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Uncertainty reaction force model of ship stern bearing based on random theory and improved transition matrix method

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Abstract. Stern bearing is a key component of marine propulsion plant. Its environment is diverse, working condition changeable, and condition severe, so that stern bearing load is of strong time variability, which directly affects the safety and reliability of the system and the normal navigation of ships. In this paper, three affecting factors of the stern bearing load such as hull deformation, propeller hydrodynamic vertical force and bearing wear are calculated and characterized by random theory. The uncertainty mathematical model of stern bearing load is established to research the relationships between factors and uncertainty load of stern bearing. The validity of calculation mathematical model and results is verified by examples and experiment yet. Therefore, the research on the uncertainty load of stern bearing has important theoretical significance and engineering practical value.

Keywords: stern bearing; uncertainty model; random theory; stochastic theory; improved transition matrix method

1. Introduction

Recent years, with the development of larger modernization and automation of ship, hull deformation changes bigger than before and shaft length and diameter is becoming much larger, engine power increasing, on the other side, the number of unstable and uncertainty factors is increasing. Therefore, safety, reliability and adaptability when the shafting and stern bearing working become more and more important(Wang 2005, Geng 2010, Murawski 2005). Stern bearing is installed in tail of the ship, affected by random propeller hydrodynamic and hull deformation. Edge load effect of the stern bearing is serious, shows randomness, influence bearing's performance and life seriously. In order to adapt to the development of ship with large size, overcome design specific problems of the ship, so that the study of ship stern bearing uncertainty reaction force has important theoretical significance and engineering value.

According to the mechanism of uncertainty, there are three type models, stochastic model (Zhu 1991), fuzzy model (Cai 1990, Feng 2002)and interval model (Su 2005). Ship stern bearing which working in diverse conditions, its reaction force shows time varying characteristics due to various

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external random forces.

The stochastic theory or method is an important branch of mathematics. Stochastic theory which based on probability theory and mathematical statistics, is a tool to solve engineering practical problems. Parameters of stocatical random model is random variable or stocatical process, research uncertainty phenomenon using the method of mathematical statistics and probability theory, random characteristics of variables such as coefficient of variation and correlation coefficient are counted according to the theory of probability. (Chen 2002, Jie 2005, Lei 2000, Benaroya 1990, Gao 2004, Elishakoff 1995). In the ship propulsion shafting statics and dynamics equations, including random parameters and random process, the output parameters of structure shows random characteristic usually. Its random parameters have random characteristic parameters and variation (Chen 2000, Elishakoff 2000). Chen (2002) put forward the random factor method in the study of truss structure stochastic dynamic problem. This method character random parameters to stochastic factor and deterministic value, the average value of random factor is 1. Chen and Che put forward a new method of random factor and transmission matrix, called expanding order transfer matrix method. And then it is proposed, used to solve chain structure system successfully(Chen 2000).

Uncertainty theory has been used in journal bearing uncertainty load. T.H.Yong(T.H. Younga 2007)put the rotor bearing system as the research object, considered the effect of axial force load uncertainty, in order to discuss the dynamic stability of the rotor bearing system, using the random theory. Based on the random theory and the optimization algorithm, Mourelatos (2004) established the calculation model of max oil film pressure of engineering bearing, representing the uncertainty of max oil film pressure in the condition of random external loading.

But all the researchers cannot establish one general uncertainty reaction force model of ship stern bearing. We will establish it including hull deformation, bearing wear, and propeller hydrodynamics.

2. Uncertainty loading model of ship stern bearing

Ship propulsion shafting is mainly composed by tail shafts, tail bearings, intermediate shafts, intermediate bearings, thrust and thrust bearings. Generally, establishing simplified model becomes necessarily, the simplified principle is that the center of mass position can't be removed. So, propulsion shafting must be simplified to the elements of the nodes and no-mass axis segments. Nodes are elected in propeller and shaft neck center, couplings, shaft section mutation and shaft end position, and be numbered in the order, from tail to head.

Model is established in Cartesian coordinate system o-xyz, the oz axis along axis direction of the shaft system, with angular velocity ω , from the end of the Z axis. For the j cross section of shaft, its status vector is Z_j, and it is composed by vertical displacement y, angle φ, bending moment M and shear Q.

$$\boldsymbol{Z}_j = \begin{bmatrix} y & \varphi & M & Q \end{bmatrix}_j^{\mathrm{T}} \tag{1}$$

2.1 Mass points force analysis

Based on ship shafting's geometrical characteristic , the shaft can be simplified to mass points and no-mass axis shafting segments. In the position of mass points, its force analysis diagram as shown in Fig.1.

where, M^{L},M^{R} are mass point's left moment and right moment , N • m； Q^{L},Q^{R} are mass point's left shear force and right shear force, N； K_{s}, C are bearing's stiffness and damping, N • s/m; G is mass point's quality, N; F_j is bearing's reaction force, N.

Assuming that the stern bearing has not eccentric in the horizontal direction, only in the vertical direction, it bears external uncertainty loading, such as the propeller hydrodynamic force F_P, bearing wear δ_2 and hull deformation δ_1, according to the principle of the force balance, the balance equations are listed as shown.

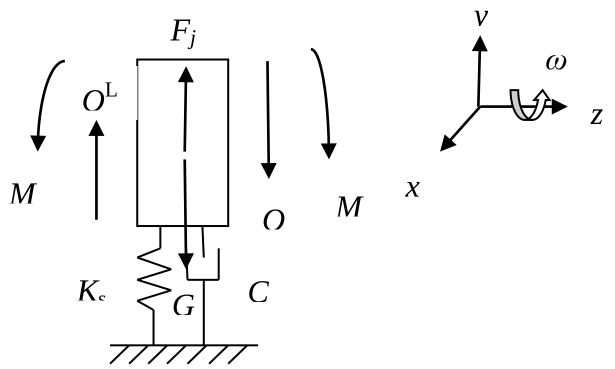


Fig. 1 Mass point force analysis diagram

$$\begin{cases} y_j^{\mathrm{R}} = y_j^{\mathrm{L}} \\ \varphi_j^{\mathrm{R}} = \varphi_j^{\mathrm{L}} \\ M_j^{\mathrm{R}} = M_j^{\mathrm{L}} \\ Q_j^{\mathrm{R}} + m_j g + F_P = Q_j^{\mathrm{L}} + m_j \omega^2 y_j + K_{\mathrm{s}j}\delta_j - K_{\mathrm{s}j} y_j \end{cases} \tag{2}$$

its matrix form is

$$\boldsymbol{Z}_j^{\mathrm{R}} = \boldsymbol{D}_j \boldsymbol{Z}_j^{\mathrm{L}} + \boldsymbol{FF}_j \tag{3}$$

$$\boldsymbol{D}_j = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ m\omega^2 - K_{\mathrm{s}} & & & 1 \end{bmatrix}_j \qquad \boldsymbol{FF}_j = \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_{\mathrm{s}}\bar{\delta} - m g - \bar{F}_P \end{bmatrix}_j$$

2.2 Uniform shaft segment's transfer matrix

Ship shaft be equivalent to non quality beams. Based on the conditions of beam's force balance and beam's deformation, including beams' stiffness EJ and shear coefficient γ, so the beam's balance equation is listed in (4).

$$\begin{cases} y_{j+1} = y_j + l_j\varphi_j + \dfrac{l_j^2}{2EJ_j}M_j + \dfrac{l_j^3}{6EJ_j}\left(1-\gamma_j\right)Q_j \\ \varphi_{j+1} = \varphi_j + \dfrac{l_j}{EJ_j}M_j + \dfrac{l_j^2}{2EJ_j}Q_j \\ M_{j+1} = M_j + l_jQ_j \\ Q_{j+1} = Q_j \end{cases} \tag{4}$$

its matrix form is

$$\boldsymbol{Z}_{j+1} = \boldsymbol{B}_j\boldsymbol{Z}_j \tag{5}$$

$$\boldsymbol{B}_j = \begin{bmatrix} 1 & l & \dfrac{l^2}{2EJ} & \dfrac{l^3}{6EJ}(1-\gamma) \\ 0 & 1 & \dfrac{l}{EJ} & \dfrac{l^2}{2EJ} \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}_j$$

where l is length of shafting segments, m; E is modulus of elasticity, Pa; J is Section moment of inertia of the shaft,m^4; D is diameter of shaft, m.

$$J_j = \frac{\pi D_j^4}{64} \qquad \gamma_j = \frac{6EJ_j}{k_t G_E A_j l_j^2}$$

where G_E is shear modulus of shafts, Pa; A is the area of section ,m^2; k_t is section coefficient.

2.3 Force analysis of mass point and shaft section

The combined parts of mass point and shaft section schematic diagram as shown in Fig. 2. Section and parts shaft are combined as research object, form point equation and segment shafting equation, and the composite part's equation is

$$\boldsymbol{Z}_{j+1}^{\mathrm{R}} = \boldsymbol{B}_j\boldsymbol{D}_j\boldsymbol{Z}_j^{\mathrm{L}} + \boldsymbol{B}_j\boldsymbol{F}\boldsymbol{F}_j \tag{6}$$

rewrite (6) as

$$\boldsymbol{Z}_{j+1}^{\mathrm{R}} = \boldsymbol{T}_j\boldsymbol{Z}_j^{\mathrm{L}} + \boldsymbol{F}_{\mathrm{B}j} \tag{7}$$

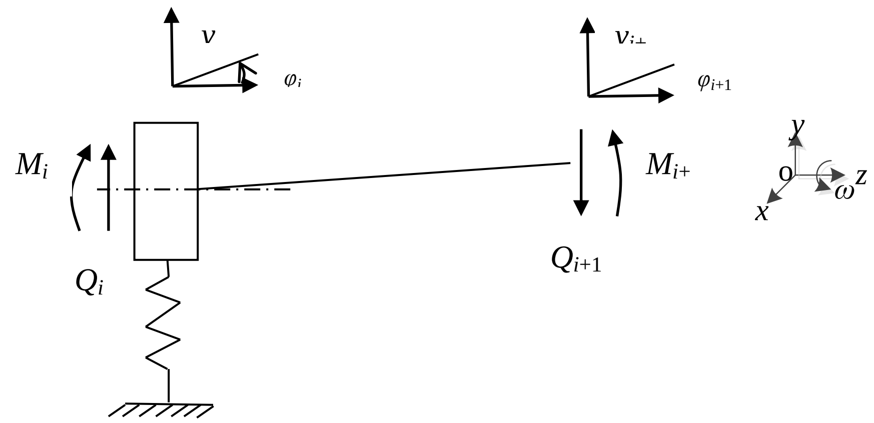


Fig. 2 Schematic diagram of mass point and shaft taking quality points with bearing stiffness and segment

and sub matrix

$$\boldsymbol{T}_j = \begin{bmatrix} \boldsymbol{T}_{11} & \boldsymbol{T}_{12} \\ \boldsymbol{T}_{21} & \boldsymbol{T}_{22} \end{bmatrix}_j \qquad \boldsymbol{F}_{\mathrm{B}j} = \begin{bmatrix} \boldsymbol{T}_e & \boldsymbol{T}_f \end{bmatrix}_j^{\mathrm{T}}$$

$$\boldsymbol{T}_{11j} = \begin{bmatrix} 1+\dfrac{l^3}{6EJ}\left(m\omega^2 - K_{\mathrm{s}}\right)(1-\gamma) & l \\ \dfrac{l^2\left(m\omega^2 - K_{\mathrm{s}}\right)}{2EJ} & 1 \end{bmatrix}_j \qquad \boldsymbol{T}_{12j} = \begin{bmatrix} \dfrac{l^2}{2EJ} & \dfrac{l^3}{6EJ}(1-\gamma) \\ \dfrac{l}{EJ} & \dfrac{l^2}{2EJ} \end{bmatrix}_j$$

$$\boldsymbol{T}_{21j} = \begin{bmatrix} l\left(m\omega^2 - K_{\mathrm{s}}\right) & 0 \\ \left(m\omega^2 - K_{\mathrm{s}}\right) & 0 \end{bmatrix}_j \qquad \boldsymbol{T}_{21j} = \begin{bmatrix} l\left(m\omega^2 - K_{\mathrm{s}}\right) & 0 \\ \left(m\omega^2 - K_{\mathrm{s}}\right) & 0 \end{bmatrix}_j$$

$$\boldsymbol{F}_{\mathrm{e}j} = \begin{bmatrix} \dfrac{l^3}{6EJ}(1-\gamma)\left(K_{\mathrm{s}}\delta - mg + F_P\right) \\ \dfrac{l^2}{2EJ}\left(K_{\mathrm{s}}\delta - mg + F_P\right) \end{bmatrix}_j \qquad F_{fj} = \begin{bmatrix} l\left(K_{\mathrm{s}}\delta - mg + F_P\right) \\ K_{\mathrm{s}}\delta - mg + F_P \end{bmatrix}_j$$

where F_{p} is stochastic variable of propeller hydrodynamic force, N; δ is stochastic variable of ship deformation and bearing wear, m.

3. Ship stern bearing's uncertainty loading model

$\boldsymbol{\Theta}$ and $\boldsymbol{P}$ are the random parameter matrix, the transfer matrix of random parameters is

$$\boldsymbol{R}_j = \boldsymbol{\Theta}_j \boldsymbol{e}_j + \boldsymbol{P}_j \tag{8}$$

$$\boldsymbol{R} = [M \quad Q]^{\mathrm{T}} \tag{9}$$

$$\boldsymbol{\Theta} = [y \quad \varphi]^{\mathrm{T}} \tag{10}$$

Due to the random parameters of F_{p} and δ, M, Q, y *and* φ are relative with F_{p} and δ, therefore $\boldsymbol{R}$ and $\boldsymbol{\Theta}$ are random parameters, their expectation and variance are $\mu(\boldsymbol{R})$, $\mu(\boldsymbol{\Theta})$, $\sigma^2(\boldsymbol{R})$, $\sigma^2(\boldsymbol{\Theta})$. Recursion formula of $\boldsymbol{\Theta}$, $\boldsymbol{P}$ *and* $\boldsymbol{e}$ *are*

$$\boldsymbol{\Theta}_{j+1} = \left[\boldsymbol{u}_{11}\boldsymbol{\Theta} + \boldsymbol{u}_{12}\right]_j \left[\boldsymbol{u}_{21}\boldsymbol{\Theta} + \boldsymbol{u}_{22}\right]_j^{-1} \tag{11}$$

$$\boldsymbol{P}_{j+1} = \left[\boldsymbol{u}_{11}\boldsymbol{P} + \boldsymbol{F}_{\mathrm{f}}\right]_j - \boldsymbol{\Theta}_{j+1}\left[\boldsymbol{u}_{21}\boldsymbol{P} + \boldsymbol{F}_{\mathrm{e}}\right]_j \tag{12}$$

$$\boldsymbol{e}_j = \left[\boldsymbol{u}_{21}\boldsymbol{\Theta} + \boldsymbol{u}_{22}\right]_j^{-1} \boldsymbol{e}_{j+1} - \left[\boldsymbol{u}_{21}\boldsymbol{\Theta} + \boldsymbol{u}_{22}\right]_j^{-1}\left[\boldsymbol{u}_{21}\boldsymbol{P} + \boldsymbol{F}_{\mathrm{e}}\right]_j \tag{13}$$

Boundary conditions:

The left boundary condition is freedom, its bending moment M and shear force Q value are zero, we know that $\boldsymbol{R}_1=0$, $\boldsymbol{e}_1 \neq 0$, so the initial value $\boldsymbol{\Theta}_1=0$,$\boldsymbol{P}_1=0$. we can calculate the value of $\boldsymbol{\Theta}_j$ and $\boldsymbol{P}_j$, j=2,3,...,n+1.

For the last right interface side, there is

$$\boldsymbol{R}_{N+1} = \boldsymbol{\Theta}_{N+1}\boldsymbol{e}_{N+1} + \boldsymbol{P}_{N+1} \tag{14}$$

When the right boundary condition of shafting is freedom

$$\boldsymbol{R}_{N+1} = 0 \tag{15}$$

$$\boldsymbol{e}_{N+1} = -\boldsymbol{\Theta}^{-1}_{N+1}\boldsymbol{P}_{N+1} \tag{16}$$

When the right boundary condition of shafting is fixed

$$\boldsymbol{e}_{N+1} = 0 \tag{17}$$

$$\boldsymbol{R}_{N+1} = \boldsymbol{P}_{N+1} \tag{18}$$

The bearings' reaction force is obtained by the comprehensive Eq. (1) to Eq. (18).

And the improved transition matrix's transfer process shows in Fig. 3.

Under the joint action of the three factors such as hull deformation, stern bearing wear and hydrodynamic vertical force of propeller, nodes load is combined by three parts: the left bending moment i-1 and the right bending moment i+1; the total mass points; the force of relative displacement of the elastic supports , so the nodes reaction force W is

$$W_i = M_{.bi} + G_i + D_{bi} \tag{19}$$

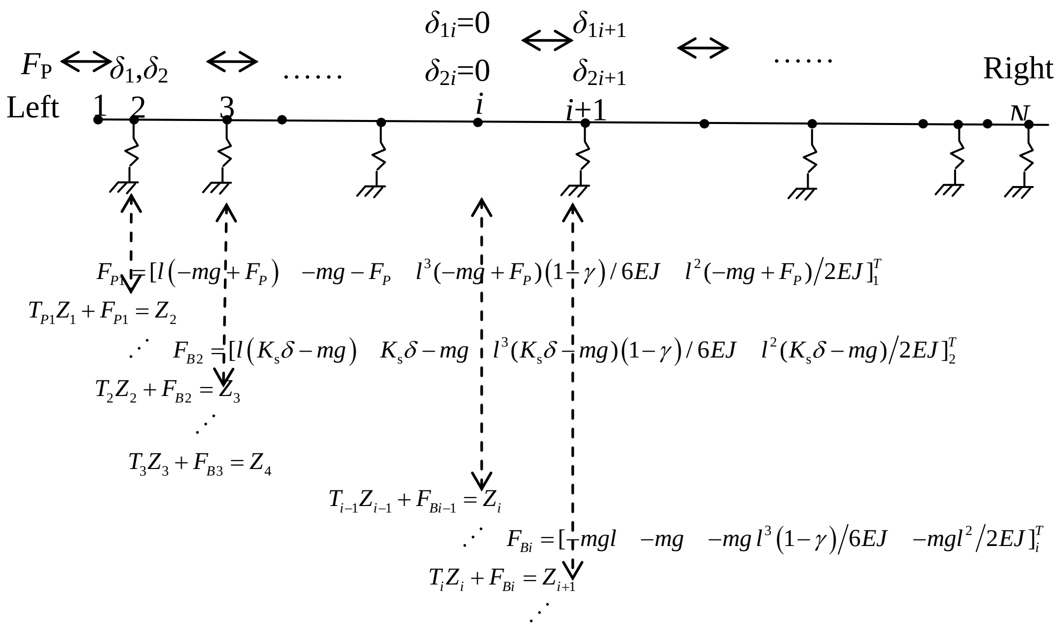


Fig. 3 Improved transition matrix's transfer process

where

$$M_{.bi}=\frac{M_i-M_{i-1}}{l_i}+\frac{M_i-M_{i+1}}{l_{i+1}}=\frac{f(1,1)_i-f(1,1)_{i-1}}{l_i}+\frac{f(1,1)_i-f(1,1)_{i+1}}{l_{i+1}} \qquad (20)$$

$$G_i=m_ig \qquad (21)$$

$$D_{bi}=K_{si}(y-\delta)_i \qquad (22)$$

4. Test verification of uncertainty loading model of ship stern bearing

Ship shafting test bed is constituted by motor, coupling, thrust bearing, thrust shaft, middle shaft, intermediate bearing, flange, tail shaft, stern bearing, tail disc, shows in Fig. 4.

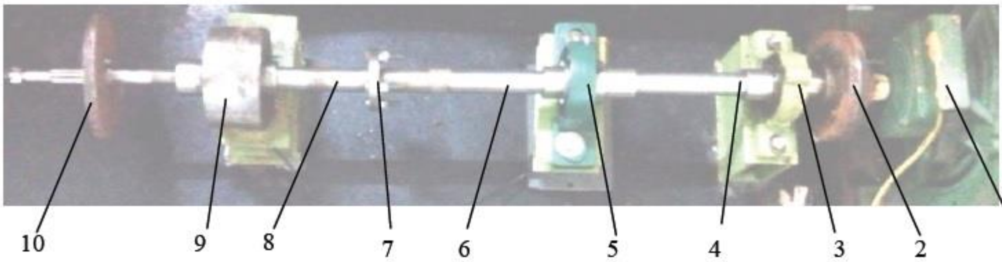
The key modeling parameter is displacement which caused by hull deformation, and the displacements are simulated by highing and reducing the intermediate bearing. And the measured parameter is reaction force of stern bearing.

The test's main geometric dimensions are shown in Fig. 5.

Other main geometric are:

Basic diameter of shaft is 25 mm ; Disc diameter:130 mm ;Stern shaft neck diameter: 48 mm, length 60 mm ; Middle shaft neck diameter 35 mm , length 60 mm ; Thrust shaft neck diameter 45 mm, length 30 mm

Taking the end stern bearing as research object, and it is oil lubrication, the end stern bearing and force sensor installation method as shown in Fig. 6.



1-Motor;2- coupling;3-thrust bearing;4-thrust shaft;5-middle shaft;
6-inermediate bearing;7-flange;8-tail shaft;9-stern bearing ;10-tail disc

Fig. 4 Ship shafting test bed

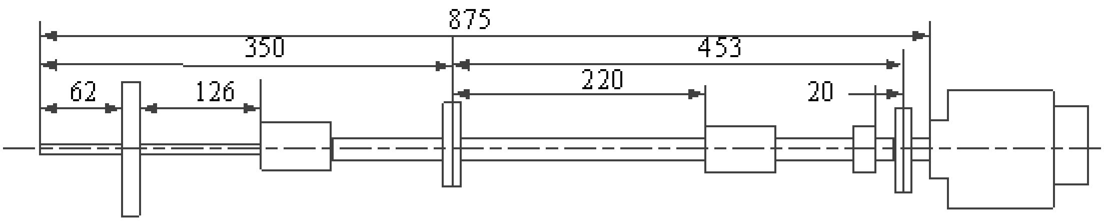
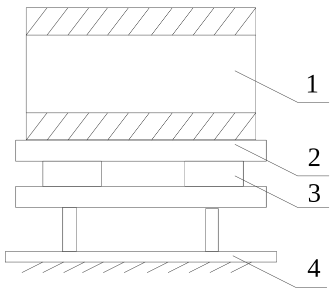


Fig. 5 Shaft's main geometric dimensions



1-End stern bearing; 2-Bearing support; 3- Force sensor; 4-Foundation

Fig. 6 Force sensor installation diagram

Table 1 Bearings reaction force list

	End stern bearing	Intermediate bearing	Thrust bearing
Theory/N	48.49	19.92	15.81
Test/N	47.73	18.44	——
Relative error	1.59%	8.03%	——

In order to simulate the influence of hull deformation and the propeller vertical force on the bearing load, the design scheme is formulated, and the steps are as follows:

(1)The end stern bearing, intermediate bearing, thrust bearing and frequency conversion motor are heighted 0.4 mm;

(2) The propulsion shaft is aligned;

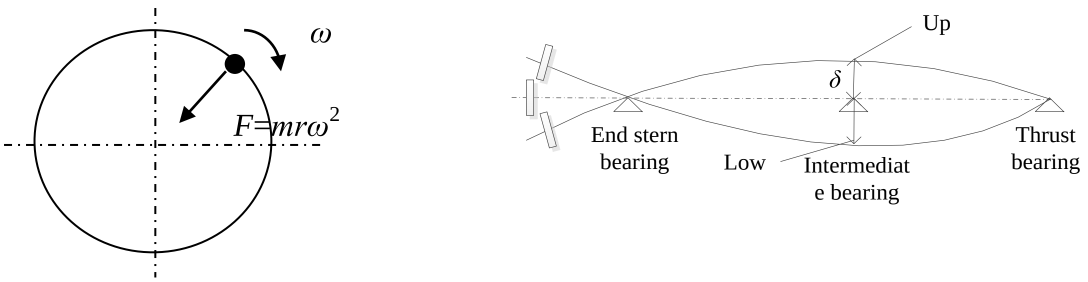
(3) Simulate the effect of propeller vertical force and hull deformation joint action, increase weight in disc and pad the intermediate bearing position.

Due to shaft alignment result, bearings' reaction force as shown in Table 1.

Calculation condition : hull deformation is up 0.4 mm and low 0.4 mm. Disk weight is 220 g, to silulate propeller’s vertical force, the test schematic diagram as shown in Fig. 7.

And the weight is 220 g, intermediate bearing up 0.4 mm, the speed of tail shaft is 100 rpm, end stern bearing load as shown in Fig. 8(a).

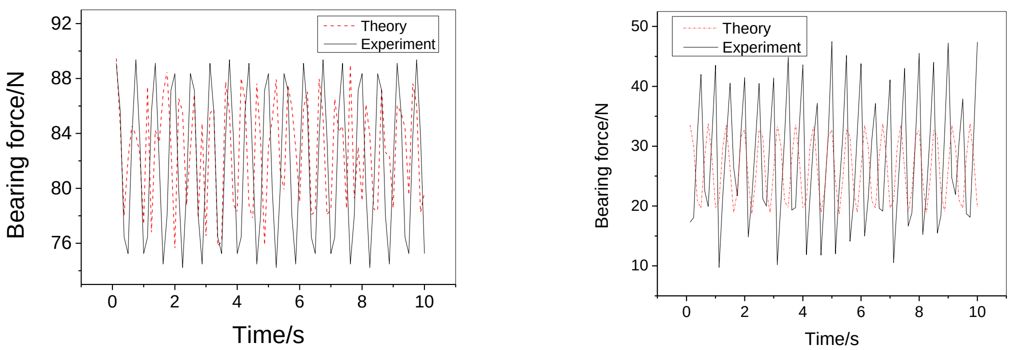
In the basic of last step , low intermediate bearing to -0.4 mm, other condition still same as last step, and the end stern bearing load as shown in Fig. 8 (b).



(a) propeller vertical force schematic diagram

(b) Hull deformation schematic diagram

Fig. 7 Test schematic diagram



(a) hull deformation -0.4 mm & propeller vertical force

(b) hull deformation 0.4 mm & propeller vertical force

Fig. 8 End stern bearing load under condition of hull deformation and propeller vertical

Table 2 Bearings reaction force statistical table

		Test cycles				Reaction force/N	
		1	2	3	4	Experiment value	Theory value
Displacement of intermediate bearing	0.4 mm	66.404	66.718	66.738	66.757	66.654	67.447
	0.3 mm	63.954	63.896	64.013	64.033	63.974	63.557
	0.2 mm	57.016	56.996	56.977	56.898	56.972	57.761
	0.1 mm	52.547	52.057	52.077	52.234	52.229	52.297
	-0.1 mm	42.590	43.394	43.806	42.767	43.139	44.014
	-0.2 mm	40.121	39.572	35.574	36.230	39.183	37.372
	-0.3 mm	36.260	38.808	38.866	38.935	36.909	36.731
	-0.4 mm	28.753	27.381	27.949	27.636	27.929	28.089
Mean value		48.456	48.602	48.250	48.186	—	—
Standard deviation		13.591	13.624	13.947	13.982	—	—

The reaction force of stern bearing is listen in Table 2

End stern bearing load shows random characteristic, because of hull deformation and propeller vertical force, the mean value changes between 48.18~48.61 N, standard value changes between 13.59~13.99 N. In the condition of hull deformation 0.4 mm and propeller vertical force, intermediate bearing position up , the force on end stern bearing is less; other condition hull deformation -0.4 mm and propeller vertical force, intermediate bearing position low, the force which on end stern bearing is increase. In the two conditions, shaft deflection increased, especially propeller shaft, add the propeller's weight, they are the main reason that reduce to end stern bearing load change larger.

5. Conclusions

(1)Based on the stochastic theory, mechanics theory and transfer matrix method, ship propulsion shafting is simplified as lumped mass with bearing base and mass less elastic shaft segments, and carrying on the force analysis of the simplified parts, listing uncertainty load equations of stern bearing by transfer matrix transformation, the ship stern bearing loading uncertainty calculation model is established. From system, the static and dynamic load uncertainty of stern bearing is studied. In the uncertainty model, three main factors, such as hull deformation, propeller hydrodynamic and bearing wear, are considered.

(2) The accuracy and rationality of the loading uncertainty representation model of the stern bearing are verified by the experiment in the marine propulsion shafting test. When shaft operation, the uncertainty loading relative error range in the 3%~9% between theory calculation value and test value. Both of the data difference is smaller, be in good agreement and verify the theoretical model of rationality and calculation results of correctness.

Acknowledgments

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