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Crack identification based on Kriging surrogate model

Hai-yang Gao*, Xing-lin Guo[a] and Xiao-fei Hu[b]

State Key Laboratory of Structural Analysis for Industrial Equipment,
Dalian University of Technology, Dalian, P.R. China

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Abstract. Kriging surrogate model provides explicit functions to represent the relationships between the inputs and outputs of a linear or nonlinear system, which is a desirable advantage for response estimation and parameter identification in structural design and model updating problem. However, little research has been carried out in applying Kriging model to crack identification. In this work, a scheme for crack identification based on a Kriging surrogate model is proposed. A modified rectangular grid (MRG) is introduced to move some sample points lying on the boundary into the internal design region, which will provide more useful information for the construction of Kriging model. The initial Kriging model is then constructed by samples of varying crack parameters (locations and sizes) and their corresponding modal frequencies. For identifying crack parameters, a robust stochastic particle swarm optimization (SPSO) algorithm is used to find the global optimal solution beyond the constructed Kriging model. To improve the accuracy of surrogate model, the finite element (FE) analysis soft ANSYS is employed to deal with the re-meshing problem during surrogate model updating. Specially, a simple method for crack number identification is proposed by finding the maximum probability factor. Finally, numerical simulations and experimental research are performed to assess the effectiveness and noise immunity of this proposed scheme.

Keywords: Kriging surrogate model; crack identification; stochastic particle swarm optimization; probability factor

1. Introduction

Fatigue damage commonly occurs in engineering structures under working conditions of high rotational velocity and impact loads, which may induce intensive stresses and dangerous damages. A significant number of methods for health monitoring and damage detection based on finite element model updating technique have been proposed and developed during recent years. A comprehensive review was given by Doebling *et al.* (1998), of most related works, as well as future research requirements.

Previous studies on damage identification can be categorized into two branches: the element-based method and the DOF-based method. For the element-based method, the model updating strategy is commonly employed, which is considered as some changes in physical properties of a structure

*Corresponding author, Ph.D. Candidate, E-mail: eric415@126.com
[a]Professor, E-mail: xlguo@dlut.edu.cn
[b]Ph.D. Candidate, E-mail: beyondhxf@163.com

such as Young's modulus, the section inertia, density and Poisson ratio of elements. Once the FE model has been constructed, the position of elements and nodes would never be changed during the identification. For the DOF-based method, damage is often described as added massless export node which connects two neighboring elements (e.g., crack). After that, the identification of crack locations and sizes is carried out. Differently from element-based method, the construction of FE model depends on damage locations and sizes which remain to be confirmed. Therefore, re-meshing techniques should be considered.

Due to its particularity and uncertainty, the crack identification problem can hardly employ the element-based method. In the early stages of the development of crack detection, the frequency contour plot method (Nandwana and Maiti 1997, Chaudhari and Maiti 2000, Lele and Maiti 2002) was one of the most favorite techniques to identify a single crack using the first few modal frequencies since these parameters can be determined by measuring at only one point of the structure. Gudmundson (1982, 1983) discussed the effect of geometrical imperfection on the eigenvalue by means of perturbation analysis. Liang *et al.* (1991) proposed a massless rotational spring model for the crack based on the Euler-Bernoulli beam theory, in which the location and size of a crack can be identified via finding the intersection point of a few frequency contour lines. Ostachowicz and Krawczuk (1991) studied the forced vibrations of the beam and the effects of the crack locations and sizes on the vibration behavior of the structure. Nandwana and Maiti (1997) developed the frequency contour plot method from constant section beam model to stepped beam and truncated wedged beam. Lele and Maiti (2002) extended the frequency contour plot method in beams based on Timoshenko beam theory. However, the frequency contour plot method suffers from the drawback that the curves of frequency contour plot might not intersect because of inaccuracies in the modeling with respect to the measured results in many cases.

Many efforts have been devoted to single crack identification in beams. However, there are few works on multiple cracks identification. In response, Hu and Liang (1993) first used the continuous damage model to identify the discrete elements of a structure. Patil and Maiti (2003) adopted the mathematical approach of Hu and Liang (1993) and applied the transfer matrix of vibration modeling to multi-crack identification. Later, Shifrin and Ruotolo (1999) presented a strategy using $n+2$ characteristic equations for a beam with n cracks. The component mode synthesis method (Kisa and Brandon 2000) and modied Fourier series method (Zheng and Fan 2001a, b) were also proposed for multi-crack identification. Unfortunately, a serious limitation is that the number of cracks present in a beam is usually not known as a *priori* in a practical damaged engineering structure.

Crack identification can be attributed to an inverse problem. Besides the direct least square method, a large volume of iterative algorithms was reported. A simple sensitivity-based method was applied by Lee (2009a, b), which relies on the minimization of certain objective function based on the residual errors between the measured and predicted frequencies; the Newton-Raphson iterative procedure was carried out to identify n cracks in a beam where $2n$ natural frequencies of the cracked beam were strictly required. In his work, the mesh of beam model was regenerated at each iterative step. Nevertheless, it is pointed out by Fang and Perera (2011) that proper selection of the initial guesses is important for the convergence of the solution. Additionally, ill-conditioning and non-uniqueness in the solution of inverse problem may appear as inevitable difficulties for a large-scale structure.

To avoid the shortcoming of sensitivity-based method, artificial intelligence techniques such as neural networks and generic algorithms have been increasingly utilized owing to their excellent pattern recognition capability and the global convergence property (Atalla and Inman 1998, Zitzler and Thiele 1999, Lee *et al.* 2005, Perera and Ruiz 2008). However, the required number of training samples would exponentially increase, resulting in considerable computational efforts.

As a combination of mathematical and statistical techniques, surrogate model might constitute a good alternative for parameter identification, which relaxes the strict requirement on the quantitative relation between inputs and outputs compared to sensitivity-based method. And it is potential to build any mathematical relationship between input parameters and output responses in both linear and nonlinear systems. Response surface model (RSM) for damage detection has drawn much attention. In the thesis by Cundy (2002), a four-step process based on RSM was developed for simple physical systems and damage identification was performed successfully, given the limited amount of "training" data used. Faravelli and Casciati (2004) and Casciati (2010) employed acceleration time histories collected under different loading conditions for identifying the presence of damage and locating cracks by a comparison of the sum of the squared errors (SSE) histograms. Moreover, a derivative of SSE, defined as ADM, was proposed and used for detecting distributed cracks in an actual masonry (Casciati 2010). Huang *et al.* (2011) pointed out that since RSM requires an understanding of the qualitative tendency of the entire design space, it is likely awkward when used for describing non-linearity commonly appeared in complex problem.

Compared to RSM, Kriging surrogate model (KSM) has more flexibility to model response data with multiple local extrema, which is already popular in the chemical and industrial design (Shyy *et al.* 2001, Gao and Wang 2008, Gao *et al.* 2008, Forrester and Keane 2009). However, to date, little research has been carried out in applying KSM to crack identification. In this work, we explore an effective scheme for multi-crack identification based on KSM. The initial KSM is constructed by the samples of crack parameters and their corresponding frequencies. To estimate the actual crack parameters, then the SPSO algorithm is employed, which is demonstrated to be effective, accurate and robust for searching for a global optimal solution. After that, the optimal solution will be used in FE analysis and inserted into the initial sample set to update the initial KSM, until the surrogate model is sufficiently accurate and the optimization process converges. For a practical damaged engineering structure, a simple procedure for crack number identification is proposed by finding the maximum probability factor. Numerical examples and experimental test have been performed on cantilever beams. Finally, some appealing merits of the proposed scheme were also discussed.

2. Finite element model

The Finite element model of a cantilever beam with double cracks is shown in Fig. 1(a). Parameters $\alpha_i = a_i/h$ and $\beta_i = s_i/L$ $(i = 1.2\ldots)$ represent the normalized size and location of the ith crack. In previous literatures, cracks are modeled as massless rotational springs or finite element equation of a beam segment based on the Euler-Bernoulli theory or Timoshenko theory. Also some empirical equations given by Ostachowicz and Krawczuk (1991) and Dimarogonas and Paipetis (1983) are employed to deal with the crack stiffness matrix for rectangular beam. However, it is somewhat discommodious when the FE models need to be re-meshed. In this study, a two-dimensional plane elastic rectangular beam is modeled by using ANSYS code to avoid this trouble instead of Euler-Bernoulli or Timoshenko beam theory, as shown in Fig. 1(b).

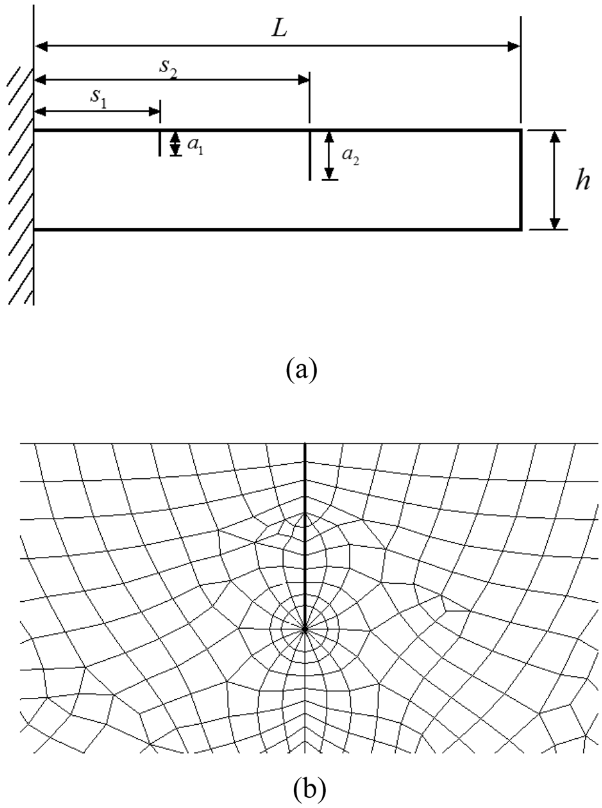

Fig. 1 Schematic: (a) cantilever beam with double cracks (b) two-dimensional mesh at crack tip

3 Basic theory of Kriging model

3.1 Sampling method

The basis of building a Kriging model is sample information. Here, a modified rectangular grid (MRG) approach presented by Gao *et al.* (2008) is adopted to ensure its uniformity within design region. Moreover, it can move some points lying on the boundary into the internal design region, which will provide more useful information for the Kriging model, and it can ensure that the points have less replicated coordinate values. A brief review is given in Fig. 2.

The advantage of this method is shown in Fig. 3 given by Gao *et al.* (2008). It can be seen that MRG can avoid the case that the sample points are spaced close to each other, which may occur using Latin Hypercube Sampling (LHS).

3.2 Construction of Kriging surrogate model

Kriging surrogate model is a statistics-based interpolated method (Sacks *et al.* 1989, Sakata *et al.* 2007). It maps the input parameters to the corresponding responses by a model function, which can be written as

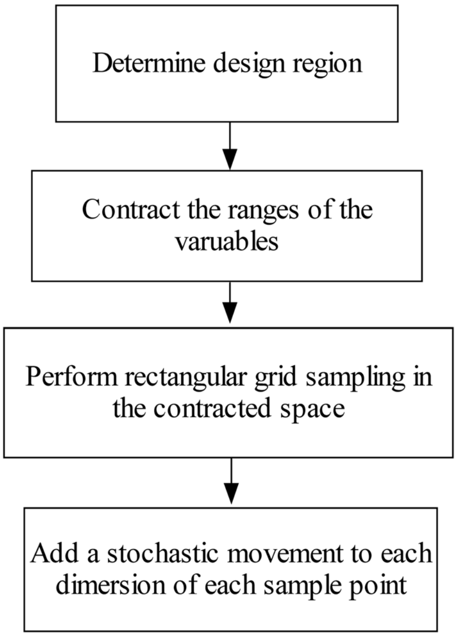

Fig. 2 Flowchart of modified rectangular grid approach

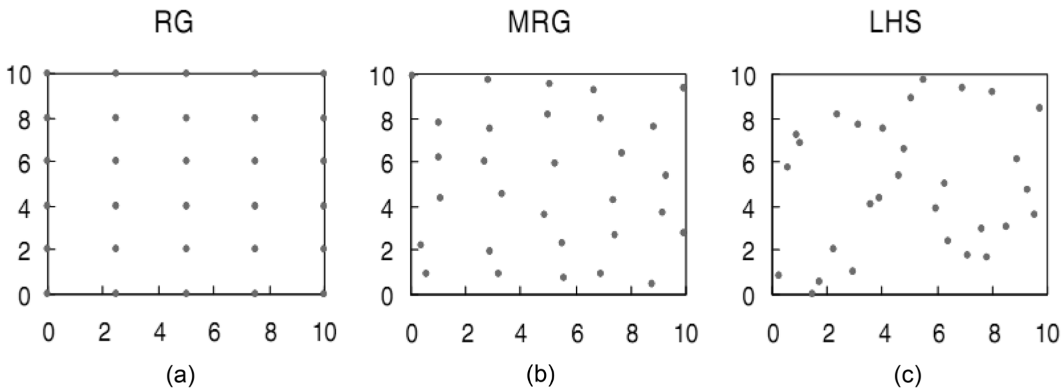

Fig. 3 Three different methods of sample distributions: (a) RG (b) MRG (c) LHS

$$\hat{\mathbf{y}}(\mathbf{x}^i) = \mathbf{f}^T(\mathbf{x}^i)\boldsymbol{\beta} + \mathbf{z}(\mathbf{x}^i) \qquad i = 1, 2, \ldots, n \tag{1}$$

where $\mathbf{x}^i = [x_1^i, x_2^i, \ldots, x_m^i]$ is the ith sample points expressed as a m-dimension variable vector, $\hat{\mathbf{y}}(\mathbf{x}^i)$ is an approximate function corresponding to a linear or nonlinear function $\mathbf{f}(\mathbf{x}^i)$. $\boldsymbol{\beta}$ denotes the matrix of regression coefficients to be estimated from the samples, and $\mathbf{z}(\mathbf{x}^i)$ represents a model of Gaussian and stationary stochastic process with mean of zero and variance of σ^2. The covariance matrix of $\mathbf{z}(\mathbf{x})$ is

$$\mathrm{Cov}[\mathbf{z}(\mathbf{x}^i), \mathbf{z}(\mathbf{x}^j)] = \sigma^2 \mathbf{R}(\theta, \mathbf{x}^i, \mathbf{x}^j) \tag{2}$$

The $n \times n$ correlation function matrix between $\mathbf{x}^i$ and $\mathbf{x}^j$ can be formed by Gaussian correlation function with only a single correlation parameter θ in this paper. It provides a smooth and infinitely differentiable surface, given by

$$\mathbf{R}(\theta, \mathbf{x}^i, \mathbf{x}^j) = \exp\left[-\theta \sum_{l=1}^{m} (x_l^i - x_l^j)^2\right] \tag{3}$$

The unknown correlation parameters $\boldsymbol{\beta}$ and σ^2 can be estimated by maximizing the log-likelihood function (Jones *et al.* 1998)

$$\ln(\boldsymbol{\beta}, \sigma^2, \theta) = -\frac{1}{2}\left[n \ln \sigma^2 + \ln|\mathbf{R}| + \frac{1}{\sigma^2}(\mathbf{Y} - \mathbf{f}^T\boldsymbol{\beta})^T \mathbf{R}^{-1}(\mathbf{Y} - \mathbf{f}^T\boldsymbol{\beta})\right] + C \tag{4}$$

where $\mathbf{Y} = [\hat{\mathbf{y}}(\mathbf{x}^1), \hat{\mathbf{y}}(\mathbf{x}^2), \ldots, \hat{\mathbf{y}}(\mathbf{x}^n)]$, denotes the response value of samples, and C denotes a constant. By differentiating the log-likelihood function with respect to $\mathbf{b}$ and σ^2, respectively, and letting them be equal to zero. We can then obtain

$$\hat{\sigma}^2 = \frac{1}{n}(\mathbf{Y} - \mathbf{F}^T\boldsymbol{\beta})^T \mathbf{R}^{-1}(\mathbf{Y} - \mathbf{F}^T\boldsymbol{\beta}) \tag{5}$$

and

$$\hat{\boldsymbol{\beta}} = (\mathbf{F}^T\mathbf{R}^{-1}\mathbf{F})^{-1}(\mathbf{F}^T\mathbf{R}^{-1}\mathbf{Y}) \tag{6}$$

where $\mathbf{F} = [\mathbf{f}(\mathbf{x}^1), \ldots, \mathbf{f}(\mathbf{x}^n)]^T$. This model leads to a best linear unbiased predictor and an associated mean squared error. The function value $\hat{\mathbf{y}}(\mathbf{x}^*)$ at new points $\mathbf{x}^*$ can be approximately predicted by

$$\hat{\mathbf{y}}(\mathbf{x}^*) = \mathbf{f}(\mathbf{x}^*)^T\hat{\boldsymbol{\beta}} + \mathbf{r}(\mathbf{x}^*)^T\mathbf{R}^{-1}(\mathbf{Y} - \mathbf{F}\hat{\boldsymbol{\beta}}) \tag{7}$$

in which

$$\mathbf{r}(\mathbf{x}^*) = [R(\theta, \mathbf{x}^1, \mathbf{x}^*), R(\theta, \mathbf{x}^2, \mathbf{x}^*), \ldots, R(\theta, \mathbf{x}^n, \mathbf{x}^*)]^T \tag{8}$$

Kriging model can not only predict responses by the new input parameters, but also estimate optimal parameters satisfied with equality or inequality response constraints in both linear and nonlinear system (Simpson *et al.* 2001). When the original surrogate model is constructed, the quality of the model can be assessed according to the accuracy of predictions. The squared multiple correlation coefficient (R^2) and the empirical integrated squared error (*EISE*) criterion (Ren and Chen 2010) can be used to select additional sample points for updating the initial KSM.

$$R^2 = 1 - \frac{\sum_{j=1}^{N}[\hat{y}_j - y_j]^2}{\sum_{j=1}^{N}[y_j - \tilde{y}]^2} \tag{9}$$

$$EISE = \frac{1}{N}\sum_{j=1}^{N}[\hat{y}_j - y_j]^2 \tag{10}$$

where $\hat{y}_j$ and y_j are the *j*th component of the response vector of surrogate model and the true value calculated via FE analysis respectively, $\tilde{y}$ is the mean of all true values, N is the length of vector $\hat{\mathbf{y}}$.

4. Optimization procedure

The identification of crack locations and sizes is actually an inverse problem. The optimal parameters can be determined by minimizing the discrepancies between the predicted responses based on KSM and measured ones from the practical structure. To construct KSM, the samples of varying crack locations and sizes are used as the input parameters and their corresponding model frequencies are adopted as the output responses. The single-objective function can be constructed by using first few frequencies, which can be expressed as

$$\min\Pi = \sum_{j=1}^{N} W_j(\hat{\lambda}_j(\mathbf{x}^*) - \lambda_j) \qquad \begin{matrix} 0 < W_j < 1 \\ \mathbf{lb} < \mathbf{x}^* < \mathbf{ub} \end{matrix} \tag{11}$$

where $\hat{\lambda}$ is a vector of frequencies predicted by KSM at sample point $\mathbf{x}^*$, and λ represents the measured frequencies caused by actual crack locations and sizes in a practical structure; W_j is weighting factor to impose to the different order of natural frequencies. In this study, all its element are set to 1. **lb** and **up** denote the lower and upper bounds.

To avoid the local optimal solution in conventional sensitivity-based method due to improper selection of the initial guesses, a SPSO algorithm (Qi *et al.* 2007) is adopted for crack identification based on the constructed Kriging model, which has been demonstrated to be effective, accurate and robust for searching a global optimal solution. When weight parameter is set to 0.0, the simple updating procedure of the PSO can be expressed as

$$X_i(t+1) = X_i(t) + c_1 r_1[P_i(t) - X_i(t)] + c_2 r_2[P_g(t) - X_i(t)] \tag{12}$$

where c_1 and c_2 are two positive constants called acceleration coefficients, P_i and P_g are local and global best locations, respectively, r_1 and r_2 are random numbers in the interval (0,1). If $P_g = P_i$, $X_i(t+1) = X_i(t)$. Hence, the particle at the global best position will stop evolution. To improve the global searching ability, this algorithm randomly generates an extra particle labeled j with position X_j to continue evolution in the search domain. It means that at least one particle is generated in the searching domain randomly to improve the global searching quality of PSO algorithm.

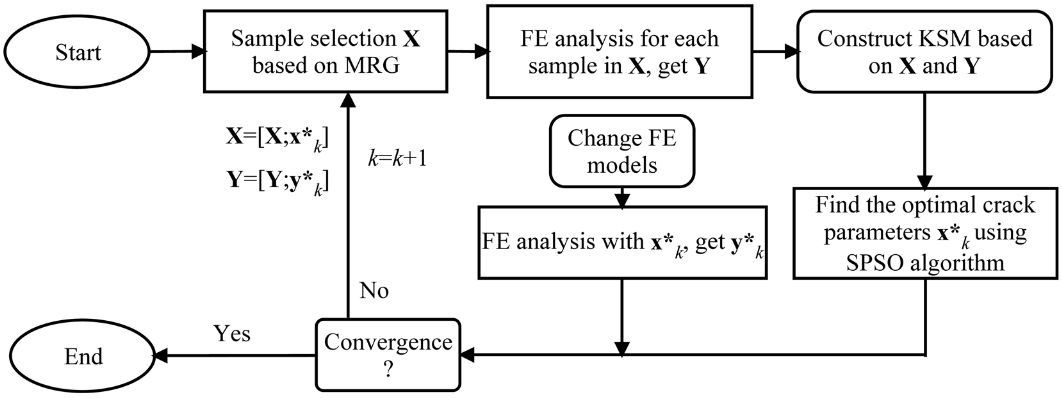

Fig. 4 Flowchart of crack identification based on Kriging surrogate model

5. Computation procedures

The proposed scheme can be organized as a surrogate model updating and optimal estimation procedure as illustrated in Fig. 4. The identification of crack locations and sizes consists of the following steps:

Step 1 Generate initial sample matrix **X** (crack locations and sizes) using MRG method and run the simulation program ANSYS to obtain the corresponding output response **Y** (say modal frequencies).

Step 2 Construct the initial Kriging surrogate model with sample parameters and output responses obtained in Step 1,

Step 3 Find the optimal parameters $\mathbf{x}^*_k$ by minimizing discrepancies between response measured in practical structures and that calculated by means of SPSO algorithm based on the initial Kriging surrogate model, and set the iterative index $k = 1$,

Step 4 Check criterions: If the current response $\mathbf{y}^*_k$ predicted by ANSYS and the one based on Kriging surrogate model satisfy the given criterions (both $R^2 > 0.98$ and $EISE < 0.01$), then stop Kriging surrogate model updating and predict the parameters with this surrogate model; else, go to Step 5,

Step 5 Add $\mathbf{x}^*_k$ and $\mathbf{y}^*_k$ behind the initial sample points generated in Step 1, then update the Kriging model with setting the iterative index $k = k + 1$,

Step 6 Loop to Step 3 and repeat the process till the criterions are satisfied.

6. Numerical examples

6.1 Single crack in a beam

First of all, four cases of single crack in a beam are performed to assess the proposed method. The followings are the properties adopted: Young's modulus $E = 210$ GPa, Poisson's ratio $\mu = 0.3$, density $\rho = 7850$ Kg/m^3, length $L = 5$ m, height $h = 0.02$ m. Finite element analysis model is built by using ANSYS code. The crack parameters (locations and sizes) are identified using SPSO algorithm

Table 1 Comparison of actual and identified crack parameters after 100 generations

Case	The number of Kriging surrogate model updating step (k)	α		β	
		Actual	Identified	Actual	Identified
A	1	0.1	0.0992	0.2	0.1996
B	1	0.2	0.1998	0.5	0.5004
C	1	0.3	0.3006	0.6	0.5998
D	1	0.4	0.4005	0.8	0.8000

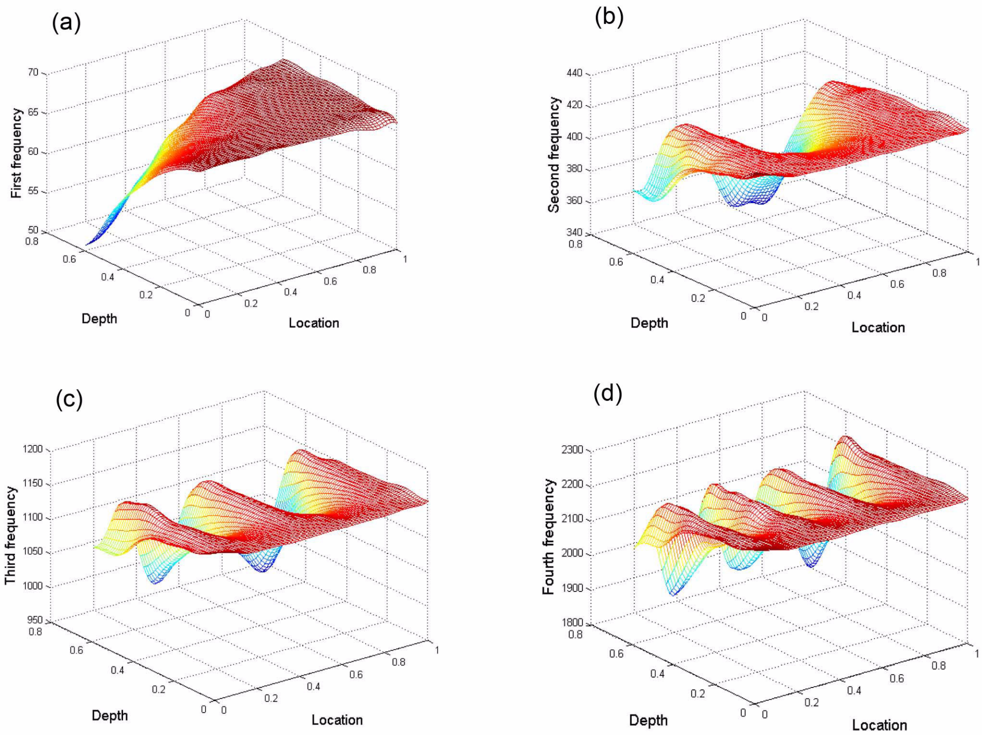

Fig. 5 Kriging model for each frequency as function of depth (size) and location after 100 generations in Case **B**

based on Kriging surrogate model. To establish the KSM, 30 groups of initial samples in the interval of (0~1) and the corresponding first four calculated model frequencies are selected as the input and output, respectively. The c_1 and c_2 of SPSO are set to 1.0. The Swarm Size and Maximum Swarm are set to 20 and 100, respectively. The inverse problem of identifying single crack in a cantilever beam is solved for four cases (**A**, **B**, **C** and **D**) as shown in Table 1.

From Table 1, it can be found that the identified crack parameters are very close to the assumed actual ones for all cases. It means that the proposed scheme has a good accuracy for single crack detection in a beam. Fortunately, the initial Kriging surrogate model ($k = 1$) is precise enough to predict the crack parameters. And more importantly, there is no limitation for initial guesses of each

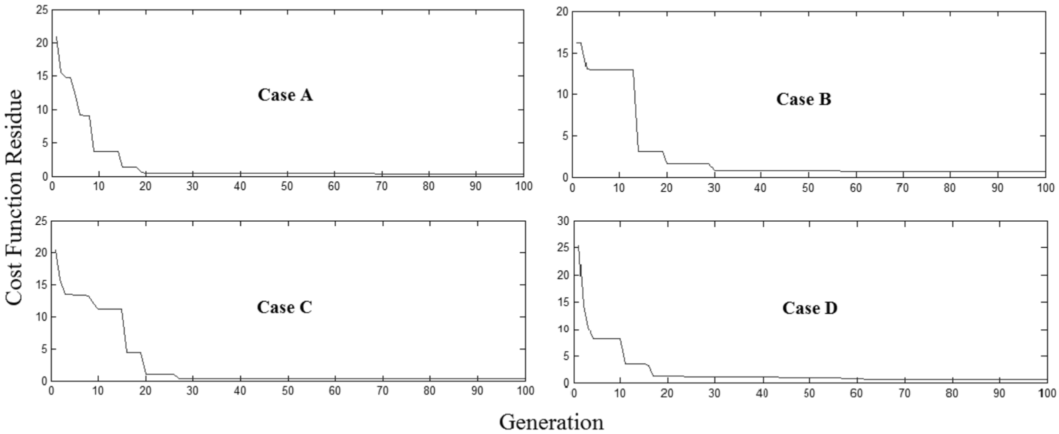

Fig. 6 The sum modal frequency residuals of each mode during SPSO for Case **A** to Case **D**

Table 2 Parameters setting of initial sample and SPSO algorithm

Parameters	Case **E**	Case **F**
Swarm Size	30	50
Maximum Swarm	300	500
c_1 and c_2	1.0	1.0
Frequency used	First 5 modes	First 6 modes
The number of sample points	200	550

Table 3 Comparison of actual and identified crack parameters in Case E and F

Case	k	α_1	α_2	α_3	α_4	α_5	β_1	β_2	β_3	β_4	β_5
E											
Actual	2	0.10	0.15	0.18	-	-	0.35	0.55	0.75	-	-
Identified		0.093	0.143	0.186	-	-	0.347	0.552	0.742	-	-
F											
Actual	3	0.08	0.10	0.15	0.18	0.20	0.20	0.30	0.40	0.50	0.60
Identified		0.075	0.106	0.142	0.169	0.211	0.198	0.310	0.392	0.496	0.611

parameter which is a common problem in sensitivity-based approach. The relations between crack parameters and first four modal frequencies in Case **B** can be extracted from Kriging model as shown in Fig. 5. Fig. 6 illustrates the convergence of cost function residue during SPSO. The horizontal axis is the number of generation while the vertical axis is the sum modal frequency residuals of each mode.

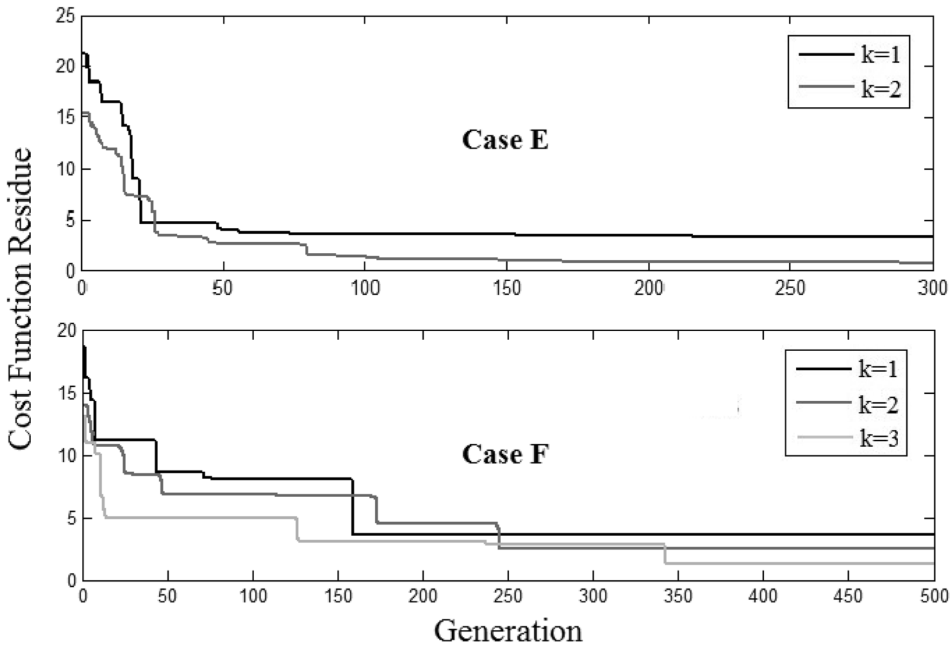

Fig. 7 The sum modal frequency residuals during SPSO algorithm and Kriging model updating for Case **E** and **F**

6.2 Multiple cracks in a beam

The presented scheme can easily be extended to a beam with multiple cracks. In this section, two cases (**E** with three cracks; **F** with five cracks) are considered to validate the effectiveness of the proposed scheme. The identification procedure is carried out with parameters setting in Table 2, and the actual and identified crack parameters are given in Table 3. The cost function values in Case **E** and **F** of each generation are plotted in Fig. 7.

It was found that the proposed scheme gave satisfactory predictions for multi-crack cases. The errors of the estimated crack locations and sizes are within 6.25 percent of the actual ones. At the same time it is interesting to remark that the identified crack locations seem to be more accurate than the identified crack sizes. One reason could be that the location parameters are more sensitive than size parameters to the model frequencies of the structure. Particularly, this method can be extended to detect any number of cracks without the strict requirement on the quantitative relation between inputs and outputs compared to sensitivity-based method. In this sense, Kriging surrogate model provides us a better choice for estimating with a moderate number of variables (less than 50) (Simpson *et al.* 2001).

To further study the effect of referring mode number on the identification result, Case F was recalculated by using 4, 5 and 7 referring modal frequencies, respectively. The results are listed in Table 4, in which the required number of Kriging model updating steps is also given. It was found that more updating steps should be made to describe a relatively accurate surrogate model when a small number of referring modes is used. All results are uniquely given by SPSO algorithm. Remarkably, the identifications with 6 and 7 modes yield very similar results due to a more prefect surrogate model compared to the ones with 4 or 5 modes.

Table 4 Identification results using different number of referring modes

Number of referring modes	k	α_1	α_2	α_3	α_4	α_5	β_1	β_2	β_3	β_4	β_5
4	7	0.071	0.114	0.152	0.162	0.245	0.192	0.322	0.444	0.475	0.632
5	4	0.075	0.104	0.162	0.178	0.188	0.202	0.292	0.395	0.502	0.610
6	3	0.075	0.106	0.142	0.169	0.211	0.198	0.310	0.392	0.496	0.611
7	3	0.076	0.106	0.145	0.170	0.210	0.198	0.311	0.392	0.496	0.611

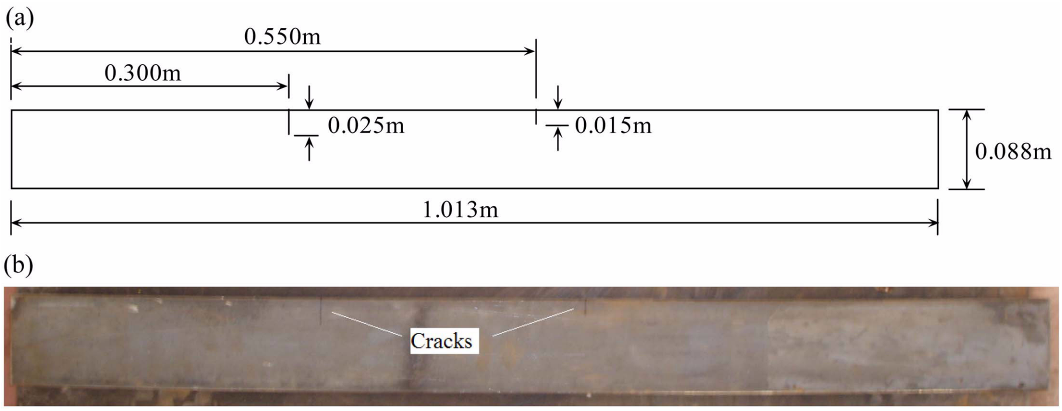

Fig. 8 (a) Dimensional drawing; (b) a steel beam with two cracks

7. Experimental study

7.1 Problem description

The validity of this scheme has been verified through simulated examples. For experimental measurements, it is expected that there would be some deviation due to noise originating from environment as well as electronic devices. In this section, a steel beam with two cracks cut by wire EDM is used to assess the feasibility and anti-noise ability of this proposed scheme, as shown in Fig. 8, which has the elastic modulus of E = 209.2 GPa, Poisson's ratio of μ = 0.25, mass density of ρ = 7830 Kg/m^3, length of L = 1.013m, width of S = 0.088 m, thickness of t = 0.009 m. A two-dimensional finite element numerical analysis model of the beam is built by using ANSYS program. As we know, frequency discrepancy between modal predictions and experimental results is usually generated due to inevitable difficulties or oversights in the design, material property and boundary condition. Therefore, model updating strategy should be employed for minimizing this discrepancy before crack identification. An undamaged beam with same material property is prepared for updating the properties of the FE model.

During the experiment, a 6-channel B&K 3050 data acquisition system with Pulse 13.1 software, an impact hammer with an accelerometer is employed as shown in Fig. 9. The acquired frequency response functions are then inputted into the modal analysis software ME'scope 4.0 to identify the first six frequencies of undamaged and cracked beams in free boundary condition, respectively, as

Fig. 9 Experimental setup for crack identification

Table 5 First six theoretical and measured frequencies

Mode	1	2	3	4	5	6
Undamaged beam						
Measured frequency (Hz)	439.66	1158.6	2152.3	2538.4	3339.1	4656.3
Theoretical frequency after model updating(Hz)	440.59	1159.7	2156.3	2537.1	3341.6	4660.7
Cracked beam						
Measured frequency (Hz)	425.61	1111.3	2138.5	2528.7	3385.4	4536.9
Theoretical frequency after model updating (Hz)	423.95	1115.7	2135.4	2535.8	3389.3	4543.8

shown in Table 5. It can be seen that relative accurate FE model can be constructed after model updating, which lays the foundation for crack identification based on the Kriging model.

7.2 An additional procedure for crack number identification

Like other methodologies, the number of cracks present in a beam is usually not known as a *priori* in a practical damaged engineering structure, which is a serious limitation in its application. Some strategies (Hu and Liang 1993, Lee 2009, Lam and Ng 2008) were presented to overcome this difficulty.

Here a simple method related to SPSO algorithm is presented for crack number identification. Different from Newton-Raphson iteration method, SPSO algorithm can provide a converged solution in most case. It means that this searching algorithm ensures the existence of solutions. The value of cost functions should be a constant in this work. When the assumed number of cracks is equal to the actual one, the value of cost function should be minimal. That is, we can find the most probable number of cracks by looking for the number which yields the minimum value of cost function. Fig.10 shows the procedure for crack number identification. Firstly, a single crack is

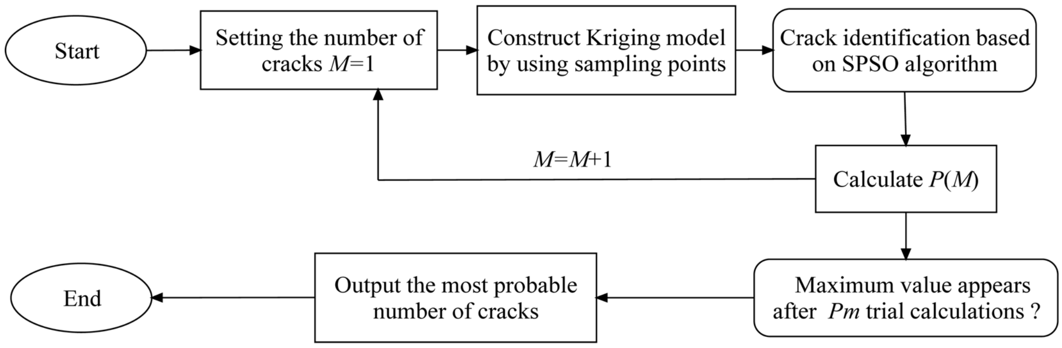

Fig. 10 Flowchart of crack number identification

Table 6 Comparison of actual and identified crack parameters in experimental case

Case	k	α_1		β_1		α_2		β_2	
		Actual	identified	Actual	identified	Actual	identified	Actual	identified
G	2	0.2841	0.2902	0.2962	0.3005	0.1705	0.1683	0.5429	0.5473

assumed and crack identification is carried out, then we calculate the cost function (CF). The values under conditions of several different numbers of cracks in the beam are calculated in the same manner. The probability factor is defined as

$$P(M) = \frac{CF(M)^{-1}}{\sum_{M=1}^{P_m} (CF(M)^{-1})} \tag{13}$$

where M denotes the crack number we assumed to calculate the CF, Pm is the maximum number of cracks to be considered. More assumptive calculations should be carried out when the largest value of $P(M)$ appears. Finally, the number of cracks can be then determined by comparing with the $P(M)$ values. The maximum value of $P(M)$ indicates the most probable number of cracks. Numerical case will be discussed in the next section.

7.3 Results and discussion

To construct the Kriging model and identify the location and size of cracks, 20 groups of sample parameters corresponding to their first four theoretical model frequencies are selected as the input and output, respectively. The bounds of crack sizes are (0~0.5), and that of the crack locations are (0~1). The c_1 and c_2 of SPSO are set to 1.0. The Swarm Size and Maximum Swarm are set to 20 and 400, respectively. The identified results are shown in Table 6. It can be found that the two cracks can be located with the errors of 1.45% and 0.81% and the observed errors of the crack sizes are 2.15% and 1.29%. The validity of Kriging surrogate model seems to be insensitive to random

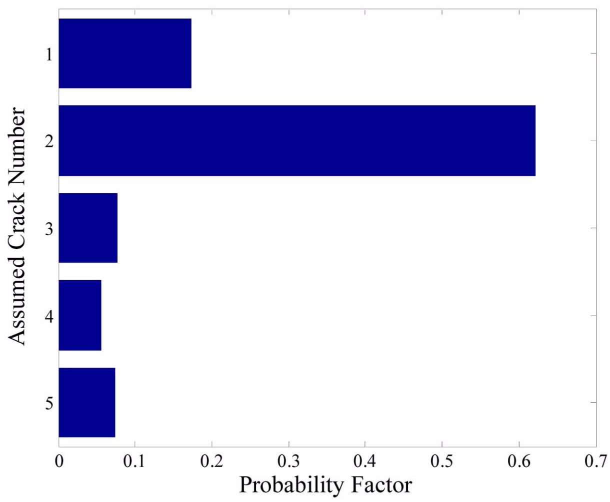

Fig. 11 Probability factor P(M) under conditions of five different assumed numbers of cracks

perturbations and noise information (Gao 2008). To check the crack number, $P(M)$ ($M = 1.2\ldots5$) is calculated under conditions of five different numbers of cracks in the beam, as shown in Fig. 11. The maximum value of $P(M)$ indicates a correct assumption of the crack number.

Although the proposed scheme shows satisfactory predictions for multi-crack identification, it should be mentioned that the natural frequency is the characteristics of the whole dynamic system. For complex structure, frequencies are not adequate since cracks result more in local features of the structure. Therefore, more responses such as model shapes, vibration amplitudes and frequency response functions might be taken into account for multiobjective SPSO. In addition, a simple method for adding sample point as illustrated in Section 5 was used to updating the Kriging model in this work. To improve the efficiency of Kriging model updating, the use of some advanced point addition criterions should be encouraged. Sakata *et al*. (2003, 2004) introduced an empirical semivariogram based semivariogram fitting approach for large-scale sampling problems on the optimization of stiffened straight cylinder and wing structure. Such an approach may serve as an excellent module for present method to improve the efficiency and accuracy of the surrogate model reconstruction.

8. Conclusions

The main aim of the present work is to provide an efficient scheme for crack identification based on Kriging surrogate model. The identification results in numerical and experimental cases show good performance of this scheme. Some appealing merits can be summarized as follow:

- Kriging surrogate model is applied to provide a simple relationship between crack parameters and corresponding modal frequencies for avoiding the expensive FE analysis at every iterative step of optimization.
- A modified rectangular grid approach was adopted to move inward the sample points on the boundary and ensure the uniformity within design region, which makes the initial Kriging model

more accurate inside the design space.

- A robust SPSO algorithm was used to solving a global optimization problem. The initial guesses of the crack parameters could be avoided, which is commonly difficult in Newton-Raphson iteration method.
- For a practical damaged engineering structure, the number of cracks is usually not known. In order to solve this issue, a simple and effective procedure is presented related to SPSO algorithm, in which the most probable number of cracks can be obtained by finding the maximum of $P(M)$ factor.

To assess the performance of the proposed scheme, Numerical studies of a cantilever beam with single and multiple cracks were carried out. The results show that the identified crack locations and sizes are in good agreement with the actual ones. Moreover, experimental research is also considered by using a cracked steel beam structure under free-free boundary condition. The proposed scheme is then proved to be effective and promising for crack identification in engineering.

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