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Critical buckling loads of carbon nanotube embedded in Kerr's medium

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Abstract. In this article, the critical buckling of a single-walled carbon nanotube (SWCNT) embedded in Kerr's medium is studied. Based on the nonlocal continuum theory and the Euler-Bernoulli beam model. The governing equilibrium equations are acquired and solved for CNTs subjected to mechanical loads and embedded in Kerr's medium. Kerr-type model is employed to simulate the interaction of the (SWNT) with a surrounding elastic medium. A first time, a comparison with the available results is made, and another comparison between various models Winkler-type, Pasternak-type and Kerr-type is studied. Effects of nonlocal parameter and aspect ratio of length to diameter of nanobeam, as well as the foundation parameters on buckling of CNT are investigated. These results are important in the mechanical design considerations of nanocomposites based on carbon nanotubes.

Keywords: Kerr's medium; Euler-Bernoulli; buckling; carbon nanotubes; nanobeam

1. Introduction

Carbon nanotube (CNT) is one of the miracles of this century its (CNTs) are molecular-scale tubes of graphitic carbon with the supreme and outstanding characteristics. The carbon nanotubes (CNTs) embedded in elastic medium broadly attract researchers' attention in recent years, have widespread applications in different fields such as nanotechnology, electronics, chemistry, physics, engineering and reinforced composite structures as well as potential uses in architectural fields (Shahsavari *et al*. 2018a, Fakhar and Kolahchi 2018, Tounsi *et al*. 2016, Karami and Janghorban 2016, Ould Youcef *et al*. 2015, Karami *et al*. 2018b, Kolahchi *et al*. 2017b, Guessas *et al*. 2018, Amnieh *et al*. 2018, Golabchi *et al*. 2018). The very unusual mechanical, electrical and thermomechanical properties of CNTs make them as one of the greatest hopeful reinforcement materials for high performance structural and multifunctional composites instead of conventional fibers, with adding them as reinforcements for polymers leaded to several important studies to estimate their mechanical properties accurately (Kolahchi and Cheraghbak 2017, Kolahchi *et al*. 2017c, Bousahla *et al*. 2014, Bouhadra *et al*. 2018, Abualnour *et al*. 2018, Madani *et al*. 2016,

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Zarei *et al*. 2017, Abdelmalek *et al*. 2017, Belmahi *et al*. 2018, Hajmohammad *et al*. 2018b, Hosseini and Kolahchi 2018, Kolahchi *et al*. 2015, Kaci *et al*. 2018, Draiche *et al*. 2016, Mahi *et al*. 2015, Zine *et al*. 2018).

At the nano scale, length dimensions are often small enough to call out the applicability of classical continuum theories because at this scale, the mechanical characteristics of structures are often different from their macroscopic behavior. Consequently, many non-local theories that consider the scale effect have been proposed such as Functionally Graded Nanotube-Reinforced Composite (Karami *et al*. 2018c), nonlocal second-order shear deformation theory (Karami *et al*. 2018d), and the nonlocal theory of elasticity (Eringen 1972), these theories take into account the influence of the screen introducing the intrinsic scale length in the constituent relations. Among the theories of the nanoscale, the theory of non-local elasticity developed by (Eringen 1983) who considered that the state of stress at a reference point in the body is considered as a function of the states of stress of all points. Recently, The theories of the nanoscale have been widely used to study the responses of nano and micro structures, such as the static (Karami *et al*. 2017, Ahouel *et al*. 2016, Karami *et al*. 2018d), the buckling and postbuckling (Arani and Kolahchi 2016, Bellifa *et al*. 2017b, Bouazza *et al*. 2015b, Larbi Chaht *et al*. 2015, Zemri *et al*. 2015, Kolahchi *et al*. 2017c, Zamanian *et al*. 2017, Khetir *et al*. 2017, Hajmohammad *et al*. 2017) and dynamic instability (She *et al*. 2018a, Belkorissat *et al*. 2015, Wu *et al*. 2018, Bouadi *et al*. 2018, Bounouara *et al*. 2016, Karami *et al*. 2018f, Youcef *et al*. 2018, Besseghier *et al*. 2017, Kolahchi and Bidgoli 2016, Ehyaei *et al*. 2017, Cherif *et al*. 2018, She *et al*. 2018a, Ebrahimi and Fardshad 2018), bending (Bouafia *et al*. 2017).

Due to difficulties encountered in experimental methods to predict the responses of nanostructures, the continuum mechanics methods are widely used to predict the responses of Visco and piezoelectric nano beam and nano plates (Kolahchi *et al*. 2016a, c, 2017e, Hajmohammad *et al*. 2018a, Shokravi 2017b, Cherif *et al*. 2018, Hajmohammad *et al*. 2018c, Kolahchi 2017). The continuum mechanics methods is one of the hypotheses of several theory such as: Euler-Bernoulli, Timoshenko, levinson and higher order shear deformation theory (Belabed *et al*. 2014, 2018, Bennoun *et al*. 2016, Bouderba *et al*. 2016, Boukhari *et al*. 2016, Bourada *et al*. 2015, Fourn *et al*. 2018, Hamidi *et al*. 2015, Hebali *et al*. 2014, Houari *et al*. 2016, Mouffoki *et al*. 2017, Tounsi *et al*. 2013b, Younsi *et al*. 2018, Zidi *et al*. 2014, 2017).

Most structures and nanostructures are subjected to axial compressive, buckling may occur, and the deflection of a structural member increase gradually with increased applied load. Buckling refers to this transition to large, often catastrophic displacements also leading to the sudden collapse of a mechanical component and structural instability, which is often called buckling. Buckling canoccur due to mechanical or thermal loads. Recently, the buckling of various structures have been widely studied (Abdelaziz *et al*. 2017, Ait Amar Meziane *et al*. 2014, Bellifa *et al*. 2017a, Chikh *et al*. 2017, El-Haina *et al*. 2017, Menasria *et al*. 2017, Bousahla *et al*. 2016, Mokhtar *et al*. 2018)

Many investigators have discussed the vibration and buckling of single-walled (SW) and multi-walled (MW) CNTs with various theories which are treated as beams, thin shells or solids in cylindrical shapes (Naceri *et al*. 2011, Wang *et al*. 2006, Zidour *et al*. 2015, Bensattalah *et al*. 2016, Bouazza *et al*. 2014, 2015b, Ehyaei *et al*. 2017, Chemi *et al*. 2018, Hamidi *et al*. 2018, Karami *et al*. 2018a, Chemi *et al*. 2015, Rakrak *et al*. 2016, Tounsi *et al*. 2013b, Dihaj *et al*. 2018, Kolahchi *et al*. 2016b). In most applications of nanocomposites, the CNT are embedded in an elastic foundation medium. The first type of elastic foundation is presented by Winkler as the "one-parameter" foundation model since it is characterized only by the vertical stiffness of the Winkler foundation springs (Murmu and Pradhan 2010, Pradhan and Reddy 2011). Karami *et al*. (2018g) analyze the wave dispersion of graphene with initial stress mounted on Winkler foundation. Several researchers (Filonenko-Borodich 1940) have been improved the Winkler model by using the second parameter to description the existence of shear stress inside the elastic medium.

In fact, both the first type of elastic foundation and the second type presented by Pasternak have been utilized by (Beldjelili *et al*. 2016, Bouderba *et al*. 2013, Yazid *et al*. 2018, Zaoui *et al*. 2019, Attia *et al*. 2018). The size-dependent dynamic deflections of viscoelastic orthotropic nanoplates under moving load embedded within visco-Pasternak substrate is analyzed by (Shahsavari *et al*. 2017). Recently, TheWinkler-Pasternak and visco-Pasternak's mediumhave been widely used to study the responses of nano and micro structures (Shahsavari *et al*. 2018a, She *et al*. 2018a).

To further improve the Pasternak model, (Kerr 1965) had added the third parameter (Kerr-type). The major role of the third parameter (Kerr-type) is to give more flexibility in controlling the grade of foundation-surface continuity between the loaded and the unloaded area of the beam-elastic system. In the Kerr-type model, the elastic medium is envisaged as consisted of lower and upper spring beds sandwiching an incompressible shear layer (Limkatanyu *et al*. 2013). However, Even though the model of Kerr-type foundation was developed since the mid-sixties, it has been found that theoretical studies of nanostructure resting or incorporated into the Kerr foundation are rare in the literature (Shahsavari *et al*. 2018b). The shear buckling lead of imperfect FG nanoplates incorporate porosities resting on an elastic Kerr foundation model environment is analyzed by (Karami *et al*. 2018f).

In this paper, the critical buckling of a CNT embedded in Kerr's medium based on the nonlocal Euler Bernoulli beam theory is studied. A comparison study with published results acquired by other investigators is presented showing an excellent agreement. The effects of nonlocal parameter, radius and length of CNT, as well as the foundation parameters on buckling of CNT embedded in an elastic medium are investigated.

2. Basic equations of carbon nanotube (CNT)

2.1 Elastic medium

Consider a CNT of length L, diameter d and thickness t as shown in Fig. 1. The CNT is surrounding with Kerr's medium. This foundation model is represented by (Van Cauwelaert *et al*. 2002)

$$f(x) = \frac{1}{1 + \dfrac{k_w}{k_c}} \left(k_w . w - k_G \frac{\partial^2 w}{\partial x^2} - \frac{D . k_G}{k_c} \frac{\partial^6 w}{\partial x^6} \right) \tag{1a}$$

Where w is the transverse displacement of CNT, the foundation model consists of two spring layer (the upper and lower springs) of constants k_w and k_c, and an intermediate shear layer with constant k_G. It is usually more convenient to use the bending stiffness D of the beam writing by

$$D = EI \tag{1b}$$

Where I and E are the moment of inertia and elastic modulus of nanotube of carbon.

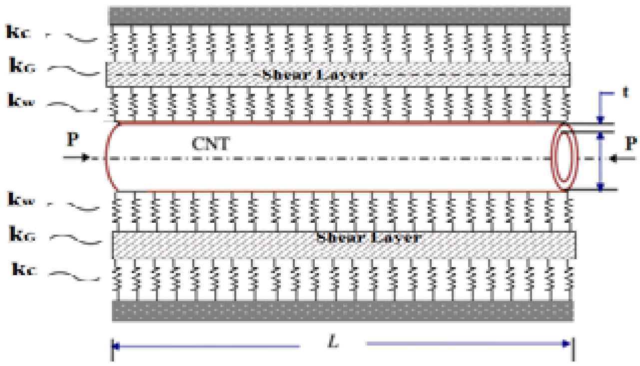

Fig. 1 Schematic diagram for simply-supported carbon nanotube embedded in Kerr's foundations

2.2 The model of nonlocal elasticity of (SWCNTs)

Based on Eringen nonlocal elasticity model Eringen (1976, 1983), the stress at a reference point in the body is considered to be a functional of the strain field at every point. When the effects of strains are neglectedat in points other than x, one get to local theory of elasticity. For homogeneous and isotropic elastic solids, the constitutive equation of non-local elasticity can be given by Eringen AC (1983). Non-local stress tensor (t) at point (x') is defined by

$$\begin{aligned} &\sigma_{ij,j} = 0 \\ &\sigma_{ij}(x) = \int K(|x - x'|,\ \tau) C_{ijkl}\varepsilon_{kl}(x')dV(x'), \quad \forall x \in V \\ &\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \end{aligned} \tag{2}$$

where (C_{ijkl}) is the classical, macroscopic stress tensor at point x', σ_{ij} and ε_{ij} are stress and strain tensors respectively. $K(|x - x'|, \tau)$ is the kernel function and ($\tau = e0a/l$) is a material constant that depends on internal and external distinctive length (such as the lattice spacing and wavelength)., where ($e0a$) is a constant proper to each material, a is an internal characteristic length, e.g., length of (C–C) bond, lattice parameter, granular distance, and (L) is an external characteristic length.

As solving of integral constitutive Eq. (2) is difficult, a simplified equation in one-dimensional differential form is used as a basis of all nonlocal constitutive formulation

$$\left(1-(e0a)^2\frac{\partial^2}{\partial x^2}\right)\sigma_x = E\,\varepsilon_x \tag{3}$$

where (E) is the Young's modulus of the material. Thus, the scale coefficient ($e0a$) in the modelling will lead to small-scale effect on the response of structures at nano size.

The expressions of the axial strain 3 can be defined by

$$\varepsilon = -y\frac{\partial^2 w}{\partial x^2} \tag{4}$$

where (w) is the transverse displacement.

The shear force and the bending moment can be defined by

$$M = \int_A z\sigma_x dA, \qquad T = \int_A \tau_{xy} dA \tag{5}$$

Based on Eqs. (3), (4) and (5) the bending moment (M) and the shear force (T) for the non-local model can be expressed as

$$\left(1-(e0a)^2 \frac{\partial^2}{\partial x^2}\right)M = EI\frac{dw}{dx} \tag{6}$$

where $\left(I = \int_A y^2 dA\right)$ is the moment of inertia, (A) is the cross-section area of the beam,

Equilibrium equations of (SWCNTs) embedded in Kerr's foundations.

Using Euler-Bernoulli beam theory, the force equilibrium equations and the moment on the one-dimensional structure for transverse vibrations of an elastic beam can be easily provided follows

$$\frac{\partial T}{\partial x} - p\frac{\partial^2 w}{\partial x^2} + f(x) = 0 \tag{7}$$

$$\frac{\partial M}{\partial x} - T = 0 \tag{8}$$

where (x) is the axial coordinate, (w) is the transverse deflection of the (SWCNT), (M) and (T) are the resultant bending moment and shear force, (ρ) is the mass density of the material, (A) is the area of the cross section of the nanotube beam, (I) the second moment of inertia,. $f(x)$ is the interaction pressure per unit axial length between the nanotube and the surrounding elastic medium,

Based on Eqs. (6), (7) and (8), the following relation can be obtained

$$M = -EI\frac{\partial \psi}{\partial x} + (e0a)^2\left[P\frac{\partial^2 w}{\partial x^2} + f(x)\right] \tag{9}$$

The derivation of Eq. (9) is as follows

$$\frac{\partial M}{\partial x} = -EI\frac{\partial^3 w}{\partial x^3} + (e0a)^2\left[P\frac{\partial^3 w}{\partial x^3} + \frac{\partial f(x)}{\partial x}\right] \tag{10}$$

Substituting Eq. (10) into Eq. (8), we can obtain

$$V = \frac{\partial M}{\partial x} = -EI\frac{\partial^3 w}{\partial x^3} + (e0a)^2\left[P\frac{\partial^3 w}{\partial x^3} + \frac{\partial f(x)}{\partial x}\right] \tag{11}$$

The derivation of Eq. (11) is as follows

$$\frac{\partial V}{\partial x} = -EI\frac{\partial^4 w}{\partial x^4} + (e0a)^2\left[P\frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 f(x)}{\partial x^2}\right] \tag{12}$$

Substituting Eqs. (1) and (12) into Eq. (7), we obtain the below governing nonlocal equation for Buckling of nanotubes based on Euler–Bernoulli beam theory

$$EI\,\frac{\partial^4 w}{\partial x^4} + \left(1-(e0a)^2\frac{\partial^2}{\partial x^2}\right)* \\ \left(P\frac{\partial^2 w}{\partial x^2} + \frac{1}{1+\frac{k_w}{k_c}}\left(k_w\, w - k_G\frac{\partial^2 w}{\partial x^2} - \frac{D\cdot k_G}{k_c}\frac{\partial^6 w}{\partial x^6}\right)\right) = 0 \tag{13}$$

Since finding an analytical solution is possible for simply supported boundary conditions for the present problem, the (SWNT) beam is assumed simply supported. As a result, the boundary conditions have the following form 13

$$w\,(x,t) = \bar{W}\,\sin(\lambda x), \qquad \lambda = N\pi/L \tag{14}$$

Where $(\bar{W})$ is the amplitude of deflection of the beam. In addition the pressure per unit axial length, acting on the outermost tube due to the surrounding elastic medium, can be described by a Kerr's model (see Eq. (1)).

Substitution of Eq. (14) into Eq. (13) gives the correspondent buckling load via nonlocal Euler-Bernoulli beam model of the CNT is embedded in a Kerr's medium ($P_{N.E.K}$) is as follows

$$P_{N.E.K} = \frac{EI\lambda^4 + \left(\frac{1}{1+\frac{k_w}{k_c}}\right)\left(\left(1+(e0a)^2\lambda^2\right)\left(\lambda^6\frac{D\cdot k_G}{k_c} + k_w\right) + k_G\,(e0a)^2\lambda^4 + k_G\lambda^2\right)}{\lambda^2\left(1+(e0a)^2\lambda^2\right)} \tag{15}$$

Note that for $k_c = \infty$, the upper spring layer vanishes and the Eq. (15) transforms into buckling load of the CNT is embedded in a Pasternak's medium ($P_{N.E.P}$)

$$P_{N.E.P} = \frac{\left(1+(e0a)^2\lambda^2\right)(k_w) + k_G\,(e0a)^2\lambda^4 + k_G\lambda^2 + EI\lambda^4}{\lambda^2\left(1+(e0a)^2\lambda^2\right)} \tag{16}$$

Further note that for ($k_c = \infty$ and $k_G = 0$), the upper spring layer, the shear layer vanishes and the Eq. (15) transforms into buckling load of the CNT is embedded in a Winkler's medium ($P_{N.E.W}$)

$$P_{N.EW} = \frac{\left(1+\left(e0a\right)^2\lambda^2\right)\left(k_w\right)+EI\lambda^4}{\lambda^2\left(1+\left(e0a\right)^2\lambda^2\right)} \tag{17}$$

3. Numerical results and discussions

3.1 Validation

The effects of nonlocal parameter ($e0a$), length (L), rod diameter (d), Kerr's foundation (k_c, k_w and k_G) and mode numbers (N) on the buckling analysis of the CNTs are written in Eq. (13). It is interesting to note that by putting $e0a = 0$, we obtain the corresponding local elasticity equation for SWCNT (Wang *et al*. 2006). Analytical solutions exist for SWCNT with nonlocal effects without any elastic medium. The analytical solution Wang *et al*. (2006) is given by

$$P_{exact} = \frac{\left(N\pi\right)^2\left(\frac{EI}{L^2}\right)}{\left(1+\left(e0a\right)^2\left(\frac{N\pi}{L}\right)^2\right)} \tag{18}$$

Table 1 show how the present results can be validated by other published literatures in buckling analysis of CNTs (Pradhan and Reddy 2011, Wang *et al*. 2006). The data adopted in generating these results are: E = 1 TPa, $G = E/[2(1+v)]$, v = 0.19, rod diameter d = 1 nm and $I = \pi d^4/64$. The various non-dimensional parameters used are

$$\overline{K}_W = \frac{k_w \cdot L^4}{D}, \quad \overline{K}_G = \frac{k_G \cdot L^2}{D}, \quad \overline{K}_C = \frac{k_c \cdot L^4}{D} \tag{19}$$

Table 1 Comparison between exact and approximate buckling loads P_{cr} (nN) for the simply supported based on nonlocal Euler-Bernoulli beam model

$e0a$ (nm)	0			1			2		
L/d	P_{cr} (exact) (Wang *et al.* 2006)	P_{cr} (DTM) (Pradhan and Reddy 2011)	P_{cr} Present	P_{cr} (exact) (Wang *et al.* 2006)	P_{cr} (DTM) (Pradhan and Reddy 2011)	P_{cr} Present	P_{cr} (exact) (Wang *et al.* 2006)	P_{cr} (DTM) (Pradhan and Reddy 2011)	P_{cr} Present
10	4.8447	4.8447	4.8447	4.4095	4.4095	4.4095	4.0460	4.0460	4.0460
12	3.3644	3.3644	3.3644	3.1486	3.1486	3.1486	2.9588	2.9588	2.9588
14	2.4718	2.4718	2.4718	2.3533	2.3533	2.3533	2.2456	2.2456	2.2456
16	1.8925	1.8925	1.8925	1.8222	1.8222	1.8222	1.7569	1.7569	1.7569
18	1.4953	1.4953	1.4953	1.4511	1.4511	1.4511	1.4094	1.4094	1.4094
20	1.2112	1.2112	1.2112	1.1821	1.1821	1.1821	1.1542	1.1542	1.1542

Table 2 Comparison of axial buckling load of the CNT embedded in Winkler, Pasternak and Kerr's medium using the present theory with those obtained by (Wang *et al.* 2006)

	$e0a$ = 0 nm		$e0a$ = 1 nm		$e0a$ = 2 nm	
	L/d = 5	L/d = 10	L/d = 5	L/d = 10	L/d = 5	L/d = 10
Without medium (Wang *et al.* 2006)	19.3789	4.8447	13.8939	4.4095	7.5137	3.4735
Without medium (Present)	19.3789	4.8447	13.8939	4.4095	7.5137	3.4735
Winkler medium (Present)	39.2733	9.8183	33.7882	9.3831	27.4081	8.44710
Pasternak medium (Present)	58.9082	14.7271	53.4232	14.2919	47.0430	13.3558
Kerr medium (Present)	45.3062	11.3265	39.8211	10.8913	33.4410	9.95530

As a validation example, (Table 1) shows a comparison of axial buckling load P of the CNT subjected to axial buckling load calculated by using the present theory with those obtained by Pradhan and Reddy (2011), and by Wang *et al.* (2006). It can be seen in Table 1 the excellent agreement of the proposed method of solution with various small scale parameters and those obtained in the literature.

Table 2 shows a comparison of axial buckling load P of the CNT embedded in Winkler, Pasternak and Kerr's medium using the present theory with those calculated by (Wang *et al.* 2006). It is clear that there is excellent agreement between the results. On the other hand, it is clearly seen from (Table 2) that the ranges of buckling load are quite different, the range is the smallest for Winkler medium, but the range is the largest for Pasternak medium. The reason for this difference is attributed to the increasing or decreasing of rigidity of elastic medium.

3.2 Effect of various models of elastic medium:

Now, to study the effect of various elastic foundation parameters on the axial buckling load P of the CNT. The lower spring modulus (k_w), the upper spring modulus parameter (k_c) and the shear layer modulus (k_G). The lower spring modulus (Winkler modulus parameter) $(\overline{K}_W)$, for the surrounding polymer matrix is varied from 0 to 400, what for L = 10 nm corresponds to the change k_w = (0÷1,96) GPa/nm, while the shear layer modulus parameter $(\overline{K}_G)$ is varied from 0 to 10, what for L = 10 nm corresponds to the change k_G = (0÷4,90) nN/nm. Similar values of modulus parameters were taken by Kitipornchai *et al.* (2006). Both, for the upper spring modulus parameter $(\overline{K}_C)$ in this present work is varied from 2 to 1000, what for L = 10 nm corresponds to the change k_c = (0,01÷205·10^3) GPa/nm. Note that for $(\overline{K}_C = 1000)$ the *results* in Kerr's foundation tend to the results in Pasternak's foundation; for $(\overline{K}_C = 1000$ and $\overline{K}_G = 0)$ the results in Kerr's foundation tend to the results in Winkler's foundation;

Fig. 2 shows a comparison of critical buckling load of SWCNT between various models of elastic medium with simply supported boundary conditions. The material properties for this case are taken as E = 1 TPa, d = 1 nm, $\overline{K}_W$ = 100, $\overline{K}_G$ = 10, $\overline{K}_C$ = 50, nonlocal parameter $e0a$ = 2 nm. It is observed that for various model of elastic medium the buckling load diminished as ratio L/d is varied from 1 to 10. In addition, it is observed that there is a significant influence of type of the elastic medium on the critical buckling loads of SWCNT.

In Fig. 3 depicts the effect of scale coefficients on the critical buckling loads of carbon nanotube embedded in Kerr's foundations. The parameter ($e0a$) values of SWCNT were taken in the range of 0–2 nm. From Fig. 3, it is observed that there is a significant influence of small scale

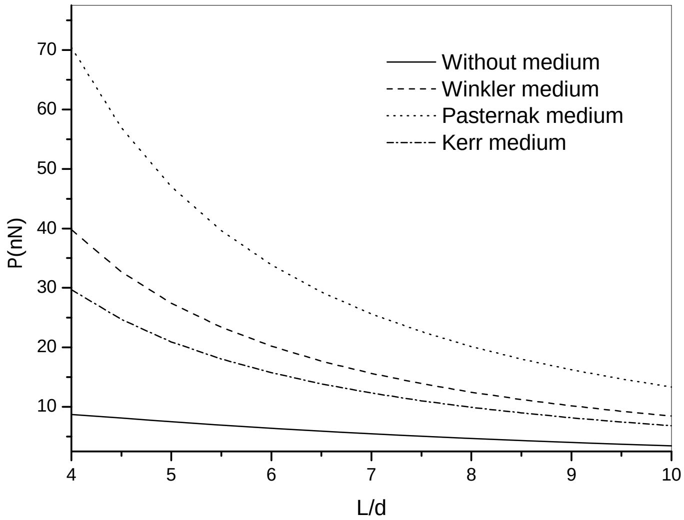

Fig. 2 Comparison of critical buckling load of SWCNT between various models of elastic medium

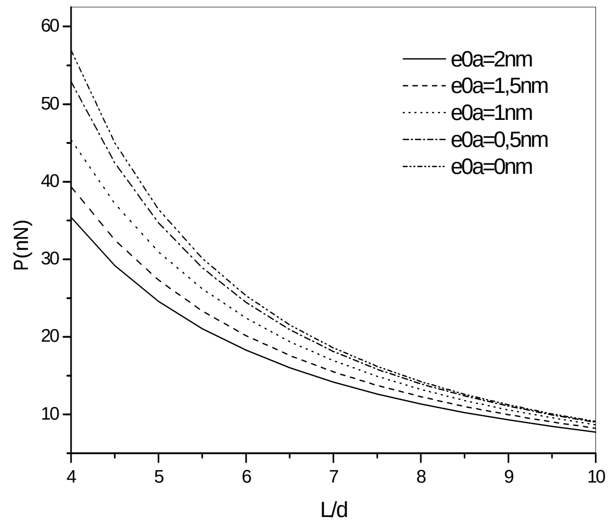

Fig. 3 Relation between the values of critical buckling loads and the aspect ratio (L/d) of SWCNT with different scale coefficients

parameter ($e0a$) on the critical buckling loads for simply-supported carbon nanotube embedded in Kerr's foundations. Considering nonlocal model are always smaller than the local (classical) model. Similar observations are made in this works (Pradhan and Reddy 2011, Wang *et al*. 2006, Zidour *et al*. 2014) for the analyses of the nano beams.

Fig. 2 shows a comparison of critical buckling load of SWCNT between various models of elastic medium with simply supported boundary conditions. The material properties for this case are taken as E = 1 TPa, d = 1 nm, $\overline{K}_W = 100$, $\overline{K}_G = 10$, $\overline{K}_C = 50$, nonlocal parameter $e0a$ = 2 nm. It is observed that for various model of elastic medium the buckling load diminished as ratio L/d is

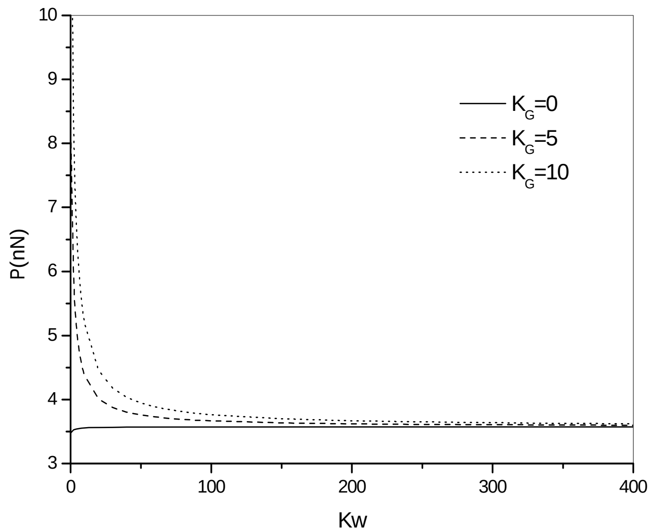

Fig. 4 Relation between the values of critical buckling loads of SWCNT embedded in Kerr's foundations and the Winkler modulus for various Shears layer modulus with $\overline{K}_C = 2$

varied from 1 to 10. In addition, it is observed that there is a significant influence of type of the elastic medium on the critical buckling loads of SWCNT.

In Fig. 3 depicts the effect of scale coefficients on the critical buckling loads of carbon nanotube embedded in Kerr's foundations. The parameter ($e0a$) values of SWCNT were taken in the range of 0–2 nm. From Fig. 3, it is observed that there is a significant influence of small scale parameter ($e0a$) on the critical buckling loads for simply-supported carbon nanotube embedded in Kerr's foundations. Considering nonlocal model are always smaller than the local (classical)

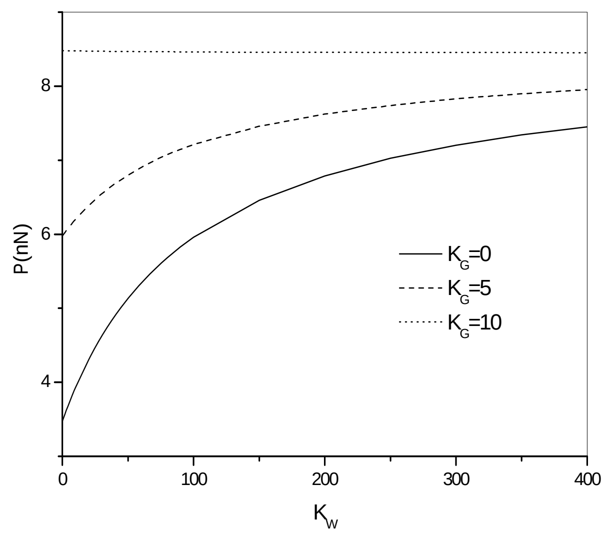

Fig. 5 Relation between the values of critical buckling loads of SWCNT embedded in Kerr's foundations and the Winkler modulus for various Shears layer modulus with $\overline{K}_C = 100$

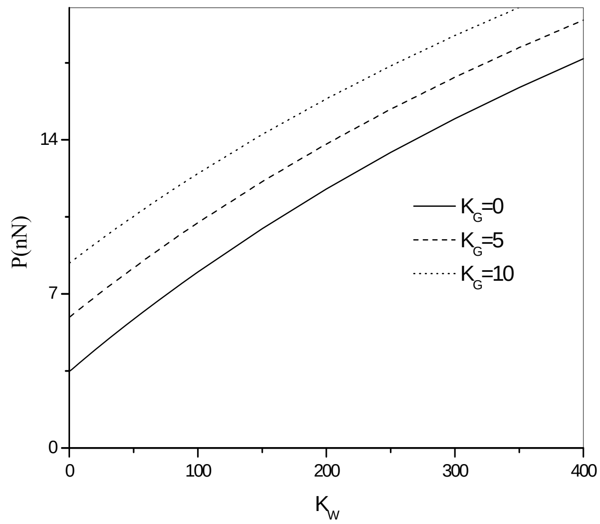

Fig. 6 Relation between the values of critical buckling loads of SWCNT embedded in Kerr's foundations and the Winkler modulus for various Shears layer modulus with $\overline{K}_C$ = 1000

model. Similar observations are made in this works (Pradhan and Reddy 2011, Wang *et al*. 2006, Zidour *et al*. 2014) for the analyses of the nano beams.

The effect of the various parameters of Kerr's foundations, on the non-local critical buckling load for short of single-walled carbon nanotubes (SWCNTs), the ratio of the length to the diameter (L/d), is 10 is presented in (Figs. 4-6). The small scale effects is considered ($e0a$ = 2 nm). It is clearly seen from figures that the ranges of the critical buckling loads for these deferent parameters of elastic medium (Kerr's foundation) are quite different, the range is the largest for Shears layer moduli $\overline{K}_G = 10$, but the range is the smallest for others. It can be clearly seen that this parameters of Kerr's foundation effect reduces or augmented the buckling loads. In other hand, it is noticed that the critical buckling loads are influenced by stiffness of the surrounding polymer elastic medium. As these parameters of elastic medium increases (soft elastic medium to hard medium),the critical buckling loads also increase.

4. Conclusions

The paper presents a embedded carbon nanotube in Kerr's foundation medium. The buckling analysis is obtained by using nonlocal continuum theory and the Euler-Bernoulli beam model. A validation example is given to show the accuracy of the present analysis. Influence of the stiffness of the elastic medium on critical buckling loads of the CNTs is shown. Winkler-type, Pasternak-type and Kerr-type models are employed to simulate the interaction of the (SWNT) with a surrounding elastic medium. The Kerr-type medium is utilized to taking into account the shear coupling between the individual Winkler-spring components through the shear-layer component. This feature is unique to the Kerr-type foundation model. The obtained numerical results show that:

- The buckling load P (nN) is rabidly decreasing as the nonlocal parameter $e0a$ decreasing.
- The minimum value of The buckling load P (nN) occurs in the case of without elastic

foundations and the maximum one occurs for the higher values of the foundation parameters $\overline{K}_C$, $\overline{K}_W$ and $\overline{K}_G$.

- The buckling load P (nN) is sensitive to the value of $\overline{K}_C$ more than these values of Winkler's parameter $\overline{K}_W$ and shear layer $\overline{K}_G$.
- The maximum value of The buckling load P (nN) occurs in the case of Pasternak elastic foundations more than these values of Winkler's and Kerr's foundations
- The buckling load P (nN) is increasing as the aspect ratio of length to diameter of the CNT decreases

The Kerr-type model results in more realistic interactive foundation forces as compared to the Winkler model.

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