Dynamic analysis of higher order shear-deformable nanobeams resting on elastic foundation based on nonlocal strain gradient theory

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Abstract.   Present investigation deals with the free vibration characteristics of nanoscale-beams resting on elastic Pasternak's foundation based on nonlocal strain-gradient theory and a higher order hyperbolic beam model which captures shear deformation effect without using any shear correction factor. The nanobeam is lying on two-parameters elastic foundation consist of lower spring layers as well as a shear layer. Nonlocal strain gradient theory takes into account two scale parameters for modeling the small size effects of nanostructures more accurately. Hamilton's principal is utilized to derive the governing equations of embedded strain gradient nanobeam and, after that, analytical solutions are provided for simply supported conditions to solve the governing equations. The obtained results are compared with those predicted by the previous articles available in literature. Finally, the impacts of nonlocal parameter, length scale parameter, slenderness ratio, elastic medium, on vibration frequencies of nanosize beams are all evaluated.

Keywords: nanobeam; dynamic; nonlocal strain gradient elasticity; pasternak foundation

1. Introduction

Over the past few years, the use of structural elements such plates and beams at macro/nano scales in various MEMS/NEMS are rising with a great speed due to the rapid progress in nanotechnology. It has been observed that at nanoscale, the physical and mechanical properties of nanostructures present obvious size dependency that make them to exhibit important mechanical, electrical and thermal performances which are better to the conventional ones at macroscale (Eltaher *et al.* 2016, Tounsi *et al.* 2013a, Benguediab *et al.* 2014b).Therefore, exploring of size-dependency is still a challenging issue in studying of nanoscale structures for reliable design. It is well known that the classical continuum theories fail to take into account the size effect due to the lack of a scale parameter. Hence, to counter this complication, nonlocal continuum theories such as nonlocal elasticity theory of Eringen (1972, 1983) and strain gradient theory (Lam *et al.* 2003) have been successfully used to capture the small-scale effects. Due to of its prominence, the

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Eringen's nonlocal elasticity theory has been used in a various studies to investigate mechanical behavior of nanoscale structures (Reddy and Pang 2008, Heireche *et al.* 2008). Peddieson *et al.* (2003) developed for the first time a version of nonlocal elasticity to study the behavior of a nonlocal Euler–Bernoulli beam model. Wang and Liew (2007) explored the static deflection of micro- and nano-micro scale structures using Euler–Bernoulli and Timoshenko beam theory. Reddy (2007) used various beam theories to study the bending, buckling and vibration of nanobeams based the nonlocal differential equations of Eringen. The generalized differential quadrature method has been developed. Murmu and Adhikari (2010) provided an analytical method to investigate the nonlocal transverse vibration of double nanobeam-system using nonlocal elasticity theory. Thai (2012) employed a nonlocal beam theory to investigate the bending, buckling, and free vibration of nanobeams based on nonlocal refined beam theory. In another work, Thai (2012), Thai and Vo (2012) employed both higher order and sinusoidal shear deformations beam models and Eringen's nonlocal elasticity theory to explore the static, buckling and dynamic behaviors of nanoscale beams. Tounsi *et al.* (2013b) investigated thermal buckling characteristic of nanobeams by employing an efficient higher-order nonlocal beam theory. Zenkour and Sobhy (2015) developed a new simple shear and normal deformations beam model to explore thermo-mechanical loading effect on bending behavior of nanoscale beams. Zhu and Li (2017a) examined the twisting statical behaviors of functionally graded (FG) nanotubes using Eringen's nonlocal integral model. Also in the recent past years the mechanical behavior of nanobeams is inspected based on various shear deformation beam theories (Berrabah *et al.* 2013, Tounsi *et al.* 2013a, b, Benguediab *et al.* 2014a, Behera and Chakraverty 2013, Besseghier *et al.* 2015, Hadj Elmerabet *et al.* 2017, Ebrahimi and Daman 2017, Thai *et al.* 2018, Yazid *et al.* 2018, Bouafia *et al.* 2017, Youcef et *al.* 2018, Belkorissat *et al.* 2015, Ahouel *et al.* 2016, Mokhtar *et al.* 2018, Khetir *et al.* 2017, Besseghier *et al.* 2017, Karami *et al.* 2017, Zemri *et al.* 2015, Al-Basyouni *et al.* 2015, Karami *et al.* 2018a, Bensaid and Bekhadda 2018).

Actually, material gradation can considerably reduce maximum stresses and change the spatial location where such maximums arise (Bounouara *et al.* 2016, Attia *et al.* 2018). This provides the opportunity of fitting material variation to reach desired stresses in a structure. Also many researchers investigated the vibration, bending and buckling of FG structures based on refined higher order shear deformation theories by including the stretching thickness effect (Hebali *et al.* 2014, Bennoun *et al.* 2016, Bousahla *et al.* 2014, Belabed *et al.* 2014, Bouhadra *et al.* 2018, Younsi *et al.* 2018, Abualnour *et al.* 2018, Draiche *et al.* 2016, Bouafia *et al.* 2017, Karami *et al.* 2018, Hamidi et *al.* 2015). In addition, the thermal and hyghro-thermal effects were studied by several authors to show their importance on mechanical behavior of FG structures (Bousahla *et al.* 2016, Bouderba *et al.* 2016, El-Haina *et al.* 2017, Menasria *et al.* 2017, Bouderba *et al.* 2013, Beldjlili *et al.* 2016, Chikh *et al.* 2017, Hamidi *et al.* 2015, Tounsi *et al.* 2013c). On the other hand, the effect of various boundary conditions on FG sandwich structures is studied by many researchers (Abdelaziz *et al.* 2017, Ait Amar Meziane *et al.* 2014, Bouderba *et al.* 2016).

To capture the size effect of nanostructures more accurately; nonlocal strain gradient theory was introduced (Lam *et al.* 2003), in which two scale parameters are suggested associated to nonlocal stress field and strain gradient stress field in order to generalize the Eringen's nonlocal model and taking into account the stiffness-hardening effect (Lim *et al.* 2015, Li et *al.* 2016) explored the free vibration behavior of an inhomogeneous nanobeams based on nonlocal strain gradient theory (NL-SGT); they showed that both nonlocal and length-scale parameters have a significant effect on the vibration frequencies. The nonlinear vibration analysis of FG nanobeams have been investigated by Simsek (2016) based on NL-SG theory and Hamiltonian approach. In

another work, Li *et al*. (2016a, b) contributed to longitudinal vibration characteristic of size-dependent rods via NL-SG theory. Ebrahimi *et al*. (2016) used the NL-SG theory for study the wave dispersion of thermally acted on heterogeneous nanoplates. Also, Ebrahimi and Barati (2017) investigated hygrothermal impacts on buckling behavior of size-dependent shear-deformable curved FG nanobeams more accurate prediction of mechanical behavior of nanostructures by using the newly developed NSGT. The NL-SG theory has been also applied to show both the stiffness-softening and hardening effects on the longitudinal dynamic and tension of nanorods, CNTs and monolayer graphene (Zhu and Li 2017b, c), they found that the nonlocal strain gradient models show a good results match well with the experimental data (or MD simulation results). Moreover, an investigation is performed using NSGT on post-buckling behavior of functionally graded nanobeams (Li and Hu 2017). Zeighampour *et al*. (2017) examined the wave propagation in fluid-conveying double-walled carbon nanotube (DWCNT) via the nonlocal strain gradient theory. Karami *et al*. (2018b) utilized the new nonlocal strain gradient 3D elasticity theory to analyze the mechanical behavior of anisotropic spherical nanoparticles. In another work, Karami *et al*. (2018c) formulated a variational approach for wave dispersion in anisotropic doubly-curved nanoshells based on a new nonlocal strain gradient higher order shell theory.

Nanobeams and nanoplates are frequently in contact with an elastic medium in many practical cases. One of the most advantageous models, which has recently been used for the analysis of structures at macro and nanoscale lying on elastic foundation, is the two-parameter elastic foundation model (Winkler-Pasternak), due to its efficiency and simplicity. Pradhan *et al*. (2009) used the Winkler and Pasternak foundation model to investigate the vibration behavior of single-walled carbon nanotubes embedded in polymer matrix, based on nonlocal Timoshenko beam theory. The Winkler-type model was used by Besseghier *et al*. (2011) to explore the thermal effect on wave propagation of double-walled carbon nanotubes embedded in an elastic medium. Aissani *et al*. (2015) studied the static, buckling and vibration behaviors of nanosize-beams embedded in an elastic medium based on a new nonlocal hyperbolic shear deformation beam theory. Chakraverty and Behera (2015) have investigated the vibration and buckling characteristics of Euler Bernoulli nanobeams embedded in an elastic medium. They used the boundary characteristic orthogonal polynomials in the Rayleigh-Ritz method to solve the governing equations. Ebrahimi and Barati (2017a, b) investigated the free vibration characteristics of the third order shear deformable FG nanobeams embedded on elastic medium. They showed that the increase of Winkler or Pasternak parameter has led to an increment in non-dimensional frequencies. In another work, Ebrahimi and Barati (2016c) and Ebrahimi and Daman (2017) employed nonlocal strain gradient theory to explore free vibration response of curved functionally graded nanobeams resting on Winkler–Pasternak foundation under various boundary conditions.

As one can observe from the works previously cited, there is no article published on vibration of homogenous nanobeams lying on Winkler–Pasternak foundation and taking into account the nonlocal strain gradient theory. Also, most of preceding works on vibration of homogenous nanoscale beams have been treated only nonlocal impacts and neglected the strain gradient effects and Pasternak elastic foundation. It is well recognized that the dynamic behavior of nanostructures is significantly affected by strain fields parameter. Motivated by this significance, this research paper investigates the dynamic behavior of nanoscale beam resting on two-parameter elastic foundation based on nonlocal strain gradient elasticity theory (NL-SGT) and higher order hyperbolic shear deformation beam model which meets the the stress-free boundary conditions on the top and bottom surfaces of the nanobeam without using shear correction factors. To capture the size effect more accurately the present model incorporates nonlocal stress and strain gradient fields.

The two parameter Winkler–Pasternak elastic foundation model is used in the present investigation. The governing equations of motion of nonlocal strain gradient nanobeam are derived by employing Hamilton's principle and solved via an analytical solution for simply boundary conditions. The obtained results are compared with those found in the literature to check the accuracy of the present solution. The effects of nonlocal parameter, length scale parameter, elastic medium parameters and slenderness ratio on the dynamic response of nanobeams are all explored.

2. Theory and formulation

Considering a nanoscaled beam lying on elastic foundation with the length L , thickness h, and rectangular cross-section $b \times h$, in which its coordinates are illustrated in Fig. 1.

2.1 Basic assumptions

The displacement field of the proposed theory is chosen based on the following assumptions:

(i) The displacements are small in comparison with the nanobeam thickness and, therefore, strains involved are infinitesimal.
(ii) The transverse displacement w includes two components of bending w_b, and shear w_s. These components are functions of coordinate x only.

$$w(x,z) = w_b(x) + w_s(x) \tag{1}$$

(iii) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x.
(iv) The displacement u in x-direction consists of bending, and shears components.

$$u = u_b + u_s \tag{2}$$

The bending component u_b is assumed to be similar to the displacement given by the classical beam theory. Therefore, the expression for u_b can be given as
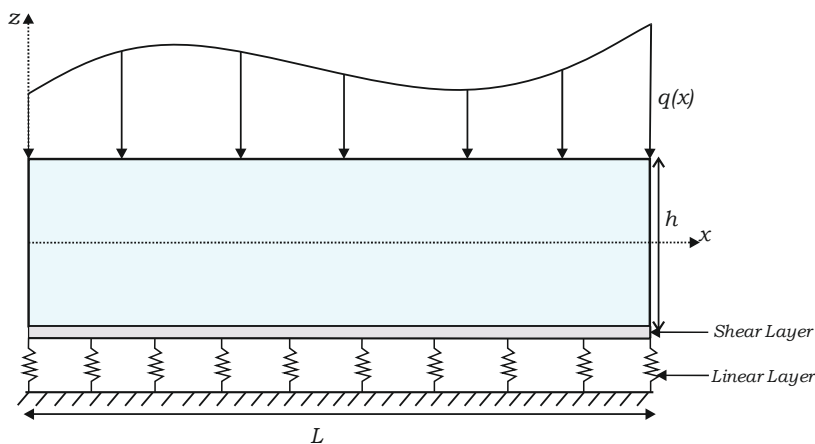

Fig. 1 Geometry and coordinate of the nanobeam on elastic foundation

$$u_b = -z\frac{\partial w_b}{\partial x} \tag{3}$$

In addition, the displacement component due to shear deformation (u_s) is supposed to be parabolic, sinusoidal, hyperbolic and exponential in nature with respect to thickness coordinate.

Hence, the shear component u_s gives rise, in conjunction with w_s, to a higher order variations of shear strain γ_{xz} and hence to shear stress τ_{xz} through the thickness of the nanobeam in such a way that shear stress τ_{xz} is zero at the top and bottom faces of the nanobeam. As a result, the expression for u_s can be given as

$$u_s = -f(z)\frac{\partial w_s}{\partial x} \tag{4}$$

2.2 Kinematic relations

There are several types of plate and beam theories for modeling of shear deformation effect (Bellifa *et al.* 2017, Mahi *et al.* 2015, Belabed *et al.* 2018, Bellifa *et al.* 2016, Ait Yahia *et al.* 2015, Fourn *et al.* 2018, Boukhari *et al.* 2016, Zine *et al.* 2018, Bensaid *et al.* 2017, Hachemi *et al.* 2017, Mouffoki *et al.* 2017, Kaci *et al.* 2018). In this research paper, the displacement field of the present refined higher order shear deformable nanobeam can be obtained as (Thai 2012, Thai and Vo 2012, Aissani *et al.* 2015)

$$u(x,z,t) = u(x,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{5a}$$

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \tag{5b}$$

Where u and w are displacements of mid-plane along axial and transversal directions, respectively, and w_b and w_s are the bending and shear components of transverse displacement. Herein, the function $f(z)$ is used to describe the distribution of the transverse shear strains and stresses through the beam thickness. It is essential that the first derivative of the shape function $f(z)$ must provide a parabolic curve in the thickness direction and satisfy the tangential zero value at $z = \pm h/2$ (Tuan et *al.* 2016). Thus the SCFs are not required and it is replaced by a shape function for estimating the distribution of shear stress through the beam thickness. So, no need to use any shear correction factor for both HSDT and RPT (El Meiche et *al.* 2011)

$$f(z) = \frac{(h/\pi)\sinh\left(\frac{\pi}{h}z\right) - z}{[\cos(\pi/2) - 1]} \tag{6}$$

The non-zero strains of the present nanobeam are given by

$$\varepsilon_x = \varepsilon_x^0 + z\kappa^b + f(z)\kappa^s \quad \text{and} \quad \gamma_{xz} = g(z)\gamma^s \tag{7}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \qquad \kappa^b = -\frac{\partial^2 w_b}{\partial x^2}, \qquad \kappa^s = -\frac{\partial^2 w_s}{\partial x^2}$$

$$\gamma^s = \frac{\partial w_s}{\partial x}, \qquad g(z) = 1 - f'(z), \quad \text{and} \quad f'(z) = \frac{df(z)}{dz} \tag{8}$$

2.3 The nonlocal strain gradient elasticity model

In the case of nonlocal elasticity theory, the stress accounts for both nonlocal elastic stress field and the strain gradient stress field. Hence, the constitutive relations for a nonlocal refined shear deformable nanobeam can be stated as (Lim *et al.* 2015, Li *et al.* 2016a, b, Ebrahimi *et al.* 2016)

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\left(\varepsilon_{xx} - \eta \frac{\partial^2 \varepsilon_{xx}}{\partial x^2} \right) \tag{9}$$

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\left(\varepsilon_{xx} - \eta \frac{\partial^2 \varepsilon_{xx}}{\partial x^2} \right) \tag{10}$$

where $\mu = (e_0 a)^2$ and $\eta = l^2$ consider the impacts of nonlocal stress field, and l indicate the material length-scale parameter, which can be used to introduce to take into account the effects of higher-order strain gradient stress field, e_0 is a constant appropriate to each material and a is an internal characteristic length. It is noted that, these length scale parameters cited above can be determined by matching with results of experiment or molecular dynamics. As usual, the size-dependent effects are supposed to be omitted in both width and thickness directions of the nanobeam. Besides the thickness effects are incorporated in the recent researches dealing with the mechanical responses of tiny structures (Li *et al.* 2018).

Furthermore, it is lately shown by Karami *et al.* (2018d, e), Shahsavari *et al.* (2018) that, there is a good agreement between molecular dynamics simulations and nonlocal strain gradient models and the general constitutive Eqs. (9) and (10) can reasonably explain size-dependent impacts on the dynamic behavior.

2.4 Variational formulation

Hamilton's principle is used here to obtain the equations of motion. The concept can be expressed in analytical form as (Aissani *et al.* 2015, Bourada *et al.* 2015, Kheroubi *et al.* 2016, Bensaid *et al.* 2017)

$$\delta \int_0^T (U + V - K) dt = 0 \tag{11}$$

where δU is the variation of the strain energy; δV represents the potential energy; and the variation of the kinetic energy is given by δK. The variation of the strain energy of the beam can be expressed by the following relation

$$\delta U = \int\limits_0^L \int\limits_A \left(\sigma_x \delta\varepsilon_x + \tau_{xz} \delta\gamma_{xz} \right) dA dx$$

$$= \int\limits_0^L \left(N \frac{d\delta u_0}{dx} - M_b \frac{d^2\delta w_b}{dx^2} - M_s \frac{d^2\delta w_s}{dx^2} + Q \frac{d\delta w_s}{dx} \right) dx \tag{12}$$

where M_b, M_s and Q represent the stress resultants and they are expressed as

$$\left(M_b, M_s \right) = \int\limits_A (z, f) \sigma_x dA \quad \text{and} \quad Q = \int\limits_A g \tau_{xz} dA \tag{13}$$

The variation of the potential energy caused by the practical loads can be given as

$$\delta V = -\int\limits_0^L (q + f_e) \delta\left(w_b + w_s \right) dx; \tag{14}$$

where q is the transverse external load. The Pasternak type model is utilized to simulate the interaction of the nanobeams with the elastic foundation as follows (Aissani *et al*. 2015, Zidi *et al*. 2014, Khalfi *et al*. 2014, Ait Amar Meziane *et al*. 2014)

$$f_e = k_w w - k_p \frac{\partial^2 w}{\partial x^2} \tag{15}$$

where k_w and k_p are the Winkler and the Pasternak parameters of the elastic foundation, respectively.

The variation of the kinetic energy can be derived as follows

$$\delta K = \int\limits_0^L \int\limits_A \rho\left[\dot{u}\delta\dot{u} + \dot{w}\delta\dot{w} \right] dA dx$$

$$= \int\limits_0^L \left\{ I_0 \left[\left(\dot{w}_b + \dot{w}_s \right) \left(\delta\dot{w}_b + \delta\dot{w}_s \right) \right] + I_2 \left(\frac{d\dot{w}_b}{dx} \frac{d\delta\dot{w}_b}{dx} \right) + K_2 \left(\frac{d\dot{w}_s}{dx} \frac{d\delta\dot{w}_s}{dx} \right) \right\} dx \tag{16}$$

$$+ J_2 \left(\frac{d\dot{w}_b}{dx} \frac{d\delta\dot{w}_s}{dx} + \frac{d\dot{w}_s}{dx} \frac{d\delta\dot{w}_b}{dx} \right) $$

Where dot-superscript sign defines the differentiation with sense to the time variable t; ρ is the mass density; and (I_0, I_2, J_2, K_2) are the mass inertias expressed as

$$\left(I_0, I_2, J_2, K_2 \right) = \int\limits_A \left(1, z^2, z, f, f^2 \right) \rho dA \tag{17}$$

The explicit equations of motion of the new proposed nonlocal strain-gradient beam model are obtained by substituting the expressions for δU, δV, and δK from Eqs. (12), (14) and (15) into Eq. (11), integrating by parts, and collecting the coefficients of δw, and $\delta\varphi$, and which are given as

follows

$$\delta w_b : \frac{d^2 M_b}{dx^2} + q - f_e = I_0 \left(\ddot{w}_b + \ddot{w}_s \right) - I_2 \frac{d^2 \ddot{w}_b}{dx^2} - J_2 \frac{d^2 \ddot{w}_s}{dx^2},$$ (18a)

$$\delta w_s : \frac{d^2 M_s}{dx^2} + \frac{dQ}{dx} = -J_2 \frac{d^2 \ddot{w}_b}{dx^2} - K_2 \frac{d^2 \ddot{w}_s}{dx^2},$$ (18b)

However, the Euler–Bernoulli beam theory can be obtained from the equilibrium equations in Eq. (18), by neglecting the shear deformation effect $w_s = 0$).

The stress resultants are obtained by substituting Eq. (7) into Eq. (10) and the subsequent results into Eq. (13), and are given as follows

$$M_b - \mu \frac{d^2 M_b}{dx^2} = \left(1 - \eta \frac{\partial^2}{\partial x^2} \right) \left(-D \frac{d^2 w_b}{dx^2} - D_s \frac{d^2 w_s}{dx^2} \right)$$ (19a)

$$M_s - \mu \frac{d^2 M_s}{dx^2} = \left(1 - \eta \frac{\partial^2}{dx^2} \right) \left(-D_s \frac{d^2 w_b}{dx^2} - H_s \frac{d^2 w_s}{dx^2} \right)$$ (19b)

$$Q - \mu \frac{d^2 Q}{dx^2} = \left(1 - \eta \frac{\partial^2}{dx^2} \right) \left(A_s \frac{dw_s}{dx} \right)$$ (19c)

where

$$(D, D_s, H_s) = \int_A \left(z^2, zf, f^2 \right) E dA, \qquad A_s = \int_A g^2 G dA$$ (20)

The nonlocal strain gradient equations of motion of the present shear deformable nano beam resting on two-parameter elastic medium, can be written in terms of displacements (w_b, w_s) by substituting Eq. (19) into Eq. (18) as

$$-D \left(1 - \eta \frac{\partial^2}{dx^2} \right) \frac{d^4 w_b}{dx^4} - D_s \left(1 - \eta \frac{\partial^2}{dx^2} \right) \frac{d^4 w_s}{dx^4} + q - \mu \frac{d^2 q}{dx^2} - f_e + \mu \frac{d^2 f_e}{dx^2} =$$

$$I_0 \left((\ddot{w}_b + \ddot{w}_s) - \mu \frac{d^2 (\ddot{w}_b + \ddot{w}_s)}{dx^2} \right) - I_2 \left(\frac{d^2 \ddot{w}_b}{dx^2} - \mu \frac{d^4 \ddot{w}_b}{dx^4} \right) - J_2 \left(\frac{d^2 \ddot{w}_s}{dx^2} - \mu \frac{d^4 \ddot{w}_s}{dx^4} \right)$$ (21a)

$$-D_s \left(1 - \eta \frac{\partial^2}{dx^2} \right) \frac{d^4 w_b}{dx^4} - H_s \left(1 - \eta \frac{\partial^2}{dx^2} \right) \frac{d^4 w_s}{dx^4} + A_s \left(1 - \eta \frac{\partial^2}{dx^2} \right) \frac{d^2 w_s}{dx^2}$$

$$= I_0 \left((\ddot{w}_b + \ddot{w}_s) - \mu \frac{d^2 (\ddot{w}_b + \ddot{w}_s)}{dx^2} \right) - J_2 \left(\frac{d^2 \ddot{w}_b}{dx^2} - \mu \frac{d^4 \ddot{w}_b}{dx^4} \right) - K_2 \left(\frac{d^2 \ddot{w}_s}{dx^2} - \mu \frac{d^4 \ddot{w}_s}{dx^4} \right)$$ (21b)

By setting the scale parameter μ and η equal to zero the equations of motion of local beam theory can be derived from Eq. (21).

3. Resolution method for S-S nanobeams

This part is dedicated to give an analytical resolution to resolve the general nonlocal governing equations for free vibration of nanosized beam resting on two-parameter elastic medium with simply supported (S-S) boundary edges.

To guarantee the boundary conditions and general governing equations of motion, the displacements fields are adopted to be of the type

$$\begin{Bmatrix} w_b \\ w_s \end{Bmatrix} = \sum_{n=1}^{\infty} \begin{Bmatrix} W_{bn} \sin(\alpha x)e^{i\omega t} \\ W_{sn} \sin(\alpha x)e^{i\omega t} \end{Bmatrix},$$ (22)

where W_n, and W_{sn} are arbitrary parameters to be determined, ω is the eigenfrequency associated with nth eigenmode, and $\alpha = n\pi/L$. The transverse applied load q is also expressed in the Fourier series as

$$q(x) = \sum_{n=1}^{\infty} Q_n \sin(\alpha x), \qquad Q_n = \frac{2}{L}\int_0^L q(x)\sin(\alpha x)dx$$ (23)

The Fourier coefficients Q_n related with some typical loads are given as follows

$$Q_n = q_0, \quad n = 1 \qquad \text{for sinusoidal load}$$ (24a)

$$Q_n = \frac{4q_0}{n\pi}, \quad n = 1,3,5.... \qquad \text{for uniform load}$$ (24b)

$$Q_n = \frac{2q_0}{L}\sin\frac{n\pi}{2}, \quad n = 1,2,3.... \quad \text{for point load } Q_0 \text{ at the midspan}$$ (24c)

By inserting the expansions of w_b, w_b sand q from Eqs. (22) and (23) into Eq. (21), respectively, leads to

$$\left\{[K]+[M]\omega^2\right\}\begin{Bmatrix} W_n \\ \phi_n \end{Bmatrix} = 0$$

In which $[K]$ and $[M]$ represent the stiffness, and mass matrixes for the nanobeam, respectively.

$$s_{11} = D\alpha^4 + \lambda\left(k_w + k_p\alpha^2\right), \quad s_{12} = D_s\alpha^4 + \lambda\left(k_w + k_p\alpha^2\right),$$
$$s_{22} = H_s\alpha^4 + A_s\alpha^2 + \lambda\left(k_w + k_p\alpha^2\right), \quad \lambda = 1 + \mu\alpha^2$$ (25)
$$m_{11} = I_0 + I_2\alpha^2, \quad m_{12} = I_0 + J_2\alpha^2, \quad m_{22} = I_0 + K_2\alpha^2$$

4. Numerical results and discussions

In this section, dynamic behavior of nonlocal strain gradient nanobeams is investigated based on a newly higher order refined beam model resting on Pasternak foundation. The present model takes into account two scale parameters associated to nonlocal and strain gradient effects for more accurate modeling of nanobeams. Configuration of nanobeam on elastic substrate is shown in Fig. 1. For all computations, the Poisson's ratio v is taken as 0.3. The length of nanobeam is considered to be $L = 10$ nm. Calculations are executed considering the non-dimensional form of natural frequencies and foundation parameters as follows

- $\overline{\omega} = \omega L^2 \sqrt{\dfrac{I_0}{EI}}$ frequency parameter ;

- $K_w = \dfrac{k_w L^4}{EI}$ Winkler parameter ;

- $K_p = \dfrac{k_p L^4}{EI}$ Pasternak parameter.

To verify the correctness of the current developed nanobeam model, the obtained results are

Table 1 Comparison of results of dimensionless frequency for S-S nanobeams ($K_w = K_p = 0$)

μ	L/h	Aissani *et al.* (2015)	Present
0	5	9.2854	9.2748
	10	9.7107	9.7076
	20	9.8289	9.8281
	100	9.8679	9.8679
1	5	8.8585	8.8484
	10	9.2643	9.2613
	20	9.3771	9.3763
	100	9.4143	9.4142
2	5	8.4856	8.4759
	10	8.87431	8.8714
	20	8.9823	8.9816
	100	9.0179	9.0179
3	5	8.1561	8.1468
	10	8.5297	8.5269
	20	8.6335	8.6328
	100	8.6678	8.6678
4	5	7.8622	7.8533
	10	8.2224	8.2197
	20	8.3225	8.3218
	100	8.3555	8.3555

compared with those obtained by Aissani *et al.* (2015), as tabulated in Table 1. It can be seen that the obtained results agree well with those presented by Aissani *et al.* (2015), for various values of nonlocal parameter μ. The strain gradient parameter has been neglected ($\eta = 0$) (Li *et al.* 2016a, b), for this case of comparison. We can see a rise in the values the nonlocal scale parameter yields to a decrement in the natural frequencies.

Fig. 2 presents the maximum magnitude of dimensionless frequency of NL-SG nanobeam against length-to-thickness ratio L/h for various nonlocal theories (NSGT, SGT, NET and CET)
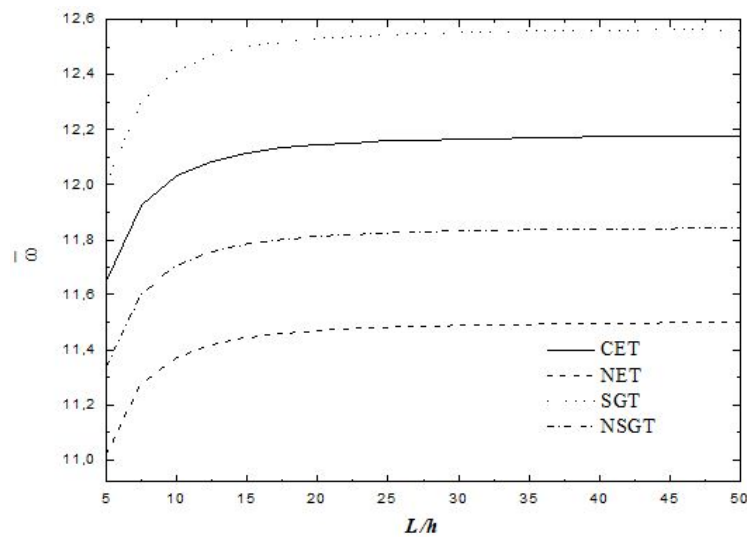

Fig. 2 Impact of slenderness ratio on nondimensional frequency of the nanobeam for various nonlocal theories ($K_w = 0$, $K_p = 0$)
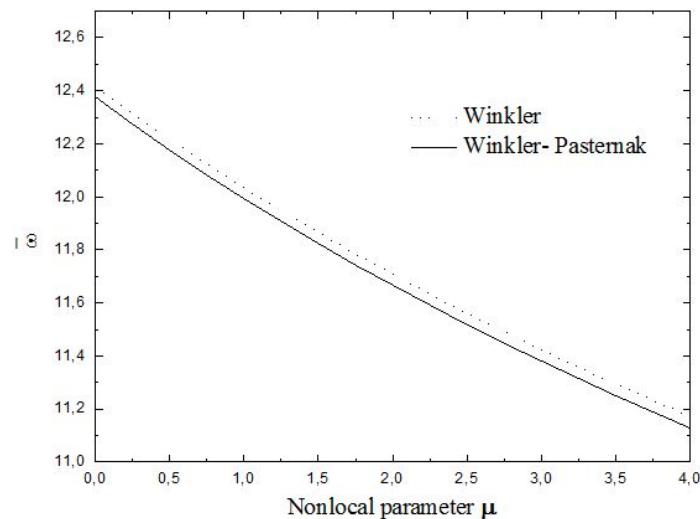

Fig. 3 Effect of nonlocal parameter on dimensionless frequency of nanobeam under various types of foundation ($L/h = 10$, $\eta = 1$)

when $K_w = K_p = 0$. It is observed that frequency is significantly influenced by slenderness ratio (L/h). But when to a certain value is exceeded, dimensionless frequency remains unchanged with increase of slenderness ratio. One we can observe that NET gives smaller frequencies than CET due to the integration of nonlocal parameter by mean that nanobeam deploy a stiffness-softening effect. In addition, NSGT provides larger frequencies compared NET by increasing the rigidity of nanobeam. This phenomenon shows that the FG nanobeam exerts a stiffness-hardening in the case where the length scale parameter is included in the model.
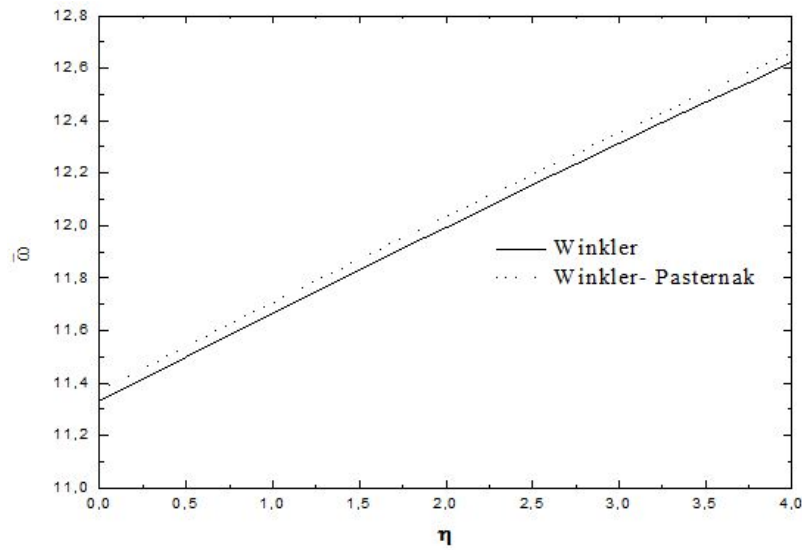

Fig. 4 Effect of length scale parameter on dimensionless frequency of nanobeam under various types of foundation ($L/h = 10, \mu = 1$)
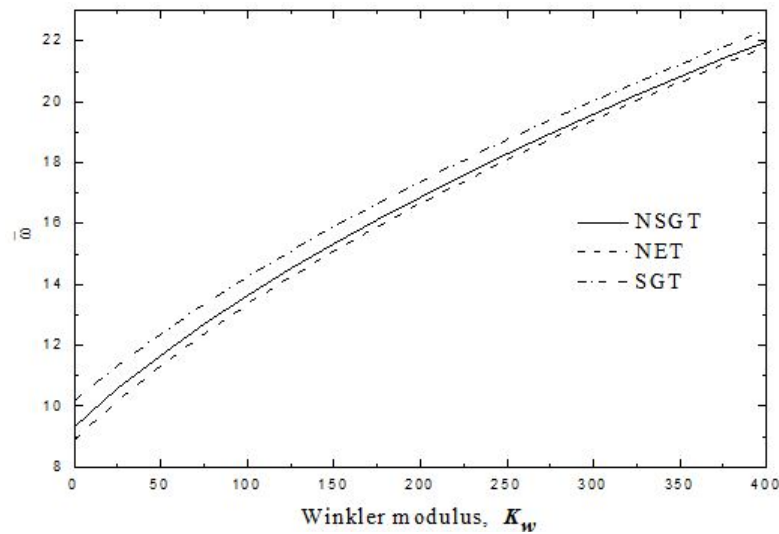

Fig. 5 Effect of Winkler modulus parameter on the dimensionless frequency of nanobeam for various nonlocal parameters ($K_p = 5, L/h = 10$)

Variation of nondimensional frequency in expressions of nonlocal parameter and length scale factor for various types of foundation at $L/h = 10$ are illustrated in Figs. 3 and 4. It is seen that a growth of the nonlocal parameter (μ) decreases the value of dimensionless frequency due to the lower stiffness of the nanobeam in this case. However, it is seen that a rise in length scale parameter (η) provides elevated frequencies values. The cause is that the nanobeam deploys stiffness-hardening impact as length scale parameter increases. It is also remarked that Winkler-Pasternak layer has more significant effect on dimensionless frequency of NGST nanobeam compared to Winkler parameter. The reason is that the combination of the two layers at ends makes the nanobeam stiffer.
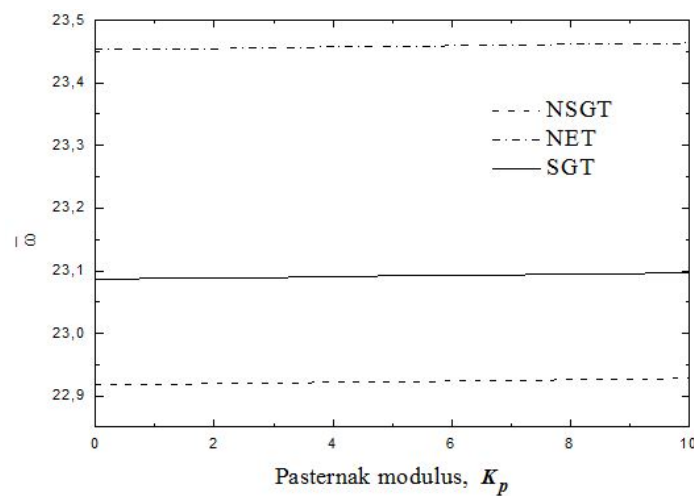

Fig. 6 Effect of Pasternak modulus parameter on the dimensionless frequency of nanobeam for various nonlocal parameters ($K_w = 100$, $L/h = 10$)
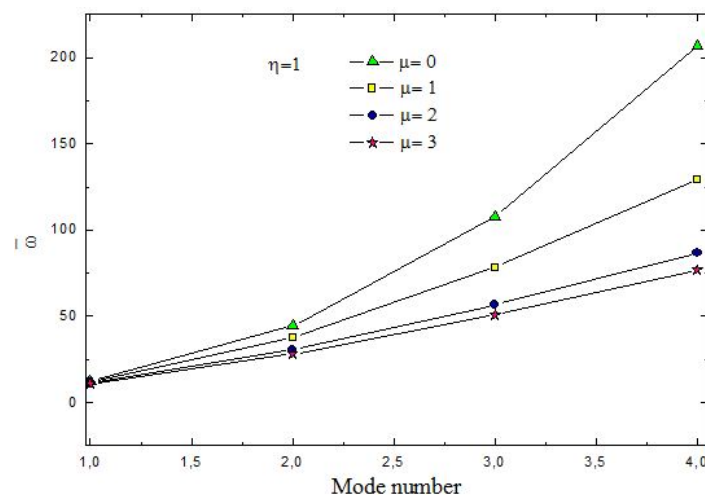

Fig. 7 Effect of mode numbers *n* on the dimensionless frequency of nanobeam for various values of nonlocal parameter ($K_w = 100$, $K_p = 5$, $L/h = 10$)
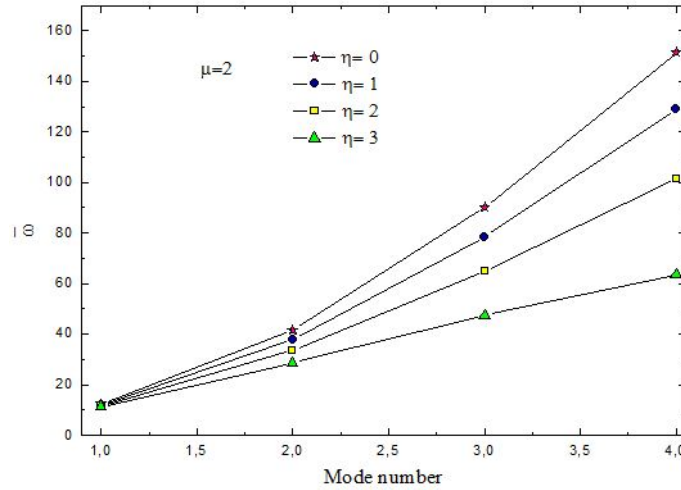
Fig. 8 Effect of mode numbers **n** on the dimensionless frequency of nanobeam for various values of
length scale parameter ($K_w = 100$, $K_p = 5$, $L/h = 10$)

Figs. 5 and 6 depict the change of maximum nondimensional frequency of nanobeam against Winkler (K_w) and Pasternak (K_p) parameters, respectively, for diverse nonlocal continuum theories at $L/h = 10$. It is observed that increasing in both (K_w) and (K_p) leads to larger frequencies. This is caused by an enhancement in the rigidity of nanobeam when it is rested on elastic foundation. In addition, one can notice that NET provides inferior frequencies compared to NSGT by neglecting the stiffness-hardening impact.

Variation in the pick values of nondimensional frequency of NSGT nanosize beam lying on elastic foundation against mode number is showed in Figs. 7 and 8, for various values of length scale and nonlocal coefficients by $L/h = 10$, $K_w = 50$ and $K_p = 10$. It is observed from these graphs that a rise in the mode number gives larger natural frequency. Also, we can observe that the effect of both length scale and nonlocal parameters become more important from second, third, fourth and fifth frequencies.

5. Conclusions

In the present work, nonlocal strain gradient higher order refined beam model is employed to investigate dynamic response of nanobeams resting on Pasternak-type foundation. The model takes into account two parameters related to nonlocal stress field and the strain gradient stress field to capture the size dependency of nanobeam more reliably. The elastic medium was simulated by using both Winkler and Pasternak-type models. The governing equations of the NSGT nanobeam are derived by applying Hamilton's principle, and then solved analytically with simply supported boundary conditions. It is shown that frequency of nanobeam decreases with the increment of nonlocal parameter. In contrast, frequency gets bigger with increase of length scale parameter which emphasizes the stiffness-hardening impact because of the strain gradients. However, effect of both nonlocal and length scale parameters on dynamic response of nanobeams are more significant at higher values of mode number. An improvement of the present model will be considered in the near future dealing on the thickness stretching effect (Bourada *et al*. 2015,

Tounsi *et al*. 2013d, Houari *et al*. 2018).

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