Low energy ultrasonic single beacon localization for testing of scaled model vehicle

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Abstract. Tracking the location (position) of a surface or underwater marine vehicle is important as part of guidance and navigation. While the Global Positioning System (GPS) works well in an open sea environment but its use is limited whenever testing scaled-down models of such vehicles in the laboratory environment. This paper presents the design, development and implementation of a low energy ultrasonic augmented single beacon-based localization technique suitable for such requirements. The strategy consists of applying Extended Kalman Filter (EKF) to achieve location tracking from basic dynamic distance measurements of the moving model from a fixed beacon, while on-board motion sensor measures heading angle and velocity. Iterative application of the Extended Kalman Filter yields *x* and *y* co-ordinate positions of the moving model. Tests performed on a free-running ship model in a wave basin facility of dimension 30 m by 30 m by 3 m water depth validate the proposed model. The test results show quick convergence with an error of few centimeters in the estimated position of the ship model. The proposed technique has application in the real field scenario by replacing the ultrasonic sensor with industrial grade long range acoustic modem. As compared with the existing systems such as LBL, SBL, USBL and others localization techniques, the proposed technique can save deployment cost and also cut the cost on number of acoustic modems involved.

Keywords: Low Energy Ultrasonic based Beacon (LEUB), Extended Kalman Filter (EKF), Autonomous Surface Vehicle/ Underwater Vehicle (ASV/UV)

1. Introduction

The general class of oceanographic exploratory research vehicles includes the Autonomous Underwater Vehicles (AUVs), Autonomous Surface Vehicles (ASVs) and Remotely Operable Vehicles (ROVs). These are widely used in scientific, military and commercial applications which include oceanographic surveying and mapping, exploration and production of oil and gas, hull inspection and mine detection (Nicholson and Healey 2008). These vehicles require reliable navigation system for trajectory tracking, geo-referencing of data acquired during their mission and for recovery of the vehicle after the mission (Leonard *et al.* 1998). There are three main types of navigation systems for the general class of marine vehicles operating at or below the surface,

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they are: Inertial Navigation System, Acoustic Navigation system and Satellite (geophysical) Based Navigation system. The inertial navigation system provides information on the vehicle with regard to its position, depth and velocity with respect to the inertial frame of reference; the acoustic based navigation system gives the position of the vehicle with respect to a beacon position and the geophysical based navigation system gives the position of the vehicle in real-time with respect to previously mapped area of the earth in fixed frame (Geyer et al. 1987). The Dead Reckoning (DR) system uses sensors like inertial measurement unit and Doppler Velocity Logger (DVL). Using estimates of velocity of the moving vehicle and the time of travel, the system predicts the vehicle position from a previous step to the next step. In the absence of knowledge of position data, Dead Reckoning relies on the integration of velocity and /or acceleration in time, and this may result in the generation of error which grows over time (Stutters et al. 2008). Recent advances in underwater navigation include Simultaneous Localization and Mapping (SLAM) and Sensors Fusion which use estimators such as Extended Kalman Filter (EKF) to perform underwater navigation (Kinsey et al. 2006). Acoustic navigation systems are generally suitable for underwater applications due to the favourable propagation characteristics of sound in water. The acoustic navigation system works based on the Time of Arrival (TOA) and/or on the Time Difference of Arrival (TDOA) of acoustic signals between acoustic transponders or beacons, to obtain the range and bearing data (Smith and Kronen 1997). In such a system, a minimum of three transponders are required to perform the trilateration and /or triangulation to estimate the position of the vehicle. Examples of such systems are Long Base Line (LBL), Short Base Line (SBL) and Ultra Short Base line (USBL). Another similar system is the GPS intelligent buoy (GIB) to estimate the position of an underwater vehicle (Alcocer et al. 2007). However, there are also disadvantages with such systems; the accuracy of estimation of the vehicle position depends on the accuracy of the beacons placement and time synchronization. There is also high operational cost related to the deployment, calibration and recovery of the beacons (Vickery 1998). A reduced cost alternative is the Single Beacon Navigation (SBN) which overcomes the shortcomings of the acoustic based navigation system. Single Beacon Range-Based localization locates the position of a moving vehicle by measuring its distance to a single beacon mounted on a GPS enabled surface vehicle. A typical underwater application is the homing of a vehicle for docking application. Homing is often performed by means of acoustic sensors, which provide the bearing of the beacon to be reached in the vehicle frame. This measurement, combined with the heading of the vehicle, is used to directly steer the AUV towards the goal (Vaganay et al. 2000). The range of the Single Beacon localization system depends on the range of the acoustic modem for distance measurement between the vehicle and the beacon. Today such modems are available in the range from 50 m to 5000 m. The application of Extended Kalman Filter (EKF) enables the estimation of the position of the vehicle with respect to the location of a single beacon (Mao et al.2007). The single beacon navigation system obtains range measurements from a single acoustic beacon combined with dead reckoning or inertial navigation system (Ferreira et al. 2010) (Santos 2015). This saves the cost not only because of reduced number of beacons but also because of reduced calibration and deployment cost.

The above background status provides the motivation for the development of the Extended Kalman Filter based Augmented Single Beacon Localization (SBL) system presented here, which uses ultrasonic based range measurement in conjunction with onboard motion sensor based heading angle and velocity measurements. The following sections illustrate the proposed method. The presentation first describes the problem formulation of localization of ASVs/UVs from measurements obtained from a single beacon. The next step is the design of the EKF and

development of the ultrasonic based range measurement system, followed by description of tests conducted on a prototype test setup and results obtained from experiments conducted with the prototype.

2. Problem formulation

The requirement is to estimate the location (x - y) coordinates of a moving autonomous surface vehicle in an indoor wave basin relative to the local frame of reference. To simplify the approach, the equation of motion is decoupled in the horizontal and vertical planes (Fossen 2011). With these constraints 2-D localization assumes that the surface vehicle moves in the horizontal plane. The kinematic equations of motion in continuous time can be written as

$$\dot{x}(t) = u(t)\cos(\psi(t))$$

$$\dot{y}(t) = u(t)\sin(\psi(t))$$

$$\dot{\psi}(t) = r(t)$$

$$\dot{r}(t) = 0$$

$$\dot{u}(t) = 0$$
(1)

Here at time t, the state variables $\{x(t), y(t)\}$ are the coordinates of the ASV, u(t) is the speed of the vehicle, $\psi(t)$ is the heading angle measured in radian and r(t) is the yaw rate measured in radians per second in the horizontal plane. The equations in (1) can be written in a discrete-time form as

$$x_{k} = x_{k-1} + \Delta T u \cos(\psi_{k-1})$$

$$y_{k} = y_{k-1} + \Delta T u \sin(\psi_{k-1})$$

$$\psi_{k} = \psi_{k-1} + \Delta T r_{k-1}$$

$$r_{k} = r_{k-1}$$

$$u_{k} = u_{k-1}$$
(2)

Where ΔT is the sampling interval between the, k^{th} and $k-1^{th}$ sample. Eq. (2) written in matrix form becomes:

$$X_{k} = \begin{bmatrix} 1 & 0 & 0 & 0 & \Delta T \cos \psi_{k-1} \\ 0 & 1 & 0 & 0 & \Delta T \sin \psi_{k-1} \\ 0 & 0 & 1 & \Delta T & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \psi_{k-1} \\ r_{k-1} \\ u_{k-1} \end{bmatrix}$$
(3)

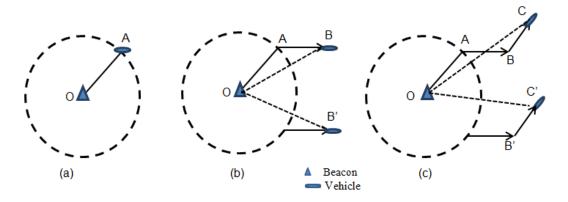


Fig. 1 Shows three different cases of single beacon localization system

The distance from the single beacon located at (x_0, y_0) to the vehicle d_k at the k^{th} instant is

$$d_k = \sqrt{(x_k - x_0)^2 + (y_k - y_0)^2} + \eta \tag{4}$$

Here η is the measurement Gaussian noise defined as N(0,R) with zero mean and variance R. Similarly, w is the process disturbance defined as N(0,Q) with zero mean and Q variance. With this, we can obtain

$$X_k = f(X_{k-1}) + w \tag{5}$$

$$d_k = h(X_k) + \eta \tag{6}$$

Where
$$f(X_{k-1}) = \begin{bmatrix} 1 & 0 & 0 & 0 & \Delta T \cos \psi_{k-1} \\ 0 & 1 & 0 & 0 & \Delta T \sin \psi_{k-1} \\ 0 & 0 & 1 & \Delta T & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
, $h(X_k) = \sqrt{(x_k - x_0)^2 + (y_k - y_0)^2}$ and

With the above set of equations, the location (x_k, y_k) of the vehicle needs to be estimated. Three different scenarios that can occur are discussed below.

i) Static vehicle: When the vehicle is static then the number of independent measurements available is far less than the number of variables to be estimated. Hence for the static condition one cannot get the location of the vehicle using a single beacon. Fig. 1(a) shows that when the beacon is at the centre, and the vehicle is stationary, there are infinite solutions with the distance known, with infinite options all lying on the circle. There is no extra information to pinpoint the location of the object.

- ii) Moving vehicle in straight line: When the vehicle moves in a straight line from A to B, the distance measurement alone gives the possibility of two solutions for the vehicle position i.e., positions indicated by B and B'. These are the only two points which have identity with the measured range OB = OB' as seen in Fig. 1(b).
- iii) Path changing maneuver: When the vehicle is made to move in circular path or in a zigzag path (For determining the vehicle dynamic characteristics (i) the turning circle and the zig-zag maneuver are to be performed), the heading information when integrated with range and velocity measurements, provides a means to discard one of the possible solution as $OC \neq OC'$ and obtain the exact solution as indicated in Fig. 1(c).

3. Extended Kalman Filter

From the above discussion, it is clear that estimation of the position of the vehicle is feasible without any ambiguity, when executing any path changing test such as the turning circle and zig-zag tests. However, here too the computation of the position is not straight forward as (i) One of the states has to be estimated as it is not measured and (ii) the functions f and h defined in Eqs. (5) and (6) are nonlinear. To overcome these challenges, the proposed method employs the Extended Kalman Filter (EKF) to estimate the unmeasured state (assumed observable) of the non-linear discrete-time controlled process by providing the inputs, measured outputs and assumptions of the variance and mean of the process and output noise (Welch & Bishop, July-2006). Fig. 2 shows the basic working of the EKF. Each iteration predicts the state using the identity

$$\hat{X}_{k|k-1} = f(\hat{X}_{k-1}, u_{k-1}) \tag{7}$$

Then the co-variance matrix is computed as

$$P_{k/k-1} = F_{k-1} P_{k-1} F_{k-1}^T + Q_{k-1}$$
(8)

Here Q is the covariance matrix that represents the process noise (w). In the next step the gain K of the Kalman filter is computed as

$$K_{k} = P_{k|k-1} H_{k}^{T} \left(H_{k} P_{k|k-1} H_{k}^{T} + R \right)^{-1}$$
(9)

Here R is the covariance matrix of the measurement noise (η). The state and co-variance estimates are updated using

$$\hat{X}_{k} = \hat{X}_{k|k-1} + K_{k} \left(d_{k} - h \left(\hat{X}_{k|k-1} \right) \right) \tag{10}$$

$$P_k = \left(I - K_k H_k\right) P_{k|k-1} \tag{11}$$

Here, I is an identity matrix and P_k represents the state error covariance matrix at the k^{th} instant. In the model update cycle shown in Fig. 2, the predicted estimates are corrected using

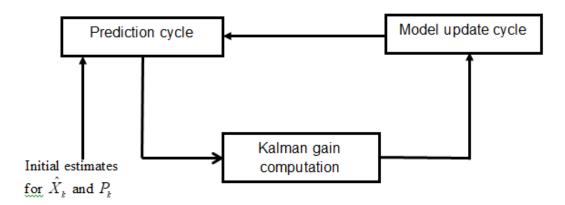


Fig. 2 Basic working of the Extended Kalman Filter (EKF)

actual measurements. This process of use of a posteriori estimate to predict the new a priori estimate to be employed in the next iteration continues endlessly.

The second problem of the functions f and h being nonlinear is handled by linearizing the F and H functions at each iteration (time step) using the Jacobian matrix of the functions as

$$F_k = \frac{\partial f}{\partial X} \bigg|_{\hat{X}_{k,1}} \tag{12}$$

$$H_{k} = \frac{\partial h}{\partial X} \bigg|_{\hat{X}_{k|k-1}} \tag{13}$$

In order to verify the feasibility of the proposed technique, the first simulation studies were conducted using Matlab code as detailed below.

3.1 Simulation studies

The initial position of the vehicle is set arbitrarily and the algorithm is executed. Fig. 3 shows the simulation result for the vehicle made to undergo a turning circle maneuver (solid blue line in Fig. 3). The beacon is placed at the origin. For this case the initial position is arbitrarily set at (400, 200). It is seen in Fig. 3 that the algorithm quickly converges and the estimated position (shown in dotted red coloured line) converges with the actual position of the vehicle in approximately 350 iterations and then onwards tracks the movement of the vehicle closely for every step. It is also seen in Fig. 3 that there is a small error between the estimated position and the actual position after the algorithm attains steady state. Fig. 4 shows another turning circle test simulation where the initial guessed position is further from the true position. Here the estimated path converges with the actual position after the estimated position goes through several loops. Fig. 5 shows the variation in the distance between the beacon and the vehicle (representing ultrasonic range measurements) as the vehicle is set to move in a circular path. The Kalman filter provides the uncertainty of the estimated states through its covariance matrix P. Larger the values of the

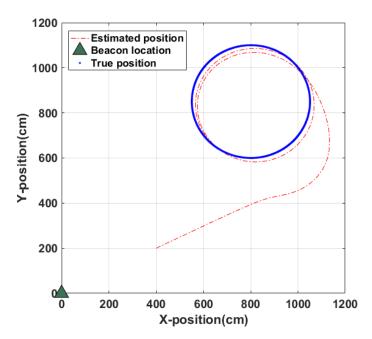


Fig. 3 Simulation result of Single Beacon based localization of unmanned surface vehicle. Initial guessed position is (400, 200) cm. Iteratively the solution converges to the true path estimation. On convergence, the beacon tracks the vehicle accurately continuously

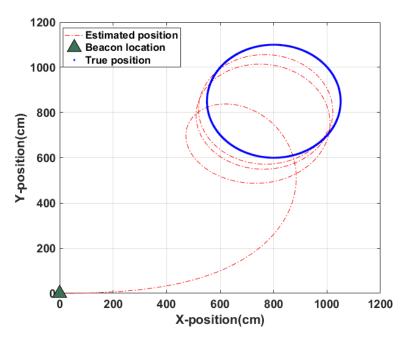


Fig. 4 Simulation result of Single Beacon based localization. Here the initial guessed position is (0,0). The estimated path converges with the actual position with more loops

element of P, the confidence on the estimated state will be less. The evolution of the error covariance P asymptotically reduces with time as shown in Fig. 6 which ensures that the confidence on estimated state is more and also the solution is converging to the true value. In this case, the time taken is 50 to 60s and the time can be further reduced by adjusting the covariance and initial guess in EKF. Thus the simulation study establishes the feasibility of the proposed technique. Physical implementation and validation of the technique was carried out by preparing a scaled down ship model of a representative prototype vessel. The tests were carried out in a laboratory wave basin.

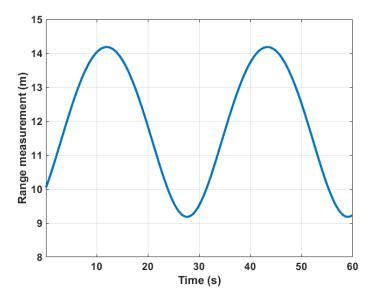


Fig. 5 Range measurement based on EKF method for simulated circular motion of vehicle

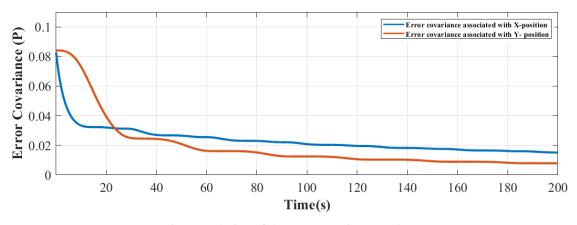


Fig. 6 Evolution of the error covariance vs time

4. Experimental verification on a model

The design of the experiment and its execution, using a scaled down model of a prototype surface vehicle, demonstrates the full implementation of the proposed augmented single beacon localization technique. The prototype vehicle is a coastal research vessel and the model is prepared for self-propulsion with details as in Table 1 below.

The model includes the following:

- i. A main propulsion motor consisting of a 440 watt brush-less DC (BLDC) motor and a speed controller. A rudder controller stepper motor with drive controls operates the twin Becker rudder steering system.
- ii. The motion sensor namely, the Motion Reference Unit (MRU) records the roll, pitch, heave and heading angle of the model vessel in dynamic mode.
- iii. A single omni-directional pulsed ultrasonic transmitter (UST) is on board (see Fig. 9) operating at a frequency of 40 kHz. An onboard Radio Frequency receiver, receiving synchronizing pulses from a shore based RF transmitter, controls the UST.
- iv. A main programmable controller cRIO9064 is placed on board to perform the t asks of controlling the propulsion motor and steering motor, acquiring data fro m the onboard MRU and transmitting acquired data in parallel to the ground st ation using Wi-Fi system. Both propulsion and steering motors operate on close d loop control and with provision to over-ride in case of requirement of open l oop operation.
- v. The main programmable controller also has an onboard USB flash drive to stor e the MRU data locally.

Fig. 7 shows the scaled down model fully fitted as described above for tests in the wave basin.

The algorithm based on the proposed technique is implemented from a shore based workstation (WS) with connection to the model via (wireless) Wi-Fi router.

Table 1 Main particulars of the coastal research vessel and on-board model systems

Particulars	Details	
Length	42.5 m	
Breadth	9.8 m	
Depth	3.75 m	
Draught	2.55 m	
Scale	1:17	
Main propulsion unit	BLDC motor 440 watts	
Steering unit	Twin Becker rudder with stepper motor	
Motion sensor	Heave, roll, pitch and heading	
Main controller	NI cRIO 9064	
Power source	Lead acid batteries	
Range measurement device	Ultrasonic based	



Fig. 7 Experimental test setup to perform Single Beacon based localization at lab scale

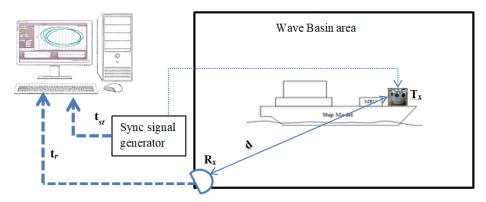


Fig. 8 Experimental setup for range measurement from model to beacon

This end-to-end communication uses client- server model, where the on-board system works as server and shares its resources with the client, namely, the base station. The number of available protocols makes the proposed client – server system scalable and operating system independent. Details of the above system are available (Dubey *et al.* 2017). The ultrasonic distance measurement is of the TOA type and is implemented with a single ultrasonic receiver (see Fig. 9) mounted at a known location on the shore of the wave basin facility and interfaced to the base WS. A sync signal generator interfaced to a Radio Frequency (RF) transmitter sends synchronizing pulses locally to the base station and through the RF to the onboard ultrasonic transmitter on the model and the base station ultrasonic





Fig. 9 Omnidirectional ultrasonic transmitter and ultrasonic receiver board

receiver units. The choice of sampling interval determined by the time delay t_s between successive sync pulses depends on the maximum distance r envisaged. Here $t_s = r/v_s$, v_s is the velocity of sound in air. The time difference between the start time t_{st} at which ultrasonic pulse is transmitted by the UST and the time t_r at which the ultrasonic pulse is received by the ultrasonic receiver at the base station, are recorded. From these values, the time of flight (TOF) as $(t_r - t_{st})$ and the distance d between the beacon and the model is calculated as $d = (t_r - t_{st}) v_s$. Fig. 8 shows the block diagram of the range measurement system, where the ultrasonic transmitter (T_x) is placed on the ship model and the ultrasonic receiver (R_x) is mounted at the shore.

4.1 Calibration of the system

To calibrate the distance measurement system prior to experiments, place the transmitter at successive exact intervals of 1.5 m and store the measured values for a duration of 2 minutes to calculate the mean and variance associated with the sensor output. During the real experiment, the measured values obtained are corrected using the linear regression model and the corrected distance is employed thereafter. Fig. 10 shows a typical plot of measured values, corrected using linear regression against the true values for a particular experiment. Table 2 compares the error of the measured and corrected values of the distance measurement. It is easily evident that the calibration process employed drastically reduces the error in distance measurement using the ultrasonic sensors.

5. Software

LabVIEW based programs implement the required tasks at both the base station and the onboard system. The proposed algorithm for distance measurement and the implementation of the EKF algorithm is performed in the base station and the closed-loop control of the model itself is implemented on the onboard controller with instructions received from the base station. The base station software facilitates user to connect to the model over Wi-Fi, connect to the beacon placed at

the shore over serial communication and to input initial value of the states $\begin{bmatrix} x & y & \psi & v \end{bmatrix}$ as well as the initial error covariance matrix P. Fig. 11 shows the screenshot of the graphical user interface of the base station. The EKF algorithm is coded in LabVIEW and runs as back process underneath the GUI shown in Fig. 11.

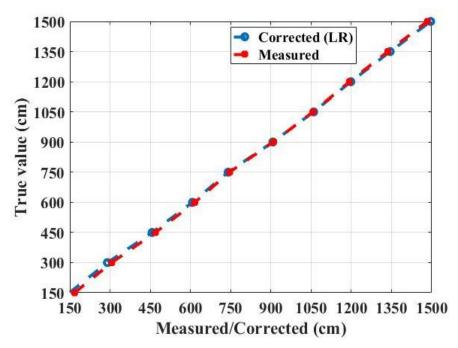


Fig. 10 Curve fitting of distance measurements during the calibration phase

Table 2 Comparison of error of the measured and corrected data

True range (set)	Initial measured range (mean)		Corrected ra	Corrected range using LR	
(cm)	(cm)	Error (%)	(cm)	Error (%)	
150.0	168.0	-12.00	148.4	1.07	
300.0	306.0	-2.00	290.1	3.30	
450.0	469.0	-4.22	457.1	-1.58	
600.0	616.0	-2.67	607.5	-1.25	
750.0	746.0	0.53	740.5	1.27	
900.0	908.0	-0.89	907.2	-0.80	
1050.0	1058.0	-0.76	1060.0	-0.95	
1200.0	1193.0	0.58	1198.0	0.17	
1350.0	1336.0	1.04	1345.0	0.37	
1500.0	1484.0	1.07	1496.0	0.27	

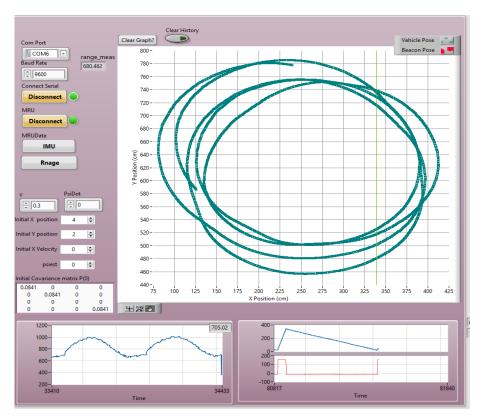


Fig. 11 Real-time vehicle location displayed on the Graphical User Interface (GUI) developed in LabVIEW

6. Results

The controlled free-running experiments for single beacon localization standardization and measurement were conducted using the candidate coastal research vessel model in a Wave Basin of size 30 m x 30 m x 3 m water depth. The model is set to selectable steady speed by controlling the propulsion motor rpm and then the rudder is set to a pre-selected fixed angle to execute a turning circle maneuver. The EKF algorithm coded in LabVIEW and running on the shore computer, takes the range measurements, estimates the location of the vehicle in real-time and displays the same. Fig. 12 shows a typical tracking of the trajectory of the vessel, where the (blue) solid line shows the true position of the vehicle and the red dotted line shows the estimated location of the vehicle using the proposed technique. Fig. 13 shows the plot of distance measurement vs. time obtained during a turning circle maneuver.

The initial position of the vehicle is set to some arbitrary value as in this case x-y coordinate is (400,400) cm and the actual model is set to turning circle maneuver with known speed of 0.5m/s. The EKF estimates the position of the vehicle based on the given speed and the initial guess. Once the range measurement from the ultrasonic sensor is made available to EKF algorithm, it updates the position of the vehicle. The solution of the proposed method converges to the actual position of the vehicle after few iterations as shown in Fig. 12. The range measurement error is around 4-5 cm

which results error in position approximately 8-10 cm. The developed measurement system has a variance of approximately 13 cm², which is the source of the measurement noise covariance matrix.

The source of the process error covariance matrix is the error associated with the motion sensor which measures the heading angle. Based on empirical estimate the value of variance associated with the motion sensor is chosen as 0.3^2 degree. The values of the necessary parameters of the EKF algorithm are given in Table 3.

Here several steady turning circles have been executed and the average turning circle diameter is obtained as 3.58m. The tests results obtained from the experiment are described in Table 4.

Table 3 Parameters necessary for the EKF algorithm

Term	Definition
Process Noise Covariance Matrix	$Q_{k-1} = \begin{bmatrix} \Delta T^2 & 0 & 0 & 0 & 0 \\ 0 & \Delta T^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 &$
Measurement Noise Covariance Matrix	$R_k = 13 \text{ cm}^2$
Assumed Initial State Vector	$X_0 = \begin{bmatrix} 4 \\ 4 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}$
Assumed Initial State Error Covariance Matrix	$P_0 = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 &$
Time Increment	$\Delta T = 0.0417$

Table 4 Results obtained from the experiment

Parameter	Value
Range Measurement Error	4-5 cm
Position Measurement Error	8-10 cm
Turning Circle Diameter	358.0 cm
Time to converge	~50 s

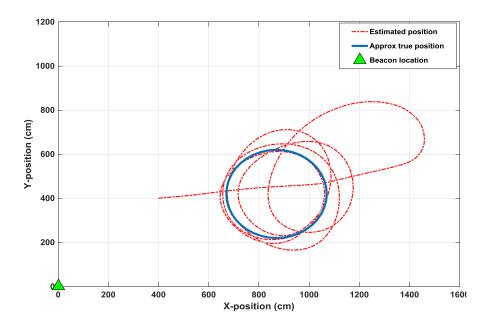


Fig. 12 Single Beacon based localization result obtained during experiment. The thin line indicates iterative convergence to the true location of the moving ship model

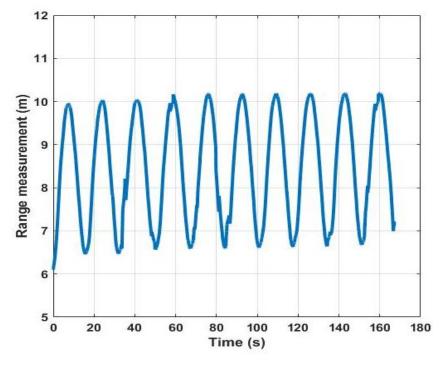


Fig. 13 Range measurement data acquired during turning circle test

7. Conclusions

This paper presents a cost-effective solution to perform and simulate augmented single beacon-based localization for an autonomous surface or underwater vehicle within the ultrasonic limiting range or in a laboratory environment. The development consists of an ultrasonic based range measurement system integrated with measurements of vehicle heading and speed. The position of the vehicle is estimated using Extended Kalman Filter. The results from simulation studies and results obtained from scaled down model of a prototype coastal research vessel, built and tested, show the practicality of the proposed technique. The worst case error in a turning circle test of 3.5 m diameter on model scale, is found to be less than 4 cm. The proposed method is applicable to underwater navigation with minimum number of sensors, homing of an underwater vehicle for docking purpose and underwater target localization from a surface vehicle. This work has potential to be extended to tracking of ship model related to motion behaviour in directional seas. The same technique can be extended to cost-effectively track underwater vehicle with the help of a surface vehicle with a known location.

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