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Prediction of compressive strength of concrete using neural networks

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Abstract. This research deals with the prediction of compressive strength of normal and high strength concrete using neural networks. The compressive strength was modeled as a function of eight variables: quantities of cement, fine aggregate, coarse aggregate, micro-silica, water and super-plasticizer, maximum size of coarse aggregate, fineness modulus of fine aggregate. Two networks, one using raw variables and another using grouped dimensionless variables were constructed, trained and tested using available experimental data, covering a large range of concrete compressive strengths. The neural network models were compared with regression models. The neural networks based model gave high prediction accuracy and the results demonstrated that the use of neural networks in assessing compressive strength of concrete is both practical and beneficial. The performance of model using the grouped dimensionless variables is better than the prediction using raw variables

Keywords: compressive strength; concrete; neural network; regression models.

1. Introduction

Under the current fast pace of construction, there is a great need of mass production of concrete with extra emphasis being laid on its conformance to the standards and specifications. The strength tests are prescribed by all specifications and 28 days compressive strength of concrete in the hardened condition is perhaps the most important parameter for the structural use of concrete (Rasa *et al.* 2009). The reliable prediction of 28 days compressive strength test results as early as possible would be of satisfaction for all parties instead of waiting for the traditional 28 days test results (Kheder *et al.* 2003).

Conventional methods of predicting 28-day compressive strength of concrete are basically based upon statistical analysis by which many linear and nonlinear regression equations have been constructed to model the prediction problem (Neville 1995, Wang *et al.* 1999a, 1999b). Usually, the early compressive strength such as 6-hour, 1-day or 3-day strength is used in a prediction equation. Furthermore, choosing a suitable regression equation involves technique as well as experience. Lee (2003) reported that, for many years, researchers have proposed various methods, which are generally based on maturity concept of concrete, to predict the concrete compressive strength. Several studies have shown that concrete strength development is determined not only by the *w/c* ratio, but that it is also influenced by the content of other ingredients. Therefore, although

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experimental data have shown the practical acceptability of this rule within wide limits, a few deviations have been reported. The current empirical equations presented in the codes and standards for estimating compressive strength are based on tests of concrete without supplementary cementitious materials, especially silica fume. The validity of these relationships for concrete with supplementary cementitious materials should, therefore, be investigated.

Over the last two decades, a different modeling method based on fuzzy logic (FL) or neural networks (NNs) has become popular and has been used by many researchers for a variety of engineering applications. NNs are a family of massively parallel architectures that solve difficult problems via the cooperation of highly interconnected but simple computing elements (or artificial neurons). Basically, the processing elements of a neural network are similar to the neurons in the brain, which consist of many simple computational elements arranged in layers (Waszczyszyn and Ziemiański 2001). The compressive strength can be calculated using the models built with NNs. It is convenient and easy to use these models for numerical experiments to review the effects of each variable on the mix proportions (Pala *et al.* 2005, Baykasoğlu *et al.* 2004, Akkurt *et al.* 2003, Oztas *et al.* 2006, Sebastia *et al.* 2003, Lai and Serra 1997, Dias *et al.* 2001).

Many research works can be found in the literature that focused on the prediction of the various properties of concrete. Oztas *et al.* (2006) proposed a neural network model, which predicts both slump and 28-day compressive strength of high strength concrete. They reported a 4-layered neural network model (NNM), which estimates compressive strength with high accuracy for both training and testing pairs. Bilim *et al.* (2009) predicted the compressive strength of ground granulated blast furnace slag concrete by using NNMs. The NNMs have also been employed for predicting the strength of concrete using non-destructive tests involving ultra sonic pulse velocity (Bilgehan and Turgut 2010, Tang *et al.* 2007). Naderpour *et al.* (2010) employed NNM for predicting the compressive strength of FRP-confined concrete. Whereas, some investigators (Ramezanianpour *et al.* 2004, Sobhani *et al.* 2010) employed adaptive network-based fuzzy inference systems (ANFIS) in prediction of the compressive strength of the concrete mix-designs. Saridemir (2009) used both neural network and ANFIS models to predict compressive strength of mortars containing metakaolin. Yeh (1999) applied nonlinear programming and NNM to design high-performance concrete mixtures. The predictor core of 28-day compressive strength of concrete of the proposed model was a trained NNM which accurately predicted the compressive strength. Ozcan *et al.* (2009) used the prediction results of NNM for long-term compressive strength of concrete containing silica fume. Recently the fuzzy polynomial neural networks (FPNN) (Park *et al.* 2002), a combination of fuzzy neural networks (FNNs) and polynomial neural networks (PNNs), has been used (Zarandi *et al.* 2008) for predicting the compressive strength of concrete.

Despite the availability of large number of models, the problem of predicting the compressive strength of concrete has remained inconclusive. It is felt that this is partly due to the complexity of the phenomenon involved and partly because of the limitations of the analytical tools commonly used, by most of the investigators. Neural networks have advantages over statistical models like their data-driven nature, model-free form of predictions and tolerance to data errors. The NN models considered in most of the studies for predicting compressive strength are not very exhaustive due to the neglect of some of the strength affecting parameters due to which more layers were considered in modeling. The objective of this study was to reanalyze the data considered in the earlier studies by employing the technique of neural networks with a view towards seeing if better predictions are possible.

The aim of this paper is to construct a NN model to predict more reliably and economically the compressive strength of concrete. The NN model with one hidden layer was constructed and the training, testing and validation stages have been performed using the available test data of 265 different concrete mix-designs (Hola and Schabowicz 2005, Kaveh and Khalegi 1998, Castro 1987, Mansour *et al.* 2004, Sobhani *et al.* 2010). Efforts were made to establish a methodology that would not only predict the compressive strength of concrete but provide an economic and rapid means for future experimental researchers as well. The NN model had eight input parameters and one output parameter. The obtained results from compressive strength tests were compared with predicted results. The sensitivity of input parameters has also been studied.

2. Aim and methodology

Although, several methods have been proposed to predict the compressive strength of normal and high strength concretes but these methods are not good enough for adoption in practice. In consequence the main aim of this paper is to assess and compare the performance of regression, and artificial neural network (ANN) models to predict the compressive strength of concrete. The paper is structured in a way that the concepts and formulation of the models are first explained. Two neural network models, one using raw variables and another using grouped dimensionless variable are trained and tested by using the gathered database. The prediction performances of the models are evaluated with determination and correlation coefficients. Finally, these models are compared to discover the most suitable one.

3. Experimental data

Table 1 shows the test data taken from Refs. (Hola and Schabowicz 2005, Kaveh and Khalegi

Table 1 Concrete mixture proportions

Source	Cement, *C* (kg/m^3)	Water, *W* (kg/m^3)	Super-plasticizer, *SP* (kg/m^3)	Fine aggre-gate, *FA* (kg/m^3)	Fineness modulus of fine aggre-gate, *FM*	Coarse aggregate, *CA* (kg/m^3)	Micro-silica, *MS* (kg/m^3)	*W/C* ratio	Max size of coarse agg, *MSCA* (kg/m^3)	Concrete comp strength (MPa)
Hola and Schab-cwicz (2005)	375.00	150.00	0.00	551.20	3.05	1379.80	0.00	0.40	16.00	24.00
	450.00	150.00	0.00	735.60	3.05	1356.40	0.00	0.33	16.00	32.00
	400.00	160.00	0.00	648.30	3.05	759.00	0.00	0.40	16.00	43.00
	400.00	160.00	0.00	1325.00	3.05	723.00	0.00	0.40	16.00	45.00
	450.00	180.00	0.00	853.00	3.05	1271.70	0.00	0.40	12.00	71.00
	450.00	146.00	9.00	507.00	3.05	1522.00	0.00	0.32	12.00	85.00
	450.00	140.00	13.50	534.80	3.05	1534.20	31.50	0.29	12.00	105.00
Kaveh and Khalegi (1998)	550.00	193.00	11.00	505.00	3.32	1199.00	50.00	0.35	12.00	60.00
	550.00	187.00	11.00	861.00	3.32	855.00	50.00	0.34	12.00	66.00
	550.00	182.00	11.00	860.00	3.32	870.00	50.00	0.33	12.00	70.00
	550.00	165.00	16.00	616.30	3.32	1152.70	50.00	0.30	12.00	75.00
	550.00	154.00	16.00	580.30	3.32	1215.70	50.00	0.28	12.00	81.00
	550.00	149.00	22.00	399.20	3.32	1409.80	50.00	0.27	12.00	84.00
	550.00	143.00	22.00	580.00	3.32	1241.00	50.00	0.26	12.00	89.00
	550.00	138.00	22.00	580.00	3.32	1255.00	50.00	0.25	12.00	94.00

Table 1 Continued

Source	Cement, *C* (kg/m^3)	Water, *W* (kg/m^3)	Super-plasticizer, *SP* (kg/m^3)	Fine aggregate, *FA* (kg/m^3)	Fineness modulus of fine aggregate, *FM*	Coarse aggregate, *CA* (kg/m^3)	Micro-silica, *MS* (kg/m^3)	*W/C* ratio	Max size of coarse agg, *MSCA* (kg/m^3)	Concrete comp strength (MPa)
Castro (1987)	600.00	358.00	0.00	580.00	2.76	1345.00	0.00	0.60	16.00	18.70
	600.00	358.00	0.00	716.80	2.76	1116.20	0.00	0.60	16.00	26.30
	550.00	230.00	0.00	529.00	2.76	1426.00	0.00	0.42	16.00	28.70
	550.00	230.00	0.00	547.50	2.76	1384.50	0.00	0.42	16.00	32.60
	550.00	216.00	0.00	724.10	2.76	1329.90	0.00	0.39	16.00	38.20
	550.00	205.00	0.00	710.60	2.76	1320.40	0.00	0.37	16.00	31.80
	600.00	358.00	0.00	605.90	2.76	1283.10	0.00	0.60	16.00	26.80
	600.00	325.00	0.00	588.20	2.76	1304.80	0.00	0.54	16.00	24.10
	480.00	235.00	0.00	685.30	2.76	1380.70	0.00	0.49	16.00	29.30
	450.00	202.00	0.00	565.85	2.76	1469.15	0.00	0.45	16.00	26.30
	420.00	175.00	0.00	565.96	2.76	1398.04	0.00	0.42	16.00	40.20
	550.00	215.00	0.00	685.30	2.76	1125.70	0.00	0.39	16.00	40.60
	550.00	205.00	0.00	565.22	2.76	1254.78	0.00	0.37	16.00	28.20
	600.00	358.00	0.00	685.30	2.76	1210.70	0.00	0.60	16.00	29.50
	600.00	358.00	0.00	521.20	2.76	1432.80	0.00	0.60	16.00	35.30
	600.00	325.00	0.00	490.30	2.76	1477.70	0.00	0.54	16.00	24.70
	450.00	220.00	0.00	580.99	2.76	1376.01	0.00	0.49	16.00	32.80
	450.00	202.00	0.00	580.00	2.76	1432.00	0.00	0.45	16.00	34.50
	420.00	175.00	0.00	399.00	2.76	1619.00	0.00	0.42	16.00	34.80
	420.00	175.00	0.00	580.00	2.76	1484.00	0.00	0.42	16.00	41.90
	550.00	216.00	0.00	580.00	2.76	1368.00	0.00	0.39	16.00	26.30
Mansour *et al.* (2004)	450.00	250.00	0.00	580.00	2.93	1393.00	0.00	0.56	16.00	20.00
	450.00	250.00	0.00	505.00	2.93	1388.00	0.00	0.56	16.00	20.00
	450.00	220.00	0.00	861.00	2.93	1205.00	0.00	0.49	16.00	24.06
	400.00	180.00	0.00	860.00	2.93	1175.00	0.00	0.45	16.00	24.41
	400.00	170.00	0.00	616.30	2.93	1368.70	0.00	0.43	16.00	23.99
	450.00	195.00	0.00	580.30	2.93	1344.70	0.00	0.43	16.00	23.79
	375.00	210.00	0.00	399.20	2.93	1531.80	0.00	0.56	16.00	24.48
	550.00	280.00	0.00	716.80	2.93	1375.20	0.00	0.51	16.00	28.13
	600.00	275.00	0.00	529.00	2.93	1391.00	0.00	0.46	16.00	27.30
	550.00	280.00	0.00	547.50	2.93	1500.50	0.00	0.51	16.00	28.75
	600.00	285.00	0.00	724.10	2.93	1095.90	0.00	0.48	16.00	28.34
	550.00	285.00	0.00	710.60	2.93	1185.40	0.00	0.52	16.00	27.72
	600.00	250.00	0.00	605.90	2.93	1348.10	0.00	0.42	16.00	33.03
	600.00	250.00	0.00	588.20	2.93	1379.80	0.00	0.42	16.00	31.44
	550.00	220.00	0.00	685.30	2.93	1349.70	0.00	0.40	16.00	35.51
	550.00	180.00	0.00	521.20	2.93	1463.80	0.00	0.33	16.00	34.61
	550.00	170.00	0.00	490.30	2.93	1434.70	0.00	0.31	16.00	37.92
	550.00	175.00	0.00	580.99	2.93	1350.01	0.00	0.32	16.00	38.13
	420.00	175.00	0.00	580.00	2.93	1512.00	0.00	0.42	16.00	35.99
	550.00	255.00	0.00	399.00	2.93	1521.00	0.00	0.46	16.00	39.03
	600.00	275.00	0.00	580.00	2.93	1240.00	0.00	0.46	16.00	39.44
	550.00	225.00	0.00	580.00	2.93	1316.00	0.00	0.41	16.00	40.20
	600.00	285.00	0.00	580.00	2.93	1374.00	0.00	0.48	16.00	24.00
	550.00	285.00	0.00	505.00	2.93	1388.00	0.00	0.52	16.00	24.00
	550.00	245.00	0.00	861.00	2.93	1205.00	0.00	0.45	16.00	25.00
	420.00	215.00	0.00	860.00	2.93	1175.00	0.00	0.51	16.00	23.00
	450.00	225.00	0.00	580.99	2.93	1404.01	0.00	0.50	16.00	30.00
	480.00	225.00	0.00	580.00	2.93	1345.00	0.00	0.47	16.00	24.00
	600.00	285.00	0.00	399.00	2.93	1532.00	0.00	0.48	16.00	25.00
	420.00	205.00	0.00	580.00	2.93	1512.00	0.00	0.49	16.00	24.00

Table 1 Continued

Source	Cement, C (kg/m^3)	Water, W (kg/m^3)	Super-plasticizer, SP (kg/m^3)	Fine aggregate, FA (kg/m^3)	Fineness modulus of fine aggre-gate, FM	Coarse aggregate, CA (kg/m^3)	Micro-silica, MS (kg/m^3)	W/C ratio	Max size of coarse agg, $MSCA$ (kg/m^3)	Concrete comp strength (MPa)
	550.00	275.00	0.00	580.00	2.93	1340.00	0.00	0.50	16.00	23.00
	550.00	270.00	0.00	580.00	2.93	1468.00	0.00	0.49	16.00	25.00
	600.00	300.00	0.00	505.00	2.93	1315.00	0.00	0.50	16.00	23.00
	600.00	295.00	0.00	861.00	2.93	1035.00	0.00	0.49	16.00	25.00
	600.00	290.00	0.00	860.00	2.93	1094.00	0.00	0.48	16.00	24.00
	450.00	225.00	0.00	616.30	2.93	1351.70	0.00	0.50	16.00	23.00
	450.00	215.00	0.00	580.30	2.93	1511.70	0.00	0.48	16.00	25.00
	420.00	215.00	0.00	735.60	2.93	1184.40	0.00	0.51	16.00	23.00
	420.00	210.00	0.00	648.30	2.93	1399.70	0.00	0.50	16.00	24.00
	550.00	295.00	0.00	1325.00	2.93	495.00	0.00	0.54	16.00	14.00
	600.00	320.00	0.00	853.00	2.93	1043.00	0.00	0.53	16.00	14.00
	480.00	265.00	0.00	724.10	2.93	1229.90	0.00	0.55	16.00	14.00
	450.00	240.00	0.00	710.60	2.93	1257.40	0.00	0.53	16.00	24.00
	560.00	230.00	0.00	605.90	2.93	1429.10	0.00	0.41	16.00	28.00
	560.00	230.00	0.00	588.20	2.93	1396.80	0.00	0.41	16.00	28.00
	550.00	230.00	0.00	685.30	2.93	1239.70	0.00	0.42	16.00	28.00
	550.00	205.00	0.00	565.85	2.93	1365.15	0.00	0.37	16.00	30.00
	450.00	200.00	0.00	565.96	2.93	1526.04	0.00	0.44	16.00	28.00
	450.00	210.00	0.00	685.30	2.93	1234.70	0.00	0.47	16.00	26.00
	450.00	205.00	0.00	565.22	2.93	1254.78	0.00	0.46	16.00	27.00
	550.00	265.00	0.00	685.30	2.93	1210.70	0.00	0.48	16.00	26.00
	420.00	205.00	0.00	521.20	2.93	1432.80	0.00	0.49	16.00	22.00
	420.00	180.00	0.00	490.30	2.93	1402.70	0.00	0.43	16.00	28.00
Mansour *et al.* (2004)	420.00	180.00	0.00	860.00	2.93	1206.00	0.00	0.43	16.00	29.00
	600.00	265.00	0.00	616.30	2.93	1337.70	0.00	0.44	16.00	27.00
	600.00	255.00	0.00	580.30	2.93	1387.70	0.00	0.43	16.00	29.00
	420.00	175.00	0.00	399.20	2.93	1692.80	0.00	0.42	16.00	30.00
	550.00	190.00	0.00	580.00	2.93	1340.00	0.00	0.35	16.00	34.00
	550.00	200.00	0.00	580.00	2.93	1468.00	0.00	0.36	16.00	33.00
	600.00	275.00	0.00	580.00	2.93	1240.00	0.00	0.46	16.00	24.00
	600.00	225.00	0.00	716.80	2.93	1179.20	0.00	0.38	16.00	33.00
	600.00	260.00	0.00	529.00	2.93	1425.00	0.00	0.43	16.00	28.00
	450.00	220.00	0.00	547.50	2.93	1383.50	0.00	0.49	16.00	24.00
	450.00	225.00	0.00	724.10	2.93	1367.90	0.00	0.50	16.00	23.00
	420.00	185.00	0.00	710.60	2.93	1209.40	0.00	0.44	16.00	27.00
	420.00	205.00	0.00	605.90	2.93	1214.10	0.00	0.49	16.00	26.00
	550.00	205.00	0.00	588.20	2.93	1307.80	0.00	0.37	16.00	34.00
	600.00	260.00	0.00	685.30	2.93	1268.70	0.00	0.43	16.00	29.00
	480.00	200.00	0.00	565.85	2.93	1327.15	0.00	0.42	16.00	30.00
	450.00	185.00	0.00	580.30	2.93	1485.70	0.00	0.41	16.00	31.00
	560.00	225.00	0.00	399.20	2.93	1554.80	0.00	0.40	16.00	31.00
	560.00	225.00	0.00	716.80	2.93	1251.20	0.00	0.40	16.00	31.00
	550.00	225.00	0.00	529.00	2.93	1563.00	0.00	0.41	16.00	31.00
	550.00	180.00	0.00	547.50	2.93	1372.50	0.00	0.33	16.00	42.00
	450.00	155.00	0.00	724.10	2.93	1323.90	0.00	0.34	16.00	42.00
	450.00	155.00	0.00	710.60	2.93	1109.40	0.00	0.34	16.00	42.00
	450.00	155.00	0.00	605.90	2.93	1290.10	0.00	0.34	16.00	42.00
	550.00	285.00	0.00	588.20	2.93	1365.80	0.00	0.52	16.00	18.00
	420.00	155.00	0.00	685.30	2.93	1349.70	0.00	0.37	16.00	34.00

Table 1 Continued

Source	Cement, *C* (kg/m^3)	Water, *W* (kg/m^3)	Super-plasticizer, *SP* (kg/m^3)	Fine aggregate, *FA* (kg/m^3)	Fineness modulus of fine aggregate, *FM*	Coarse aggregate, *CA* (kg/m^3)	Micro-silica, *MS* (kg/m^3)	*W/C* ratio	Max size of coarse agg, *MSCA* (kg/m^3)	Concrete comp strength (MPa)
Mansour *et al.* (2004)	420.00	155.00	0.00	521.20	2.93	1463.80	0.00	0.37	16.00	34.00
	420.00	185.00	0.00	490.30	2.93	1434.70	0.00	0.44	16.00	29.00
	600.00	240.00	0.00	580.99	2.93	1350.01	0.00	0.40	16.00	30.00
	600.00	260.00	0.00	580.00	2.93	1512.00	0.00	0.43	16.00	28.00
	420.00	190.00	0.00	580.00	2.93	1340.00	0.00	0.45	16.00	26.00
	550.00	240.00	0.00	716.80	2.93	1331.20	0.00	0.44	16.00	28.00
	550.00	235.00	0.00	529.00	2.93	1291.00	0.00	0.43	16.00	28.00
	600.00	350.00	0.00	547.50	2.93	1348.50	0.00	0.58	16.00	14.00
	600.00	350.00	0.00	724.10	2.93	1229.90	0.00	0.58	16.00	13.00
	600.00	340.00	0.00	710.60	2.93	1257.40	0.00	0.57	16.00	17.00
	450.00	200.00	0.00	605.90	2.93	1429.10	0.00	0.44	16.00	26.00
	450.00	200.00	0.00	588.20	2.93	1396.80	0.00	0.44	16.00	27.00
	420.00	200.00	0.00	685.30	2.93	1239.70	0.00	0.48	16.00	26.00
	420.00	180.00	0.00	565.85	2.93	1365.15	0.00	0.43	16.00	28.00
	550.00	215.00	0.00	565.96	2.93	1526.04	0.00	0.39	16.00	32.00
	600.00	265.00	0.00	685.30	2.93	1234.70	0.00	0.44	16.00	26.00
	480.00	200.00	0.00	565.22	2.93	1254.78	0.00	0.42	16.00	27.00
	450.00	175.00	0.00	685.30	2.93	1210.70	0.00	0.39	16.00	31.00
	560.00	280.00	0.00	521.20	2.93	1432.80	0.00	0.50	16.00	20.00
	560.00	235.00	0.00	490.30	2.93	1402.70	0.00	0.42	16.00	28.00
	550.00	200.00	0.00	565.22	2.93	1500.78	0.00	0.36	16.00	37.00
	550.00	220.00	0.00	685.30	2.93	1349.70	0.00	0.40	16.00	31.00
	450.00	265.00	0.00	521.20	2.93	1463.80	0.00	0.59	16.00	13.00
	450.00	165.00	0.00	490.30	2.93	1434.70	0.00	0.37	16.00	33.00
	450.00	155.00	0.00	580.99	2.93	1350.01	0.00	0.34	16.00	33.00
	550.00	195.00	0.00	580.00	2.93	1512.00	0.00	0.35	16.00	33.00
	420.00	170.00	0.00	399.00	2.93	1521.00	0.00	0.40	16.00	30.00
	420.00	180.00	0.00	580.00	2.93	1468.00	0.00	0.43	16.00	28.00
	420.00	145.00	0.00	580.00	2.93	1240.00	0.00	0.35	16.00	34.00
	600.00	200.00	0.00	580.00	2.93	1316.00	0.00	0.33	16.00	34.00
	600.00	285.00	0.00	505.00	2.93	1449.00	0.00	0.48	16.00	24.00
	420.00	165.00	0.00	861.00	2.93	1107.00	0.00	0.39	16.00	32.00
	550.00	300.00	0.00	860.00	2.93	1232.00	0.00	0.55	16.00	18.00
	550.00	200.00	0.00	616.30	2.93	1303.70	0.00	0.36	16.00	34.00
	600.00	350.00	0.00	580.30	2.93	1251.70	0.00	0.58	16.00	18.00
	600.00	350.00	0.00	399.20	2.93	1444.80	0.00	0.58	16.00	17.00
	600.00	200.00	0.00	716.80	2.93	1103.20	0.00	0.33	16.00	38.00
	450.00	155.00	0.00	534.80	2.93	1361.20	0.00	0.34	16.00	38.00
	450.00	180.00	0.00	505.00	2.93	1449.00	0.00	0.40	16.00	30.00
	420.00	160.00	0.00	861.00	2.93	1107.00	0.00	0.38	16.00	32.00
	420.00	220.00	0.00	860.00	2.93	1175.00	0.00	0.52	16.00	21.00
	550.00	255.00	0.00	616.30	2.93	1368.70	0.00	0.46	16.00	23.00
	600.00	275.00	0.00	580.30	2.93	1344.70	0.00	0.46	16.00	24.00
	480.00	250.00	0.00	399.20	2.93	1531.80	0.00	0.52	16.00	20.00
	450.00	240.00	0.00	580.00	2.93	1512.00	0.00	0.53	16.00	20.00
	560.00	295.00	0.00	580.00	2.93	1340.00	0.00	0.53	16.00	20.00
	560.00	295.00	0.00	580.00	2.93	1240.00	0.00	0.53	16.00	20.00
	550.00	200.00	0.00	716.80	2.93	1179.20	0.00	0.36	16.00	38.00
	550.00	200.00	0.00	529.00	2.93	1425.00	0.00	0.36	16.00	38.00
	450.00	165.00	0.00	547.50	2.93	1345.50	0.00	0.37	16.00	38.00
	450.00	160.00	0.00	724.10	2.93	1341.90	0.00	0.36	16.00	38.00
	450.00	160.00	0.00	710.60	2.93	1324.40	0.00	0.36	16.00	38.00

Table 1 Continued

Source	Cement, *C* (kg/m³)	Water, *W* (kg/m³)	Super-plasticizer, *SP* (kg/m³)	Fine aggregate, *FA* (kg/m³)	Fineness modulus of fine aggre-gate, *FM*	Coarse aggregate, *CA* (kg/m³)	Micro-silica, *MS* (kg/m³)	*W/C* ratio	Max size of coarse agg, *MSCA* (kg/m³)	Concrete comp strength (MPa)
	550.00	200.00	0.00	605.90	2.93	1379.10	0.00	0.36	16.00	38.00
	420.00	155.00	0.00	588.20	2.93	1336.80	0.00	0.37	16.00	38.00
	420.00	160.00	0.00	685.30	2.93	1245.70	0.00	0.38	16.00	33.00
	420.00	165.00	0.00	565.85	2.93	1526.15	0.00	0.39	16.00	32.00
	600.00	225.00	0.00	565.96	2.93	1354.04	0.00	0.38	16.00	33.00
	600.00	225.00	0.00	685.30	2.93	1362.70	0.00	0.38	16.00	33.00
	420.00	155.00	0.00	547.50	2.93	1345.50	0.00	0.37	16.00	33.00
	550.00	195.00	0.00	724.10	2.93	1341.90	0.00	0.35	16.00	34.00
	550.00	195.00	0.00	710.60	2.93	1324.40	0.00	0.35	16.00	34.00
	600.00	215.00	0.00	605.90	2.93	1379.10	0.00	0.36	16.00	34.00
	600.00	215.00	0.00	588.20	2.93	1336.80	0.00	0.36	16.00	34.00
	600.00	215.00	0.00	685.30	2.93	1245.70	0.00	0.36	16.00	34.00
	450.00	165.00	0.00	521.20	2.93	1570.80	0.00	0.37	16.00	34.00
	450.00	165.00	0.00	490.30	2.93	1429.70	0.00	0.37	16.00	35.00
	420.00	160.00	0.00	580.99	2.93	1467.01	0.00	0.38	16.00	35.00
	420.00	160.00	0.00	580.00	2.93	1405.00	0.00	0.38	16.00	35.00
	550.00	210.00	0.00	399.00	2.93	1526.00	0.00	0.38	16.00	35.00
	600.00	230.00	0.00	580.00	2.93	1351.00	0.00	0.38	16.00	35.00
	480.00	180.00	0.00	580.00	2.93	1512.00	0.00	0.38	16.00	35.00
	450.00	175.00	0.00	580.00	2.93	1340.00	0.00	0.39	16.00	35.00
	560.00	200.00	0.00	505.00	2.93	1315.00	0.00	0.36	16.00	36.00
	560.00	290.00	0.00	861.00	2.93	1035.00	0.00	0.52	16.00	20.00
	550.00	285.00	0.00	580.99	2.93	1312.01	0.00	0.52	16.00	20.00
	450.00	240.00	0.00	399.00	2.93	1636.00	0.00	0.53	16.00	21.00
	550.00	235.00	0.00	580.00	2.93	1345.00	0.00	0.43	16.00	29.00
Mansour *et al.* (2004)	420.00	165.00	0.00	685.30	2.93	1207.70	0.00	0.39	16.00	33.00
	600.00	265.00	0.00	490.30	2.93	1544.70	0.00	0.44	16.00	29.00
	420.00	210.00	0.00	580.00	2.93	1345.00	0.00	0.50	16.00	23.00
	550.00	270.00	0.00	399.00	2.93	1532.00	0.00	0.49	16.00	23.00
	550.00	270.00	0.00	580.00	2.93	1512.00	0.00	0.49	16.00	23.00
	600.00	295.00	0.00	580.00	2.93	1340.00	0.00	0.49	16.00	23.00
	600.00	300.00	0.00	580.00	2.93	1468.00	0.00	0.50	16.00	23.00
	600.00	210.00	0.00	505.00	2.93	1315.00	0.00	0.35	16.00	37.00
	450.00	165.00	0.00	861.00	2.93	1035.00	0.00	0.37	16.00	37.00
	450.00	160.00	0.00	860.00	2.93	1094.00	0.00	0.36	16.00	37.00
	420.00	165.00	0.00	616.30	2.93	1351.70	0.00	0.39	16.00	32.00
	420.00	165.00	0.00	580.30	2.93	1511.70	0.00	0.39	16.00	32.00
	550.00	210.00	0.00	399.20	2.93	1520.80	0.00	0.38	16.00	32.00
	600.00	225.00	0.00	716.80	2.93	1115.20	0.00	0.38	16.00	32.00
	480.00	180.00	0.00	529.00	2.93	1315.00	0.00	0.38	16.00	32.00
	450.00	175.00	0.00	547.50	2.93	1272.50	0.00	0.39	16.00	32.00
	560.00	210.00	0.00	724.10	2.93	1171.90	0.00	0.38	16.00	32.00
	560.00	210.00	0.00	710.60	2.93	1243.40	0.00	0.38	16.00	32.00
	550.00	260.00	0.00	399.20	2.93	1568.80	0.00	0.47	16.00	28.00
	550.00	260.00	0.00	580.00	2.93	1455.00	0.00	0.47	16.00	28.00
	450.00	245.00	0.00	580.00	2.93	1405.00	0.00	0.54	16.00	20.00
	450.00	245.00	0.00	580.00	2.93	1345.00	0.00	0.54	16.00	20.00
	450.00	245.00	0.00	716.80	2.93	1214.20	0.00	0.54	16.00	20.00
	550.00	290.00	0.00	529.00	2.93	1563.00	0.00	0.53	16.00	20.00
	420.00	230.00	0.00	547.50	2.93	1372.50	0.00	0.55	16.00	20.00
	420.00	230.00	0.00	724.10	2.93	1107.90	0.00	0.55	16.00	20.00
	420.00	235.00	0.00	710.60	2.93	1133.40	0.00	0.56	16.00	20.00

Table 1 Continued

Source	Cement, *C* (kg/m^3)	Water, *W* (kg/m^3)	Super-plasticizer, *SP* (kg/m^3)	Fine aggregate, *FA* (kg/m^3)	Fineness modulus of fine aggregate, *FM*	Coarse aggregate, *CA* (kg/m^3)	Micro-silica, *MS* (kg/m^3)	*W/C* ratio	Max size of coarse agg, *MSCA* (kg/m^3)	Concrete comp strength (MPa)
Mansour *et al.* (2004)	600.00	325.00	0.00	605.90	2.93	1214.10	0.00	0.54	16.00	20.00
	600.00	325.00	0.00	588.20	2.93	1307.80	0.00	0.54	16.00	20.00
	600.00	300.00	0.00	685.30	2.93	1268.70	0.00	0.50	16.00	25.00
	480.00	260.00	0.00	565.85	2.93	1402.15	0.00	0.54	16.00	20.00
	450.00	245.00	0.00	565.96	2.93	1469.04	0.00	0.54	16.00	20.00
	560.00	275.00	0.00	685.30	2.93	1299.70	0.00	0.49	16.00	25.00
	560.00	300.00	0.00	565.22	2.93	1359.78	0.00	0.54	16.00	19.00
	550.00	290.00	0.00	685.30	2.93	1245.70	0.00	0.53	16.00	20.00
	600.00	305.00	0.00	521.20	2.93	1570.80	0.00	0.51	16.00	23.00
	600.00	320.00	0.00	490.30	2.93	1429.70	0.00	0.53	16.00	21.00
	600.00	320.00	0.00	580.99	2.93	1454.01	0.00	0.53	16.00	21.00
	450.00	245.00	0.00	580.00	2.93	1405.00	0.00	0.54	16.00	20.00
	450.00	240.00	0.00	399.00	2.93	1526.00	0.00	0.53	16.00	22.00
	420.00	205.00	0.00	580.00	2.93	1351.00	0.00	0.49	16.00	25.00
Sobhani *et al.* (2010)	350.00	95.20	0.00	575.90	3.20	1273.00	0.00	0.27	16.00	61.10
	350.00	98.50	0.00	558.20	3.20	1325.40	0.00	0.28	16.00	54.00
	339.50	97.70	0.00	655.30	3.20	1273.00	0.00	0.28	16.00	65.70
	339.50	97.60	0.00	535.00	3.20	1247.00	0.00	0.28	16.00	62.20
	336.00	97.60	0.00	535.00	3.20	1247.00	0.00	0.28	16.00	54.50
	332.50	97.70	0.00	655.30	3.20	1273.00	0.00	0.28	16.00	63.10
	329.00	97.60	0.00	535.00	3.20	1247.00	0.00	0.28	16.00	52.20
	325.50	97.70	0.00	655.30	3.20	1273.00	0.00	0.28	16.00	64.10
	410.00	117.80	0.00	491.20	3.20	1273.00	0.00	0.29	16.00	59.90
	350.00	100.90	0.00	460.30	3.20	1419.80	0.00	0.29	16.00	61.90
	350.00	102.60	0.00	535.00	3.20	1247.00	0.00	0.29	16.00	64.20
	332.50	105.60	0.00	535.00	3.20	1247.00	17.50	0.30	16.00	62.20
	380.00	118.10	0.00	354.20	3.20	1440.60	0.00	0.31	16.00	60.50
	350.00	107.60	0.00	535.00	3.20	1247.00	0.00	0.31	16.00	61.50
	325.50	107.80	0.00	535.00	3.20	1247.00	24.50	0.31	16.00	65.00
	343.00	107.60	0.00	535.00	3.20	1247.00	0.00	0.31	16.00	61.20
	320.00	97.70	0.00	671.80	3.20	1247.00	0.00	0.31	16.00	63.20
	346.00	115.60	0.00	484.00	3.20	1289.00	27.30	0.31	16.00	76.70
	380.00	121.10	0.00	502.50	3.20	1325.40	0.00	0.32	16.00	67.40
	320.00	102.20	0.00	679.10	3.20	1259.70	0.00	0.32	16.00	62.80
	320.00	103.20	0.00	665.60	3.20	1234.20	0.00	0.32	16.00	60.30
	350.00	120.40	0.00	526.20	3.20	1325.40	0.00	0.34	16.00	63.50
	350.00	119.00	0.00	710.60	3.20	1121.50	0.00	0.34	16.00	59.60
	350.00	120.00	0.00	623.30	3.20	1208.70	0.00	0.34	16.00	61.10
	252.60	95.00	0.00	828.00	3.20	1206.00	19.60	0.35	16.00	66.70
	345.20	129.90	0.00	482.00	3.20	1282.00	27.10	0.35	16.00	71.20
	375.00	134.00	0.00	1300.00	3.20	600.00	0.00	0.36	16.00	64.00
	332.50	129.90	0.00	509.80	3.20	1325.40	17.50	0.37	16.00	61.40
	343.00	136.90	0.00	480.00	3.20	1278.00	27.00	0.37	16.00	71.20
	252.60	103.40	0.00	836.00	3.20	1063.00	19.60	0.38	16.00	62.70
	258.90	98.40	0.00	835.00	3.20	1083.00	0.00	0.38	16.00	55.00
	350.00	139.70	0.00	591.30	3.20	1145.50	0.00	0.40	16.00	58.30

Table 2 Percentile of parameters for the data of concrete compressive strength (265 data points)

S. No.	Parameter	Percentile values				
		0%	25%	50%	75%	100%
	Basic parameters					
1.	Cement, *C* (kg/m^3)	252.6	420.0	480.0	560.0	600.0
2.	Fine aggregate, *FA* (kg/m^3)	354.2	535.0	580.0	685.3	1325.0
3.	Coarse aggregate, *CA* (kg/m^3)	495.0	1245.7	1341.9	1425.0	1692.8
4.	Micro-silica, *MS* (kg/m^3)	0.0	0.0	0.0	0.0	50.0
5.	Water, *W* (kg/m^3)	95.0	165.0	210.0	255.0	358.0
6.	Superplasticizer, *SP* (kg/m^3)	0.0	0.0	0.0	0.0	22.0
7.	Maximum size of coarse aggregate, *MSCA* (mm)	12	16	16	16	16
8.	Fineness modulus of fine aggregate, *FM*	2.76	2.93	2.93	2.93	3.32
	Additional non-dimensional parameters					
1.	*FA/C*	0.67	1.03	1.24	1.44	3.47
2.	*CA/C*	0.90	2.32	2.73	3.21	4.77
3.	*MS/C*	0.00	0.00	0.00	0.00	0.09
4.	*W/C*	0.25	0.37	0.42	0.49	0.60
5.	*SP/C*	0.00	0.00	0.00	0.00	0.04
6.	*MSCA/MSFA*	6.00	8.00	8.00	8.00	8.00

1998, Castro 1987, Mansour *et al.* 2004, Sobhani *et al.* 2010) for the determination of compressive strength of normal and high strength concrete (NSC, HSC) mixes using cylinders (150 × 300 mm). Portland cement Type-I was used in the data reported in Table 1. The data consists of eight input parameters viz. the quantities of cement, fine aggregate, coarse aggregate, micro-silica, water and super-plasticizer, maximum size of coarse aggregate, fineness modulus of fine aggregate and concrete compressive strength of concrete, an output parameter. The first, second and third quartile values of different basic as well as the non-dimensional variables are given in Table 2. The values of variables corresponding to the 0% and 100% percentile given in the table are obviously the lower and upper limits of variables.

4. Regression models

A number of research efforts have concentrated on using multivariable regression models to improve the accuracy of predictions. Statistical models have the attraction that once fitted they can be used to perform predictions much more quickly than other modeling techniques and are correspondingly simpler to implement in software.

Popovics augments Abrams model, a widely accepted equation relating the water cement ratio *w/c* of concrete to its strength with additional variables such as slump and uses least square regression to determine equation coefficients (Popovics and Ujhelyi 2008). Apart from its speed, statistical modeling has the advantage over other techniques in that it is mathematically rigorous and can be used to define confidence interval for the predictions. This is especially true when comparing statistical modeling with artificial intelligence techniques. Statistical analysis can also provide

Table 3 Regression models for predicting compressive strength of concrete

Reference	Compressive strength prediction model
Abram's law (Popovics and Ujhelyi 2008)	$f'_c = \frac{b_0}{b_1^{W/C}}$
Popovics and Ujhelyi (2008)	$f'_c = b_o + b_1 W/C$
Lyse (1932)	$f'_c = b_0 + \frac{b_1}{W/C}$
Multivariable linear model: *M*1	$f'_c = 49.36 + 0.029C - 0.012FA - 0.008CA + 0.164MS - 0.149W$ $-0.409SP - 6.309MSCA + 40.744FM$
Multilinear power model: *M*2	$f'_c = C^{1.11} FA^{0.180} CA^{0.304} MS^{-0.014} W^{-1.343} SP^{-0.0001} MSCA^{0.320} FM^{1.452}$
Popovics and Ujhelyi (2008): *M*3	$f'_c = b_o + b_1 W/C + b_2 CA + b_3 FA + b_4 C$ $f'_c = 16635 - 127.628 W/C - 0.0313CA - 0.0292FA - 0.0365C$

f'_c = compressive strength of concrete in MPa; *W*, *C*, *CA*, *FA* and *MS* = quantities of water, cement, coarse aggregate, fine aggregate and micro-silica in kg/m^3 respectively; *MSCA* = Maximum size of coarse aggregate in mm; *FM* = Fineness modulus of fine aggregate; *W/C* = water-cement ratio; and b_0, b_1, b_2, b_3, b_4 = model parameters.

insight into the key factors influencing 28 days compressive strength through correlation analysis. For these reasons statistical analysis was the chosen technique for the strength prediction in this study.

A large number of regression models, mostly empirical and some semi-empirical, based on mechanics, are available for the prediction of 28-day compressive strength of concrete. The most popular regression equation used in the prediction of compressive strength of a given mix proportion are given in Table 3. Multivariable linear and power models (*M*1 and *M*2) developed for the data used in this study are also given in the table. For the sake of comparison, the parameters of model *M*3 of Popovics and Ujhelyi (2008) have also been determined by regression using the method of least squares for the data used in this study.

5. Neural network model

Because strengthening of concrete is a complex non-linear process dependent on many variables, it is a problem well suited to the artificial intelligence concept known as artificial neural networks (ANNs). Much of the current research into concrete strength prediction recognizes that neural nets are appropriate for the problem.

In the last few years, artificial neural networks (ANN) technology, a sub-field of artificial intelligence, is being used to solve a wide variety of problems in civil engineering applications (Topcu and Saridemir 2007, 2008a, 2008b, Pala *et al.* 2007, Adhikary and Mutsuyoshi 2006). The most important property of ANN in civil engineering problems is their capability of learning directly from examples.

The manner in which the data is presented for training is the most important aspect of the neural network method. Often this can be done in more than one way; the best configuration being determined by trial-and-error. It can also be beneficial to examine the input/output patterns or data sets that the network finds difficult to learn. This enables a comparison of the performance of the neural network model for these different combinations of data. In order to map the causal relationship related to the compressive strength of concrete, two separate input/output schemes (called Model-$A1$ and Model-$A2$) were employed, where the first took the input of raw causal parameters while the second utilized their non-dimensional groupings. This was done in order to see if the use of the grouped variables produced better results. The Model-$A1$ thus takes the input in the form of causative factors namely, C, FA, CA, MS, W, SP, $MSCA$ and FM and yields the output, which is the concrete compressive strength, f'_c

$$\text{Model-}A1\text{: } f'_c = f(C,\ FA,\ CA,\ MS,\ W,\ SP,\ MSCA \text{ and } FM) \tag{1}$$

where f'_c = compressive strength of concrete in MPa; W, C, CA, FA and MS = quantities of water, cement, coarse aggregate, fine aggregate and micro-silica in kg/m^3 respectively; $MSCA$ = maximum size of coarse aggregate in mm and FM = fineness modulus of fine aggregate.

The matrix of dimensions for the variables involved is

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & -3 & -3 & -3 & -3 & -3 & -3 & 1 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The columns of the above matrix correspond to the variables in the order in which they appear in Eq. (1) and the rows of the matrix correspond to the three fundamental dimensions viz. M (mass), L (Length) and T (Time). Though the number of fundamental dimensions involved in the model is three but the rank of the above dimensional matrix is 1, thus according to the Buckingham-PI theorem, the number of dimensionless parameters required for modeling would be 8 ($9 - 1 = 8$). The independent PI terms obtained from the nullity theorem are: FA/C, CA/C, MS/C, W/C, SP/C, $MSCA/MSFA$ and FM whereas the corresponding dimensionless output is $f_c'/CgMSCA$, where g is the acceleration due to gravity. The Model $A2$ employing these dimensionless variables is thus given by

$$\text{Model-}A2\text{: } \frac{f'_c}{CgMSCA} = g\left(\frac{FA}{C},\ \frac{CA}{C},\ \frac{MS}{C},\ \frac{W}{C},\ \frac{SP}{C},\ \frac{MSCA}{MSFA},\ FM\right) \tag{2}$$

where $MSFA$ is the maximum size of fine aggregate. The current study used the data described above (265 data points) for the prediction of compressive strength of concrete. The training of the above two models was done using 67% of the data (178 data points) selected randomly. Validation and testing of the proposed models was made with the help of the remaining 33% of observations (87 data points), which were not involved in the derivation of the model.

Three neuron models namely, tansig, logsig and purelin, have been used in the architecture of the network with the back propagation algorithm. In the back propagation algorithm, the feed-forward (FFBP), cascade-forward (CFBP) and Elman back propagation (EBP) type network were considered. Levenberg-Marquardt nonlinear least square fitting method (1963) was employed for training the networks. Each input is weighted with an appropriate weight and the sum of the weighted inputs and the bias forms the input to the transfer function. The neurons employed use the following differentiable transfer function to generate their output

$$y_j = f \cdot \left(\sum_i W_{ij} x_i + \phi_j \right) = \frac{1}{1 + e^{-\left(\sum_i W_{ij} x_i + \phi_j \right)}} \tag{3}$$

Tan-sigmoid transfer function

$$y_j = f \cdot \left(\sum_i W_{ij} x_i + \phi_j \right) = \frac{2}{1 + e^{-2\left(\sum_i W_{ij} x_i + \phi_j \right)}} - 1 \tag{4}$$

Linear transfer function

$$y_j = f \cdot \left(\sum_i W_{ij} x_i + \phi_j \right) = \sum_i W_{ij} x_i + \phi_j \tag{5}$$

The weight, *w*, and biases, *f*, of these equations are determined in such a way as to minimize the energy function. The sigmoid transfer functions generate output between 0 and 1 or -1 and +1 as the neuron's net input goes from negative to positive infinity depending upon the use of log or tan sigmoid. When the last layer of a multilayer network has sigmoid neurons (log or tan) then the output of the network is limited to a small range, whereas, the output of linear output neurons can take on any value.

The optimal architecture was determined by varying the number of hidden neurons. The optimal configuration was based upon minimizing the difference between the neural network predicted value and the desired output. In general, as the number of neurons in the layer is increased, the prediction capability of the network increases in beginning and then becomes stationary.

The performance of all neural network model configurations was based on the mean percent error (MPE), mean absolute deviation (MAD), root mean square error (RMSE), correlation coefficient (CC) and coefficient of determination, R^2, of the linear regression line between the predicted values from the neural network model and the desired outputs.

The training of the neural network models was stopped when either the acceptable level of error was achieved or when the number of iterations exceeded a prescribed maximum. The neural network model configuration that minimized the MAE and RMSE and optimized the R^2 was selected as the optimum and the whole analysis was repeated several times.

6. Sensitivity analysis

Sensitivity tests were conducted to determine the relative significance of each of the independent parameters (input neurons) on the compressive strength of concrete (output) in both of the models given by Eqs. (1) and (2). In the sensitivity analysis, each input neuron was in turn eliminated from the model and its influence on the prediction of compressive strength of concrete was evaluated in terms of the MPE, MAD, RMSE, CC and R^2 criteria. The network architecture of the problem considered in the present sensitivity analysis consists of one hidden layer with 12 neurons and the value of epochs has been taken as 100.

The results in Table 4 show that for Model-*A*1, the variables in the order of decreasing level of sensitivity are: *W*, *C*, *MSCA*, *FM*, *MS*, *CA*, *SP* and *FA*. It is thus seen that the last three parameters

Table 4 Sensitivity analysis of Model *A*1 with feed-forward back propagation for different sets of input variables

Input variables	MPE	MAD	RMSE	CC	R^2
All (Eq. (1))	-1.09	7.13	3.18	0.98	0.96
No *C*	3.50	12.60	4.62	0.96	0.92
No *FA*	0.53	6.64	2.97	0.98	0.97
No *CA*	1.10	6.88	3.17	0.98	0.96
No *MS*	2.20	7.54	3.43	0.98	0.95
No *W*	6.22	17.73	5.88	0.93	0.87
No *SP*	2.15	7.55	3.12	0.98	0.96
No *MSCA*	1.00	7.86	3.77	0.97	0.94
No *FM*	0.57	8.63	3.74	0.97	0.95

Note: MPE, mean percent error; MAD, mean absolute deviation; RMSE, root mean square error; CC, correlation coefficient; R^2, coefficient of determination.

Table 5 Sensitivity analysis of Model *A*2 with feed-forward back propagation

Input variables	MPE	MAD	RMSE	CC	R^2
All (Eq. (2))	-1.11	7.62	0.04	0.99	0.98
No *FA/C*	1.20	9.05	0.05	0.99	0.98
No *CA/C*	3.46	12.30	0.06	0.98	0.97
No *MS/C*	1.05	8.21	0.05	0.99	0.98
No *W/C*	6.81	18.38	0.08	0.97	0.94
No *SP/C*	1.45	7.78	0.05	0.99	0.98
No *MSCA/MSFA*	1.26	8.11	0.05	0.99	0.97
No *FM*	2.71	11.10	0.07	0.98	0.96

have least significant effect when taken independently. The elimination of *W* is found to have the most significant effect as it reduces the value of R^2 from 0.96 to 0.87. The earlier models that ignored some of the useful parameters appeared to be less accurate.

Table 5 gives the results of sensitivity analysis for Model-*A*2. The variables in the order of decreasing level of sensitivity are: *W/C*, *FM*, *MSCA/MSFA*, *CA/C*, *FA/C*, *MS/C* and *SP/C*. The results presented in Tables 4 and 5 indicate that the models incorporating only limited number of the available parameters like *C*, *W/C*, *FA* and *CA* are not good enough for achieving the desired accuracy and reliability in the estimation of compressive strength of concrete. These findings are consistent with existing understanding of the relative importance of the various parameters on compressive strength of concrete.

Model-*A*2 using the dimensionless variables is found to be better than Model-*A*1 involving raw parameters. The sensitivity study of Model-*A*2, gives the impression that elimination of some of the variables has only marginal influence on the resulting concrete compressive strength. However considering the limitations and uncertainties in the data a full-fledged network involving all input variables would be desirable.

7. Analysis and interpretation of test results

The preprocessing of the network training set was performed by normalizing the inputs and targets so that their mean is zero and standard deviations as unity. Similarly, all weights and bias values were initialized to random numbers. While the numbers of input and output nodes are fixed, the hidden nodes in the case of FFBP were subjected to trials and the one producing the most accurate results (in terms of the CC) was selected. The optimization of the training procedure automatically fixes the hidden nodes in the case of the CFBP. The training of these networks was stopped after reaching the minimum mean square error between the network yield and true output over all the training patterns.

The information on number of nodes required to achieve minimum error taken in the case of each training scheme used (i.e., FFBP, CFBP and EBP) is shown in Table 6 for Model-*A*1 and *A*2. As a matter of general information, which is not of real significance in this study, it can be seen that the cascade correlation algorithm, designed for efficient training, trained the network with fewer epochs

Table 6 Network architecture

Model	Algorithm	Network configuration			Learning rate	Momentum function
		I	*H*	*O*		
Model *A*1	FFBP	8	12	1	0.5	0.7
	CFBP	8	12	1	0.5	0.7
	EBP	8	12	1	0.5	0.7
Model *A*2	FFBP	7	12	1	0.5	0.7
	CFBP	7	16	1	0.5	0.7
	EBP	7	20	1	0.5	0.7

Note: *I*, *H* and *O* indicate number of input, hidden and output nodes, respectively; FFBP, feed-back propagation; CFBP, cascade-forward back propagation; RBF, radial basis function; EBP, Elman back propagation network.

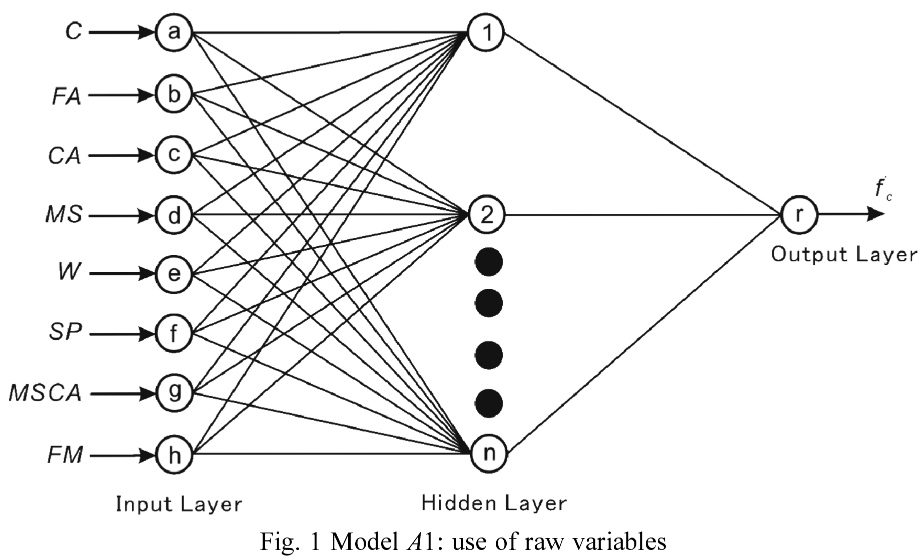

Fig. 1 Model *A*1: use of raw variables

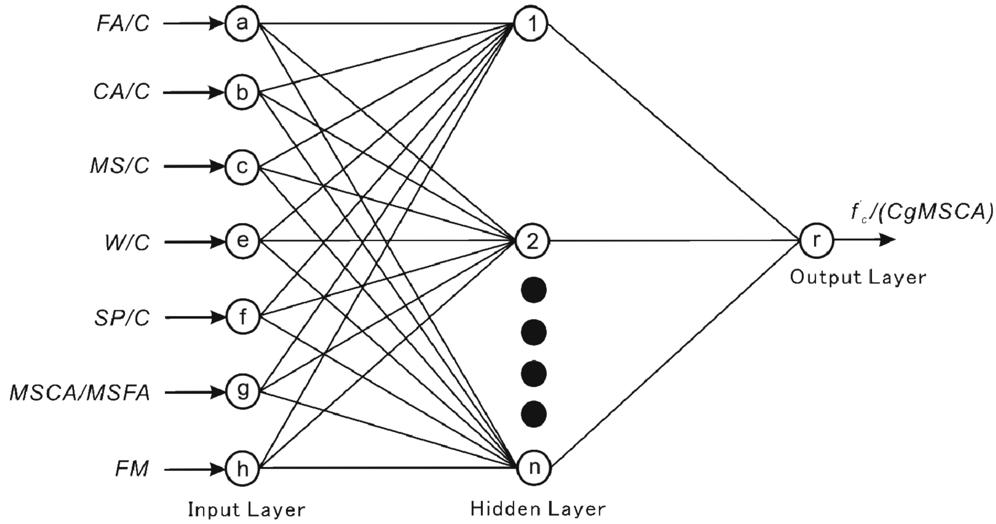

Fig. 2 Model *A*2: use of non-dimensional variables

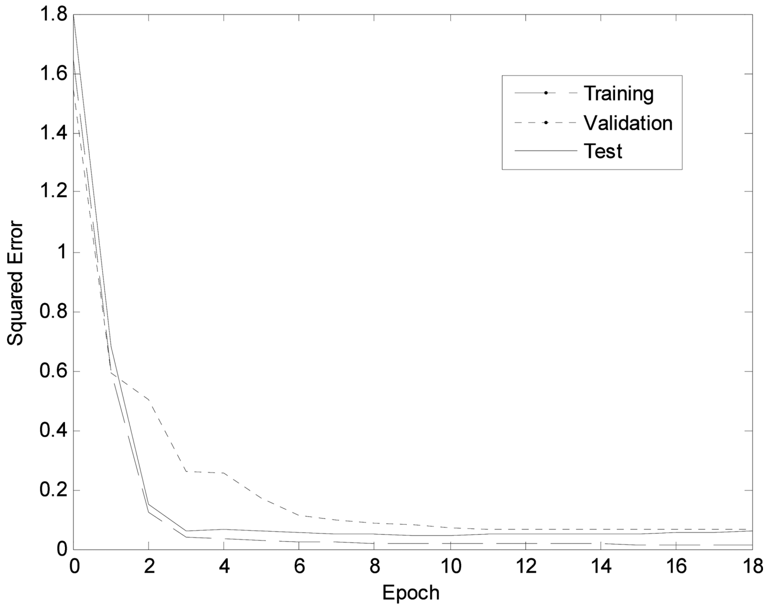

Fig. 3 Epochs versus squared error of raw variables (Model-*A*1) by back propagation

than the FFBP network.

The network architecture of the two models, given by Eqs. (1) and (2), is given in Figs. 1 and 2 respectively for BP training scheme. The error estimation parameters (MPE, MAD, RMSE, CC and R^2), on the basis of which the performance of a model is assessed, are already given in Tables 4 and 5.

The training and validation of the two models is shown in Figs. 3 and 4. The trained values of connecting weights and bias for the two models are given in Tables 7 and 8 obtained from FFBP training scheme.

The error estimates for the two ANN models and three regression models are summarized in

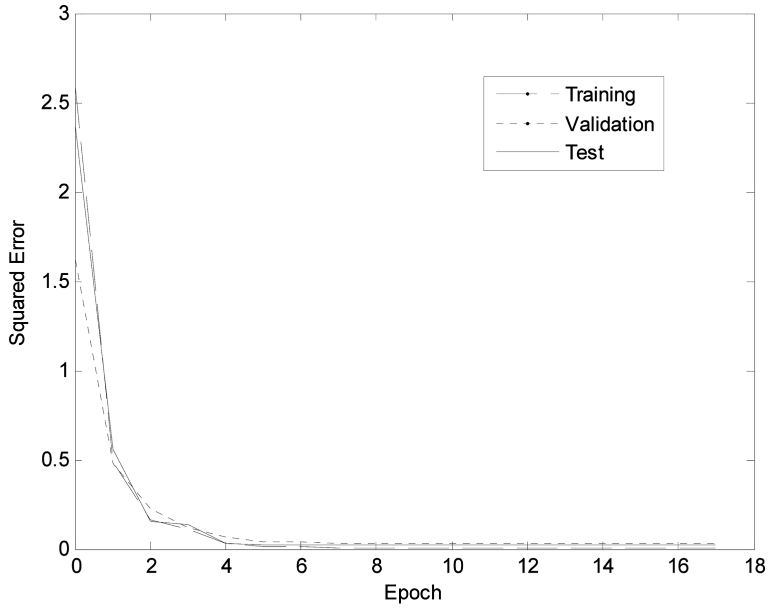

Fig. 4 Epochs versus squared error of grouped variables (Model-*A*2) by back propagation

Table 7 Connection weights and biases for Model-*A*1 (refer to Fig. 1)
(output bias = 1.4101 and *R* = 0.982)

No. of neurons	Input weights								Output weights (*r*)	Input biases
	C	*FA*	*CA*	*MS*	*W*	*SP*	*MSCA*	*FM*		
1	-0.557	0.133	1.493	1.152	0.799	1.532	1.413	0.297	-0.063	2.347
2	0.212	0.023	-0.063	1.173	1.216	-0.354	-0.531	0.289	0.883	-2.382
3	-0.329	1.363	0.208	-0.497	-0.451	-0.116	1.101	-0.537	-0.004	2.701
4	0.980	-0.955	-0.188	-0.544	-2.002	0.480	-0.453	0.977	-0.062	0.282
5	0.551	-0.117	0.972	-0.332	1.134	0.931	1.202	0.548	-0.053	0.129
6	0.225	-1.493	0.268	2.220	1.266	1.410	0.094	-0.278	0.597	0.464
7	0.291	0.829	-0.124	-0.868	0.908	0.140	-0.842	0.808	-0.370	0.506
8	-0.844	-0.140	-0.273	0.551	0.540	-0.730	0.541	0.201	-0.225	-0.886
9	-0.586	1.961	-0.550	-1.053	1.437	0.542	0.632	0.028	0.374	0.968
10	-0.864	-0.306	0.147	0.301	-0.517	-0.291	0.189	-0.254	-1.742	1.437
11	-1.720	-0.311	-1.147	1.776	-0.008	0.780	0.148	0.927	0.058	1.415
12	-0.796	0.764	-0.213	-0.510	-0.179	0.389	-0.859	0.042	0.260	2.548

Table 9. Besides the five error estimates considered above for the ANN models, two additional estimates (viz. percent data for error within 10% range and percentage of error enveloping 90% of the data) for judging the performance of models have been considered. The histograms of error in the prediction of the compressive strength of concrete for Model-*A*1 and Model-*A*2 are plotted in Fig. 5. The error in the three regression models of Kheder *et al.* (2003) is also plotted in Fig. 5. The percentage error in the prediction of the concrete compressive strength for different data sets is

Table 8 Connection weights and biases for Model-$A2$ (refer to Fig. 2)
(output bias = 0.4913 and R = 0.992)

No. of neurons	Input weights							Output weights (r)	Input biases
	FA/C	*CA/C*	*MS/C*	*W/C*	*SP/C*	*MSCA/ MSFA*	*FM*		
1	1.018	-0.063	-0.342	0.284	-0.487	0.330	0.080	-0.923	-2.408
2	-2.674	-0.280	-1.133	-0.903	2.368	-0.856	0.198	1.051	-1.801
3	-0.121	-1.263	-2.192	-2.422	-1.175	0.175	-0.187	0.002	-1.484
4	-0.899	-0.259	0.686	1.925	-0.023	-0.949	0.011	0.173	-0.574
5	-0.777	-1.813	-1.028	-0.603	-0.913	-0.252	1.656	-0.419	-2.217
6	-0.367	-0.690	-0.225	0.184	-0.031	-0.458	0.814	-1.013	1.361
7	0.529	1.923	0.032	-0.694	1.231	-0.992	0.242	-0.373	-1.267
8	-0.142	1.463	-0.260	-0.902	-0.906	0.558	0.641	0.238	0.252
9	0.294	0.791	0.348	0.224	-0.260	-1.309	-1.751	-0.445	2.208
10	-0.098	-0.279	-0.089	0.177	0.248	0.246	0.361	-0.657	-2.682
11	-0.716	-0.005	-0.267	-0.079	0.108	-0.578	0.450	0.640	2.122
12	-0.173	1.157	0.244	-0.324	1.113	-0.228	0.114	-1.325	2.821

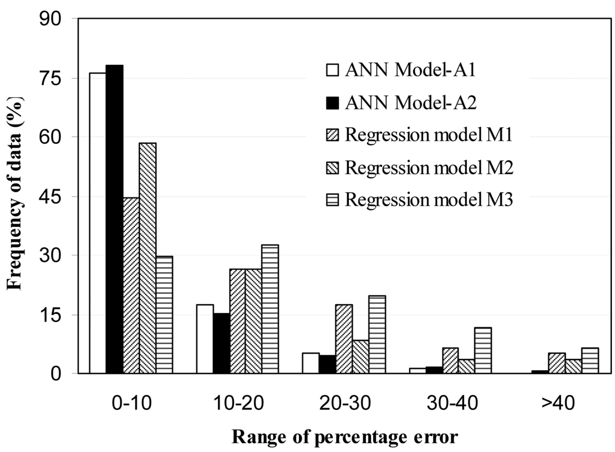

Fig. 5 Histogram of percentage error for different models

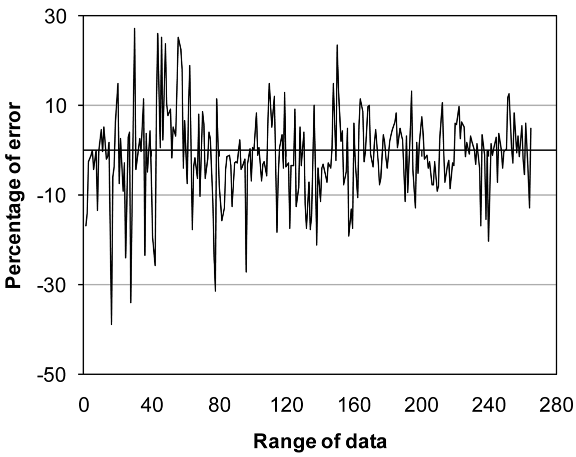

Fig. 6 Percentage error in prediction of compressive strength by Model-$A1$ for individual data points

plotted in Figs. 6 and 7 for Model-$A1$ and Model-$A2$ respectively. The predicted value of the compressive strength of concrete has been plotted against its observed value in Figs. 8 and 9 for the Model-$A1$ and Model-$A2$ respectively.

Among the three regression models viz. $M1$ to $M3$, the model $M2$ is the best with a mean error of 11.4% (Table 9). Whereas the mean errors in neural network Model-$A1$ and Model-$A2$ are only 7.13 and 7.62%, respectively. It is observed from Fig. 5 and Table 9 that ANN Model-$A2$ is slightly better than Model-$A1$. A comparison of ANN Model-$A2$ with the best regression model $M2$ shows that more than 78.1% of the data has error less than 10% for Model-$A2$ (Table 9) whereas, only 58.5% of the data has the same percentage of error for the best regression model. It was also

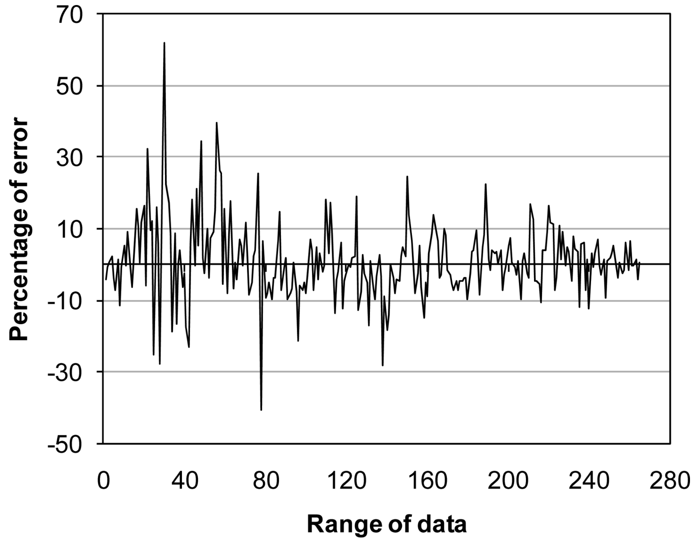

Fig. 7 Percentage error in prediction of compressive strength by Model-$A2$ for individual data points

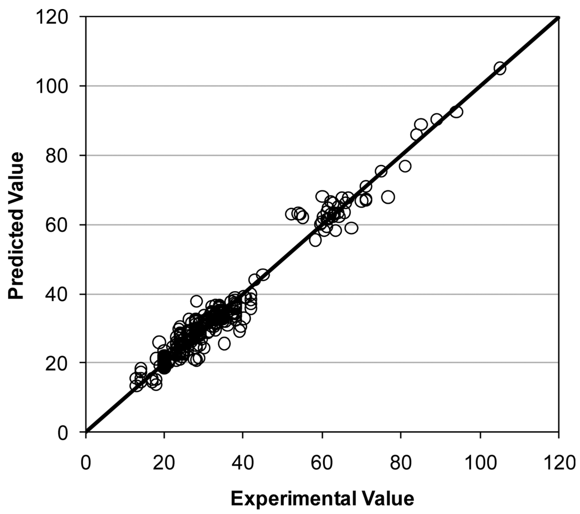

Fig. 8 Observed versus predicted compressive strength for Model-$A1$

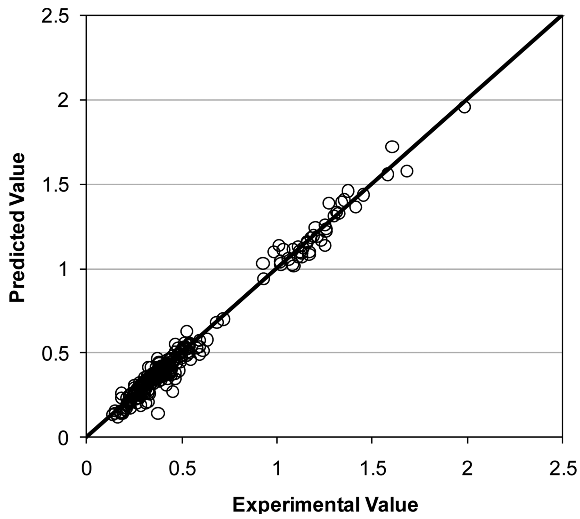

Fig. 9 Observed versus predicted compressive strength for Model-$A2$

observed from Fig. 5 and Table 9 that for about 85% of the data, the percentage error is less than 13.9% for the two ANN models, whereas the percentage error in the best regression based model $M2$ for the same percentage of data is about 20.1%. This clearly indicates the supremacy of the neural network models over the regression models.

It is observed that the use of non-dimensional grouped variables (i.e., Model-$A2$) may be more beneficial than that of the raw variables as input (i.e., Model-$A1$), provided an appropriate training scheme is chosen. The most suitable network, FFBP Model-$A2$, has the highest CC = 0.99 and R^2 = 0.98; lowest RMSE = 0.04; and the values of MPE and MAD are comparable with Model-$A1$. Whereas

Table 9 Error estimates for different models

Parameter for error estimate	ANN Model *A*1	ANN Model *A*2	Regression model *M*1	Regression model *M*2	Regression model *M*3
Mean percent error (MPE)	-1.09	-1.11	3.39	1.27	-3.77
Mean absolute deviation in percent (MAD)	7.13	7.62	15.69	11.39	18.37
Root mean square error (RMSE)	3.08	0.04	7.00	6.37	9.59
Coefficient of correlation (CC)	0.98	0.99	0.90	0.92	0.80
Coefficient of determination, R^2	0.96	0.98	0.81	0.84	0.64
Percent data for error within 10%	76.2	78.1	44.5	58.5	29.8
Percentage error enveloping 85% data	13.1	13.9	27.9	20.1	32.6

the values of these parameters for the best regression model *M*2 are: CC = 0.92, R^2 = 0.84, MPE = 1.27, MAD = 11.39 and RMSE = 6.37, which indicates the supremacy of ANN models. The ANN models featured small RMSE during training; however, the value was slightly higher during validation. The models showed consistently good correlation throughout the training and testing.

The network configuration (FFBP Model-*A*2) along with corresponding weight and bias matrix given in Table 7 is thus recommended for general use in order to predict the compressive strength of concrete.

8. Conclusions

A generalized model for predicting the compressive strength of concrete using neural network has been developed. The network predictions were generally more satisfactory than those given by traditional regression equations because of low errors and high correlation coefficients. Predictions based on grouped dimensionless form of the data (*FA/C*, *CA/C*, *MS/C*, *W/C*, *SP/C*, *MSCA/MSFA* and *FM*) were better than those based on the raw data (*C*, *FA*, *CA*, *MS*, *W*, *SP*, *MSCA* and *FM*). The neural network with one hidden layer was selected as the optimum network to predict the compressive strength of concrete. The network configuration of Model-*A*2 with FFBP is recommended for general use in order to predict the compressive strength of concrete. On the basis of sensitivity analysis, it is observed that the water, *W*, cement, *C*, maximum size of coarse aggregate, *MSCA*, fineness modulus, *FM* and micro-silica, *MS* are the five most significant parameters for the prediction of compressive strength of concrete. From the study of sensitivity of Model-*A*1 and Model-*A*2 as well as keeping in view the variability in the outcome resulting from application of different analytical schemes, it is felt that the network which requires all input quantities may be followed for generality. The neural network model is far better than the regression models in the prediction of the compressive strength of concrete.

In view of the variability in the outcome resulting from application of different analytical schemes, it is felt that the network which requires all input quantities may be followed for generality.

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