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Influence of porosity and axial preload on vibration behavior of rotating FG nanobeam

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Abstract. In this paper, a nanobeam connected to a rotating molecular hub is considered. The vibration behavior of rotating functionally graded nanobeam based on Eringen's nonlocal theory and Euler–Bernoulli beam model is investigated. Furthermore, axial preload and porosity effect is studied. It is supposed that the material attributes of the functionally graded porous nanobeam, varies continuously in the thickness direction according to the power law model considering the even distribution of porosities. Porosity at the nanoscopic length scale can affect on the rotating functionally graded nanobeams dynamics. The equations of motion and the associated boundary conditions are derived through the Hamilton's principle and generalized differential quadrature method (GDQM) is utilized to solve the equations. In this paper, the influences of some parameters such as functionally graded power (FG-index), porosity parameter, axial preload, nonlocal parameter and angular velocity on natural frequencies of rotating nanobeams with pure ceramic, pure metal and functionally graded materials are examined and some comparisons about the influence of various parameters on the natural frequencies corresponding to the simply-simply, simply-clamped, clamped-clamped boundary conditions are carried out.

Keywords: vibration; functionally graded nanobeam; porosity; rotation; Eringen's nonlocal elasticity; GDQ method

1. Introduction

Nanotechnology is capable to create new materials with immense range of applications such as, medicine, electro mechanical systems and energy production. Lately, carbon nanotubes and nanobeams have received a special consideration from scholars in nanotechnology community. In order to analysis of the mechanical features of structures, continuum and semi-continuum models and atomistic models were provided. Semi-continuum and atomistic models are not qualified for analyzing large scale systems and need heavy computations. On the other hand, the classical continuum theories are not suitable for analysis of nanostructures. Therefore, nonlocal elasticity theory of Eringen which considers the size effect is utilized for nanostructures analysis.

The nonlocal elasticity theory presents an excellent tool for prediction of the nanostructures behavior and is a modified form of classical elasticity theory. Actually the stress at a reference

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point is a function of strains at all points in the body. Eringen and Edelen (1972) presented the nonlocal continuum theory to take the size-dependent effect into account. Afterwards, multitude of literatures has appeared with considering the application of this theory in nanostructures oscillations.

Peddieson *et al*. (2003) utilized this theory to study the static deformations of beams. Sudak (2003) investigated the buckling behavior of multiwalled carbon nanotubes using the Eringen's nonlocal theory and understood that the small-scale effect contribute to the reduction of critical axial buckling. Zhang *et al*. (2004b) reported a multiple shell model for the analysis of the axial buckling of multiwalled carbon nanotubes under axial compression using the nonlocal continuum mechanics and the influence of the small-scale effect on the axial buckling strain was discussed. Murmu and Adhikari (2010a) utilized the nonlocal elasticity theory to study the free vibration analysis of a nonlocal double-elastic beam. Thai (2012), Thai and Vo (2012) investigated the buckling and vibration characteristics of nanobeam using the Eringen's nonlocal theory. Kiani (2010) utilized this theory to study free transverse vibration analysis of embedded single-walled carbon nanotubes (SWCNTs) with various boundary conditions via meshless model. Zhang *et al*. (2007) reported the transverse vibration of double-walled carbon nanotubes including the thermal effects with utilization of nonlocal elasticity theory. Kiani and Mehri (2010) utilized the nonlocal elasticity theory to study vibration characteristics of a nanotube subjected to a moving nanoparticle. Ansari and Sahmani (2012) used this theory to study the small scale effect on vibration behavior of SWCNTs with arbitrary boundary conditions.

Space planes require high-performance heat-resistant materials which can withstand ultrahigh temperatures and extremely large temperature gradients. To meet these needs, functionally gradient materials (FGMs) were proposed in 1985, Koizumi and Niino (1995).

Commonly, FGMs are made of a combination of ceramics and a mixture of various metals, Zhao *et al*. (2012) and Ebrahimi (2013). Zenkour and Abouelregal (2015) considered thermoelastic interaction in functionally graded nanobeams subjected to time-dependent heat flux. Ebrahimi and Barati (2016) presented an exact solution for buckling analysis of embedded piezo- electro-magnetically actuated nanoscale beams. Sankar (2001) reported an elasticity solution for functionally graded Euler-Bernoulli beam subjected to static transverse loads. In recent years, FGMs have received a special attention from researchers in industrial community. Such as, optics, chemical, nuclear, mechanical, electronics, Ebrahimi (2013). Literatures show that increasing attentions exists for dynamic and static analysis of FG beams, Larbi *et al*. (2013), Chakraborty and Gopalakrishnan (2003), Aydogdu and Taskin (2007), Ying *et al*. (2008), Kapuria *et al*. (2008), Yang and Chen (2008), Li (2008), Yang *et al*. (2008), Akgöz and Civalek (2014) and Barretta *et al*. (2015b). Pradhan and Chakraverty (2013) investigated the vibration characteristics of functionally graded beams based on the classical and first-order shear deformation beam theories by using the Rayleigh–Ritz method.

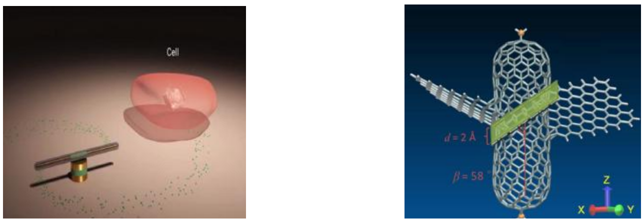
Barretta *et al*. (2015a) used a nonlocal thermodynamic method and new nonlocal elastic model to study bending of functionally graded Bernoulli-Euler nanobeams. They utilized the nonlocal expressions of the free energy and reported that the nonlocal behavior for new nonlocal model is based on a participation factor and a small length-scale parameter, it should be noted that suitable choices of the participation factors can make the functionally graded Bernoulli-Euler nanobeam stiffer or more flexible. Barretta *et al.* (2016a) reported a modified gradient nonlocal elasticity model for functionally graded Timoshenko nanobeams via a nonlocal thermodynamic method by considering the first derivatives of the shear and axial strains. They investigated the influence of the gradient components which are usually disregarded in the unmodified Eringen model on the bending treatment of functionally graded Timoshenko nanobeams. Ghadiri *et al*. (2016a) utilized the nonlocal elasticity theory and examined free vibration behavior of Euler-Bernoulli FG nanobeam accompanied by rotation and surface effects through differential quadrature method.

Eringen differential model provides vanishing size effects in nanobeams subjected to point loads and is not appropriate for describing the treatment of a cantilever nanobeam under a concentrated load at free-end, Barretta *et al*. (2016b). In recent paper Barretta *et al*. (2016b) utilized the Eringen differential law together with an additional term involving the derivative of the axial stress and presented a modified version of Eringen differential law. Jin and Wang (2015) used the weak form quadrature element method and studied the free vibration analysis of functionally graded beams based on the classical beam theory. Eltaher *et al*. (2012) used the finite element method to study the free vibration analysis of functionally graded size-dependent for the Euler-Bernoulli beam theory.

The porosity effect has significant role, and it should not be ignored in the vibrational study of functionally graded beams, because the porosities occur inside functionally graded materials during fabrication. Ebrahimi and Mokhtari (2015) used the differential transform method to investigated the free vibration characteristics of rotating porous beam with functionally graded microstructure based on Timoshenko beam theory. Then, Ebrahimi and Zia (2015) used the multiple scales and Galerkin's methods to study the nonlinear transverse vibration properties of functionally graded porous Timoshenko beams. Şimşek (2016) investigated the nonlinear free vibration of a FG nanobeam using the nonlocal strain gradient and Euler-Bernoulli beam theories and a novel Hamiltonian approach with considering the von- Kármán's geometric nonlinearity.

In recent years, the scholars found that many nanodevices have a rotating motion and therefore, they edicate their researches to design the rotating nanomachines. As rotation effect is an important design factor in the study of nanostructures, it is necessary to understand the vibration behavior of the rotary nanodevices. Many researchers dedicate their studies to design of rotating nanomachines that can generate controllable rotation. Such as, biological molecular motors, unprecedented chemical synthesis, Chen *et al*. (2012), Tierney *et al*. (2011), Lubbe *et al*. (2011), Goel and Vogel (2008), Bath and Turberfield (2007) and Van Delden *et al*. (2005), turnstiles, Bedard and Moore (1995), ratchets, Serreli *et al*. (2007), artificial muscles, Liu *et al*. (2005), cars, Khatua *et al*. (2009) and Kudernac *et al*. (2011). Zhang *et al*. (2004a) reported a model for analysis a double-walled CNT as rotational bearings. Narendar (2011) presented an atomistic model for a rotating SWCNT using the nonlocal elasticity theory. Aranda-Ruiz *et al*. (2012) investigated the bending vibration characteristics of nonuniform rotating with clamp-free boundary conditions using the nonlocal elasticity theory via differential quadrature method (DQM). Pradhan and Murmu (2010) utilized the nonlocal elasticity theory and examined flapwise bending vibration characteristics of a rotating nanobeam with clamp-free boundary conditions via differential quadrature method (DQM). Narendar (2012) investigated flapwise bending vibration analysis of SWCNT with consideration of rotation effect and rotary inertia and transverse shear deformation via differential quadrature method (DQM). Guo *et al*. (2015) reported a model for micromotors made by a patterned Au/Ni/Cr nanodisk as bearing and a nanowire as rotor and they investigated the rotation characteristics of the micromotors (Fig. 1(a)).

Li *et al*. (2014) used the molecular dynamics simulations and reported a model for nano-turbine was made by a CNT and graphene nanoplates (Fig. 1(b)). They used the gromacs software package and studied the rotating motion of a nano-turbine and and grapheme nanoplates. They comprehend that the thermal fluctuations at the nanoscopic length scale are very important. Ilkhanin and Hosseini-Hashemi (2016) used the modified couple stress theory to study the size



(a) (b)

Fig. 1 (a) One of the applications of nanomotors presented by Guo *et al*. (2015); (b) Schematic of the nano-turbine presented by Li *et al*. (2014)

dependent free vibrations of rotating nanobeams with considering the effects of Coriolis force and tangential load and understood that the influence of the scale parameter on natural frequency of the rotating nano-tube was stronger than tangential load. Ghadiri and Shafiei (2016) used the nonlocal elasticity theory to examine the flapwise bending vibrations of a nano-turbine blade. They investigated vibration characteristic of a cantilever and propped cantilever rotary nanoplate by DQM. Benvenuti and Simone (2013) investigated the equivalence between nonlocal and gradient elasticity models of one-dimensional boundary value problem. The local/nonlocal and fully nonlocal stress-strain laws was applied and corresponding equilibrium equations of a tensile rod were obtained by Benvenuti and Simone (2013). Then, corresponding closed-form solutions were obtained for the local/nonlocal and the fully nonlocal models. Romano *et al*. (2017) represented paradoxes in solving nonlocal elastic problems of simple beams, such as the cantilever under end-point loading. They found that the elastic beam problem relevant to Eringen's nonlocal integral law does not avouch existence being the paradoxes. For this purpose Romano *et al*. (2017) considered the local/nonlocal constitutive mixture and it was found that the local elastic fraction of the mixture impels well-posedness. Claim to find the exact solution for bending of Timoshenko and Euler-Bernoulli beams using Eringen's nonlocal integral model was presented by Tuna and Kirca (2016). Then, comment on Tuna and Kirca (2016) work was reported by Romano and Barretta (2016). Romano and Barretta (2016) showed that the claim to find the exact solution by Tuna and Kirca (2016) is not valid. A comparison of the superiority of stress-driven nonlocal integral model versus the strain-driven nonlocal integral model was presented by Romano and Barretta (2017). It was found that the small-scale effects in Euler-Bernoulli nanobeams formulated according to stress-driven nonlocal integral model can be efficiently simulated. Also, it was understood that the stress-driven nonlocal integral model removes the indispensable difficulties represented by the strain-driven nonlocal integral model.

It should be noted that, none of the previous articles have considered the porosity and axial preload effects on a rotary functionally graded nanobeam based on the nonlocal elasticity theory for various boundary conditions. This paper will be practical for engineers who are designing nanoactuators, nanosensors, nano-turbines, molecular nano-motors and nano-electro-mechanical machines.

2. Problem formulation

Consider a functionally graded nanobeam with the length L, uniform thickness h and cross sectional area A. The coordinate system is shown in Fig. 2 and the nanobeam rotates about z-axis with constant angular velocity Ω and hub radius is R.

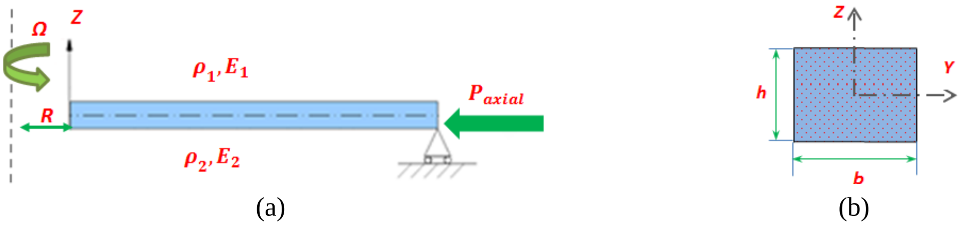


Fig. 2 (a) Rotating functionally graded nanobeam; (b) Cross section area of the porosed functionally graded nanobeam with even distribution of porosities

It is supposed that the material attributes of the functionally graded nanobeam, such as Young's modulus, mass density, vary continuously in the thickness direction according to the power law which are expressed as follows

$$V_1(z)=\left(\frac{z}{h}+\frac{1}{2}\right)^k \tag{1}$$

$$V_2(z)=1-\left(\frac{z}{h}+\frac{1}{2}\right)^k \tag{2}$$

According to Eqs. (1)-(2), it can be seen that the top and bottom surfaces of the nanobeam are made by pure ceramic and pure metal, respectively. Using the rule of mixture, distribution of a material feature *p*, in the thickness direction is defined as follows, Şimşek and Yurtcu (2013)

$$p(z)=p_1V_1+p_2V_2 \tag{3}$$

Where V_1 and V_2 are the volume fractions at the upper and lower surfaces of the nanobeam and these are related by the following equation

$$V_1+V_2=1 \tag{4}$$

The material specifications at the upper surface of the nanobeam is p_1 and the material specifications at the lower surface of the nanobeam is p_2. According to Eqs. (1)-(4) and considering the even distribution of porosities, the effective material properties of the functionally graded nanobeam can be acquired as below, Shafiei *et al*. (2016a)

$$p(z)=(p_1-p_2)\left(\frac{z}{h}+\frac{1}{2}\right)^k+p_2-(\alpha/2)(p_1+p_2) \tag{5}$$

where k is a positive number which determines the material distribution in the z direction and α indicates the porosity volume fraction. For example density and Young's modulus for functionally graded nanobeam with considering the porosity effect can be defined as

$$\rho(z)=(\rho_1-\rho_2)\left(\frac{z}{h}+\frac{1}{2}\right)^k+\rho_2-(\alpha/2)(\rho_1+\rho_2) \tag{6a}$$

$$E(z) = (E_1 - E_2)\left(\frac{z}{h} + \frac{1}{2}\right)^k + E_2 - (\alpha/2)(E_1 + E_2) \tag{6b}$$

Here, ρ_1 and ρ_2 are the mass density for upper and lower surfaces, respectively. Also, E_1 and E_2 indicates the Young's modulus for upper and lower surfaces of the nanobeam, respectively.

3. The Euler-Bernoulli beam theory

Displacement field (u_1, u_2, u_3) at any point of the Euler-Bernoulli beam (x, z) can be defined as follows

$$u_1(x,z,t) = u(x,t) - z\frac{\partial w(x,t)}{\partial x} \tag{7a}$$

$$u_2(x,z,t) = 0 \tag{7b}$$

$$u_3(x,z,t) = w(x,t) \tag{7c}$$

Where t is time, u and w are displacement components. The nonzero strain–displacement relations for the Euler-Bernoulli beam are acquired as follows

$$\varepsilon_{xx}^E = \frac{\partial u(x,t)}{\partial x} - z\frac{\partial^2 w(x,t)}{\partial x^2} \equiv \varepsilon_{xx}^0 - z\kappa^0 \tag{8}$$

Here ε_{xx}^0 and κ^0 are the extensional strain and the bending strain respectively. On the other hand, if the material of functionally graded beam obeys Hooke's law, the normal stress can be determined as follows

$$\sigma_{xx} = E(z)\varepsilon_{xx}^E \tag{9}$$

The strain energy can be obtained by

$$\delta U = \int_V \sigma_{ij}\delta\varepsilon_{ij}dV \tag{10}$$

Substituting Eq. (8) and Eq. (9) into Eq. (10) yields

$$\delta U = \int_0^L \left(N\left(\delta\varepsilon_{xx}^0\right) - M\left(\delta\kappa^0\right)\right)dx \tag{11}$$

Where, N and M are the axial force and the bending moment respectively and are defined as

$$N = \int_A \sigma_{xx}dA \tag{12}$$

$$M = \int_A z.\sigma_{xx} dA \tag{13}$$

Also, the kinetic energy T and Variation of kinetic energy δT for Euler-Bernoulli beam can be determined as follows

$$T = \frac{1}{2}\int_0^L \int_A \rho(z)\left(\left(\frac{\partial u_x}{\partial t}\right)^2 + \left(\frac{\partial u_y}{\partial t}\right)^2 + \left(\frac{\partial u_z}{\partial t}\right)^2\right) dAdx \tag{14a}$$

$$\delta T = \int_0^L \left[m_0\left(\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t} + \frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t}\right) - m_1\left(\frac{\partial u}{\partial t}\frac{\partial^2 \delta w}{\partial t \partial x} + \frac{\partial \delta u}{\partial t}\frac{\partial^2 w}{\partial t \partial x}\right) + m_2\left(\frac{\partial^2 w}{\partial t \partial x}\frac{\partial^2 \delta w}{\partial t \partial x}\right)\right] \tag{14b}$$

Here, m_0, m_1 and m_2 are the mass moments of inertia and can be defined as follows

$$m_i = \int_A \rho(z) z^i dz \tag{15}$$

Also, the work is done by external forces W^{ext} and the first variation of the external forces δW^{ext} for Euler-Bernoulli beam can be written as

$$W^{ext} = \frac{1}{2}\int_0^1 \left(\frac{\partial w}{\partial x}\right)^2 \bar{N} dx \tag{16a}$$

$$\delta W^{ext} = \int_0^L \left(\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x}\right) \bar{N} dx \tag{16b}$$

where $\bar{N}$ is

$$\bar{N} = N^{rotation} - P^{axial} \tag{16c}$$

Here, $N^{rotation}$ denotes the axial rotation force and P^{axial} is the initial axial force.

On the other hand, the Hamilton's principle can be written as

$$\int_0^t \delta\left(T - U + W^{ext}\right) dt = 0 \tag{17}$$

Substituting Eqs. (8)-(16), into Eq. (17), the following Euler–Lagrange equations can be determined as follows

$$\frac{\partial N}{\partial x} = m_0 \frac{\partial^2 u}{\partial t^2} - m_1 \frac{\partial^3 w}{\partial x \partial t^2} \tag{18}$$

$$\frac{\partial^2 M}{\partial x^2} - \frac{\partial}{\partial x}\left(\bar{N}\frac{\partial w}{\partial x}\right) = m_0 \frac{\partial^2 w}{\partial t^2} + m_1 \frac{\partial^3 u}{\partial x\, \partial t^2} - m_2 \frac{\partial^4 w}{\partial x^2 \partial t^2} \tag{19}$$

4. Nonlocal theory

Nonlocal theory states that the stress at a reference point is a function of strains at all points in the body. Accordingly, for a homogeneous, isotropic, nonlocal elastic solid, $\sigma_{ij}(x)$ at any point is defined as

$$\sigma_{ij}(x) = \int_V \tilde{\alpha}\left(\left|x - x'\right|, \tau\right) C_{ijkl} \varepsilon_{kl}(x') \, dV(x') \tag{20}$$

Here, σ_{ij}, C_{ijkl} and u_i are the stress tensor, elastic modulus tensor and the displacement vector, respectively. In addition, $\tilde{\alpha}(|x - x'|, \tau)$ is the nonlocal kernel and $|x - x'|$ and $\tilde{\alpha}$ are the euclidean distance and nonlocal modulus, respectively. The strain tensor, ε_{ij}, can be written as

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{21}$$

Here, $\tau = \frac{(e_0.a)}{l}$ is a constant which represents external specification length l and internal specification length a and constant e_0 which depends on each material. It should be noted that the length scale coefficient $e_0 = \frac{\sqrt{\pi^2 - 4}}{2\pi} \cong 0.39$ was given by Eringen (1983). Also, Zhang *et al*. (2005) found $e_0 = 0.82$ by matching the nonlocal theoretical results of SWCNTs given by Zhang *et al*. (2004) with molecular dynamics simulations results given by Sears and Batra (2004).

Eq. (21) is arduous to solve. Therefore, according to nonlocal theory, the spatial integrals given by Eq. (21) can be converted to equal differential constitutive equations under certain conditions.

The constitutive relation was obtained by modified bessel function is as follows

$$\left(1 - (e_0.a)^2 \nabla^2\right)\sigma = C : \varepsilon \tag{22}$$

Here ∇^2 denotes the Laplacian operator and C is fourth order elasticity tensor.

The small-scale parameter ($e_0.a$) depends on the boundary conditions, nature of motion, chirality, geometric sizes, number of walls and mode shapes. A conservative estimation of the small-scale coefficient is smaller than 2.1 nm for SWCNTs if the value of frequency is assessed to be greater than 10 THz, Wang (2005). Also, Wang *et al*. (2007) showed that the non-dimensional nonlocal parameter $\frac{(e_0.a)}{l}$ is smaller than 0.6126 and 0.6138 for Euler–Bernoulli and Timoshenko cantilever nanobeams, respectively. Hereunto, there is no meticulous study made on predicting the magnitude of the length scale coefficient to simulate vibration behavior of functionally graded nanobeams, Eltaher *et al*. (2012), Ebrahimi and Salari (2015). Therefore, researchers investigated the mechanical behaviours of functionally graded nanobeams based on the nonlocal theory of Eringen by changing the value of the small scale parameter. In this study, the non-dimensional nonlocal parameter is considered to be in the range of 0-0.5.

For nonlocal FG beams, the nonlocal stress-strain relation may be simplified as follows, Ebrahimi and Salari (2015)

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E(z)\varepsilon_{xx} \tag{23}$$

Where ($\mu = (e_0.a)^2$)

Pursuant the nonlocal elasticity theory of Eringen, the force-strain and the moment-strain relations for the Euler-Bernoulli beam theory can be written as

$$N - \mu \frac{\partial^2 N}{\partial x^2} = A_{xx} \frac{\partial u}{\partial x} - B_{xx} \frac{\partial^2 w}{\partial x^2} \tag{24}$$

$$M - \mu \frac{\partial^2 M}{\partial x^2} = B_{xx} \frac{\partial u}{\partial x} - C_{xx} \frac{\partial^2 w}{\partial x^2} \tag{25}$$

In relations above, (A_{xx}, B_{xx}, C_{xx}) are the axial, coupling and bending stiffnesses, respectively and are defined as follows

$$(A_{xx}, B_{xx}, C_{xx}) = \int_A E(z)(1, z, z^2) dA \tag{26}$$

Substituting Eqs. (24)-(25), into Eqs. (18)-(19), axial normal force N and bending moment M, can be obtained as below

$$N = A_{xx} \frac{\partial u}{\partial x} - B_{xx} \frac{\partial^2 w}{\partial x^2} + \left(e.a\right)^2 \left(m_0 \frac{\partial^3 u}{\partial x\, \partial t^2} - m_1 \frac{\partial^4 w}{\partial x^2 \partial t^2} \right) \tag{27}$$

$$M = B_{xx} \frac{\partial u}{\partial x} - C_{xx} \frac{\partial^2 w}{\partial x^2} + \left(e.a\right)^2 \left(m_0 \frac{\partial^2 w}{\partial t^2} + m_1 \frac{\partial^3 u}{\partial x\, \partial t^2} - m_2 \frac{\partial^4 w}{\partial x^2 \partial t^2} + (N^{rotation} - P^{axial}) \frac{\partial^2 w}{\partial x^2} \right) \tag{28}$$

Finally, by substituting Eqs. (27)-(28) and Eqs. (18)-(19), the governing equations of motion for Euler-Bernoulli FG nanobeam including rotation effects and axial initial preload and porosity effect can be stated as

$$A_{xx} \frac{\partial^2 u}{\partial x^2} - B_{xx} \frac{\partial^3 w}{\partial x^3} + \left(e.a\right)^2 \left(m_0 \frac{\partial^4 u}{\partial t^2 \partial x^2} - m_1 \frac{\partial^5 w}{\partial t^2 \partial x^3} \right) - m_0 \frac{\partial^2 u}{\partial t^2} + m_1 \frac{\partial^3 w}{\partial x\, \partial t^2} = 0 \tag{29}$$

$$\begin{aligned} & B_{xx} \frac{\partial^3 u}{\partial x^3} - C_{xx} \frac{\partial^4 w}{\partial x^4} - (N^{rotation} - P^{axial}) \frac{\partial^2 w}{\partial x^2} - m_0 \frac{\partial^2 w}{\partial t^2} - m_1 \frac{\partial^3 u}{\partial t^2 \partial x} + m_2 \frac{\partial^4 w}{\partial t^2 \partial x^2} + \\ & \left(e.a\right)^2 \left(m_0 \frac{\partial^4 w}{\partial t^2 \partial x^2} + m_1 \frac{\partial^5 u}{\partial t^2 \partial x^3} - m_2 \frac{\partial^6 w}{\partial t^2 \partial x^4} + (N^{rotation} - P^{axial}) \frac{\partial^4 w}{\partial x^4} \right) = 0 \end{aligned} \tag{30}$$

According to Fig. 2, the uniform beam rotates about an axis parallel to the z direction with a constant counter clockwise rotational speed, Ω. The rotation force is created as follows, Aranda-Ruiz *et al*. (2012) and Pradhan and Murmu (2010)

$$N^{rotation} = \int_x^L \iint_A \rho dA \Omega^2 \left(\xi + R\right) d\xi \tag{31}$$

The boundary conditions for the Euler-Bernoulli functionally graded nanobeam can be obtained by following relations, Ebrahimi and Salari (2015)

$$N = 0 \quad or \quad u = 0 \qquad at \qquad x = 0 \;\; and \;\; x = l \tag{32}$$

$$\frac{\partial M}{\partial x} - m_1 \frac{\partial^2 u}{\partial t^2} + m_2 \frac{\partial^3 w}{\partial x \partial t^2} = 0 \quad or \quad w = 0 \qquad at \qquad x = 0 \;\; and \;\; x = l \tag{33}$$

$$M = 0 \quad or \quad \frac{\partial w}{\partial x} = 0 \qquad at \qquad x = 0 \;\; and \;\; x = l \tag{34}$$

5. Solution procedure

Bellman and Casti (1971), Bellman *et al*. (1972) and Ansari *et al*. (2010) presented the differential quadrature (DQ) method. Then, Shu (2000) and Shu and Richards (1992) modified the computation of weighting coefficients in this method. This method has high possiblity in solving equilibrium equations and has good accuracy to solve the partial differential equations.

Because of easy formulation and good accuracy of this method some scholars preferred to work on the analysis of nanostructures through differential quadrature method (DQM), Aranda-Ruiz *et al*. (2012), Pradhan and Murmu (2010), Shafiei *et al*. (2016a), Murmu and Pradhan (2009), Wang and Wang (2014), Pourasghar *et al*. (2015), Vosoughi *et al*. (2012) and Ghadiri *et al*. (2016a, b). The most notable stage in this method is detecting the weight coefficients. In this way the partial derivatives are calculated using these coefficients. The *r*th order derivative of $f(x_i)$ is approximated as a weighted linear summation as follows, Shu (2012)

$$\left.\frac{\partial^r f(x)}{\partial x^r}\right|_{x=x_i} = \sum_{j=1}^{n} C_{ij}^{(r)} f(x_i) \tag{35}$$

And $M(x)$, $C_{ij}^{(1)}$ is expressed as

$$M(x_i) = \prod_{j=1, i\neq j}^{n} (x_i - x_j) \qquad i = j \tag{36}$$

$$C_{ij}^{(1)} = \frac{M(x_i)}{(x_i - x_j) M(x_j)} \qquad i, j = 1, 2, ..., n \quad \text{and} \quad i \neq j \tag{37}$$

$$C_{ij}^{(1)} = -\sum_{j=1, i\neq j}^{n} C_{ij}^{(1)} \qquad i = j \tag{38}$$

The weighing coefficients along *x* direction can be written as

$$C_{ii}^{(r)} = -\sum_{j=1, i\neq j}^{n} C_{ij}^{(r)} \qquad i, j = 1, 2, ..., n \quad \text{and} \quad 1 \le r \le n-1 \tag{39}$$

$$C_{ij}^{(r)} = r\left[C_{ij}^{(r-1)} C_{ij}^{(1)} - \frac{C_{ij}^{(r-1)}}{\left(x_i - x_j\right)} \right] \qquad i,\, j = 1, 2, ..., n, i \neq j \quad \text{and} \quad 2 \leq r \leq n-1 \tag{40}$$

In the equations above, superscript r is the order of the derivative and n is the number of grid points. By implementation of generalized differential quadrature method into governing equation Eqs. (29)-(30), the following equation is expressed as

$$\begin{aligned} & A_{xx} \sum_{k=1}^{n} C_{i,k}^{(2)} u_i - B_{xx} \sum_{k=1}^{n} C_{i,k}^{(3)} w_i + (e.a)^2 \left(m_0 \frac{\partial^2}{\partial t^2} \sum_{k=1}^{n} C_{i,k}^{(2)} u_i - m_1 \frac{\partial^2}{\partial t^2} \sum_{k=1}^{n} C_{i,k}^{(3)} w_i \right) \\ & - m_0 \frac{\partial^2 u_i}{\partial t^2} + m_1 \frac{\partial^2}{\partial t^2} \sum_{k=1}^{n} C_{i,k}^{(1)} w_i = 0 \end{aligned} \tag{41}$$

$$\begin{aligned} & B_{xx} \sum_{k=1}^{n} C_{i,k}^{(3)} u_i - C_{xx} \sum_{k=1}^{n} C_{i,k}^{(4)} w_i - \sum_{k=1}^{n} C_{i,k}^{(1)} \left((N^{rotation} - P^{axial}) \sum_{k=1}^{n} C_{i,k}^{(1)} w_i \right) \\ & + (e.a)^2 \left(\begin{aligned} & m_0 \frac{\partial^2}{\partial t^2} \sum_{k=1}^{n} C_{i,k}^{(2)} u_i + m_1 \frac{\partial^2}{\partial t^2} \sum_{k=1}^{n} C_{i,k}^{(3)} u_i - m_2 \frac{\partial^2}{\partial t^2} \sum_{k=1}^{n} C_{i,k}^{(4)} w_i + \\ & \sum_{k=1}^{n} C_{i,k}^{(2)} \left[\sum_{k=1}^{n} C_{i,k}^{(1)} \left((N^{rotation} - P^{axial}) \sum_{k=1}^{n} C_{i,k}^{(1)} w_i \right) \right] \end{aligned} \right) \\ & - m_0 \frac{\partial^2 w_i}{\partial t^2} - m_1 \frac{\partial^2}{\partial t^2} \sum_{k=1}^{n} C_{i,k}^{(1)} u_i - m_2 \frac{\partial^2}{\partial t^2} \sum_{k=1}^{n} C_{i,k}^{(2)} w_i = 0 \end{aligned} \tag{42}$$

In the present analysis, in order to obtain a better convergence, Gauss–Chebyshev technique is used and stated as follows

$$\zeta_i = \frac{1}{2}\left(1 - \cos\left(\frac{(i-1)}{(n-1)} \pi \right) \right) \qquad i = 1, 2, 3, \, \dots \, , n \tag{43}$$

Substituting $w = We^{i\omega t}$ into Eqs. (41)-(42) and Eqs. (32)-(34) into Eqs. (41)-(42), the problem will be transformed into the eigen value problem and Eq. (44) will be solved, Shafiei *et al*. (2016b).

$$[\mathrm{K}]_{total} \{w\} = \Psi^4 \{w\} \tag{44}$$

It should be noted that the way to apply the boundary conditions by the GDQM is mentioned in Appendix A.

6. Numerical results

In this paper, the numerical results are studied for vibration analysis of the rotating Euler-Bernoulli FG nanobeam, accompanied by porosity and axial preload effects for simply-simply,

simply-clamped, clamped-clamped boundary conditions. In order to the generalize and simplify the solution of the governing equation, non-dimensional parameters are defined as bellow

$$\Psi^2 = \left(\frac{m_0}{EI}\right)_{ceramic} L^4\omega^2 \tag{45}$$

$$\Phi^2 = \left(\frac{m_0}{EI}\right)_{ceramic} L^4\Omega^2 \tag{46}$$

$$\mu = \frac{e_0 a}{L} \tag{47}$$

$$\delta = \frac{r}{L} \tag{48}$$

$$\zeta = \frac{x}{L} \tag{49}$$

Here, Ψ and Φ are the non-dimensional parameters for frequency and angular velocity, respectively. Also, μ and δ are the non-dimensional nonlocal parameter and non-dimensional hub radius, respectively. In Appendix B, variation of the non-dimensional fundamental frequency with respect to the sufficient number of grid points for rotating nanobeam related to simply-simply, simply-clamped and clamped-clamped boundary conditions are presented. It shows that, to get converged to exact results for GDQ method, 11 grid points are necessary. It should be noted that, for exact evaluating the second, third and the fourth modes of non-dimensional frequency, 19 grid points are enough.

6.1 Results and discussion

In this section, we have examined the vibration behavior of rotating Euler-Bernoulli FG nanobeam accompanied by porosity and axial preload effects for simply-simply, simply-clamped, clamped-clamped boundary conditions.

Table 1 presents the first and second non-dimensional natural frequencies of rotating Euler-Bernoulli nanobeam corresponding to simply-simply, simply-clamped, clamped-clamped boundary conditions (with L/d = 10, without porosity) for various values of μ, in comparison with results presented by Wang *et al*. (2007).

Figs. 3-5 demonstrate the variation of non-dimensional fundamental frequencies with respect to the non-dimensional angular velocity (varying from 0 to 5) for pure ceramic material, pure metal material and functionally graded material (with k = 1,5) for different values of the nonlocal parameters regarding to simply-simply, simply-clamped, clamped-clamped boundary conditions, respectively. Here, nonlocal parameter can be considered 0, 0.2, 0.4.

According to Figs. 3-5, it is found that when FG index power of the rotating nanobeam increases, the influence of nonlocal parameter on the non-dimensional fundamental frequences decreases (specially for simply-clamped, clamped-clamped boundary conditions). This means that, for pure ceramic and functionally graded materials with low FG-index, the nonlocal parameter has

Table 1 Comparison of non-dimensional fundamental frequencies and non-dimensional second frequencies for simply-simply (S-S), simply-clamped (S-C), clamped-clamped (C-C) boundary conditions (with L/h = 10) for various values of μ with respect to results presented by Wang *et al*. (2007)

B.C.	Frequency	μ = 0		μ = 0.1		μ = 0.3		μ = 0.5	
		Present	Wang *et al.* (2007)	Present	Wang *et al.* (2007)	Present	Wang *et al.* (2007)	Present	Wang *et al.* (2007)
S-S	Fundamental frequency	3.1415920	3.1416	3.0685301	3.0685	2.6799956	2.68	2.3022302	2.3022
	Second frequency	6.2831801	6.2832	5.7816627	5.7817	4.3013395	4.3013	3.4603981	3.4604
S-C	Fundamental frequency	3.9266013	3.9266	3.820890	3.8209	3.2828384	3.2828	2.7899265	2.7899
	Second frequency	7.0685764	7.0686	6.4648773	6.4649	4.7667505	4.7668	3.8324967	3.8325
C-C	Fundamental frequency	4.7300395	4.73	4.5944554	4.5945	3.9183654	3.9184	3.3153208	3.3153
	Second frequency	7.853197	7.8532	7.1402411	7.1402	5.1963037	5.1963	4.1560694	4.1561

Table 2 Comparison of non-dimensional fundamental frequencies with various values of nonlocal parameters and porosity volume fraction parameters for simply-simply, simply-clamped, clamped-clamped boundary conditions. (FG_Index = 0.5, angular velocity = 3, axial preload = 10)

Fundamental frequency		α = 0	α = 0.1	α = 0.2	α = 0.3	α = 0.4	α = 0.5
Simply supported	μ = 0	12.24254	12.34991	12.48741	12.66989	12.92384	13.3019
	μ = 0.1	12.01192	12.11663	12.25075	12.42874	12.67649	13.04538
	μ = 0.2	11.49955	11.59832	11.72486	11.89283	12.12669	12.47503
	μ = 0.3	10.97922	11.07188	11.19061	11.34826	11.56782	11.89501
	μ = 0.4	10.56802	10.65578	10.76825	10.91761	11.1257	11.4359
	μ = 0.5	10.27032	10.35448	10.46234	10.60562	10.80528	11.10302
Clamped - Simply Supported	μ = 0	15.68866	15.83332	16.01784	16.26149	16.5985	17.09632
	μ = 0.1	15.5097	15.6505	15.83017	16.06755	16.39611	16.88189
	μ = 0.2	15.08639	15.21921	15.38885	15.61323	15.92425	16.38494
	μ = 0.3	14.63395	14.75933	14.91963	15.13187	15.42647	15.86362
	μ = 0.4	14.27038	14.39027	14.54365	14.74691	15.02935	15.44903
	μ = 0.5	14.00789	14.12398	14.27257	14.46962	14.74364	15.15123
Clamped	μ = 0	20.59938	20.78791	21.02761	21.34286	21.77682	22.41402
	μ = 0.1	20.47686	20.66067	20.89454	21.20244	21.62678	22.25086
	μ = 0.2	20.19383	20.36789	20.58974	20.88243	21.28687	21.88368
	μ = 0.3	19.90291	20.06799	20.27874	20.55735	20.94335	21.51482
	μ = 0.4	19.67755	19.83608	20.03875	20.30711	20.67964	21.23258
	μ = 0.5	19.51915	19.67323	19.87039	20.13176	20.4951	21.03539

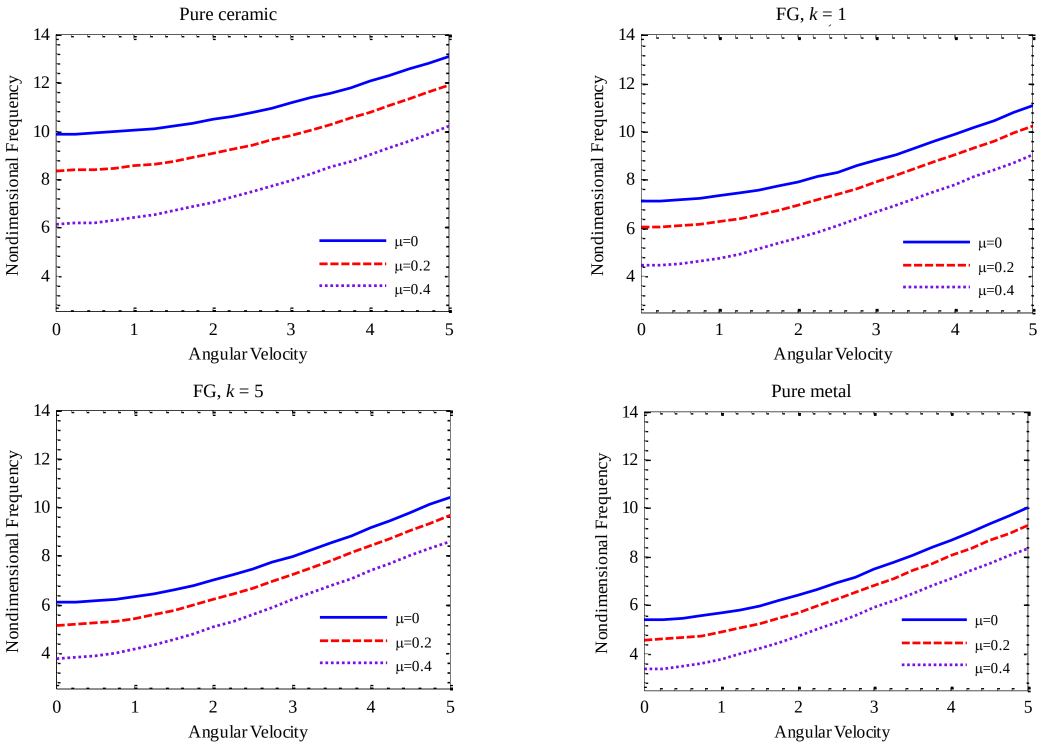


Fig. 3 Variation of the non-dimensional fundamental frequency parameter with respect to the non-dimensional angular velocity parameter for pure ceramic, pure metal and functionally graded materials with k = 1 and 5 for various values of the nonlocal parameter regarding to simply-simply boundary condition

a remarkable role, and in the vibrational behavior of nanobeams and it should not be ignored. Also, for porous FG nanobeam with simply-clamped, clamped-clamped boundary conditions, when the angular velocity of the rotating nanobeam increases, the influence of nonlocal parameter on the non-dimensional fundamental frequences decreases.

Figs. 6-8 show the non-dimensional fundamental frequency with respect to the axial preload (varying from 0 to 30) for pure ceramic material, pure metal material and functionally graded material (with k = 0.1 and 5) for different values of the porosity volume fraction parameters α, regarding to simply-simply, simply-clamped, clamped-clamped boundary conditions.

Here, the angular velocity and nonlocal parameter are 1 and 0.1, respectively. Also, porosity volume fraction parameter can be considered 0, 0.25 and 0.5. According to Figs. 6-8, for pure ceramic and functionally graded materials with low FG-index (for example k = 0.1), when the porosity parameter is increased, the non-dimensional frequencies increase too, as were reported by Ebrahimi and Hashemi (2016) for tapered FG rotating Euler-Bernoulli nanobeam considering even distribution of porosity effect. The reason for this issue is due to this fact that by increasing the porosity parameter, the stiffness of the nanobeam decreases, Shafiei *et al*. (2016a). But, for pure metal and functionally graded materials with high FG-index (for example k = 5), smaller amount of porosity parameter, causes greater non-dimensional frequencies and the influence of porosity parameter on the frequeny increases. Similarly, this result has been reported by Ebrahimi and

Hashemi (2016) for tapered FG rotating Euler-Bernoulli nanobeam considering even distribution of porosity effect. Also, with the increase in axial tensile preload parameter, the stiffness of the rotating porose FG nanobeam increases and so the non-dimensional frequencies have greater values, as were reported by Murmu and Adhikari (2010b) for rotating Euler-Bernoulli nanobeam without porosity effect.

From Figs. 6-8, it is observed that, when the FG-index of the rotating nanobeam increases, the influence of porosity parameter on the non-dimensional frequences decreases for simply-simply, simply-clamped, clamped-clamped boundary conditions. In this case, an opposite dynamical response was reported by Ebrahimi and Hashemi (2016) for tapered FG rotating Euler-Bernoulli nanobeam considering even distribution of porosity effect. Ebrahimi and Hashemi (2016) revealed the influence of porosity parameter on the fundamental frequencies of tapered FG rotating nanobeam increases with the increase in the FG-index. Also, for the rotating nanobeam with pure ceramic material, the influence of porosity on the frequences is greater than the rotating nanobeam with pure metal material. Figs. 9-11, show the variation of the non-dimensional fundamental frequency Ψ with respect to the axial preload (varying from 0 to 70) and angular velocity (Φ = 0, 2, 4, 8) for various values of the nonlocal parameter μ regarding to simply-simply, simply-clamped, clamped-clamped boundary conditions, respectively.

Here, the FG-Index and porosity parameters are 0.1 and 0.25, respectively. Also, the nonlocal

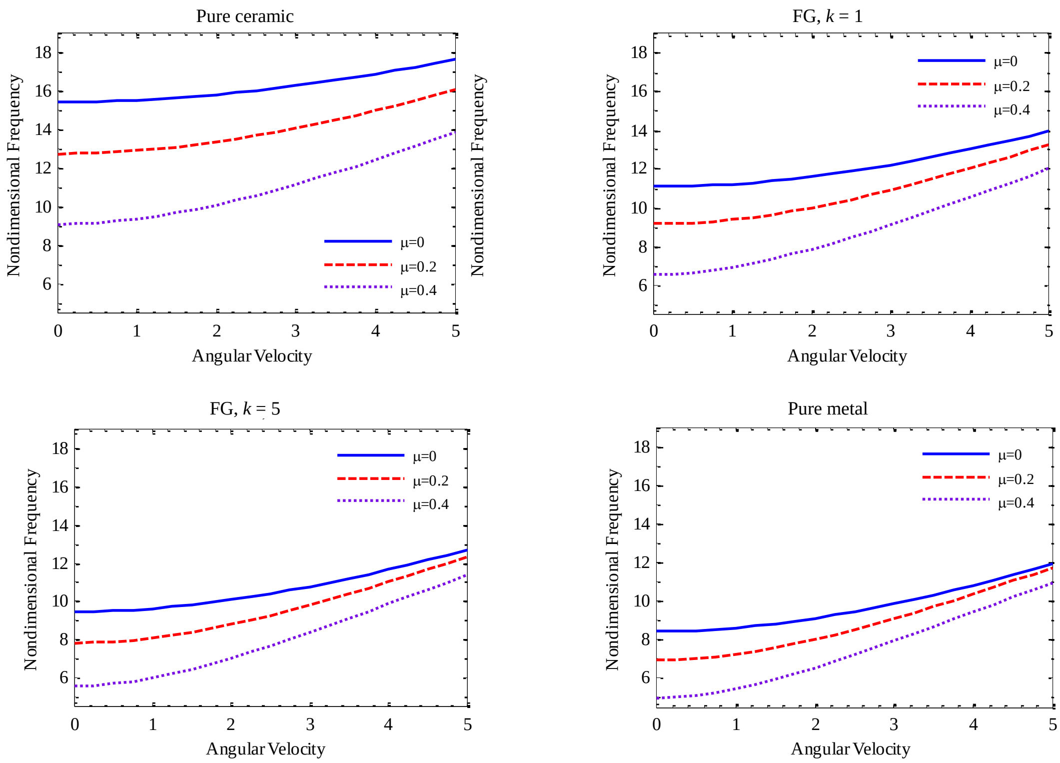


Fig. 4 Variation of the non-dimensional fundamental frequency parameter with respect to the non-dimensional angular velocity parameter for pure ceramic, pure metal and functionally graded materials with k = 1 and 5 for different values of the nonlocal parameter regarding to simply-clamped boundary condition

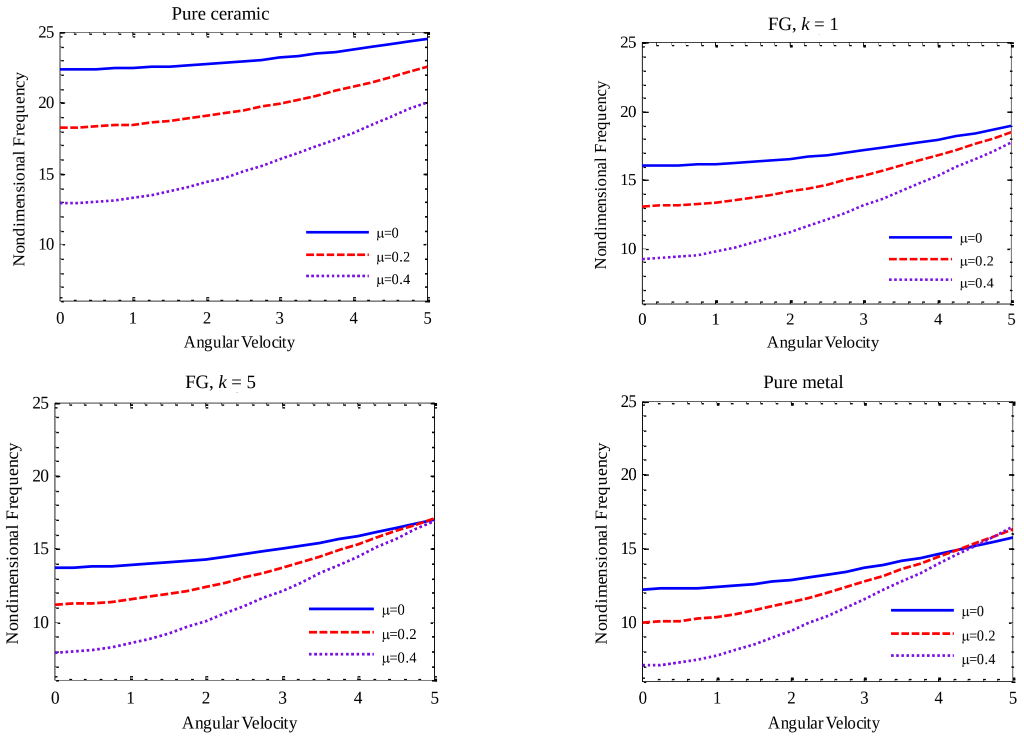


Fig. 5 Variation of the non-dimensional fundamental frequency parameter with respect to the non-dimensional angular velocity parameter for pure ceramic, pure metal and functionally graded materials with k = 1 and 5 for different values of the nonlocal parameter regarding to clamped-clamped boundary condition

parameter can be considered 0, 0.15 and 0.4. Table 2 shows the non-dimension fundamental frequencies for various values of nonlocal parameters and porosity volume fraction parameters corresponding to simply-simply, simply-clamped, clamped-clamped boundary conditions, respectively.

Here, the FG-Index and angular velocity and axial preload are 0.5, 3 and 10, respectively.

According to Figs. 3-5 and Figs. 9-11 and Tables 1 and 2, when the nonlocal parameter increases, non-dimensional fundamental frequencies decrease for simply-simply, simply-clamped, clamped-clamped boundary conditions. The substantial reason for this issue is due to this fact that by increasing the nonlocal parameter, the stiffness of the nanobeam decreases and so the value of non-dimensional fundamental frequency reduces, Ebrahimi and Salari (2015) and Ghadiri *et al*. (2016b).

In this case, a reverse behavior of nonlocal parameter was reported by Shafiei *et al*. (2016b). According to their work the non-dimensional fundamental frequencies of rotating tapered axially FG nanobeam increases with the increase in nonlocal parameter.

From Table 2, it is observed that the influence of nonlocal parameter on the frequencies of simply supported nanobeam is greater than simply-clamped and clamped-clamped nanobeam. It should be noted that, according to Table 2, the influence of porosity parameter on the fundamental frequencies of clamped-clamped nanobeam is greater than simply-clamped and simply supported nanobeam.

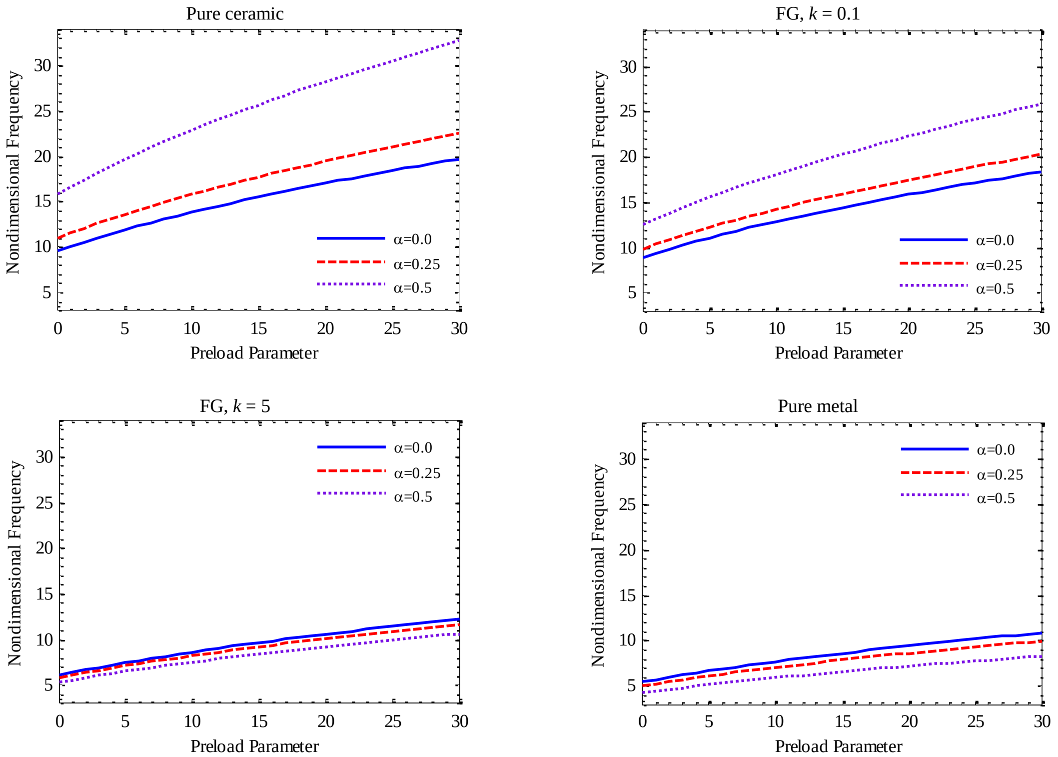


Fig. 6 Variation of the non-dimensional fundamental frequency parameter with respect to the axial preload for pure ceramic, pure metal and functionally graded materials with k = 1 and 5 for different values of the porosity volume fraction parameter regarding to simply-simply boundary condition

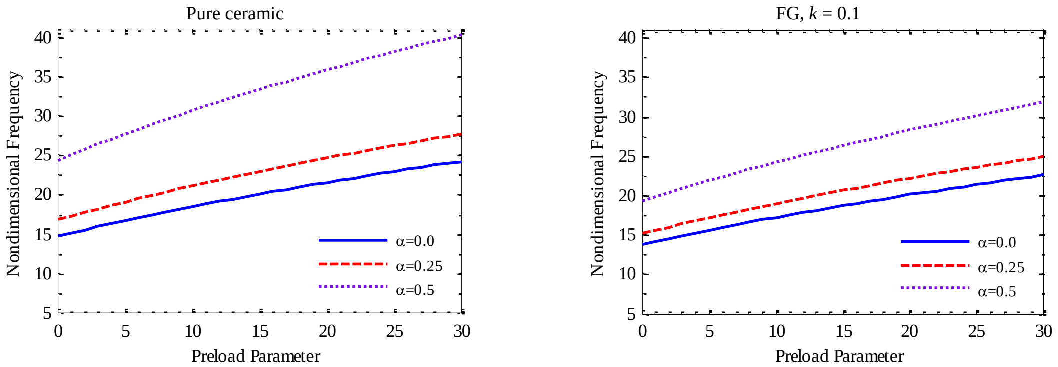


Fig. 7 Variation of the non-dimensional fundamental frequency parameter with respect to the axial preload for pure ceramic, pure metal and functionally graded materials with k = 1 and 5 for different values of the porosity volume fraction parameter regarding to simply-clamped boundary condition

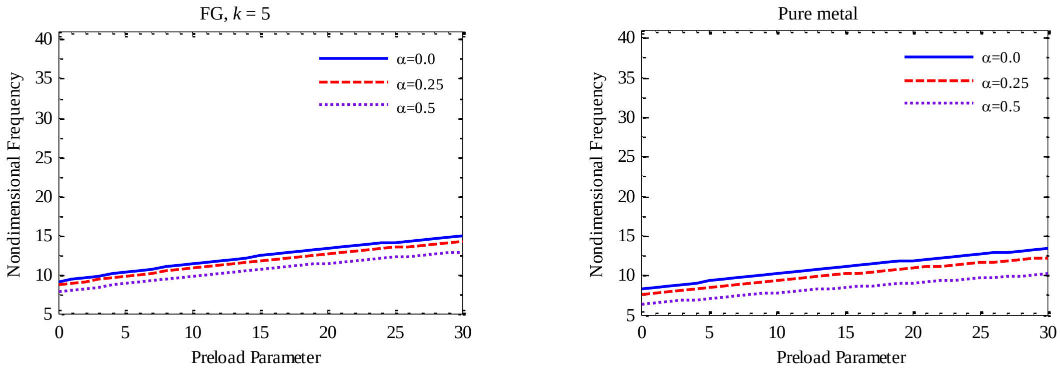


Fig. 7 Continued

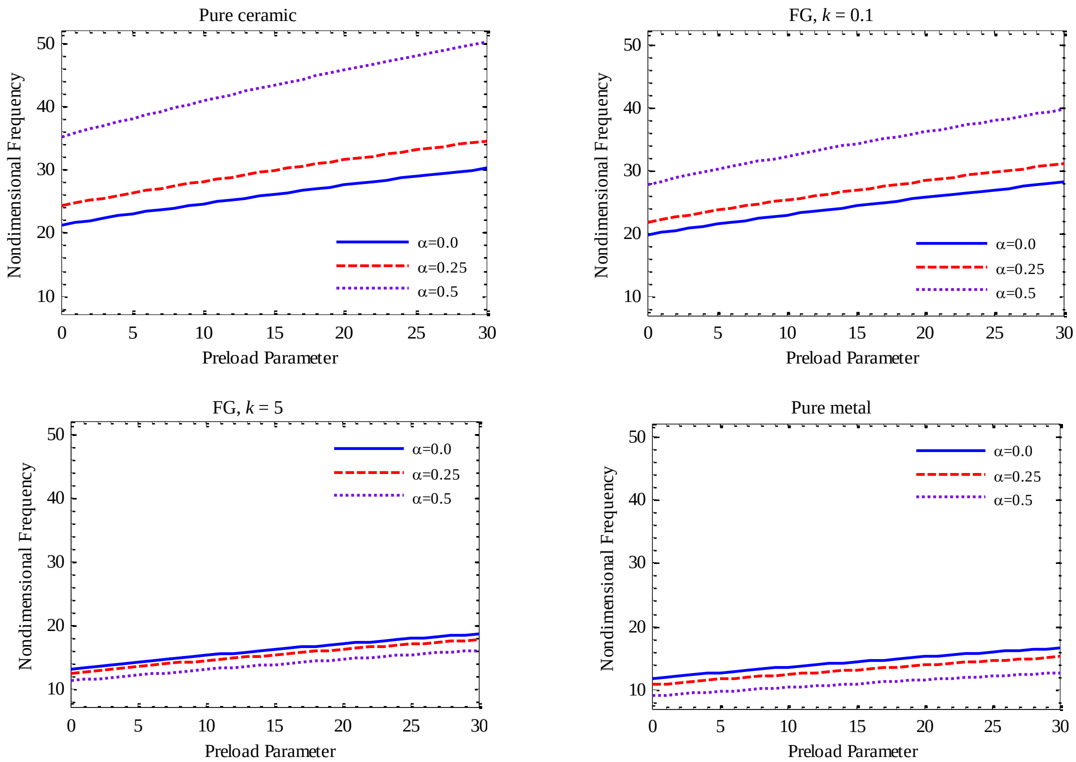


Fig. 8 Variation of the non-dimensional fundamental frequency parameter with respect to the axial preload for pure ceramic, pure metal and functionally graded materials with k = 1 and 5 for different values of the porosity volume fraction parameter regarding to clamp ed-clamped boundary condition

Figs. 3-5 and Figs. 9-11 show that when the angular velocity of the rotating porose FG nanobeam is increased, the non-dimensional frequences increase for all given nonlocal parameters, as were reported by Murmu and Adhikari (2010b) for rotating Euler-Bernoulli nanobeam without porosity effect. The reason is that as the nanobeam rotates, the stiffness of the nanobeam increases and hence non-dimensional frequencies increase.

According to Figs. 3-11, order of non-dimensional fundamental frequencies for pure ceramic, pure metal and functionally graded materials can be obtained as:

pure ceramic material> functionally graded material (k = 1) > functionally graded material (k = 5) > pure metal material. The reason is that the FG-index tends to increase the weight and decrease the stiffness of the microbeam and so decreases the values of natural frequency.

7. Conclusions

The nonlocal Euler-Bernoulli beam theory was employed to discuss about the free vibration of rotating functionally graded nanobeams accompanied by the porosity and rotary effects and axial preload. The governing equations of motion and the related boundary conditions were obtained usind the Hamilton's principle. Afterward, generalized differential quadrature method (GDQM) was used to discretize the governing differential equations related to simply-simply, simply-clamped, clamped-clamped boundary conditions. In this research, the influences of the various parameters such as functionally graded power (FG-index), porosity parameter, nonlocal parameter, axial preload and angular velocity on natural frequencies of rotating FG nanobeams are investigated.

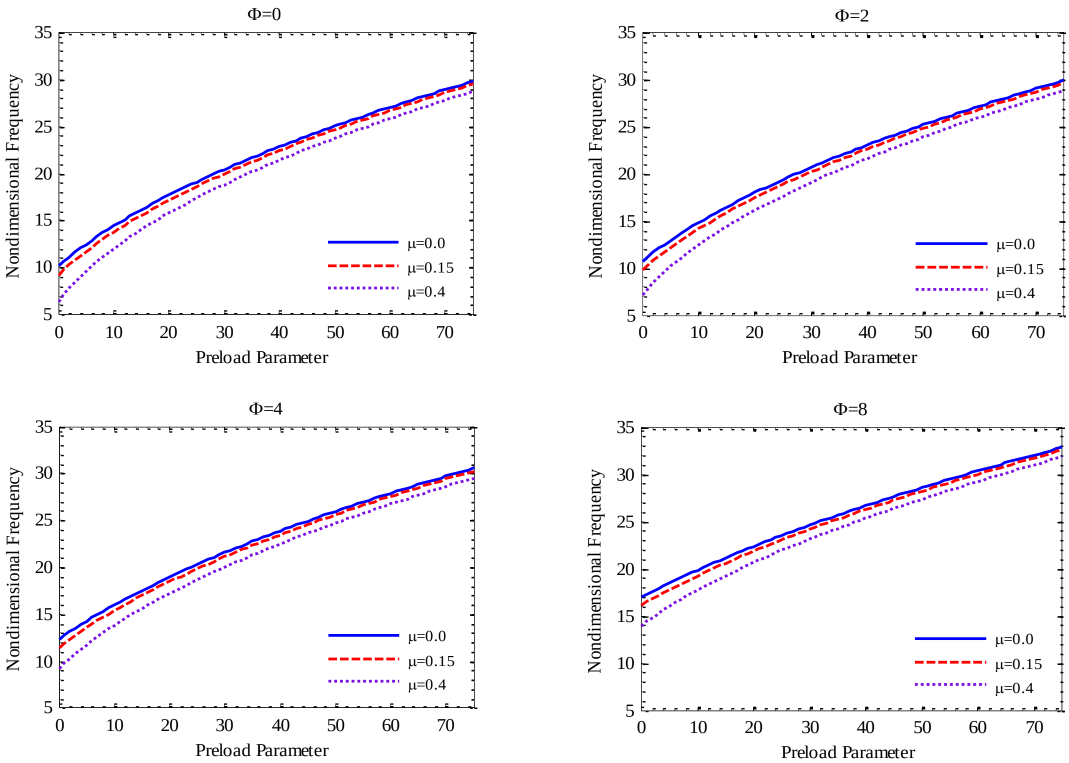


Fig. 9 Variation of the non-dimensional fundamental frequency parameter with respect to the axial preload and angular velocity for different values of the nonlocal parameter regarding to simply-simply boundary condition

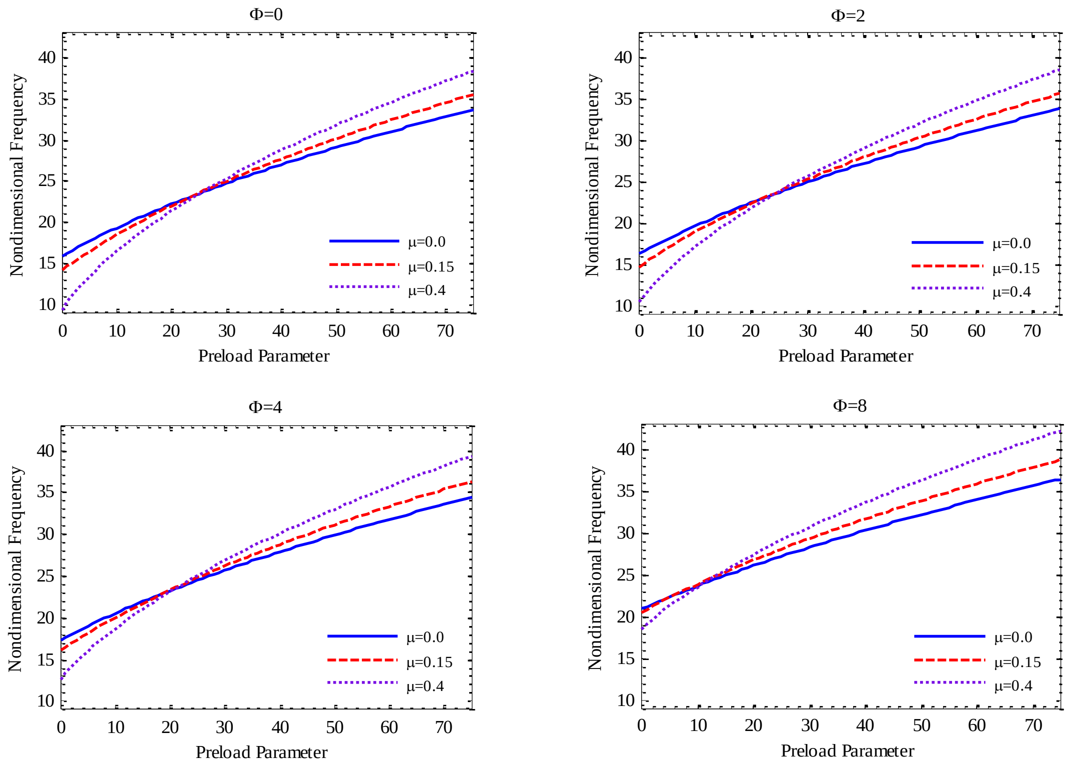


Fig. 10 Variation of the non-dimensional fundamental frequency parameter with respect to the axial preload and angular velocity for different values of the nonlocal parameter regarding to simply-clamped boundary condition

The following results could be highlighted from this research:

(1) In rotating FG nanobeams with considering the porosity effect and axial preload, the influence of porosity parameter over the fundamental frequencies of clamped-clamped nanobeam is more than the simply-clamped and simply supported nanobeam.

(2) The influence of the nonlocal parameter over the fundamental frequencies of simply supported nanobeam is more than the simply-clamped and clamped-clamped nanobeam for rotating FG nanobeam with considering the porosity effect and axial preload.

(3) In rotating porose FG nanobeams with axial preload, comparison of fundamental frequencies for pure ceramic, pure metal and functionally graded materials is as follows:

(4) pure ceramic material > functionally graded material with volume fraction index (k = 1) > functionally graded material with volume fraction index (k = 5) > pure metal material

(5) The influence of porosity on the frequencies for the rotating nanobeam with pure ceramic material is more than the rotating nanobeam with pure metal material.

(6) The influence of porosity on the frequencies decreases with the increase in the FG-index of the rotating nanobeam for simply-simply, simply-clamped, clamped-clamped boundary conditions.

(7) In rotating porose FG nanobeams with high FG-index (materials with more amount of metal), when porosity parameter increases, the non-dimensional frequencies decrease. But,

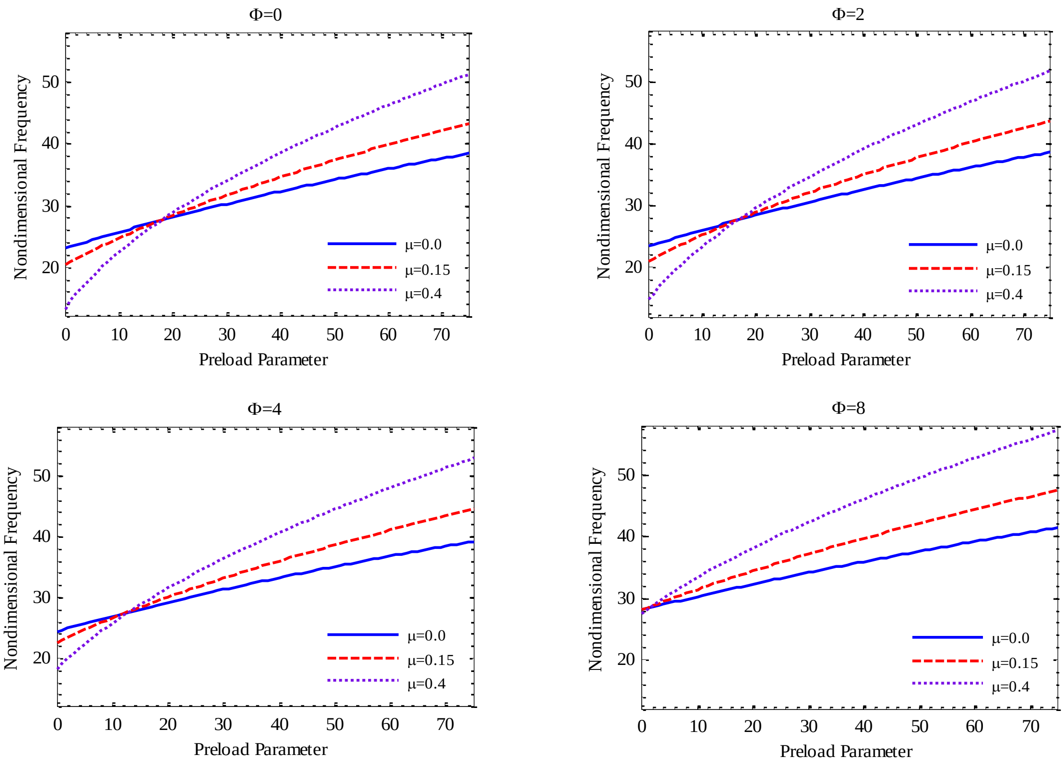


Fig. 11 Variation of the non-dimensional fundamental frequency parameter with respect to the axial preload and angular velocity for different values of the nonlocal parameter regarding to clamped-clamped boundary condition

for nanobeams with low FG-index (materials with more amount of ceramic) with the increase in porosity parameter, the non-dimensional frequencies increase.

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CC

Appendix A

Using generalized differential quadrature method, Eqs. (29)-(30) are simplified as follows

$$K_d = \omega^2 M_d$$
$$\left\{\begin{bmatrix} [KA]_{n\times n} U_i & [KB]_{n\times n} W_i \\ [KC]_{n\times n} U_i & [KD]_{n\times n} W_i \end{bmatrix} = \omega^2 \begin{bmatrix} [MA]_{n\times n} U_i & [MB]_{n\times n} W_i \\ [MC]_{n\times n} U_i & [MD]_{n\times n} W_i \end{bmatrix}\right\}_{2n\times 2n} e^{i\omega t} \tag{50}$$

Here, *d* index denotes the domain and *KA*, *KB*, *MA*, *MB*, *KC*, *KD*, *MC* and *MD* are expressed as

$$\begin{aligned}
[KA] &= A_{xx}\sum_{k=1}^{n} C_{i,k}^{(2)} \\
[KB] &= -B_{xx}\sum_{k=1}^{n} C_{i,k}^{(3)} \\
[KC] &= B_{xx}\sum_{k=1}^{n} C_{i,k}^{(3)} \\
[KD] &= -C_{xx}\sum_{k=1}^{n} C_{i,k}^{(4)} - \left(N^{rotation} - P^{axial}\right)\sum_{k=1}^{n} C_{i,k}^{(2)} + \left(e_0 a\right)^2 \left(N^{rotation} - P^{axial}\right)\sum_{k=1}^{n} C_{i,k}^{(4)} \\
[MA] &= m_0 [I] - \left(e_0 a\right)^2 m_0 \sum_{k=1}^{n} C_{i,k}^{(2)} \\
[MB] &= -m_1 \sum_{k=1}^{n} C_{i,k}^{(1)} + \left(e_0 a\right)^2 m_1 \sum_{k=1}^{n} C_{i,k}^{(3)} \\
[MC] &= m_1 \sum_{k=1}^{n} C_{i,k}^{(1)} - \left(e_0 a\right)^2 m_1 \sum_{k=1}^{n} C_{i,k}^{(3)} \\
[MD] &= m_0 [I] - m_2 \sum_{k=1}^{n} C_{i,k}^{(2)} - \left(e_0 a\right)^2 \left(m_0 \sum_{k=1}^{n} C_{i,k}^{(2)} - m_2 \sum_{k=1}^{n} C_{i,k}^{(4)} \right)
\end{aligned} \tag{51}$$

So, by employing the GDQM to the boundary conditions equations, the matrix of boundary conditions can be derived similar to Eq. (50) as

$$K_b = \omega^2 M_b$$
$$\left\{\begin{bmatrix} [KA_b]_{1\times n} U_i & [KB_b]_{1\times n} W_i \\ [KC_b]_{1\times n} U_i & [KD_b]_{1\times n} W_i \\ [KE_b]_{1\times n} U_i & [KF_b]_{1\times n} W_i \\ [KG_b]_{1\times n} U_i & [KH_b]_{1\times n} W_i \\ [KI_b]_{1\times n} U_i & [KJ_b]_{1\times n} W_i \\ [KK_b]_{1\times n} U_i & [KL_b]_{1\times n} W_i \end{bmatrix} = \omega^2 \begin{bmatrix} [MA_b]_{1\times n} U_i & [MB_b]_{1\times n} W_i \\ [MC_b]_{1\times n} U_i & [MD_b]_{1\times n} W_i \\ [ME_b]_{1\times n} U_i & [MF_b]_{1\times n} W_i \\ [MG_b]_{1\times n} U_i & [MH_b]_{1\times n} W_i \\ [MI_b]_{1\times n} U_i & [MJ_b]_{1\times n} W_i \\ [MK_b]_{1\times n} U_i & [ML_b]_{1\times n} W_i \end{bmatrix}\right\}_{6\times 2n} e^{i\omega t} \tag{52}$$

Here, b index denotes the boundary and KA_b, KB_b ... KK_b and KL_b are expressed as bellow (for example $N=0, \frac{\partial M}{\partial x} - m_1 \frac{\partial^2 u}{\partial t^2} + m_2 \frac{\partial^3 u}{\partial x \partial t^2} = 0$ and M = 0)

$$
\begin{aligned}
&[KA_b] = A_{xx} \sum_{k=1}^{n} C_{1,k}^{(1)} \\
&[KB_b] = [0]_{1\times n} \\
&[KC_b] = [0]_{1\times n} \\
&[KD_b] = -C_{xx} \sum_{k=1}^{n} C_{1,k}^{(3)} - (e_0 a)^2 \sum_{k=1}^{n} C_{1,k}^{(1)} \left[\sum_{k=1}^{n} C_{1,k}^{(1)} \left(N^{rotatio} + P^{axial} \right) \sum_{k=1}^{n} C_{1,k}^{(1)} \right] \\
&[KE_b] = [0]_{1\times n} \\
&[KF_b] = -C_{xx} \sum_{k=1}^{n} C_{1,k}^{(2)} - (e_0 a)^2 \sum_{k=1}^{n} C_{1,k}^{(1)} \left(N^{rotatio} + P^{axial} \right) \sum_{k=1}^{n} C_{1,k}^{(1)} \\
&[KG_b] = A_{xx} \sum_{k=1}^{n} C_{n,k}^{(1)} \\
&[KH_b] = [0]_{1\times n} \\
&[KI_b] = [0]_{1\times n} \\
&[KJ_b] = -C_{xx} \sum_{k=1}^{n} C_{n,k}^{(3)} - (e_0 a)^2 \sum_{k=1}^{n} C_{1,k}^{(1)} \left[\sum_{k=1}^{n} C_{n,k}^{(1)} \left(N^{rotatio} + P^{axial} \right) \sum_{k=1}^{n} C_{n,k}^{(1)} \right] \\
&[KK_b] = [0]_{1\times n} \\
&[KL_b] = -C_{xx} \sum_{k=1}^{n} C_{1,k}^{(2)} - (e_0 a)^2 \sum_{k=1}^{n} C_{n,k}^{(1)} \left(N^{rotatio} + P^{axial} \right) \sum_{k=1}^{n} C_{n,k}^{(1)}
\end{aligned}
\tag{53}
$$

Moreover, MA_b, MB_b ... MK_b and ML_b are calculated as bellow

$$
\begin{aligned}
&[MA_b] = (e_0 a)^2 m_0 \sum_{k=1}^{n} C_{1,k}^{(1)} \\
&[MB_b] = [0]_{1\times n} \\
&[MC_b] = [0]_{1\times n} \\
&[MD_b] = (e_0 a)^2 \left(m_0 \sum_{k=1}^{n} C_{1,k}^{(1)} - m_2 \sum_{k=1}^{n} C_{1,k}^{(3)} \right) \\
&[ME_b] = [0]_{1\times n} \\
&[MF_b] = (e_0 a)^2 \left(m_0 - m_2 \sum_{k=1}^{n} C_{1,k}^{(2)} \right)
\end{aligned}
\tag{54}
$$

$$
\begin{aligned}
&\left[MG_b\right]=\left(e_0a\right)^2 m_0\sum_{k=1}^{n}C_{n,k}^{(1)}\\
&\left[MH_b\right]=\left[0\right]_{1\times n}\\
&\left[MI_b\right]=\left[0\right]_{1\times n}\\
&\left[MJ_b\right]=\left(e_0a\right)^2\left(m_0\sum_{k=1}^{n}C_{n,k}^{(1)}-m_2\sum_{k=1}^{n}C_{n,k}^{(3)}\right)\\
&\left[MK_b\right]=\left[0\right]_{1\times n}\\
&\left[ML_b\right]=\left(e_0a\right)^2\left(m_0-m_2\sum_{k=1}^{n}C_{n,k}^{(2)}\right)
\end{aligned}
\tag{54}
$$

In order to solve the governing equations coupled with boundary conditions, the following matrix equation can be evaluated, Shu (2012).

$$
\left[K^*\right]\left\{\lambda_i\right\}=\omega^2\left[M^*\right]\left\{\lambda_i\right\}
\tag{55}
$$

Here, λ and ω are the mode shape and the natural frequency. Also, K^* and M^* are expressed as

$$
\begin{aligned}
&\left[K^*\right]=\begin{bmatrix} K_d & {K_b}^T \\ K_b & \left[0\right] \end{bmatrix}\\
&\left[M^*\right]=\begin{bmatrix} M_d & {M_b}^T \\ M_b & \left[0\right] \end{bmatrix}
\end{aligned}
\tag{56}
$$

Eventually, with solving the eigenvalue problem, the eigenvector and natural frequencies will be obtained.

Appendix B

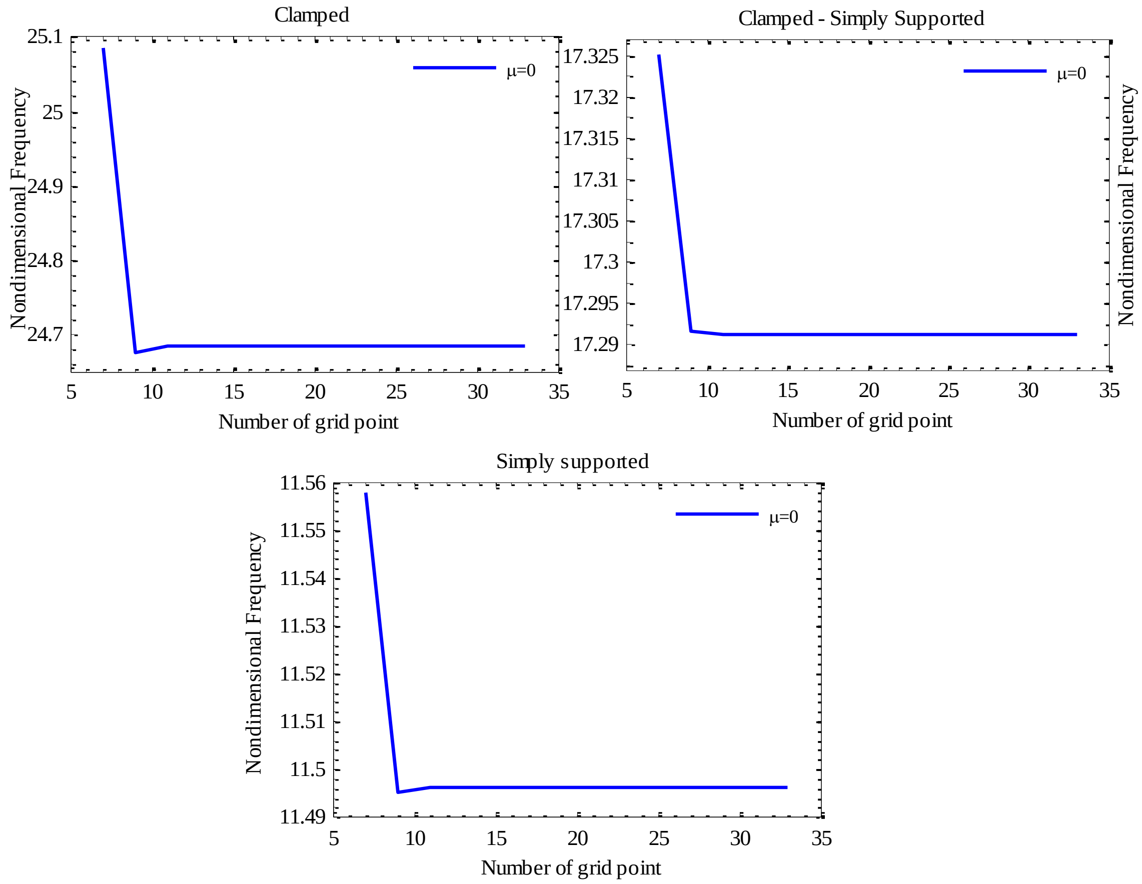

Fig. 12 Variation of the non-dimensional fundamental frequency with respect to the sufficient number of grid points for rotating nanobeam related to simply-simply, simply-clamped and clamped-clamped boundary conditions. (Φ = 1, μ = 0, δ = 1, α = 0.25 and P^{axial} = 1)