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Modeling and multiple performance optimization of ultrasonic micro-hole machining of PCD using fuzzy logic and taguchi quality loss function

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Abstract. Polycrystalline diamond is an ideal material for parts with micro-holes and has been widely used as dies and cutting tools in automotive, aerospace and woodworking industries due to its superior wear and corrosion resistance. In this research paper, the modeling and simultaneous optimization of multiple performance characteristics such as material removal rate and surface roughness of polycrystalline diamond (PCD) with ultrasonic machining process has been presented. The fuzzy logic and taguchi's quality loss function has been used. In recent years, fuzzy logic has been used in manufacturing engineering for modeling and monitoring. Also the effect of controllable machining parameters like type of abrasive slurry, their size and concentration, nature of tool material and the power rating of the machine has been determined by applying the single objective and multi-objective optimization techniques. The analysis of results has been done using the MATLAB 7.5 software and results obtained are validated by conducting the confirmation experiments. The results show the considerable improvement in *S/N* ratio as compared to initial cutting conditions. The surface roughness of machined surface has been measured by using the Perthometer (M4Pi, Mahr Germany).

Keywords: fuzzy logic; micro-machining; surface roughness; modeling; optimization; ultrasonic

1. Introduction

Modern materials such as high-strength metals and ceramics that are developed to meet the needs of advanced industries are typically strong, hard and brittle. There has been the introduction of many new materials such as tungsten and titanium carbides, polycrystalline diamonds, rubies, sapphire, hard steels, magnetic alloys and corundum. Polycrystalline diamond is having high thermal conductivity, high wear resistance, high hardness, high electrical conductivity and high resistance to corrosion. The material removal rate and surface roughness are important parameters in ultrasonic machining process. While technologically desirable, these characteristics often render the materials difficult and sometimes impossible to shape by machining processes into useful components and parts.

The use of hard and brittle materials has become increasingly more extensive. However, it is not

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feasible to machine these materials with the application of traditional metal-cutting techniques. The processing of such materials for the part fabrication has become a challenging problem. However, if ultrasonic energy is applied to the machining process and coupled with the use of hard abrasive grits, extremely hard and brittle materials can be effectively machined. The methods of optimization can be classified into two approaches namely reliability based and robust design based methods. The objective of robust design is to optimize the mean and minimize the variability that results from uncertainty represented by noise factors. The robust design of a vibration absorber with mass and stiffness uncertainty in the main system is used to demonstrate the robust design approach in dynamics as reported by Kumar and Khamba (2006). The circularity, cylindricity, surface roughness and hole oversize of the ultrasonically and conventionally drilling of Inconel 738-LC were measured and compared by Azarhoushang and Akbari (2007). The on-line tool wear monitoring during ultrasonic machining using tool resonance frequency was determined by Hocheng and Kuo (2002). The effects of various parameters of ultrasonic drilling of two-dimensional carbon fiber-reinforced silicon carbide(C/SiC) composites including abrasives, volume ratio, electric current and down-force on the material removal rate, hole clearance, edge quality and tool wear were studied by Hocheng *et al*. (2000). The optimum parameters for multi-performance characteristics in drilling using grey relational analysis were determined by Tosun (2006). The design optimization of cutting parameters for side milling operations with multiple performance characteristics was done by Chang and Lu (2007). The experiments were conducted to understand the tool wear mechanism in rotary ultrasonic machining (RUM) of silicon carbide (SiC). The topography of the end face and lateral face of a diamond tool in RUM of SiC was observed under digital microscope by Zeng *et al*. (2005). The laser processing of polycrystalline diamond, tungsten carbide and a related composite material was done by Harrison and Henry (2006). The parametric optimization of ultrasonic machining of Co-based super alloy using the taguchi multi-objective approach has been done by Kumar and Khamba (2009). The robust design method is essential for improving engineering productivity as reported by Roy (1990). The statistical analysis of experimental parameters in ultrasonic machining of tungsten carbide using taguchi approach has been done by Kumar and Khamba (2008). The grey taguchi method was applied to optimize the milling parameters by Tsao (2009). The effect of various machining parameters during machining of PCD was studied by Tso and Lin (2002). From the literature review it has been concluded that the modeling and multi-objective optimization of parameters involved in ultrasonic micro machining of PCD has not been done by the previous authors. Therefore as the polycrystalline diamond is an ideal material for parts with micro-holes and has been widely used as dies and cutting tools in automotive, aerospace and woodworking industries due to its superior wear and corrosion resistance. In view of the extensive applications, there was the need of this type of research work to be done. In the present study, the multiple performance characteristics have been optimized simultaneously and more informed analysis has been made by conducting less expensive experiments for design improvement. Also the modeling and simultaneous optimization of the results has been done using the fuzzy logic and taguchi quality loss function.

2. Taguchi analytical methodology

Taguchi's method of experimental design provides a simple, efficient and systematic approach to determine optimal machining parameters as studied by Gaitonde *et al*. (2007). Taguchi has recommended orthogonal arrays (OA) for the designing of experiments. In taguchi method, the results of experiments are analyzed to achieve one or more of the objectives as to establish the best or the optimum condition for a product or process, to estimate the contribution of individual parameters and interactions and to estimate the response under the optimum condition. The optimum condition for hole roundness in deep holes has been found by Deng and Chin (2005).

Analysis of variance (ANOVA) is the statistical treatment applied to the results of the experiments in determining the percent contribution of each parameter against a stated level of confidence. The study of ANOVA table for a given analysis helps to determine which of the parameters need control and which do not. Taguchi suggested two different routes to carry out the complete analysis. First, the standard approach; where the results of a single run or the average of repetitive runs are processed through main effect and analysis of variance. The second approach, which taguchi strongly recommends for multiple runs, is to use signal-to-noise ratio (*S/N*) for the same steps in the analysis. The *S/N* ratio is a concurrent quality metric linked to the loss function as reported by Phadke (1989).

Design of experiment (DOE) methods result in an efficient experimental schedule and produce a statistical analysis to determine easily as to which parameters have the most significant effects on the final results. The use of signal-to-noise ratio (*S/N*) in system analysis provides a quantitative value for response variation comparison. The requirement to test multiple factors means that a full factorial experimental design that describes all possible conditions would result in a large number of experiments. After conducting the experiments, the data from all experiments has to be evaluated to determine the optimum levels of the design variables using the analysis of variance (ANOVA) and the analysis of mean (ANOM) of the *S/N* ratio. There are several *S/N* ratios available depending on the types of characteristics; lower is better (*LB*), nominal is best (*NB*) and higher is better (*HB*).

Lower-the-better type problem

$$(S/N)_L = -10\log_{10}\left(1/n\sum_{i=1}^{n} y_i^2\right) \tag{1}$$

where $(S/N)_L$ is signal-to-noise ratio for lower-the-better type problem, n is the number of repetitions for each trial, independent of the values assigned to noise factors, and y_i is the value of the response obtained in the i^{th} repetition of the trial.

Higher-the-better type problem

In this type of problem, the quality characteristic is again continuous and non-negative and it is to be made as large as possible. There is no adjustment factor to be used in this case as well and one is interested in maximizing the objective function expressed as

$$(S/N)_H = -10\log_{10}\left\{\frac{1}{n}\cdot\sum_{i=1}^{n}\frac{1}{y_i^2}\right\} \tag{2}$$

Where $(S/N)_H$ is signal-to-noise ratio for higher-the-better type problem.

Nominal-the-best type problem

In the nominal-the-best type problem, the quality characteristic is continuous and non-negative, but its target value is non zero and assumes some finite value. For these types of problems, if the mean becomes zero the variance also tends to become zero. A scaling factor can be used as an adjustment factor to shift the mean closer to the target for such type of problems. The objective function that is to be maximized can be expressed as

$$(S/N)_N = 10\log_{10}(\mu^2/\sigma^2) \tag{3}$$

Where $(S/N)_N$ is signal-to-noise ratio for Nominal-the-best type problem

$$\mu = \frac{1}{n}\cdot\sum_{i=1}^{n} y_i^2 \tag{4}$$

$$\sigma = \frac{1}{(n-1)\cdot\sum_{i=1}^{n}(y_i-\mu)^2} \tag{5}$$

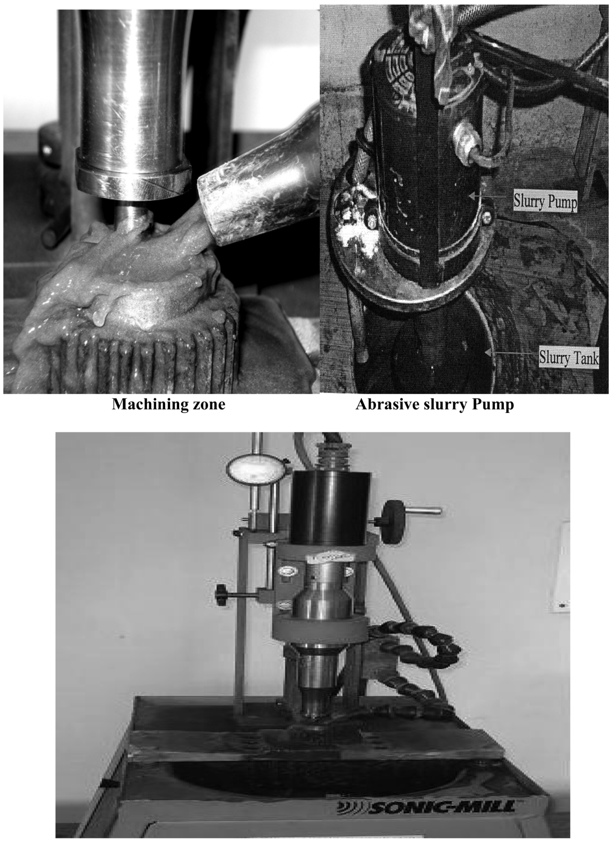
3. Tool design for experimentation

In ultrasonic machining, the mass and dimensions of the tool constitute a very important design problem to economize the machining operation. As the tool materials selected for the experimentation possess different densities, the designing of the tools is needed to be done with a consideration that the mass of each tool should be same to the maximum possible extent. From the pilot experimentation, it was concluded that mass of the tool should be within the critical limit of 50 gm. All the tools were made as single piece unit by machining on a centre lathe. The tip of tool contains unified threads and is tightened to the horn manually. The horn is of 25.4 mm size and it contains internal threads. The length of the tool tip in ultrasonic machining process needs to be restricted and maintained within the limits of 15-20 mm for optimum results. The use of a length more than 20 mm resulted in over stressing of the tool and shortened tool life.

While designing the tools, the dimensions for each tool were decided to ensure that the mass of each tool is same and is equal to 50 gm with a permissible variation of one gm. The tool finish is another important factor that is found to affect the surface finish of the machined surface. Hence, the surface finish of tool face was maintained at a level of 4.5 microns before starting a new experiment. The tool face also tends to gain a convex shape as a result of uneven distribution of the abrasive particles under the tool face while machining takes place. This alters the contour of the machined surface as well as the material removal rate. To rectify this problem, facing operation was performed on each tool on center lathe after a particular experimental run was executed. This helped to ensure a perfectly flat surface of the tool which is responsible for machining and thus the undesirable effect on the shape and size of the cavity produced is also controlled. Further, in order to deal with the problems of fatigue failure of the tools while machining and other problems pertaining to the overheating and stress loading of the tools, a number of tools were prepared for each tool material. This also helped to maintain the continuity of the experimentation.

4. Experimentation

The experiments were performed on a Sonic-Mill, 500 *W* (Albuquerque, *NM*) as shown in Fig. 1. The machining of work material was performed using different input parameters the tool material being three different titanium alloys (TITAN12, TITAN15 and TITAN31). The frequency was varied from 18 to 22 kHz. The three different values of power rating taken were (25, 50 and 75)



Machining zone **Abrasive slurry Pump**

Ultrasonic Machine Tool (Type: Sonic Mill, USA)

Fig. 1 Pictorial view of the experimental set-up

Table 1 Representation of control variables and their levels

S.No	Control variables	Levels	Level 1	Level 2	Level 3
A	Tool material	3	TITAN12	TITAN15	TITAN31
B	Abrasive slurry	3	Al_2O_3	SiC	B_4C
C	Slurry concentration (%)	3	20	25	30
D	Abrasive grit size	3	220	320	500
E	Power rating (%)	3	25	50	75

percent. The three different abrasive slurries (Al_2O_3, SiC and B_4C), each of three grit sizes (220, 320 and 500) were adopted with percentage concentrations by volume with water (20, 25 and 30).

The Table 1 shows the control variables and their levels. There was no withdrawal of the tool during the tests. Abrasive slurry feed circulation and frequency amplitude was maintained constant. The frequency measurement was performed with the help of a frequency meter. The trials were carried out under maximum material removal rate (MRR) conditions with a tool rotation of 350 rpm. All the experiments were repeated four times; hence four trials were conducted at each experimental run. The output variables were recorded for each trial and then the results for each experimental run were averaged out to obtain the mean value of response variable (MRR) for that particular experiment. The analysis of results has been performed using the MATLAB 7.5 software.

5. Analysis and verification of results

5.1 Selection of orthogonal arrays

The orthogonal array based on the taguchi concept was utilized to arrange the discrete variables and robust solutions for unconstrained optimization problems were found. In this investigation, the five machining parameters, tool material, abrasive slurry, slurry concentration, grit size and power rating were taken with three different levels of each. Thus a total of 243 ($3\times3\times3\times3\times3$) different combinations were considered. However, according to taguchi, the samples could be organized into only 18 groups and if they were to be considered separately still it yield results with the same

Table 2 *S/N* ratios for MRR and SR in single quality optimization

Exp. No	Average MRR (μm^3/min)	Average SR(μm)	*S/N* ratio(dB)	
			MRR	SR
1	0.186	0.795	14.609	1.992
2	0.200	1.075	13.979	0.628
3	0.218	0.995	13.230	0.043
4	0.229	0.755	12.803	2.441
5	0.229	1.01	12.803	0.086
6	0.218	0.995	13.230	0.043
7	0.245	0.49	12.216	6.196
8	0.267	1.175	11.469	1.400
9	0.238	1.185	12.468	1.185
10	0.232	1.16	12.690	1.474
11	0.229	0.895	12.803	0.895
12	0.254	1.627	11.903	0.963
13	0.259	1.175	11.734	1.400
14	0.254	0.825	11.903	1.670
15	0.250	1.085	12.041	0.708
16	0.255	1.285	11.869	2.178
17	0.123	0.555	18.201	5.114
18	0.132	0.555	17.588	5.114

confidence. The *S/N* ratios of MRR and surface roughness in single quality optimization according to the arrangement of the samples into 18 groups; L_{18} according to taguchi is shown in Table 2. The numbers (1, 2 and 3) represents the various experimental levels of the different factors. The initial parameter settings of the experiment "A1B1C1D1E1" was decided from the pilot experimentation done to determine the significant parameters.

5.2 Determination of quality loss for each quality characteristics

The material removal rate is larger-the-better type and surface roughness is the smaller-the better type. A quality loss or mean square deviation (MSD) function is used to calculate the deviation between the experimental value and the desired value. The mean square deviation is different for the different types of problems.

Smaller-the better type

$$\mathrm{MSD} = \frac{y_1^2 + y_2^2 + y_3^2 + y_4^2 + \ldots + y_n^2}{n} \tag{6}$$

Table 3 Computational results of the parameters

Quality loss		Normalized quality loss		Total normalized quality loss and multiple *S/N* ratio	
MRR	SR	MRR	SR	TNQL	MSNR(dB)
28.905	0.632	0.437	0.238	0.357	4.473
25.000	1.155	0.378	0.436	0.401	3.968
21.042	0.990	0.318	0.374	0.340	4.685
19.069	0.570	0.288	0.215	0.258	5.883
19.069	1.020	0.288	0.385	0.326	4.867
21.042	0.990	0.318	0.374	0.340	4.685
16.659	0.240	0.252	0.090	0.187	7.281
14.027	1.380	0.212	0.521	0.335	4.749
17.654	1.404	0.267	0.530	0.372	4.294
18.579	1.345	0.281	0.508	0.371	4.306
19.069	0.801	0.288	0.302	0.293	5.331
15.50	2.647	0.234	1.000	0.540	2.676
14.907	1.380	0.225	0.521	0.343	4.647
15.50	0.680	0.234	0.256	0.242	6.161
16.0	1.177	0.242	0.444	0.322	4.921
15.378	1.651	0.232	0.623	0.388	4.111
66.098	0.308	1.000	0.116	0.646	1.938
57.392	0.308	0.868	0.116	0.567	2.464
				Mean MSNR(n_m)	4.524

Higher-the better type

$$\text{MSD} = \frac{1/y_1^2 + 1/y_2^2 + 1/y_3^2 + 1/y_4^2 + \ldots + 1/y_n^2}{n} \tag{7}$$

Where y_1, y_2, y_3, y_4 and y_n represents the responses of experiments, n is the number of repetitions. The quality loss values for each quality characteristics are shown in Table 3.

5.3 Determination of normalized quality loss for each quality characteristic

The normalized quality loss has been determined by using following formula

$$y_{ij} = \frac{L_{ij}}{L_{im}} \tag{8}$$

Where y_{ij} represents the normalized quality loss, L_{ij} is quality loss for the i^{th} quality characteristic at the j^{th} run in the experiment design matrix and L_{im} is maximum quality loss for the i^{th} quality characteristic among all the experimental runs. The normalized quality loss values for each quality characteristics are shown in Table 3.

5.4 Determination of total normalized quality loss

The total normalized quality loss has been determined by using following formula

$$Y_j = \sum_{i=1}^{k} z_i y_{ij} \tag{9}$$

Where Y_j represents the total normalized quality loss, y_{ij} is normalized quality loss for the i^{th} quality characteristic at the j^{th} run in the experiment design matrix, z_i is the weighting factor for the i^{th} quality characteristic and k is number of quality characteristics. Here k=2 and assuming weighting factors z for MRR and SR as 0.6 and 0.4. The total normalized quality loss (TNQL) and multiple *S/N* ratio (MSNR) are shown in Table 3.

5.5 Determination of multiple S/N ratio, factor effects and optimum combinations

The multiple *S/N* ratios as given in Table 4 have been determined from the following formula

$$\eta_j = -10\log_{10}(Y_j) \tag{10}$$

The optimum combinations corresponding to maximum average effect are considered. The optimum combination of parameters is A1B3C1D3E3.

5.6 Performing the confirmatory experiments

The confirmatory experiments has been performed with optimum settings of the factors and levels as determined to verify the optimum conditions. The multiple *S/N* ratio at optimum level has been determined by applying the following formula

Table 4 Effect of factor levels on multiple *S/N* ratio

Factors	Mean MSNR(dB)		
	Level 1	Level 2	Level 3
Tool material	5.116*	4.502	3.954
Abrasive slurry	4.473	4.317	4.782*
Slurry concentration	5.065*	4.384	4.122
Abrasive grit size	3.861	4.512	5.197*
Power raring	4.430	4.516	4.626*

*Optimum level

Table 5 Confirmatory experiments results (Multi-objective optimization)

		Predicted	Experimental
Level	A1B1C1D1E1	A1B3C1D3E3	A1B3C1D3E3
MRR	0.186	-	0.201
SR	0.795	-	0.522
MSNR(dB)	4.473	4.874	5.954

Improvement of MSNR: 1.481 dB

$$\eta_{op} = \eta_m + \sum_{i=1}^{p}(\eta_i - \eta_m) \tag{11}$$

Where η_m represents the average value of multiple *S/N* ratios in all experimental runs, η_i are multiple *S/N* ratios corresponding to optimum factor levels and p is the number of factors.

The predicted multiple *S/N* ratio and that from the confirmatory experiments is shown in Table 5. The improvement in multiple *S/N* ratio at the optimum combination is found to be 1.481 dB. The values of material removal rate and surface roughness at this optimum combination are 0.201 (μm^3/min) and 0.522 (μm) in comparison to 0.186 (μm^3/min) and 0.795 (μm) for initial setting of parameters.

5.7 Comparison of multi-objective and single objective optimization results

The results of single quality optimization for MRR and surface roughness are summarized in Tables 6 and 7. The confirmatory experiments results of single objective optimization are shown in Table 8. The results of multi-objective optimization (MOO) and single objective optimization (SOO) using taguchi quality function has been compared in Table 9. The results shows that the quality values at optimum settings are different in each case. The results of MOO basically depend on weights assigned to quality values. As in the present research work, the most important quality assumed was MRR with weight 0.6 and the optimum MRR value in SOO is more as compared to MRR obtained from MOO. The result is almost same to that of optimum SR (obtained SOO) while. Therefore chance of quality loss is always there, when the objective is to optimize the multiple

Table 6 *S/N* response table for MRR in single quality optimization

Factors	Mean *S/N* ratio(dB)		
	Level 1	Level 2	Level 3
Tool material	12.698	13.376*	11.426
Abrasive slurry	13.820*	13.370	12.398
Slurry concentration	13.724*	12.344	13.521
Abrasive grit size	13.524	13.571*	12.493
Power rating	12.592	13.726*	13.271

*Optimum level

Table 7 *S/N* response table for SR in single quality optimization

Factors	Mean *S/N* ratio(dB)		
	Level 1	Level 2	Level 3
Tool material	2.613*	1.632	1.342
Abrasive slurry	1.906	2.397*	1.624
Slurry concentration	2.651*	1.059	1.550
Abrasive grit size	1.818	2.085*	1.684
Power rating	1.345	2.360*	1.882

*Optimum level

Table 8 Confirmatory experiments results (single objective optimization)

		Predicted	Experimental
Level	A1B1C1D1E1	A1B3C1D3E3	A1B3C1D3E3
MRR	0.186	-	0.224
SR	0.795	-	0.498
S/N(dB) for MRR	14.609	16.242	16.044
S/N(dB) for SR	1.992	2.142	2.032

Improvement of *S/N* for MRR: 1.435 dB Improvement of *S/N* for SR: 0.04 dB

Table 9 Comparison of results from single and multi-objective optimization

	SOO results		MOO results	Quality loss (%)
	MRR	SR	MRR & SR	
Level	A2B1C1D2E2	A1B2C1D2E2	A1B3C1D3E3	
MRR	0.224	-	0.201	11.443
SR	-	0.498	0.522	4.597

Table 10 Analysis of variance (MRR)

Effect	*SS*	*F*	*P*-value
Tool material	5.17	0.70	0.549
Abrasive slurry	19.32	2.62	0.188
Slurry concentration	1.78	0.24	0.796
Abrasive grit size	5.49	0.74	0.531
Power rating	4.03	0.55	0.617
Tool material X Abrasive slurry	29.67	2.01	0.258
Tool material X SC	7.79	0.53	0.72
Abrasive slurry X SC	3.77	0.26	0.892
Residual error	14.76		

Table 11 Analysis of variance (SR)

Effect	*SS*	*F*	*P*-value
Tool material	6.50	0.62	0.584
Abrasive slurry	15.52	1.47	0.332
Slurry concentration	2.31	0.22	0.813
Abrasive grit size	6.50	0.62	0.585
Power rating	19.06	1.81	0.276
Tool material X Abrasive slurry	16.84	0.80	0.584
Tool material X SC	63.31	3.00	0.156
Abrasive slurry X SC	2.58	0.12	0.967
Residual error	21.11		

quality characteristics simultaneously. The multi-objective optimization is useful in the sense that at the same optimum parameter level, one can get the optimum quality value of multiple quality characteristics at the same time rather than a single optimum quality characteristic. The ANOVA for MRR and SR is shown in Tables 10 and 11.

6. Modeling of results using fuzzy logic approach

Fuzzy logic is an effective tool for dealing with complex nonlinear systems. Fuzzy logic is based on imprecision and is similar to the way people make decisions based on imprecise and non numerical information. Fuzzy logic modeling is based on mathematical theory combining multivalued logic, probability theory and artificial intelligence methods. Fuzzy modeling is based on fuzzy set theory in which the linguistic statements are expressed mathematically and corresponds to the analysis of a human expert. The inputs and outputs in fuzzy systems are in the form of linguistic variables. The variables are then tested with IF-THEN rules, which produce one or more responses depending on which rules are asserted. Each rule has an antecedent part and a consequent part. The antecedent part is a collection of conditions connected by AND, OR, NOT logic operators and consequent part represents its action. In fuzzy inference engine, the truth value for the premise of each rule is computed and applied to conclusion part of each rule. This results in one fuzzy subset being assigned to each output variable for each rule. The response of each rule is weighed according to the degree of membership of its inputs and the centroid of the responses is calculated to generate the appropriate output.

The concept of fuzzy reasoning for three input one output fuzzy logic unit is described as follows. The fuzzy rule base consists of a group of IF-THEN statements with three inputs x_1,x_2,x_3 and one output y; that is,

Rule 1: if x_1 is A_1 and x_2 is B_1 and x_3 is C_1 then y is D_1; else
Rule 2: if x_1 is A_2 and x_2 is B_2 and x_3 is C_2 then y is D_2; else
Rule 3: if x_1 is A_3 and x_2 is B_3 and x_3 is C_3 then y is D_3; else
Rule n: if x_1 is A_n and x_2 is B_n and x_3 is C_n the y is D_n;

A_i, B_i, C_i and D_i are fuzzy subsets defined by corresponding membership functions; that is μ_A, μ_B, μ_C and μ_D.

Eighteen rules were developed based on experimental conditions. By taking max-min compositional operation, the fuzzy reasoning of these rules yields a fuzzy output. Suppose that x_1,x_2 and x_3 are the three input variables of the fuzzy logic unit, the membership function of the output of fuzzy reasoning can be expressed as

$$\mu_{D_0}(y) = [\mu_{A_1}(x_1)\Lambda\mu_{B_1}(x_2)\Lambda\mu_{C_1}(x_3)\Lambda\mu_{D_1}(y)V\ldots\times\mu_{A_n}(x_1)\Lambda\mu_{B_n}(x_2)\Lambda\mu_{C_n}(x_3)\Lambda\mu_{D_n}(y)] \quad (12)$$

Where Λ is the minimum operator and V is the maximum operator.

The membership functions can be of different forms like triangular, trapezoidal, Gaussian, sigmoid etc. In this study, triangular and trapezoidal membership functions are considered. The triangular shaped membership function for input is specified by three parameters {a,b,c} as follows

$$triangle(x;\, a, b, c) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a} & a \le x \ge b \\ \dfrac{c-x}{c-b} & b \le x \ge c \\ 0, & c \le x \end{cases} \quad (13)$$

By using min and max, an alternate expression for the proceeding equation is

$$triangle(x;\, a, b, c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right) \quad (14)$$

Where a, b, c stand for the triangular fuzzy triplet and determine the x coordinates of the three corners of the underlying triangular membership function.

The input-output numerical values are correlated by linguistic variables. This was obtained through the design of membership functions consisting of fuzzy set values. The linguistic values such as LOW, MEDIUM and HIGH are used to represent the input variables slurry concentration, grit size and power rating. The output numerical values are also correlated in a similar manner, by means of membership functions such as LOWEST, LOWER, LOW, LOW MEDIUM, MEDIUM, HIGH MEDIUM, HIGHER, HIGH and HIGHEST. The membership functions used in this work

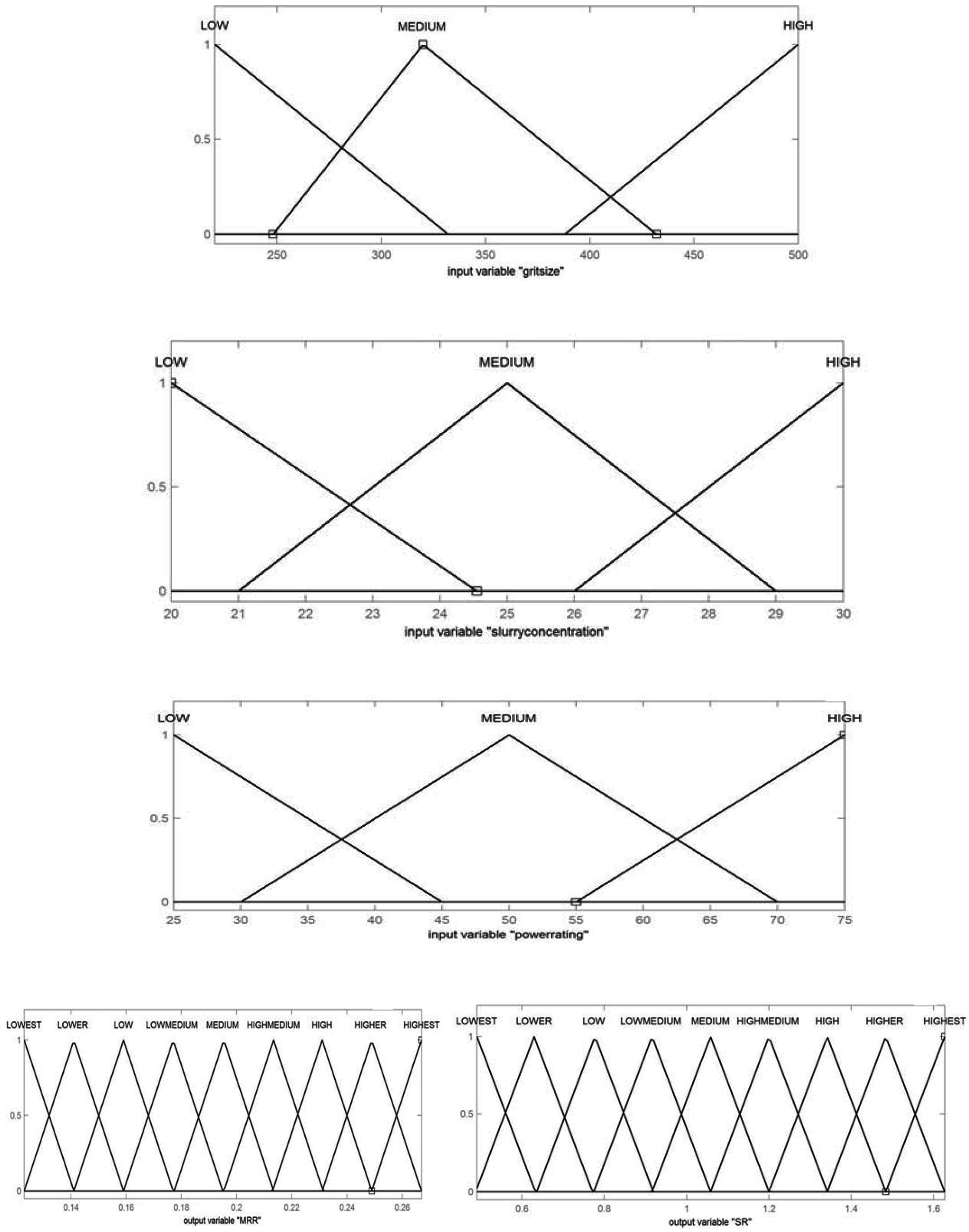


Fig. 2 Membership functions for input and output parameters using triangular membership function (slurry concentration, grit size and power rating)

using a triangular membership function for input parameters slurry concentration, grit size and power rating and the output parameters material removal rate (MRR) and surface roughness are represented in Fig. 2. From the figure, one can infer that the experimental values and fuzzy values are very close to each other and hence the fuzzy rule based modeling technique can be effectively used for prediction of MRR and surface roughness.

Similarly, the trapezoidal shaped membership function for input is specified by four parameters as follows

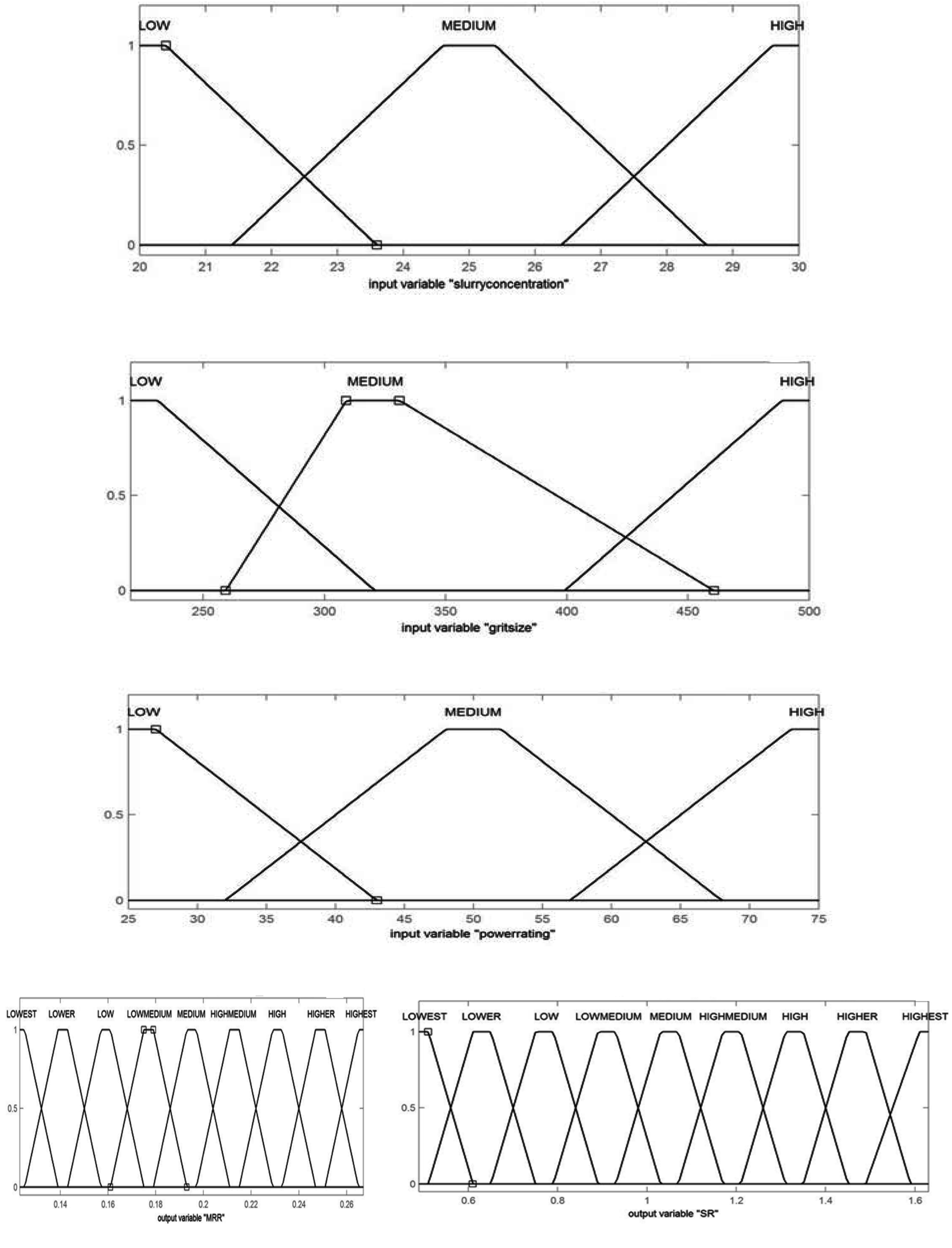


Fig. 3 Membership functions for input and output parameters using trapezoidal membership function (slurry concentration, grit size and power rating)

$$trapezoid(x;\, a, b, c, d) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c} & c \le x \ge d \\ 0, & d \le x \end{cases} \tag{15}$$

An alternative expression using min and max is

$$trapezoid(x;\, a, b, c, d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) \tag{16}$$

The membership functions used in this work using a triangular membership function for input parameters slurry concentration, grit size and power rating and the output parameters material removal rate (MRR) and surface roughness are represented in Fig. 3. He parameters [*a*, *b*, *c*, *d*] determine the *x* coordinates of the four corners of the underlying trapezoidal membership function. Finally, a defuzzification method is used. Defuzzification is an important operation in the theory of fuzzy sets. It transforms fuzzy set information into numeric information. In the present study, the centroid defuzzification method has been selected, because it produces the center of area of possibility distribution of the inference output and is a more frequently used defuzzification method for calculating the centroid of the area under the membership function

$$y_0 = \frac{\sum y \mu_{D_o}(y)}{\sum \mu_{D_o}(y)} \tag{17}$$

The non fuzzy value y_0 gives the output value in numerical form. The comparison between the

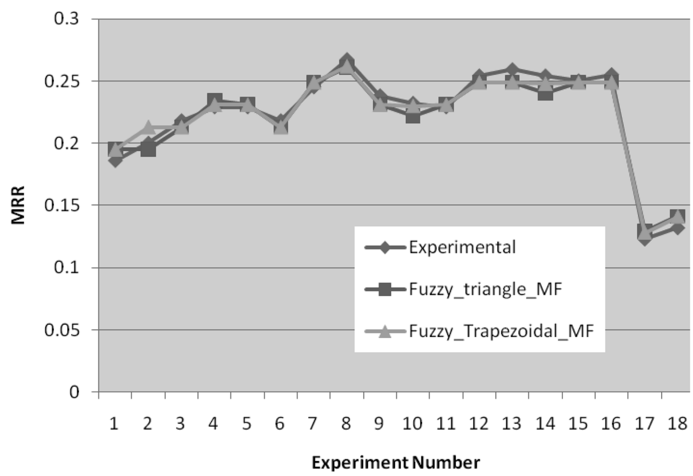


Fig. 4 Comparison between experimental results and fuzzy results for MRR (μm^3/min)

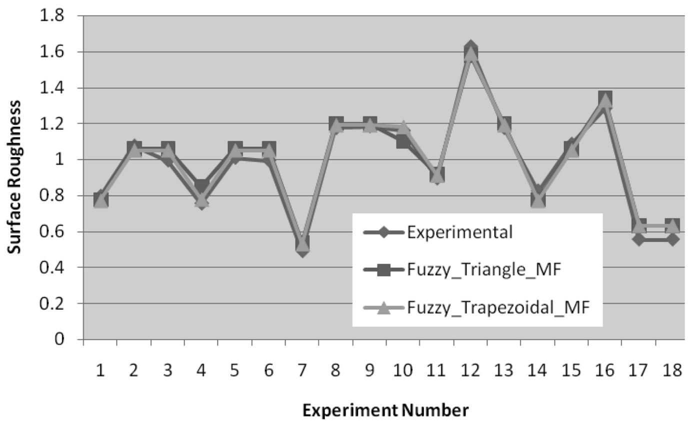


Fig. 5 Comparison between experimental results and fuzzy results for surface roughness (μm)

experimental and the fuzzy model prediction values for material removal rate and surface roughness is presented in Figs. 4 and 5. From the figure, one can infer that the experimental values and fuzzy values are very close to each other and hence the fuzzy rule based modeling technique can be effectively used for prediction of MRR and surface roughness in ultrasonic drilling of Titanium alloys.

7. Results and discussion

The material removal rate is of primary importance in rough ultrasonic machining of polycrystalline diamond. This study confirms that there exists an optimum condition for precision machining of PCD although the condition may vary with the composition of the material, the accuracy of the machine and other external factors. The taguchi quality function has been applied because high material removal rate and low surface roughness are conflicting goals, which cannot be achieved simultaneously with a particular combination of control settings.

It was observed that for the PCD, MRR tend to increase with a corresponding increase in the coarseness of the slurry used irrespective of the abrasive used for preparation of the slurry. This is because the coarser grit causes more extensive damage to the material during the abrasive impact. When the size of the abrasive particle becomes comparable with the tool amplitude, maximum MRR is obtained. Any further increase in grit number decreases the grit size considerably, resulting in several layers of abrasive particles which results in less effective machining. Also, the MRR obtained with different tool materials (TITAN12, TITAN15 and TITAN31) are significantly different when all other input parameters are controlled and remain fixed. Thus, the tool material properties such as hardness and toughness also have been found to control the machining characteristics in USM of PCD.

The optimum combination of design parameters is A1B3C1D3E3 as shown in Table 4. The test results reveal the following as optimum operating conditions: a tool material of TITAN12, a slurry concentration of 20%, a grit size of 500, abrasive slurry of B_4C and a power rating of 75% (375 *W*).

A mathematical model of the material removal rate using fuzzy logic appraoch has been formulated by identifying the physical parameters that affect the process of material removal and surface roughness in ultrasonic machining process. The calculated results from the model show good agreement when compared to the experimental findings.

8. Conclusions

1. The fuzzy logic rule based models for material removal rate and surface roughness were developed for the experimental data using two different membership functions, viz. triangular and trapezoidal. The predicted fuzzy output values and measured values are fairly close to each other, which indicate that the fuzzy logic model can be effectively applied to predict the material removal rate and surface roughness in ultrasonic machining of polycrystalline diamond.
2. In fuzzy rule based modeling, the trapezoidal membership functions perform better than triangular membership functions.
3. The taguchi quality loss function can be used to optimize the multiple quality characteristics. The quality characteristics experimental values of material removal rate and surface roughness at optimum conditions (0.201 μm^3/min, 0.522 μm) have been improved considerably in comparison to initial parameter settings of the experiment (0.186 μm^3/min, 0.795 μm). The improvement in MSNR at the optimum combination found to be 1.481 dB.
4. The optimum parameter values in the present operating conditions found to be are tool material: Titan 12, abrasive slurry: B_4C, slurry concentration: 20%, abrasive grit size: 500 and power rating: 375 *W*.
5. The material removal rate and surface roughness have been affected by using the different types of abrasive slurries. It could be concluded that use of boron carbide slurry results in better material removal rate for same process conditions in comparison to silicon carbide and aluminum oxide. This can be attributed to the higher hardness and cutting ability of boron carbide in comparison to silicon carbide and aluminum oxide abrasives.
6. The loss of quality is always possible during optimization of multiple quality characteristics at a time. The deviation of quality from its optimum value depends mainly on the weight assigned to it. Therefore a careful selection of weights for different quality values plays a crucial role in multi-objective optimization.

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