2	0.184	$3.905 \cdot 10^3$	3.37
$A_c \times l$	0.011	4	0.00263	55.647	2.74
$A_c \times A_s$	0.047	4	0.012	2.941	2.74
$l \times A_s$	0.00056	4	0.000139	249.77	2.74
Erro	0.00038	8	0.0000475	-	-
Total	2.204	26	-	-	-

Table 16 ANOVA method results for analytical analyses with concrete f_c = 30.0 MPa and $F = 3Fr$

Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical,\ 0{,}05}$ $N = 26$
A_c	0.508	2	0.254	506.76	3.37
l	0.0228	2	0.011	22.74	3.37
A_s	0.223	2	0.111	222.40	3.37
$A_c \times l$	0.031	4	0.0077	15.34	2.74
$A_c \times A_s$	0.028	4	0.0069	1.07	2.74
$l \times A_s$	0.00215	4	0.00054	13.73	2.74
Erro	0.004	8	0.0005	-	-
Total	0.818	26	-	-	-

where, the Eq. (26) is related to concrete with f_c = 30 MPa and the Eq. (27) is related to concrete with f_c = 30.8 MPa. The values are expressed by kN.m for the M_r and cm^4 for I_C.

In Figs. 10 and 11 are illustrated the relationships for I_c vs. M_r for the beams analyzed using the Brazilian Technical Code.

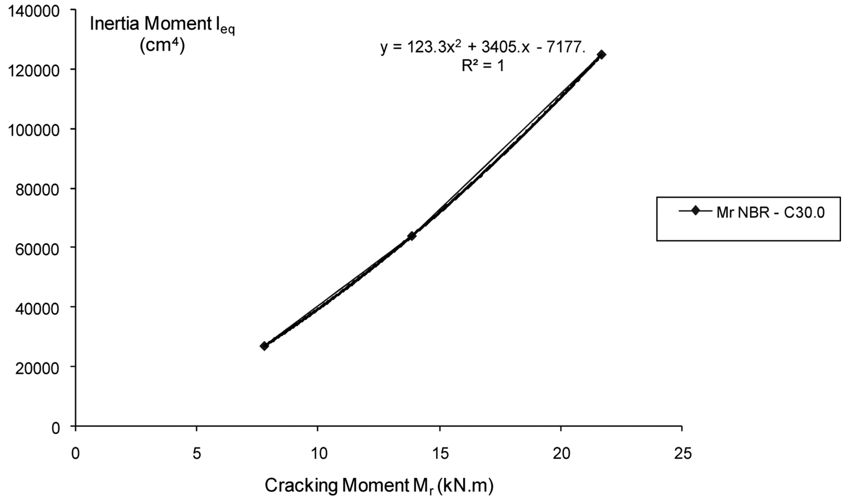

Fig. 10 Inertia moment vs. cracking moment for concrete with $f_c = 30.0$ MPa

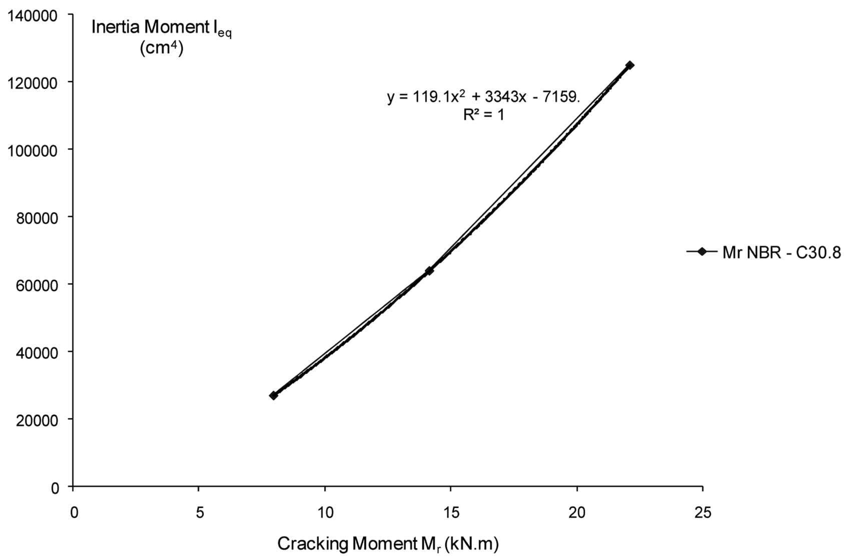

Fig. 11 Inertia moment vs. cracking moment for concrete with $f_c = 30.8$ MPa

6. Conclusions

In this work a damage model for the concrete proposed by Pituba (2006a) has been used in the estimative of displacements in RC beams in order to show its potentiality for employment in practical cases of the structural engineering.

Some numerical tests have been made as well as analytical tests using the NBR Procedure and some interesting conclusions can be related.

The cracking moment used by the Brazilian Technical Code does not consider the contribution of the reinforcement bars in the cross section of the RC beams. It is supplied an only one value of M_r

that it is independent of the reinforcement distribution in the beam. This fact does not happen in the numerical analyses whose M_r values change according with reinforcement distribution.

Besides, it is noted that the displacements obtained by analytical analyses are larger than the displacements obtained by numerical ones. This fact is due to NBR Procedure estimate an average stiffness for the whole beam based on model proposed by Branson (1968), whose formulation leads to high displacement values. In the other hand, the damage model degrades the stiffness in a selective way. Therefore, it is possible to take account the contribution of the concrete between cracks in order to resist to tension stress in that zone (tension stiffness), Murthy *et al.* (2008) and Na and Kwak (2011). However, the existence of a certain safety reservation in the estimative of displacements is necessary.

It is interesting to mention the work developed by Greco and Pau (2011) that follows the same procedure adopted in the present work in order to deal with detection of damage in arch structures using measures of numerical and experimental displacements.

Finally, in general way, the ANOVA method has been evidenced that the cross section and effective span length are the most important variables in the problem when the beam is submitted to low value loads. However, when the loads increase, the cross section of the beam remains the most important, but the reinforcement distribution is more important than the effective span length because in this stage the concrete is very damaged.

In order to proposal a more realistic procedure to estimate displacements in RC beams, the results of this work together other ones in future works will be used. For this goal be achieved, other parameters of the problem will be changed, such as: beam support conditions and more different values of compression strength of the concretes. It is possible to perform a parametric study with concretes belong different compression strength classes *C20*, *C25*, *C30*, *C35*, *C40*, *C45* and *C50* in order to formulate equations for the estimative of the cracking moment in beams based on the numerical results.

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NB

Notations

M_r	Cracking moment
D_T	Fourth-order damage tensor in tension regimes
D_C	Fourth-order damage tensor in compression regimes
D_i	Scalar damage variables
I	Second-order identity tensor
A	Second-order tensor related to transverse isotropy symmetry
E_T	Constitutive tensor in tension regimes
E_C	Constitutive tensor in compression regimes
λ_0, μ_0	Lamè constants
λ_{ij}^{+}	Damage functions related to damage tensor in tension regimes
λ_{ij}^{-}	Damage functions related to damage tensor in compression regimes
μ_i	Damage functions related to shear behaviour of the concrete
$g(\varepsilon)$	Hyperplane in the strain space
N	Unite vector perpendicular to hyperplane $g(\varepsilon)$
γ_i	Damage functions related to hyperplane $g(\varepsilon)$
$Y_{T,C}$	Associated variables in tension or compression regimes
A_i, B_i, Y_{0i}	Parameters of the damage model
I_{eq}	Equivalent inertia moment in the critical cross section
M_a	Active moment in the critical cross section
I_C	Inertia moment of the undamaged cross section considering only concrete area
a	Cross section shape factor
f_t	Tension strength of the concrete
f_c	Compression strength of the concrete
F_q	Accidental loading
F_g	Permanent loading
F_r	Applied loading when the cracking process is initiated
g	Gravitational loading
l	Span length
E_S	Elasticity module of reinforcement bar
A_C	Cross section area
A_S	Reinforcement area in the cross section