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Scour below pipelines due to random waves alone and random waves plus currents on mild slopes

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Abstract. This paper provides a practical stochastic method by which the maximum equilibrium scour depth below a pipeline exposed to random waves plus a current on mild slopes can be derived. The approach is based on assuming the waves to be a stationary narrow-band random process, adopting the Battjes and Groenendijk (2000) wave height distribution for mild slopes including the effect of breaking waves, and using the empirical formulas for the scour depth on the horizontal seabed by Sumer and Fredsøe (1996). The present approach is valid for wave-dominant flow conditions. Results for random waves alone and random wave plus currents have been presented and discussed by varying the seabed slope and water depth. An approximate method is also proposed, and comparisons are made with the present stochastic method. For random waves alone it appears that the approximate method can replace the stochastic method, whereas the stochastic method is required for random waves plus currents. Tentative approaches to related random wave-induced scour cases for random waves alone are also suggested.

Keywords: scour depth; pipeline; mild slope; random waves; current; stochastic method

1. Introduction

The present work addresses the scour below a pipeline on a mild-sloped seabed due to random waves alone and random waves plus a current. A pipeline resting on the seabed is situated within the boundary layer of the flow close to the bed. In deep water, the flow can be considered as steady, while in shallow and intermediate water depths, there is commonly combined wave-current flow. A typical design condition for a pipeline in the vicinity of the seafloor in, e.g., the North Sea is that the flow is wave-dominated and that the seabed consists of fine sand. When a scour hole develops, this may have considerable effect on the dynamic behaviour and the on-bottom stability of the pipeline. After installation, for example, on a plane or sloped seabed consisting of fine sand, it may experience different seabed conditions, e.g., the seabed may be flat or rippled. This is mainly due to the complicated flow generated by the interaction between the incoming flow, the pipeline, and the seabed. The result will depend on the incoming flow velocity, the geometry of the bed and the bed material, as well as on the ratio between the near-bed oscillatory fluid particle excursion amplitude

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and the pipeline diameter. Additional details on the background and complexity as well as reviews of the problem are given in, e.g., Whitehouse (1998) and Sumer and Fredsøe (2002). Myrhaug and Ong (2011a) gave a review of the authors' studies on two-dimensional (2D) random wave-induced equilibrium scour characteristics around marine structures including comparison with data from random wave-induced scour experiments. Recently the authors have also provided practical stochastic methods for calculating the maximum equilibrium scour depth around vertical piles on horizontal beds (Myrhaug and Ong 2013 a, b, Ong *et al*. 2013); below pipelines on horizontal beds (Myrhaug and Ong 2011b) due to 2D and three-dimensional (3D) nonlinear random waves; around vertical piles on mild slopes due to 2D and 3D nonlinear random waves alone (Ong *et al*. 2016a) and due to 2D and 3D nonlinear random waves plus current (Ong *et al*. 2016b). To our knowledge, no studies are available in the open literature dealing with random wave-induced scour below pipelines on mild slopes.

The purpose of this study is to provide an engineering approach by which the maximum equilibrium scour depth below a pipeline exposed to random waves alone and random waves plus a current, respectively, on mild slopes can be derived. The approach is based on assuming the waves to be a stationary narrow-band random process, adopting the Battjes and Groenendijk (2000) wave height distribution for mild slopes including the effect of breaking waves, and using the empirical formulas for the scour depth by Sumer and Fredsøe (1996). Wave-dominant flow conditions are considered in this study. Results are presented and discussed by varying the seabed slope and water depth. An approximate method is proposed and compared with the present stochastic method. Tentative approaches to related random wave induced scour cases for random waves alone are also suggested.

2. Scour in regular waves alone and regular waves plus currents

The 2D scour below a fixed pipeline on a horizontal seabed in regular waves was investigated in laboratory tests by Sumer and Fredsøe (1990). They obtained the following empirical formula for the equilibrium scour depth S below the pipeline with diameter, D (see Fig. 1)

$$\frac{S}{D} = 0.1KC^{0.5} \tag{1}$$

where the Keulegan-Carpenter number KC is defined as

$$KC = \frac{UT}{D} \tag{2}$$

Here U is the undisturbed linear near-bed orbital velocity amplitude, T is the wave period, and Eq. (1) is based on data for which $2 \leq KC \leq 1000$. The threshold of sediment motion should be exceeded for scouring to occur, which may not be the case for small values of KC.

Eqs. (1) and (2) are valid for live-bed scour, for which $\theta > \theta_{cr}$ where θ is the undisturbed Shields parameter defined by

$$\theta = \frac{\tau_w}{\rho g(s-1)d_{50}} \tag{3}$$

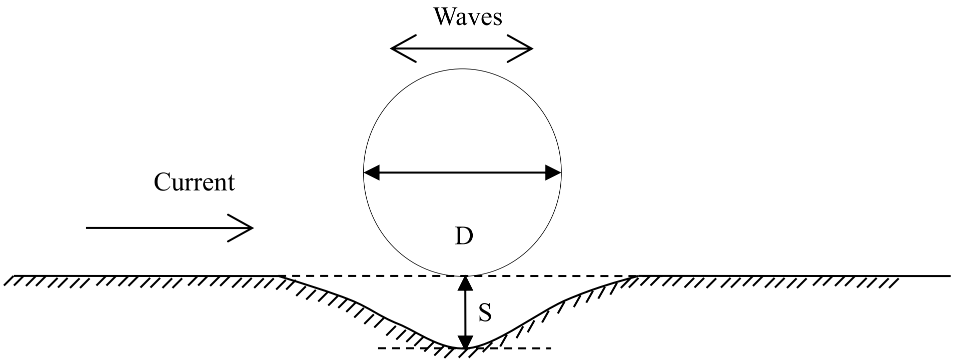


Fig. 1 Definition sketch of the scour depth (*S*) below a pipeline with diameter (*D*)

where τ_w is the maximum bottom shear stress under the waves, ρ is the density of the fluid, *g* is the acceleration due to gravity, *s* is the sediment density to fluid density ratio, d_{50} is the median grain size diameter, and θ_{cr} is the critical value of the Shields parameter corresponding to the initiation of motion at the bed, i.e., $\theta_{cr} \approx 0.05$. It should be noted that this is only correct for high grain Reynolds numbers (see Soulsby (1997, Ch. 6.4) for more details). One should note that the scour process attains its equilibrium stage through a transition period. Thus, the approach is valid when it is assumed that the storm generating random waves has lasted longer than the time-scale of the scour. Further details on the time-scale of the scour are given in Sumer and Fredsøe (1996).

The maximum bottom shear stress within a wave cycle is taken as

$$\frac{\tau_w}{\rho} = \frac{1}{2} f_w U^2 \tag{4}$$

where f_w is the friction factor, which here is taken from Myrhaug *et al.* (2001), and is valid for waves plus currents for wave-dominated situations (see Myrhaug *et al.* (2001), Table 3)

$$f_w = c\left(\frac{A}{z_0}\right)^{-d} \tag{5}$$

$$(c,d) = (18,1) \text{ for } 20 \leq A/z_0 < 200 \tag{6}$$

$$(c,d) = (1.39, 0.52) \text{ for } 200 \leq A/z_0 < 11000 \tag{7}$$

$$(c,d) = (0.112, 0.25) \text{ for } 11000 \leq A/z_0 \tag{8}$$

Table 1 Parameters for the four locations

	Location 1	Location 2	Location 3	Location 4
***x* (m)**	0	200	400	600
$k_p h$	0.77	0.70	0.64	0.57
KC_{rms}	12.73	13.33	13.91	14.57

where $A = U/\omega$ is the near-bed orbital displacement amplitude, $\omega = 2\pi/T$ is the angular wave frequency, and $z_0 = d_{50}/12$ is the bed roughness (see e.g., Soulsby (1997)). The advantage of using this friction factor for rough turbulent flow is that it is possible to derive the stochastic approach analytically.

It should be noted that the KC number can alternatively be expressed as

$$KC = \frac{2\pi A}{D} \tag{9}$$

Moreover, A is related to the linear wave amplitude a by

$$A = \frac{a}{\sinh kh} \tag{10}$$

where h is the water depth, and k is the wave number determined from the dispersion relationship $\omega^2 = gk \tanh kh$.

Sumer and Fredsøe (1996) presented results of an experimental study on scour below pipelines subject to combined colinear irregular waves and currents acting on a perpendicularly oriented pipeline with KC ranging from 5 to about 50, and sand with $d_{50} = 0.16$ mm. Therefore the proposed method is strictly not applicable to other orientations between waves, current and pipeline. Sumer and Fredsøe (1996) found that their empirical formula for the equilibrium scour depth for regular waves given in Eqs. (1) and (2) can be used for irregular waves provided that the KC number is calculated by $KC_{rms} = U_{rms}T_p/D$. Here $U_{rms} = \sqrt{2}\sigma_u$ is the root-mean-square (*rms*) value of the near-bed wave induced velocity amplitude, and T_p is the spectral peak period. Moreover, $\sigma_u^2 = \int_0^\infty S_u(\omega)d\omega$ where $S_u(\omega)$ is the spectrum of the instantaneous near-bed wave induced velocity $u(t)$. It should be noted that U_{rms} corresponds to U_m defined by Sumer and Fredsøe (2002).

Based on their data Sumer and Fredsøe (1996) found the following empirical expressions for the scour depth S below pipelines exposed to random waves plus currents

$$\frac{S}{D} = \frac{S_{cur}}{D} F \tag{11}$$

Here $S_{cur}/D = 0.6$ is the non-dimensional scour depth for current alone with a non-dimensional standard deviation $\sigma/D = 0.2$ i.e., reflecting the scatter in the data, and F is given by the following empirical equations

$$F = \frac{5}{3} KC_{rms}^{a} \exp(2.3b) \;\; \text{for} \;\; 0 \le U_{cwrms} \le 0.7 \tag{12}$$

$$F = 1 \;\; \text{for} \;\; 0.7 < U_{cwrms} \le 1 \tag{13}$$

where

$$U_{cwrms} = \frac{U_c}{U_c + U_{rms}} \tag{14}$$

and U_c is the current velocity.

For $0 \le U_{cwrms} \le 0.4$ the coefficients *a* and *b* are given by

$$a = 0.557 - 0.912(U_{cwrms} - 0.25)^2 \tag{15}$$

$$b = -1.14 + 2.24(U_{cwrms} - 0.25)^2 \tag{16}$$

For $0.4 \le U_{cwrms} \le 0.7$ the coefficients *a* and *b* are given by

$$a = -2.14U_{cwrms} + 1.46 \tag{17}$$

$$b = 3.3U_{cwrms} - 2.5 \tag{18}$$

It is noticed that Eqs. (11), (12), (15) and (16) reduce to Eq. (1) for random waves alone, i.e., for $U_{cwrms} = 0$. Moreover, also notice that Eqs. (15) and (17) as well as Eqs. (16) and (18) are discontinuous at $U_{cwrms} = 0.4$. However, since wave-dominated flow will be considered here, Eqs. (17) and (18) are not used, and thus this does not affect the present results.

The stochastic method proposed here for random waves plus currents is valid for wave-dominated flow. Moreover, it is based on assuming that Eqs. (11), (12), (15), (16) are also valid for regular waves if KC_{rms} and U_{rms} are replaced by *KC* and *U*, respectively, i.e., if *F* is given by

$$F = \frac{5}{3} KC^{a} \exp(2.3b) \tag{19}$$

where U_{cwrms} is replaced by

$$U_{cw} = \frac{U_c}{U_c + U} \tag{20}$$

and the coefficients *a* and *b* are given by

$$a = 0.557 - 0.912(U_{cw} - 0.25)^2 \tag{21}$$

$$b = -1.14 + 2.24(U_{cw} - 0.25)^2 \tag{22}$$

The dispersion relationship for regular waves plus currents at an angle φ to the direction of the wave propagation is $\omega = kU_c \cos\varphi + \left(gk \tanh kh\right)^{1/2}$ (see e.g., Soulsby (1997)), which determines the wave number k for given values of ω, U_c and h. However, for wave-dominated situations the effect of U_c on k is small, i.e., k is determined from $\omega^2 = gk\tanh kh$, as previously given for waves alone.

It should be noted that since Eq. (1) appears to be physically sound for KC=0, i.e., S equals zero for $KC = 0$, the formula can be taken to be valid from KC=0. This extension of Eq. (1) relies on the threshold of motion being exceeded, which for small values of KC may not be the case. It should also be noted that the assumption of transferring the results for random waves plus currents to regular waves plus currents is not obvious, although it is to some extent justified here by referring to the results for random and regular waves alone.

3. Scour in random waves alone and random waves plus currents on mild slopes

Here a tentative stochastic approach will be outlined following the approach presented in Myrhaug *et al*. (2009) and Myrhaug and Ong (2011b), except for the modification performed by adopting the Battjes and Groenendijk (2000) wave height distribution. As a first approximation, it is assumed that the scour formulas for the case of a horizontal bed described in Section 2 can be applied for the case of mild slopes as well. Fig. 2 shows the definition sketch of the scour below a pipeline on a mild slope.

As mentioned the Battjes and Groenendijk (2000) distribution includes the effects of breaking waves. This will be discussed in Section 4.2, demonstrating that the effects of wave breaking on mild slopes are of minor importance.

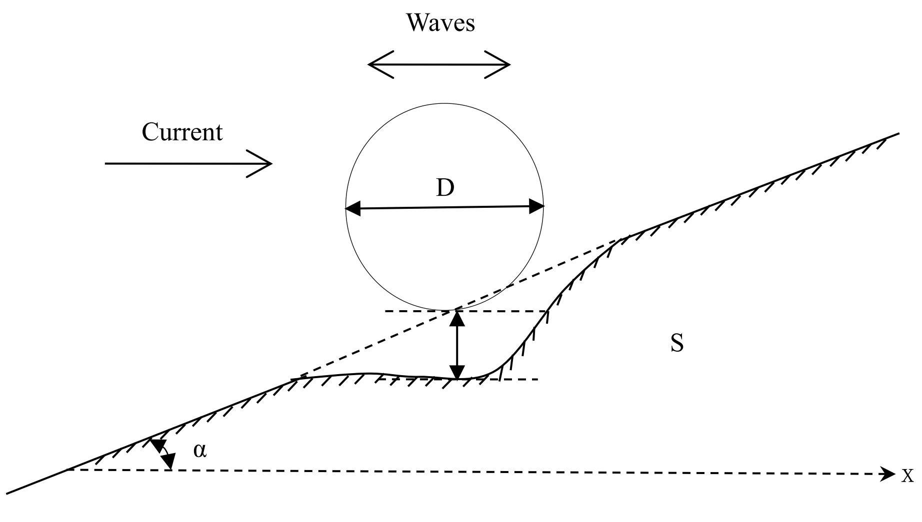


Fig. 2 Definition sketch of the scour depth (S) below a pipeline with diameter (D) on a mild slope (α)

3.1 Theoretical background

At a fixed point in a sea state with stationary narrow-band random waves consistent with regular linear waves in finite water depth h and wave height $H = 2a$, the near-bed orbital displacement amplitude, A, and the near-bed horizontal orbital velocity amplitude, U, can be taken as, respectively

$$A = \frac{H}{2\sinh k_p h} \tag{23}$$

$$U = \omega_p A = \frac{\omega_p H}{2\sinh k_p h} \tag{24}$$

where $\omega_p = 2\pi / T_p$ is the spectral peak frequency, T_p is the spectral peak period, and k_p is the wave number corresponding to ω_p determined from the dispersion relationship

$$\omega_p^2 = g k_p \tanh k_p h \tag{25}$$

Moreover, A and U are made dimensionless by taking $\hat{A} = A/A_{rms}$ and $\hat{U} = U/U_{rms}$, respectively, where

$$A_{rms} = \frac{H_{rms}}{2\sinh k_p h} \tag{26}$$

$$U_{rms} = \omega_p A_{rms} = \frac{\omega_p H_{rms}}{2\sinh k_p h} \tag{27}$$

and H_{rms} is the rms value of H. By combining Eqs. (23), (24), (26) and (27) it follows that

$$\omega_p = \frac{U}{A} = \frac{U_{rms}}{A_{rms}} \tag{28}$$

and consequently

$$\frac{U}{U_{rms}} = \frac{A}{A_{rms}} = \frac{H}{H_{rms}} \tag{29}$$

Here the Battjes and Groenendijk (2000) parametric wave height distribution based on laboratory experiments on shallow foreshores is adopted. This cumulative distribution function (*cdf*) is composed of two two-parameter Weibull distributions of the non-dimensional wave height $\hat{H} = H/H_{rms}$

$$P(\hat{H}) = \begin{cases} P_1(\hat{H}) = 1 - \exp[-(\frac{\hat{H}}{\hat{H}_1})^{k_1}]; \hat{H} < \hat{H}_{tr} \\ P_2(\hat{H}) = 1 - \exp[-(\frac{\hat{H}}{\hat{H}_2})^{k_2}]; \hat{H} \geq \hat{H}_{tr} \end{cases} \tag{30}$$

where $k_1 = 2$, $k_2 = 3.6$, $\hat{H}_1 = H_1 / H_{rms}$, $\hat{H}_2 = H_2 / H_{rms}$, $\hat{H}_{tr} = H_{tr} / H_{rms}$. The values of H_1 and H_2 can either be read from Table 2 in Battjes and Groenendijk (2000), or they can be solved by an iteration procedure solving two equations (see Eqs. (37) and (38)). Furthermore, H_{tr} is the transitional wave height corresponding to the change of wave height where there is a change of the distribution associated with depth-induced wave breaking, given by

$$H_{tr} = (0.35 + 5.8 \tan \alpha) h \tag{31}$$

where α is the slope angle, and H_{rms} is related to the zeroth spectral moment m_0 by

$$H_{rms} = (2.69 + 3.24\sqrt{m_0} / h)\sqrt{m_0} \tag{32}$$

It should be noted that the deep water value of Eq. (32) is $H_{rms} = 2.69\sqrt{m_0}$, corresponding to a 5% reduction of the factor 2.83 obtained from the Rayleigh distribution, accounting for finite bandwidth effects obtained by Goda (1979).

The Battjes and Groenendijk (2000) distribution is a so-called point model, i.e., depending on local parameters regardless of the history of the waves in deeper water. It should be noted that the effect of the bottom slope is of a secondary nature compared to the effect of water depth (see Battjes and Groenendijk (2000) for more details). Although the *cdf* in Eq. (30) is a continuous function of H, but with an abrupt change of its shape at $H = H_{tr}$ (i.e., the derivative and thus the *pdf* is discontinuous at this point), which is physically unrealistic, this feature is acceptable since all the integral statistical properties of the wave height are well defined. This change in the *cdf* (and *pdf*) is related to depth-induced breaking, and thus H_{tr} is expressed as the limiting wave height for non-breaking waves (i.e., defined as for purely depth-limited breaking, by excluding the steepness effect on wave breaking). Thus the effect of wave breaking is inherent in the *cdf* for H larger than H_{tr} (see Battjes and Groenendijk (2000) for more details).

The zeroth spectral moment, m_0, is obtained as

$$m_0 = \int_0^\infty S_\eta(\omega, h) d\omega \tag{33}$$

where $S_\eta(\omega, h)$ is the wave spectrum in finite water depth, which can be obtained by multiplying the deep water wave spectrum $S_\eta(\omega)$ with a depth correction factor $\psi(\omega, h)$ as

$$S_\eta(\omega, h) = \psi(\omega, h) S_\eta(\omega) \tag{34}$$

where, according to Young (1999)

$$\psi(\omega, h) = \frac{[k(\omega, h)]^{-3} \partial k(\omega, h) / \partial \omega}{\{[k(\omega, h)]^{-3} \partial k(\omega, h) / \partial \omega\}_{kh \to \infty}} \tag{35}$$

ensuring that the frequency part of the wave spectrum becomes proportional to k^{-3} irrespectively of the water depth (see Young (1999) for more details). From Eq. (35) it follows that (see Jensen 2002)

$$\psi(\omega,h)=\frac{\omega^6}{(gk)^3[\tanh kh+kh(1-\tanh^2 kh)]} \tag{36}$$

For given h, α, and m_0, the values of $\hat{H}_1$ and $\hat{H}_2$ can be either read from Table 2 in Battjes and Groenendijk (2000), or they can be determined by solving the following two equations:

1) The distribution function has to be continuous, i.e.

$$P_1(\hat{H})=P_2(\hat{H}) \tag{37}$$

2) The mean square normalized wave height, or the second moment of the probability density function (*pdf*) of $\hat{H}$, has to equal unity, i.e.,

$$\hat{H}^2_{rms}=\int_0^{\hat{H}_{tr}}\hat{H}^2 p_1(\hat{H})d\hat{H}+\int_{\hat{H}_{tr}}^{\infty}\hat{H}^2 p_2(\hat{H})d\hat{H}=1 \tag{38}$$

where p_1 and p_2 are the *pdf*s of $\hat{H}$ and defined as $p_1=dP_1/d\hat{H}$ and $p_2=dP_2/d\hat{H}$, as given in Eqs. (48) and (49), respectively.

3.2 Outline of stochastic method

The highest among random waves in a stationary narrow-band sea-state is considered, as it is reasonable to assume that it is mainly the highest waves which are responsible for the scour response. This is based on earlier comparisons between the stochastic method (Myrhaug and Rue 2003 and Myrhaug *et al.* 2009) and the corresponding Sumer and Fredsøe (1996) experimental data. It is also assumed that the sea state has lasted long enough to develop the equilibrium scour depth. The highest waves considered here are those exceeding the probability $1/n$, $\hat{H}_{1/n}$ (i.e., $1-P(\hat{H}_{1/n})=1/n$).

The parameter of interest is the expected (mean) value of the maximum equilibrium scour characteristics caused by the $(1/n)^{\text{th}}$ highest waves, which is given as

$$E\left[S(\hat{H})\middle|\hat{H}>\hat{H}_{1/n}\right]=n\int_{\hat{H}_{1/n}}^{\infty}S(\hat{H})p(\hat{H})d\hat{H} \tag{39}$$

where $S(\hat{H})$ represents the scour characteristics, and $p(\hat{H})$ is the *pdf* of $\hat{H}$. More specifically, the present approach is based on the following assumptions: (1) the free surface elevation is a stationary narrow-band process with zero expectation, and (2) the scour response formula for regular waves plus currents given in the previous section (see Eqs. (11) and (19) to (22)), are valid for irregular waves as well. These assumptions are essentially the same as those given in e.g., Myrhaug *et al.* (2009), where further details are found.

For a narrow-band process $T=T_p$ where $T_p=2\pi/\omega_p=2\pi A_{rms}/U_{rms}$ and $k=k_p$. Then by referring to Eq. (29) it follows that

$$\hat{U}=\frac{U}{U_{rms}}=\frac{A}{A_{rms}}=\frac{H}{H_{rms}}=\hat{H} \tag{40}$$

By substituting Eq. (40) in Eqs. (11) and (19) to (22), Eqs. (11) and (19) to (22) can be re-arranged to

$$\hat{S} \equiv \frac{S}{D} = \frac{S_{cur}}{D} F(\hat{H}) \tag{41}$$

where $F(\hat{H})$ for $0 \le U_{cwrms} \le 0.4$ (i.e., U_{cwrms} as given in Eq. (14)) is given by

$$F(\hat{H}) = \frac{5}{3}(KC_{rms}\hat{H})^a \exp(2.3b) \tag{42}$$

$$KC_{rms} = \frac{U_{rms}T_p}{D} = \frac{2\pi A_{rms}}{D} \tag{43}$$

$$U_{cw}(\hat{H}) = \left(\frac{U_c}{U_{rms}}\right) / \left(\frac{U_c}{U_{rms}} + \hat{U}\right) = \left(\frac{U_c}{U_{rms}}\right) / \left(\frac{U_c}{U_{rms}} + \hat{H}\right) \tag{44}$$

$$\mathrm{a}(\hat{U}) = 0.557 - 0.912(U_{cw} - 0.25)^2 \tag{45}$$

$$\mathrm{b}(\hat{U}) = -1.14 + 2.24(U_{cw} - 0.25)^2 \tag{46}$$

Let S denote $\hat{S}$ given in Eqs. (41) to (46). Then the mean of the maximum equilibrium scour depth caused by the $(1/n)^{\text{th}}$ highest waves follows from Eq. (39) as

$$E\left[S(\hat{H}) \middle| \hat{H} > \hat{H}_{1/n}\right] = n \int_{\hat{H}_{1/n}}^{\infty} S(\hat{H}) p(\hat{H}) d\hat{H} \tag{47}$$

where the *cdf* of $\hat{H}$ is given in Eq. (30), and $p\left(\hat{H}\right)$ is the *pdf* of $\hat{H}$, i.e., $p_1 = dP_1 / d\hat{H}$ and $p_2 = dP_2 / d\hat{H}$, given as follows

$$p_1(\hat{H}) = \frac{k_1}{\hat{H}}(\frac{\hat{H}}{\hat{H}_1})^{k_1} \exp[\ (\frac{\hat{H}}{\hat{H}_1})^{k_1}]; \hat{H} < \hat{H}_{tr} \tag{48}$$

$$p_2(\hat{H}) = \frac{k_2}{\hat{H}}(\frac{\hat{H}}{\hat{H}_2})^{k_2} \exp[\ (\frac{\hat{H}}{\hat{H}_2})^{k_2}]; \hat{H} \ge \hat{H}_{tr} \tag{49}$$

Moreover, $\hat{H}_{1/n}$ is obtained by solving the equation $1 - P(\hat{H}_{1/n}) = 1/n$.

It should be noted that the formulation in Section 3 is general, i.e., valid for a finite water depth.

4. Results and discussion

To the authors' knowledge no data exist in the open literature for random wave-induced scour below pipelines on mild slopes.

4.1 Prediction of parameters

In the present study, the effect of mild slopes on scour below pipelines in random waves alone and random waves plus currents is investigated. Four bed slopes, 1/50, 1/100, 1/150 and 1/250, are considered for this purpose.

The case with the bed slope $\alpha = 1/100$ is exemplified to show the procedure of calculating all the required parameters. Fig. 3 shows the seabed conditions with α =1/100. The water depth at the seaward location (i.e., x = 0 m) is 15 m; the horizontal length of the sloping seabed is 600 m; the diameter of the pipeline D is set to be 1m for all the cases.

The wave spectrum in finite water depth $S_\eta(\omega,h)$ can be obtained from the spectrum in deep water $S_\eta(\omega)$, see Eq. (34). Hence, the random waves with a standard JONSWAP spectrum (γ = 3.3) and significant wave height H_{m0} = 8 m and spectral peak period Tp = 11.1 s are assumed to describe the sea state in deep water. Fig. 4 shows some results of the wave spectra at the four locations transformed from the deep water according to Eqs. (33) - (36). The water depth at each location, as well as the corresponding values of KC_{rms} and k_ph at each location are presented in Table 1. It is clearly seen in Fig. 4 that the wave energy decreases as the water depth decreases.

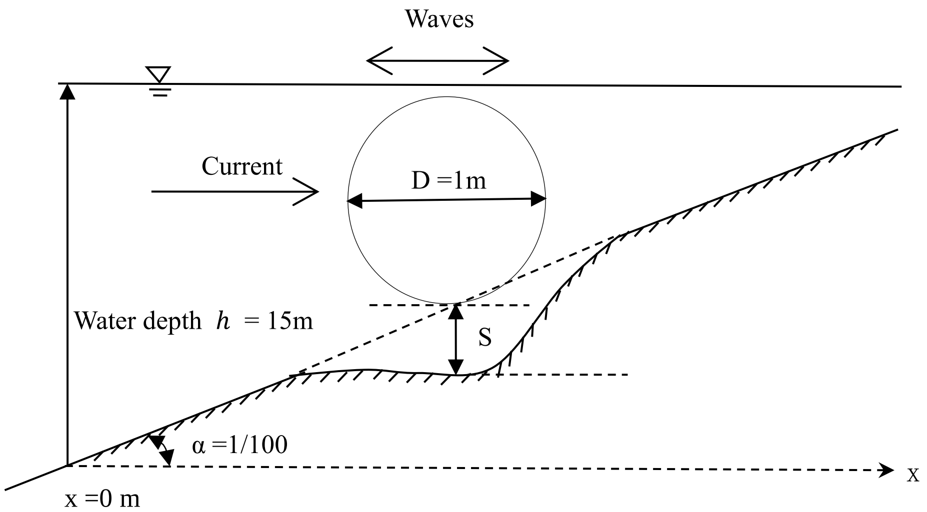


Fig. 3 Definition sketch of the seabed conditions with α =1/100

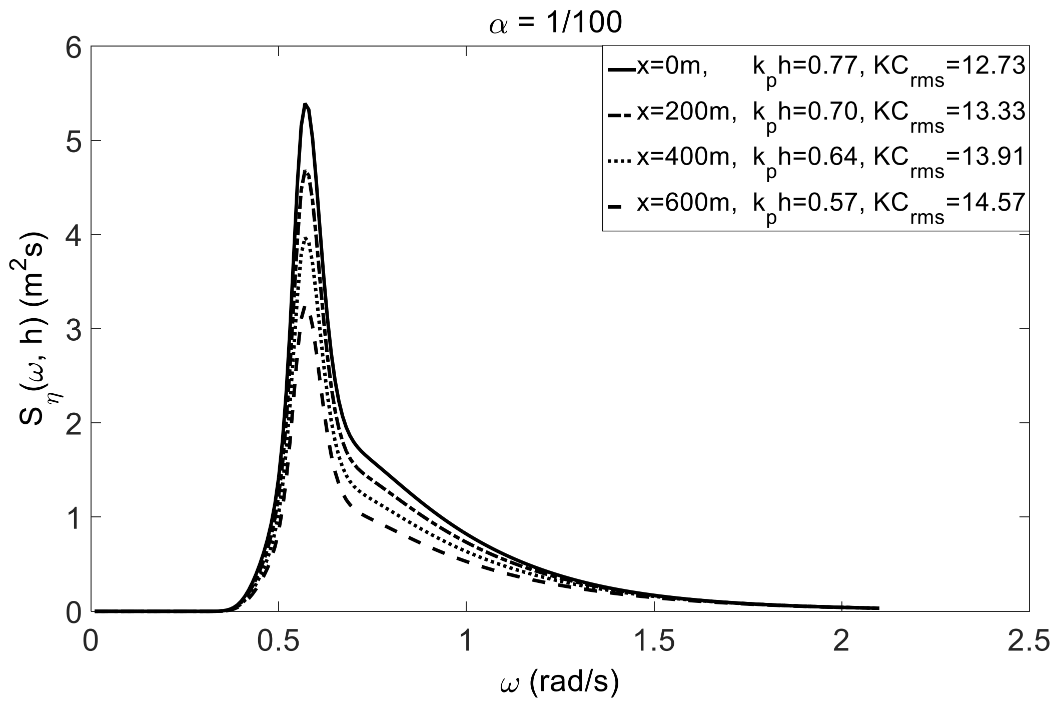


Fig. 4 The transitional wave spectra in finite water depth $S_\eta(\omega, h)$ versus ω at four locations for slope α =1/100

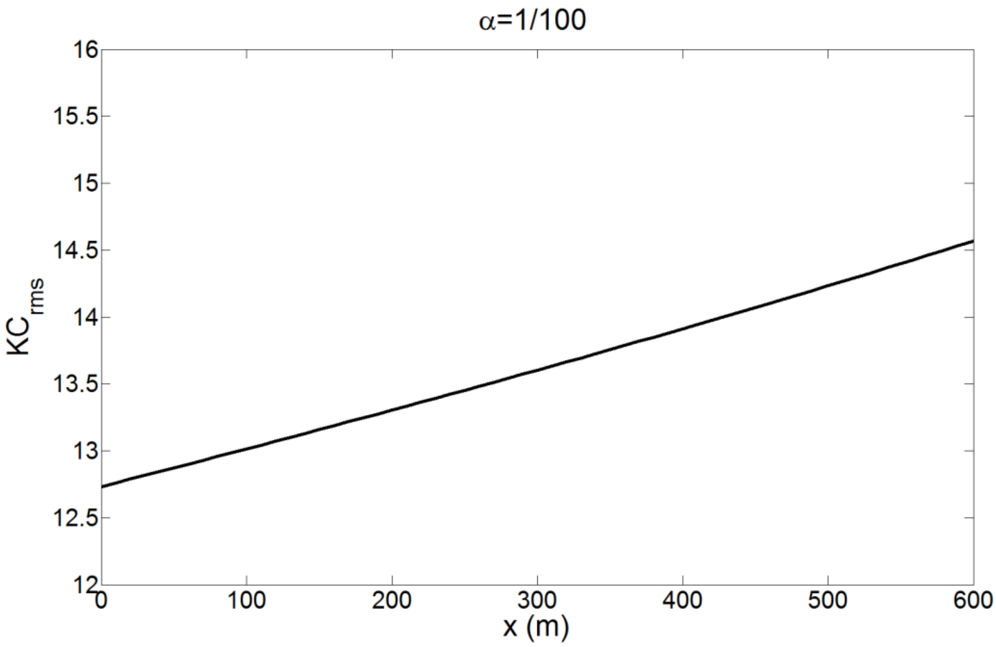


Fig. 5 KC_{rms} versus x in finite water depth for slope α =1/100

With the values of m_0 along x, H_{rms} can be determined by Eq. (32) and therefore KC_{rms} by Eqs. (26) and (43). Fig. 5 shows KC_{rms} versus x for the slope α=1/100. It appears that KC_{rms} increases from 12.73 to 14.57 as the water depth decreases (for x: 0 m→ 600 m). It needs to be noted that although H_{rms} decreases as x approaches to 600 m, $k_p h$ decreases because of the finite water depth. These effects increase KC_{rms} along x.

4.2 Random waves alone

The *pdf* of the Battjes and Groenendijk (2000) wave height distribution of $\hat{H}$ at the four locations referred to earlier (see Table 1) are shown in Fig. 6. The discontinuous points in Fig. 6 are due to the transitional wave height $\hat{H}_{tr}$, representing the limiting wave height for non-breaking waves. The figure shows that from location 1 to location 4 (as x increases from 0 m to 600 m), $\hat{H}_{tr}$ decreases from 1.78 to 1.30, reflecting that the influence of breaking waves on the distributions becomes more significant as the water depth decreases. It should be noted that the area under each *pdf* curve is equal to one.

Fig. 7 shows the predictions of S/D for the $(1/10)^{th}$ highest waves $(S/D_{1/10})$ along x for the slope α = 1/100. It should be noted that the use of the $(1/10)^{th}$ value is justified based on the results in Myrhaug and Rue (2003) and Myrhaug *et al*. (2009) showing that this value can be taken to represent the upper value for the random wave-induced data in Sumer and Fredsøe (1996). It appears that $S/D_{1/10}$ increases slightly as the water depth decreases (i.e., as x increases from 0 m to 600 m). This effect may be attributed to the increase of KC_{rms} (Fig. 5) along the sloping bed since large KC_{rms} induces more scour. The reason is that for pipelines the scour is caused by lee-wake erosion. For waves alone the downstream lee-wake vortex system occurs on both sides of the pipeline, and as KC_{rms} increases the lee-wake vortex system is enhanced leading to more scour.

Four different bed slopes (α = 1/50, 1/100, 1/150, 1/250) are considered in the present study. The seabed configuration is illustrated in Fig. 8. Fig. 9 shows KC_{rms} versus x for the four slopes; KC_{rms} increases as the water depth decreases for all the slopes. Futhermore, it appears that KC_{rms} increases as the slope increases at a given location x. Fig. 10 shows $S/D_{1/10}$ for the different slopes. Deeper scour hole is encountered when the slope becomes steeper. For all slopes, $S/D_{1/10}$ increases slightly as x approaches to 600 m; at a given location x, it appears that $S/D_{1/10}$ increases as the slope increases. These results are physically sound and consistent with those observed in Fig. 9. It is noted that the shape of the curve for the slope 1/50 in Fig. 10 differs from those for the other slopes, reflecting an inherent feature of the model.

An assessment of breaking waves is feasible by using the surf parameter $\xi_p = (H_s/((g/2\pi)\, T_p^2))^{-1/2} \tan\alpha$ defined in terms of the significant wave height in deep water H_s = 8m and the spectral peak period T_p = 11.1s, giving ξ_p = (0.020, 0.033, 0.049, 0.098) for the slopes (1/250, 1/150, 1/100, 1/50). For individual waves the surf parameter is defined as $\xi = (H/((g/2\pi)T^2))^{-1/2} \tan\alpha$, where H is the deep water wave height. Types of breaking waves are defined in terms of this surf parameter (see e.g., Battjes (1974)); spilling if $0 < \xi < 0.5$ and plunging if $0.5 < \xi < 3$. Thus, if breaking occurs in this example there will most likely be spilling breakers; and therefore the effect of breaking waves on scour is of minor importance.

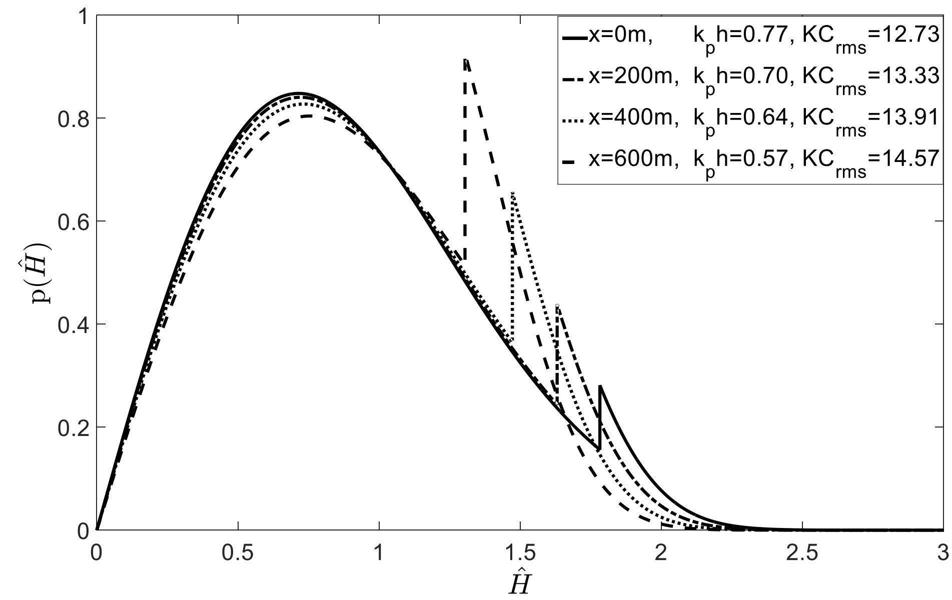


Fig. 6 *pdf* of $\hat{H}$ at four locations for slope α =1/100

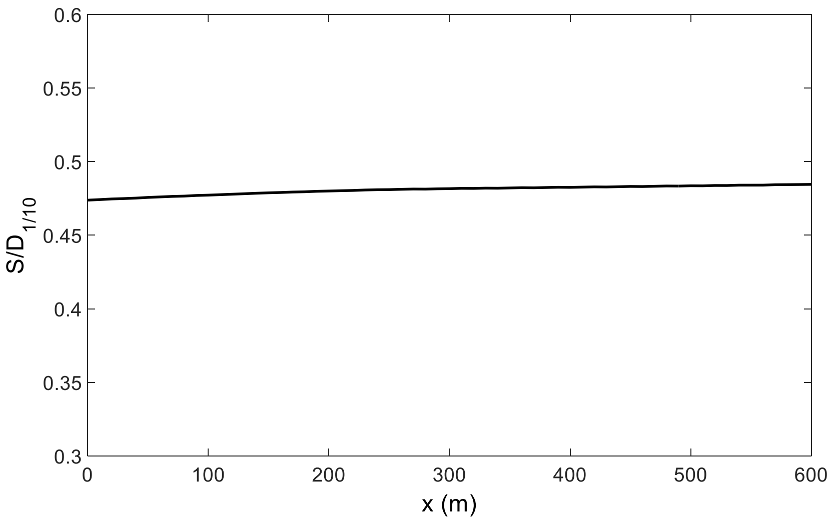


Fig. 7 $S/D_{1/10}$ versus x in finite water depth for slope α =1/100

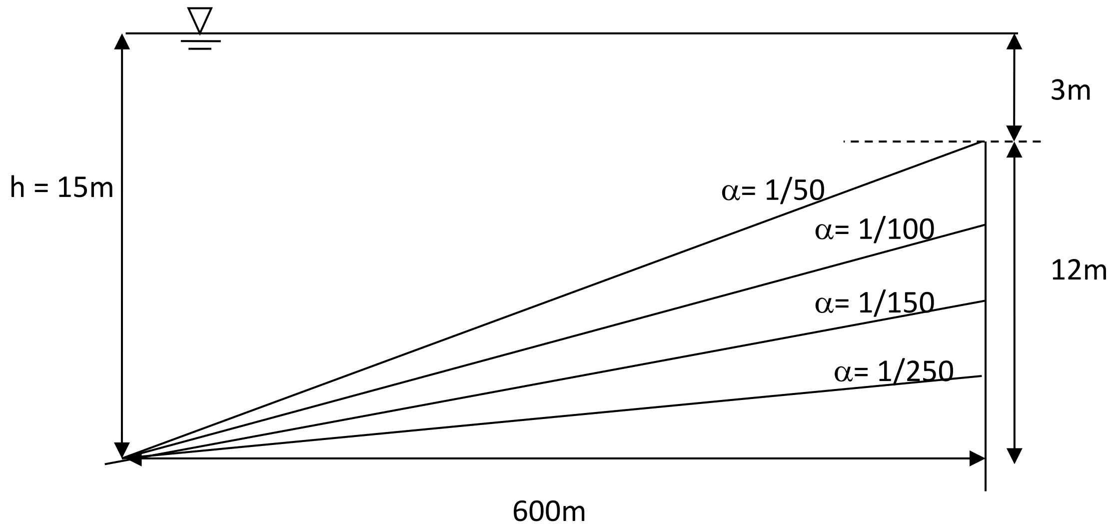


Fig. 8 Sketch of the seabed conditions for four slopes α = (1/50, 1/100, 1/150, 1/250). The total horizontal length of the sloping bed is 600 m, and the water depth at the seaward direction is 15 m

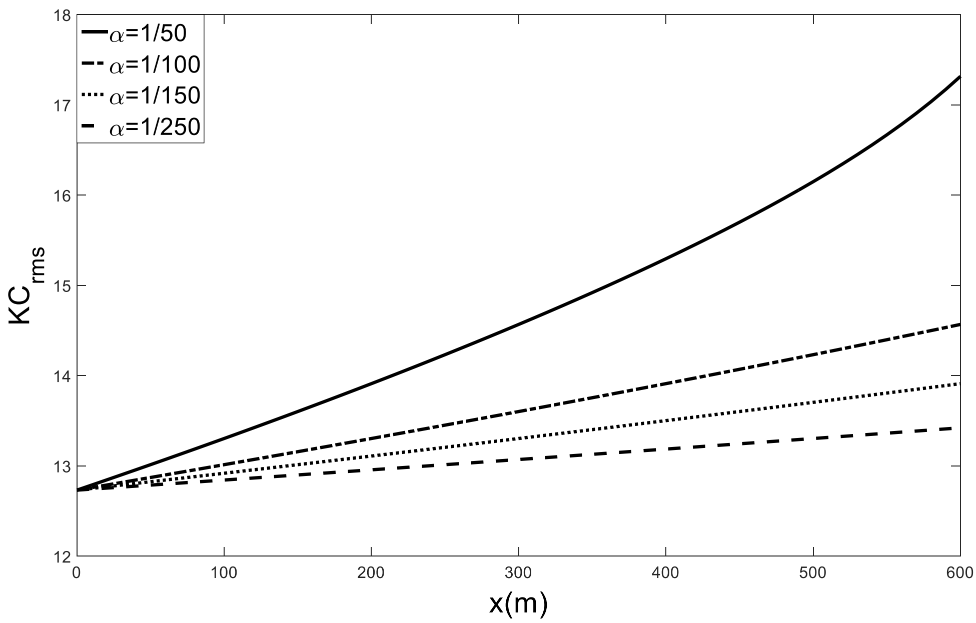


Fig. 9 KC_{rms} versus x for α = (1/50, 1/100, 1/150, 1/250)

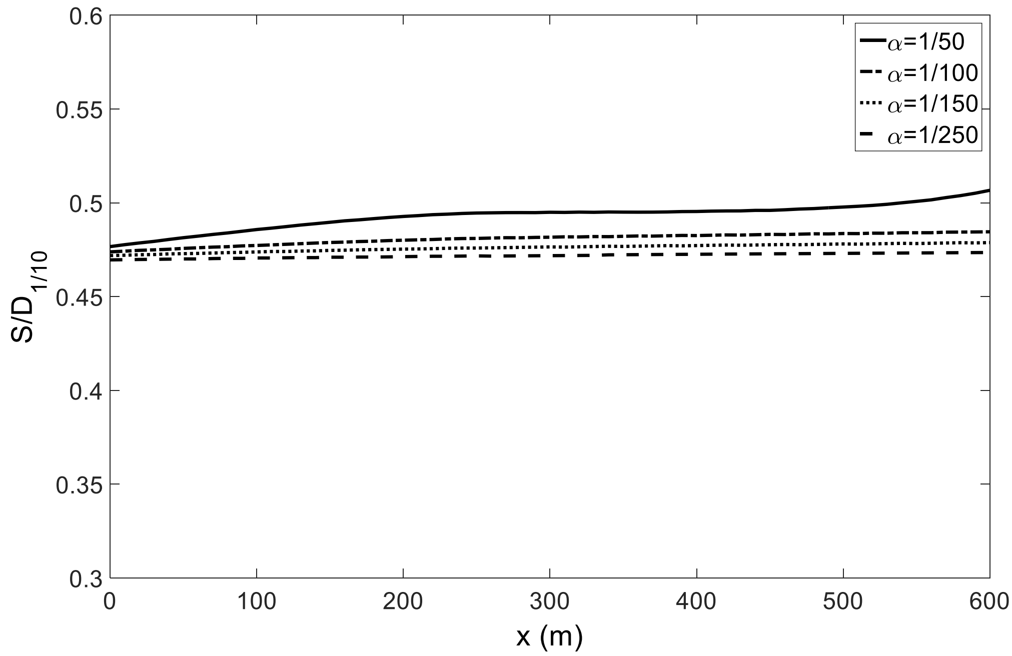


Fig. 10 $S/D_{1/10}$ versus x for α = (1/50, 1/100, 1/150, 1/250)

4.3 Random waves plus currents

The effect of the current on random wave-induced scour below a pipeline is investigated in this section. This can be achieved by changing U_c in U_{cw} (see Eq. (44)).

Fig. 11 shows $S/D_{1/10}$ versus U_{cwrms} in combined random waves plus current for α =1/100 at the four locations along the seabed (see Table 1). It should be noted that the results for $S/D_{1/10}$ for U_{cwrms} = 0 (random waves alone) are the same as those presented in Fig. 7. The increase of $S/D_{1/10}$ from location 1 to location 4 for U_{cwrms} = 0 shows that the scour depth increases as the water depth decreases. It is clearly seen in Fig. 11 that the effect of the current is to increase the scour depth compared with that for random waves alone, and the effect is enhanced as U_{cwrms} increases. More specifically, for all the locations, $S/D_{1/10}$ for U_{cwrms} = 0.4 is approximately 1.2 times larger than for random waves alone. The reason is that by adding a current to random waves the lee-wake vortex system occurs more downstream than upstream, causing a larger scour depth.

4.4 Alternative view of random wave-induced scour: Approximate method

An alternative pragmatic view of the scour process below pipelines and around a single vertical pile under random waves is that of Sumer and Fredsøe (1996, 2001) referred to in Section 2. They looked for which parameters of the random waves to represent the scour variable, finding by trial and error that the use of H_{rms} and T_p in an otherwise deterministic approach gave the best agreement with data.

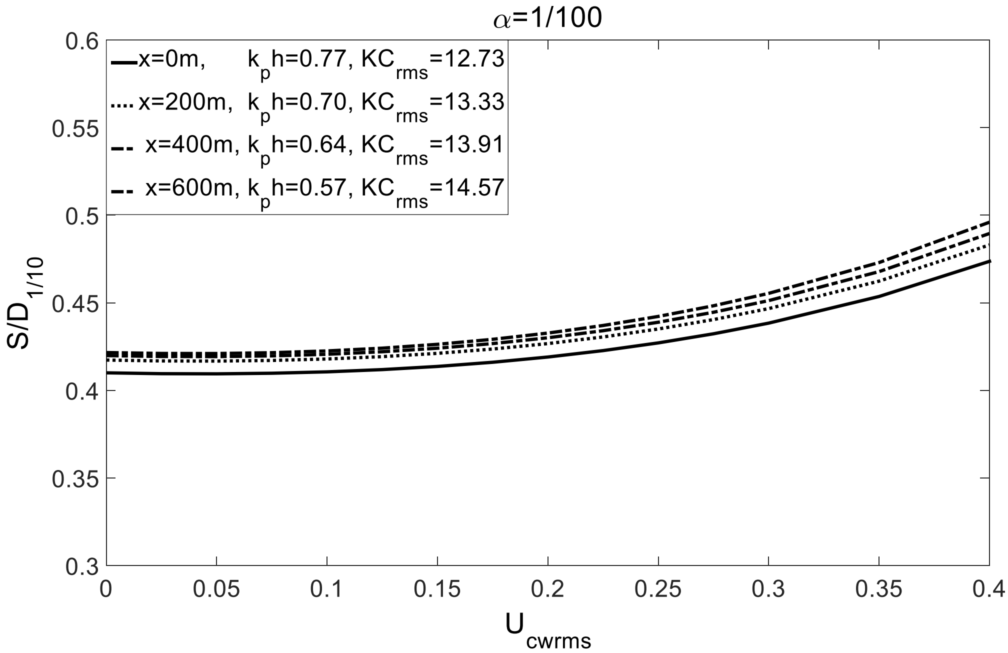


Fig. 11 $S/D_{1/10}$ versus U_{cwrms} at four locations for slope α =1/100

Here the alternative view of the scour process is considered using the results of the present stochastic method. The question is how well the mean scour depth caused by the $(1/n)^{\text{th}}$ highest waves, $E[S(H)|H > H_{1/n}]$ (see Eq. (47)), can be represented by using the mean of the $(1/n)^{\text{th}}$ highest waves in the scour depth formula for regular waves, i.e., $S(E[H_{1/n}])$.

An alternative *KC* number for random waves in the approximate method can be defined as

$$KC_{1/n} = \frac{E[U_{1/n}]T_p}{D} = \frac{2\pi E[A_{1/n}]}{D} \tag{50}$$

Based on the narrow-band assumption, $E[U_{1/n}]$ and $E[A_{1/n}]$ can be defined as

$$E[A_{1/n}] = \frac{E[H_{1/n}]}{2\sinh k_p h} \tag{51}$$

$$E[U_{1/n}] = \omega_p E[A_{1/n}] = \frac{\omega_p E[H_{1/n}]}{2\sinh k_p h} \tag{52}$$

where $E[A_{1/n}]$, $E[U_{1/n}]$ and $E[H_{1/n}]$ are the mean values of the $(1/n)^{\text{th}}$ largest values of the near-bed orbital displacement amplitude, velocity and wave height, respectively.

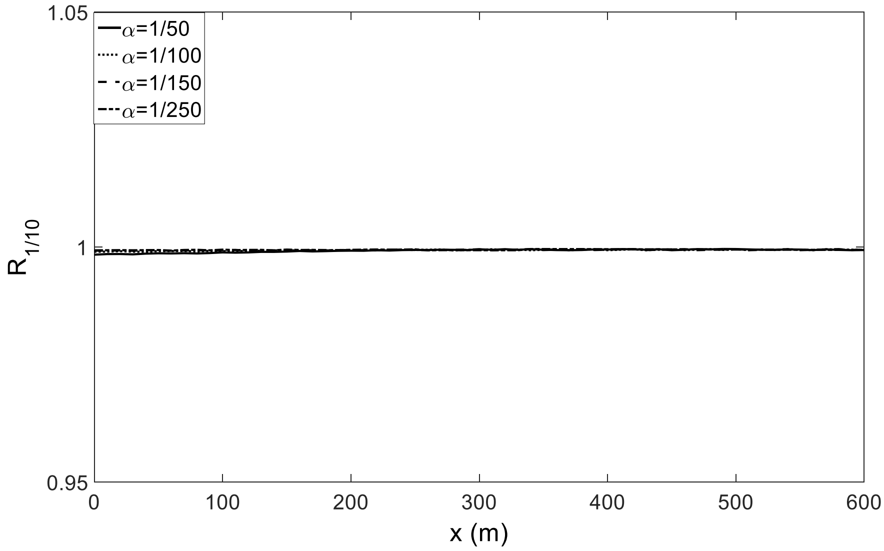


Fig. 12 Random waves alone: The stochastic to approximate method ratio $R_{1/10}$ versus x for four slopes α = (1/50, 1/100, 1/150, 1/250)

The scour depth below a pipeline for random waves alone can be obtained by replacing KC with $KC_{1/n}$ in Eq. (1), given by

$$\frac{S}{D} = 0.1KC_{1/n}^{0.5} \tag{53}$$

For random waves plus currents, the approximate model can be obtained by replacing KC with $KC_{1/n}$ in Eqs. (41), (42), (44) – (46)

$$\hat{S} = \frac{S}{D} = \frac{S_{cur}}{D}\frac{5}{3}KC_{1/n}^{a}\exp(2{,}3b) \tag{54}$$

where

$$a = 0.557 - 0.912(E[U_{cw1/n}] - 0.25)^2 \tag{55}$$

$$b = -1.14 + 2.24(E[U_{cw1/n}] - 0.25)^2 \tag{56}$$

$$E\left[U_{cw1/n}\right] = \frac{U_c}{U_c + E\left[U_{1/n}\right]} \tag{57}$$

For the case of random waves alone, the results of the stochastic to approximate method ratio of the scour depth for the four slopes are shown in Fig. 12, denoted by $R_{1/10}$ for n = 10. It is interesting to note that the approximate method gives almost the same values as that of the stochastic method for all slopes. This is also the case for $R_{1/3}$, (not shown here). Thus, it appears that the approximate method can replace the stochastic method for random waves alone.

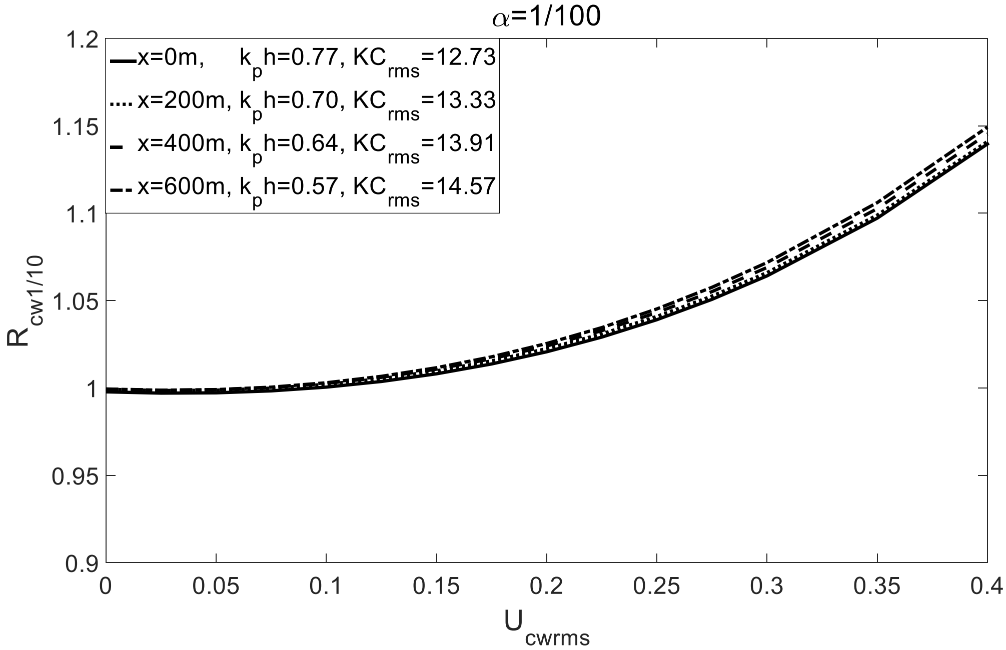


Fig. 13 Random waves plus current: The stochastic to approximate method ratio $R_{cw1/10}$ versus U_{cwrms} at four locations for slope α =1/100

For random waves plus currents, Fig. 13 shows the result of the stochastic to approximate method ratio of the scour depth, denoted by $R_{cw\ 1/10}$ for n = 10 and α =1/100. It appears that for all locations, the values of $R_{cw\ 1/10}$ are larger than one. The difference between the stochastic method and the approximate method increases as U_{cwrms} increases, suggesting that the effect of current increases the difference between the two methods. Overall, it appears that the stochastic method cannot be replaced by the approximate method for estimating the scour depth for random waves plus current.

4.5 Shields parameter

As described in Section 2, the scour prediction model in Eq. (1) is valid for live-bed scour, for which $\theta > \theta_{cr}$, where θ is the undisturbed Shields parameter defined in Eq. (3).

For a sloping bed the gravity gives a force component on the grain which may increase or decrease the threshold shear stress required from the flow. The threshold Shields parameter, $\theta_{\alpha cr}$, for initiation of motion of the grains at a bed sloping at an angle α to the horizontal in upsloping flows is related to the value θ_{cr} for the same grains on a horizontal bed by (see e.g., Soulsby (1997, Section 6.4))

$$\frac{\theta_{\alpha cr}}{\theta_{cr}} = \cos\alpha\left(1+\frac{\tan\alpha}{\tan\phi_i}\right) \tag{58}$$

where ϕ_i is the angle of repose of the sediment.

Following Myrhaug (1995) and Myrhaug and Holmedal (2002), the non-dimensional maximum

Shields parameter for individual narrow-band random waves near a horizontal bed, $\hat{\theta} = \theta / \theta_{rms}$, is equal to the non-dimensional maximum bottom shear stress for individual narrow-band random waves, $\hat{\tau} = \tau_w / \tau_{wrms}$. Here θ_{rms} is defined as

$$\theta_{rms} = \frac{\tau_{wrms} / \rho}{g(s-1)d_{50}} \tag{59}$$

where, by definition

$$\frac{\tau_{wrms}}{\rho} = \frac{1}{2} c \left(\frac{A_{rms}}{z_0} \right)^{-d} U_{rms}^2 \tag{60}$$

and θ is defined in Eq. (3). By using this and following Myrhaug and Holmedal (2002, Eq. (21)), $\hat{\theta}$ is given as

$$\hat{\theta} = \hat{H}^{2-d} \tag{61}$$

For random waves it is not obvious which value of the Shields parameter to use to determine the conditions corresponding to live-bed scour. However, it seems to be consistent to use corresponding statistical values of the scour depth and the Shields parameter, e.g., given by

$$E\left[\hat{\theta}(\hat{H}) \middle| \hat{H} > \hat{H}_{1/n}\right] = n\Gamma\left(2 - \frac{d}{2}, \ln n\right) \tag{62}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function (see Abramowitz and Stegun (1972, Ch. 6.5, Eq. (6.5.3)). This is used in conjunction with Eq. (58) when the bed is sloping.

5. Tentative approaches to related cases for random waves alone

5.1 Effect of pipe position in scour

Sumer and Fredsøe (1990) also did some tests where the scour depth was measured below pipelines fixed at different *e*-levels relative to the undisturbed horizontal bed (where *e* is the clearance between the pipeline and the undisturbed seabed, see Fig. 14). Based on these results Sumer and Fredsøe (2002) proposed the following empirical formula for the equilibrium scour depth for regular waves

$$\frac{S}{D} = 0.1 KC^{0.5} \exp(-0.6 \frac{e}{D}) \; ; \; 0 \le \frac{e}{D} \le 2 \tag{63}$$

This equation is valid for live-bed scour in regular waves. By rearranging the equation, it is noticed that this is a generalization of Eq. (1) namely

$$\frac{S}{D} / \exp(-0.6 \frac{e}{D}) = 0.1 KC^{0.5} \tag{64}$$

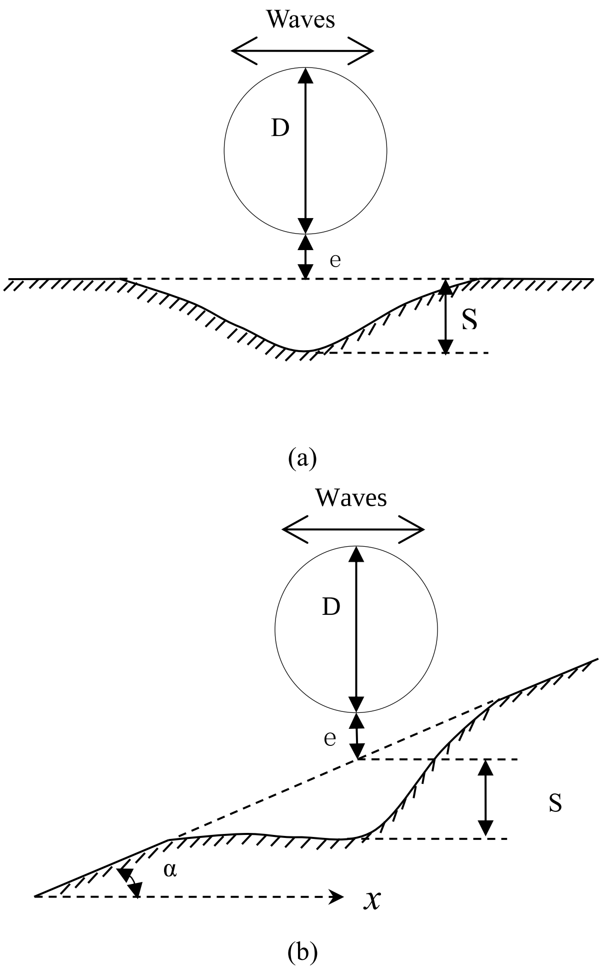
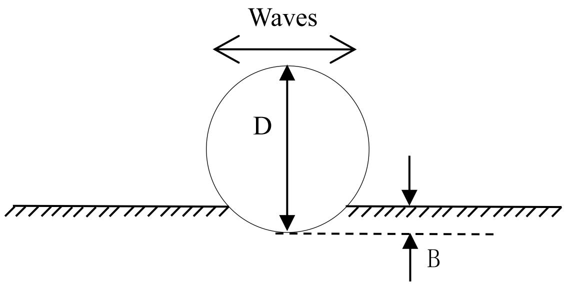


Fig. 14 Definition sketch of scour below pipelines with a pipe position e above the undisturbed bed: (a) on a horizontal bed; (b) on a mild slope

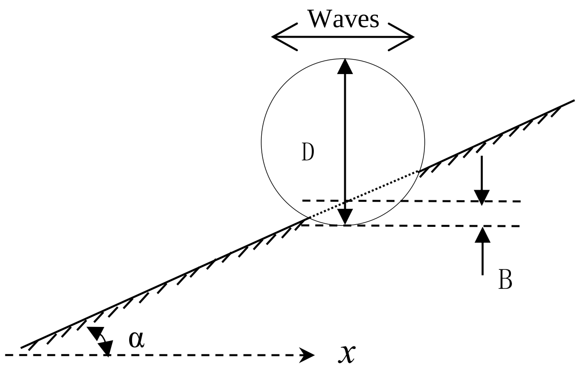
Thus, all the results given for the scour depth in random waves below pipelines initially resting on an undisturbed bed with a mild slope can be used in this case by multiplication of the factor $\exp(-0\cdot 6e/D)$. No data exist for scour in random waves on mild slopes.

5.2 Self-burial of pipelines at span shoulder

Sumer *et al.* (2001) summarized the results of an experimental study on the onset of scour below pipelines and self-burial of pipelines on a horizontal bed in currents and regular waves. They found that the self-burial at span shoulders was the same as the scour depth in the case of a fixed pipeline with an initial zero gap. Thus, the self-burial depth at span shoulder *B* (see Fig. 15) is given by Eq. (1) where *B* replaces *S*, namely



(a)



(b)

Fig. 15 Definition sketch of self-burial depth B at span shoulder of pipelines: (a) on a horizontal bed; (b) on a mild slope

$$\frac{B}{D} = 0 \cdot 1 KC^{0.5} \;;\; KC \leq 100 \tag{65}$$

This equation is valid for live-bed in regular waves. Thus, all the results given for the scour depth in random waves below pipelines initially resting on an undisturbed bed with a mild slope can be used in this case. No data exist for self-burial of span shoulders in random waves on mild slopes.

6. Conclusions

A practical method for estimating the scour depth below pipelines exposed to random waves alone and random waves plus currents on mild slopes for wave-dominated flow conditions with

$0 \leq U_c/(U_c + U_{rms}) \leq 0.4$ is provided.

The main conclusions are:

1. The Battjes and Groenendijk (2000) wave height distribution for mild slopes is applied to describe the random wave condition on mild slopes including the effect of breaking waves. A method for transformation of the wave spectrum from deep water to finite water depth is presented. Then a method is derived for calculating the random wave-induced scour below a pipeline based on assuming the waves to be a stationary narrow-band random process.
2. For random waves alone, the present results reveal that the effect of a mild slope increases the scour depth compared with that at the seaward location. Moreover, a larger bed slope causes more scour at a fixed location.
3. The present results show that the effect of a current increases the random wave-induced scour depth. This effect becomes more pronounced as the current increases. The scour depth for random waves plus currents ranges up to about 1.3 times that for random waves alone.
4. The results suggest that for random waves alone the approximate method can replace the stochastic method, whereas the stochastic method is required for random waves plus currents.
5. Tentative approaches to related random wave-induced scour cases are also suggested, such as the effect of pipe position in scour on mild slopes, and self-burial of pipelines at span shoulder on mild slopes.

Although the methodology is simple, it should be useful as a first approximation to represent the stochastic properties of the scour depth below pipelines under both random waves alone and random waves plus current on mild slopes for wave-dominated flow conditions. However, comparisons with data are required before a conclusion regarding the validity of this method can be given. In the meantime the method should be useful as an engineering tool for the assessment of scour and in scour protection work of pipelines on mild slopes.

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