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Design of bars in tension or compression exposed to a corrosive environment

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Abstract. This study is devoted to the optimal design of compressed bars under axial tensile or compressive forces and exposed to a corrosive environment. Dolinskii's linear stress corrosion model is adopted for analysis. Analytical and numerical results are derived for optimal variation of the cross-sectional area of the bar along its axis.

Keywords: corrosive; environment; design; bars; tension; compression

1. Introduction

According to Shaw and Kelly (2006), "corrosion is degradation of materials' properties due to interactions with their environments, and corrosion of most metals (and many materials for that matter) is inevitable. While primarily associated with metallic materials, all material types are susceptible to degradation."Moreover, "several studies over the past 30 years have shown that the annual direct cost of corrosion to an industrial economy is approximately 3.1% of the country's Gross national product. In the United States, this amounts over $276 B per year," in accordance to the Historic Congressional Study (2002).

Hansson (2011) writes: "Unfortunately,…corrosion resistance is often relegated to a lower priority when materials are selected on the basic of those properties essential to the specific application, such as strength, stiffness, and electrical conductivity. The result is that corrosion is ubiquitous, occurring in all forms of engineering materials from microelectronics to orthopedic implants to major civil infrastructures and to everyday objects in our lives…"

Interaction of the stress level and the corrosion phenomenon was introduced by Dolinskii (1967) who suggested a linear model, whereas Gutman and Zainullin (1984) resorted to an exponential model. These models have been revisited recently by Elishakoff and Miglis (2011, 2012). Extension to the case of random applied loading was conducted by Elishakoff and Soret (2012) whereas Fridman and Elishakoff (2013), Fridman (2014) studied the deterministic optimization problem.

In this paper we deal with design specifics of the bar in tension subjected to corrosive

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environment with attendant optimization.

2. Problem formulation

We consider a bar whose cross sectional area represents an annulus, with inner radius $\underline{R}$ and outer radius $\overline{R}$. The bar is subjected to a tensile load *Q*. The normal stress *σ(t)* in the cross-section reads

$$\sigma(t) = \frac{Q}{\pi\left[\overline{R}^2(t) - \underline{R}^2\right]} \tag{1}$$

Outer surface of the bar is in contact with a corrosive environment. Therefore, through the corrosion process, the outer radius decreases whereas the inner radius remains constant. We express the outer radius from Eq. (1) as

$$\overline{R}(t) = \sqrt{Q/\pi\sigma(t) + \underline{R}^2} \tag{2}$$

Note that since outer radius is a function of time, so is the resulting stress *σ(t)*. We assume that the outer radius varies as a function of the corrosion velocity *v(t)* also referred as the corrosion rate as follows

$$\overline{R}(t) = \overline{R}_0 - \int_0^t v(x)dx \tag{3}$$

where $\overline{R}_0$ is the initial value of the outer radius and t is the time instant when the outer radius is recorded. The problem consists in evaluating the durability of the structure and the associated time-dependent reliability.

3. Linear relationship between corrosion rate and stress

We consider the simplest possible relationship between corrosion rate and stress. According to Dolinskii (1967), the corrosion velocity *v(t)* linearly depends on stress value *σ(t)*

$$v(t) = v_0 + m\sigma(t) \tag{4}$$

where *m* is a coefficient dependent on both the material and the corrosive environment. The expression in the right hand side must be equated in conjunction with the value of outer radius given in Eq. (2). The following equation is obtained

$$\overline{R}_o - v_o t - \int^t m\sigma(x)dx = \sqrt{Q/\pi\sigma(t) + \underline{R}^2} \tag{5}$$

Differentiation with respect to time *t* yields the first order differential equation with variable coefficients

$$-\nu_o - m\sigma(t) = -\frac{Q}{2\pi\sigma^2(t)\sqrt{Q/\pi\sigma(t) + \underline{R}^2}} \frac{d\sigma(t)}{dt} \tag{6}$$

We integrate Eq. (6) to get

$$\int_0^t dt = \frac{Q}{2\pi}\int_0^t \frac{d\sigma(t)}{\sigma(t)^2(\nu_0 + m\sigma(t))\sqrt{Q/\pi\sigma(t) + \underline{R}^2}} \tag{7}$$

Performing intégration in Eq. (7), we derive”

$$t = \left(\frac{mQ}{2\pi\nu_0^2}\frac{1}{\sqrt{\underline{R}^2 - Qm/\pi\nu_0}}\ln\frac{\sqrt{Q/\pi\sigma(t) + \underline{R}^2} - \sqrt{\underline{R}^2 - Qm/\pi\nu_0}}{\sqrt{Q/\pi\sigma(t) + \underline{R}^2} + \sqrt{\underline{R}^2 - Qm/\pi\nu_0}} - \frac{\sqrt{Q/\pi\sigma(t) + \underline{R}^2}}{\nu_0}\right)\Bigg|_0^t \tag{8}$$

This expression can be simplified by making the following substitutions

$$r_t = \sqrt{Q/\pi\sigma(t) + \underline{R}^2} = \overline{R}(t), \quad p = \sqrt{\underline{R}^2 - Qm/\pi\nu_0}\ ; \quad n = \frac{mQ}{2\pi\nu_0^2} \tag{9}$$

As a result, we arrive at

$$t = \frac{n}{p}\left(\ln\frac{r_t - p}{r_t + p} - \ln\frac{r_0 - p}{r_0 + p}\right) - \frac{r_t - r_0}{\nu_0} \tag{10}$$

4. Design of annularcross- section

In the final moment T of the operation of the structure we obtain the corresponding expression of structure’s life-time, or durability

$$T = \frac{n}{p}\left(\ln\frac{r_T - p}{r_T + p} - \ln\frac{r_0 - p}{r_0 + p}\right) - \frac{r_T - r_0}{\nu_0} \tag{11}$$

where

$$r_T = \sqrt{Q/\pi\sigma(T) + \underline{R}^2} = \overline{R}_T\ ; \quad r_0 = \sqrt{Q/\pi\sigma_0 + \underline{R}^2} = R_0 \tag{12}$$

where $\sigma(T)$ depends on the problem at hand.

Eq. (10) should be used, for example to assess durability of the structural element during its operation, when the specified cross-sectional dimensions and the parameters of aggressive medium are known. The design phase requires determining the dimensions of the cross-sectional design element for a pre-specified period of its operation. To solve this problem, we rewrite (10) as follows

$$\left(t + \frac{r_t - r_0}{\nu_0}\right)\frac{p}{n} = \ln\frac{(r_t - p)(r_0 + p)}{(r_t + p)(r_0 - p)} \tag{12}$$

We transform this expression to get

$$\ln\{\exp[(\ t+\frac{r_t-r_0}{v_0})\frac{p}{n}]\}=\ln\frac{(r_t-p)(r_0+p)}{(r_t+p)(r_0-p)}$$

Hence

$$\exp[(t+\frac{r_t-r_0}{v_0})\frac{p}{n}]=\frac{(r_t-p)(r_0+p)}{(r_t+p)(r_0-p)}$$

which could also be directly derived from Eq. (12) by definition of the natural logarithm. As a first approximation, we set $e^z \approx 1+z$. Thus, we get

$$1+(t+\frac{r_t-r_0}{v_0})\frac{p}{n}=\frac{(r_t-p)(r_0+p)}{(r_t+p)(r_0-p)}$$

At $t=T$, we obtain

$$1+(T+\frac{r_T-r_0}{v_0})\frac{p}{n}=\frac{(r_T-p)(r_0+p)}{(r_T+p)(r_0-p)} \tag{13}$$

From (13), using (11) we find $\bar{R}_0$

$$\bar{R}_0=-B+\sqrt{B^2-C} \tag{14}$$

where

$$B=\frac{1}{2}\left[\frac{n v_0\left(\bar{R}_T-\sqrt{\underline{R}^2-n_1}\right)}{\sqrt{\underline{R}^2-n_1}\left(\bar{R}_T+\sqrt{\underline{R}^2-n_1}\right)}-\sqrt{\underline{R}^2-n_1}-\frac{n v_0}{\sqrt{\underline{R}^2-n_1}}-T v_0-\bar{R}_T\right]$$

$$C=\frac{n v_0\left(\bar{R}_T-\sqrt{\underline{R}^2-n_1}\right)}{\bar{R}_T+\sqrt{\underline{R}^2-n_1}}+n v_0+T v_0\sqrt{\underline{R}^2-n_1}+\bar{R}_T\sqrt{\underline{R}^2-n_1} \quad ; \quad n_1=\frac{mQ}{\pi v_0}$$

5. Stretched/compressed elements

In the case of tension members, $\sigma(T)$ in (11) can be made equal to σ_y, the yield stress. With this in mind, we obtain

$$\bar{R}_T=\sqrt{\frac{Q}{\pi\sigma_y}+\underline{R}^2} \tag{15}$$

The critical stress for the struts at the final time T are accepted by Euler's formula

$$\sigma_{cr} = \frac{P_{cr}}{A_T} = \frac{\dfrac{\pi^2 EI}{l^2}}{A_T} \tag{16}$$

For circular cross-section moment of inertia of the cross-sectional area at the time T is defined as

$$I = \pi\left(\overline{R}_T^4 - \underline{R}^4\right)/4\,; \quad A_T = \pi\left(\overline{R}_T^2 - \underline{R}^2\right)$$

Finally, we get

$$\sigma_{cr} = \frac{\pi^2 E\left(\overline{R}_T^2 + \underline{R}^2\right)}{4l^2} \tag{17}$$

The lower limit of the external radius of the final time T is the inequality $\sigma_{cr} \geq \sigma_T = \dfrac{Q}{A_T}$, with (17) as

$$\overline{R}_T \geq \sqrt[4]{\frac{4Ql^2}{\pi^3 E} + \underline{R}^4} \tag{18}$$

The condition of applicability of the formulas (16) and (17) is limited by the inequality

$$\sigma_{cr} \geq \sigma_T = \frac{Q}{A_T} \leq \sigma_y \varphi \tag{19}$$

where the coefficient φ depends on the conditional flexibility $\overline{\lambda}$

$$\overline{\lambda} = \lambda\sqrt{\sigma_y / E}\,; \quad \lambda = l/i\,; \; i = \sqrt{I_T / A_T} = \sqrt{\frac{\pi\left(\overline{R}_T^4 - \underline{R}^4\right)}{4\pi\left(\overline{R}_T^2 - \underline{R}^2\right)}} = \frac{1}{2}\sqrt{\overline{R}_T^2 + \underline{R}^2}$$

In the latter formula l and I are bar'slength and the radius of inertia of its cross section, respectively. As a result, we get

$$\overline{\lambda} = \frac{2l}{\sqrt{\overline{R}_T^2 + \underline{R}^2}}\sqrt{\sigma_y / E} \tag{20}$$

The values of the coefficient of φ according to [B] (SNIP II-23-81, building regulations of the former Soviet Union) are determined by the following expressions

$$\text{for} \quad 0 \leq \overline{\lambda} \leq 2.5, \; \varphi = 1 - \left(0.073 - 5.53\sigma_y / E\right)\overline{\lambda}\sqrt{\overline{\lambda}}$$

$$\text{for} \quad 2.5 > \overline{\lambda} \leq 4.5 \quad \begin{array}{l} \varphi = 1.47 - 13\sigma_y / E - \left(0.371 - 27.3\sigma_y / E\right)\overline{\lambda} + \\ + \left(0.0275 - 5.53\sigma_y / E\right)\overline{\lambda}^2 \end{array}$$

$$\text{for} \quad \bar{\lambda} > 4.5 \;\; \varphi = \frac{332}{\bar{\lambda}^2\left(51 - \bar{\lambda}\right)} \tag{21}$$

6. Optimization problem

In general, at the design stage optimization problem usually reduces to minimization of the initial cross-sectional area of the structural element working in a corrosive environment for a predetermined period of operation. In our case, the goal of the optimization problem is to find the inner and outer (primary) radii, wherein the initial cross section area of the annulus ought to attain a minimum, i.e.

$$A_0 = \pi(R_0^2 - \underline{R}^2) \to \min \tag{22}$$

In the case of tension members, taking into account that $\bar{R}_0$ is defined by (14), and $\bar{R}_T$ by (15) the optimization problem reduces to finding a single parameter, namely the inner radius $\underline{R}$.

Regarding the columns, with substitution of Eq. (21) into (19) with (20) and taking into account the expression for A_T , we observe that it is analytically unfeasible to find a relation ship $\bar{R}_T$ from *R*, similar to (18). In this case, the optimization problem(as opposed to tension members) reads as follows

Find a vector of designs

$$\bar{X} = \{\underline{R}, \bar{R}_T\}^T \tag{23}$$

such that

$$A_0 = \pi(\bar{R}_0^2 - \underline{R}^2) \to \min$$

value $\bar{R}_0$ being determined by Eq. (14).

To solve the problems of nonlinear mathematical programming (22) and (23) one can resort to the random-search algorithm, for example.

7. Numerical results

As an illustration of the numerical optimization we consider stretched and compressed elements of annular cross-section with the following data: $\nu_0 = 6 \cdot 10^{-4}$ м/year; *T*=10 years; $\sigma_y = 235$ MPa; $l = 1$ м (compressed element). By varying the load (Q_1=50 kN; Q_2 =100 kN; Q_3 =150 kN; Q_4 =200 kN). We consider two options for different values of the v dependence of corrosion coefficient *m*. The values of the latter we set at either of two values (a) $m = 17 \cdot 10^{-10}$ м/MPa; (b) $m = 17 \cdot 10^{-7}$ м/MPa. Similar calculations were carried out with the introduction of additional restrictions on the annular cross-section

$$\delta \le D/10 \tag{24}$$

where

$$\delta = \overline{R}_0 - \underline{R}R\,; \quad D = 2(\frac{\overline{R}_0 - \underline{R}}{2} + \underline{R}) = \overline{R}_0 + \underline{R} \tag{25}$$

The results of numerical analysis for a tension member are shown in Table 1-4. Variations in cross-sectional area A_0 versus the load for 4 four versions are shown in Fig. 1. The corresponding results for the calculation of the compressed element are listed in Tables 5-8 (see also Fig. 2).

Table 1 [$m = 17 \cdot 10^{-10}$ м/MPa]

Q (kN)	$\underline{R}$ (cm)	$\bar{R}_0$ (cm)	$\bar{R}_T$ (cm)	A_0 (cm^2)	A_T (cm^2)	σ_0 (MPa)	σ_T (MPa)
50	0	1.43	0.827	6,38	2.13	78	235
100	0	1.77	1,167	9,79	4,26	102	235
150	0	2,02	1,43	12,9	6,38	116	235
200	0	2,25	1,65	15,85	8,51	126	235

Table 2 [$m = 17 \cdot 10^{-7}$ м/MPa]

Q (kN)	$\underline{R}$ (cm)	$\bar{R}_0$ (cm)	$\bar{R}_T$ (cm)	A_0 (cm^2)	A_T (cm^2)	σ_0 (MPa)	σ_T (MPa)
50	0,71	1.85	1,08	9,14	2.13	55	235
100	0,99	2,32	1,52	13,86	4,26	72	235
150	1,23	2,69	1,88	18,01	6,38	83	235
200	1,37	2,97	2,14	21,84	8,51	92	235

Table 3 [$m = 17 \cdot 10^{-10}$ м/MPa]

Q (kN)	$\underline{R}$ (cm)	$\bar{R}_0$ (cm)	$\bar{R}_T$ (cm)	A_0 (cm^2)	A_T (cm^2)	σ_0 (MPa)	σ_T (MPa)
50	3,186	3,891	3,29	15,67	2.13	32	235
100	3,56	4,35	3,75	19,51	4,26	51	235
150	3,86	4,72	4,12	23,05	6,38	65	235
200	4,19	5,1	4,5	26,63	8,51	75	235

Table 4 [$m = 17 \cdot 10^{-7}$ м/MPa]

Q (kN)	$\underline{R}$ (cm)	$\bar{R}_0$ (cm)	$\bar{R}_T$ (cm)	A_0 (cm^2)	A_T (cm^2)	σ_0 (MPa)	σ_T (MPa)
50	3,4	4,15	3,5	17,81	2.13	28	235
100	3,87	4,72	4,04	23,0	4,26	43	235
150	4,22	5,16	4,46	27,56	6,38	54	235
200	4,53	5,54	4,82	31,83	8,51	63	235

Table 5 [$m = 17 \cdot 10^{-10}$ м/MPa]

Q (kN)	$\underline{R}$ (cm)	$\bar{R}_0$ (cm)	$\bar{R}_T$ (cm)	A_0 (cm^2)	A_T (cm^2)	σ_0 (MPa)	σ_T (MPa)	λ	$\bar{\lambda}$	φ	$\sigma_y\varphi$ (MPa)
50	3,38	4,21	3,61	19,82	5,08	25	98.5	121	4,06	0.42	99.8
100	3.68	4.64	4.04	25.0	8.63	40	116	110	3.67	0.49	116
150	4.14	5.15	4.55	29.46	11.18	51	134	98	3.26	0.57	134
200	5.03	6,01	5.4	33.96	12.45	59	161	81	2,72	0.69	161

Table 6 [$m = 17 \cdot 10^{-7}$ м/MPa]

Q (kN)	$\underline{R}$ (cm)	$\bar{R}_0$ (cm)	$\bar{R}_T$ (cm)	A_0 (cm^2)	A_T (cm^2)	σ_0 (MPa)	σ_T (MPa)	λ	$\bar{\lambda}$	φ	$\sigma_y\varphi$ (MPa)
50	3.06	4.0	3.36	20.84	5.93	24	84	132	4.42	0.37	86
100	3.57	4.62	3.95	27.03	9.0	37	111	113	3.77	0.47	112
150	4.11	5.22	4.53	32.47	11.45	46	131	98	3.28	0.57	134
200	4.42	5.59	4.89	36.89	13.82	54	145	91	3.04	0.62	145

Table 7 [$m = 17 \cdot 10^{-10}$ м/MPa]

Q (kN)	$\underline{R}$ (cm)	$\bar{R}_0$ (cm)	$\bar{R}_T$ (cm)	A_0 (cm^2)	A_T (cm^2)	σ_0 (MPa)	σ_T (MPa)	λ	$\bar{\lambda}$	φ	$\sigma_y\varphi$ (MPa)
50	3.66	4.46	3.86	20.47	4.77	24	105	113	3.77	0.47	111
100	4.31	5.17	4.57	25.76	7.4	39	135	95	3.2	0.59	137
150	4.44	5.4	4.8	29.76	10.53	50	142	92	3.07	0.61	144
200	4.92	5.92	5.32	33.91	12.74	59	157	83	2,77	0.68	159

Table 8 [$m = 17 \cdot 10^{-7}$ м/MPa]

Q (kN)	$\underline{R}$ (cm)	$\bar{R}_0$ (cm)	$\bar{R}_T$ (cm)	A_0 (cm^2)	A_T (cm^2)	σ_0 (MPa)	σ_T (MPa)	λ	$\bar{\lambda}$	φ	$\sigma_y\varphi$ (MPa)
50	3.75	4.58	3.93	21.58	4.36	23	114	110	3.69	0.49	115
100	4.26	5.2	4.54	28.08	7.7	36	130	96	3.22	0.58	136
150	4.66	5.68	5.0	33.08	10.15	45	148	88	2.94	0.64	150
200	5.0	6.07	5.37	37.56	12.47	53	160	82	2,74	0.68	162

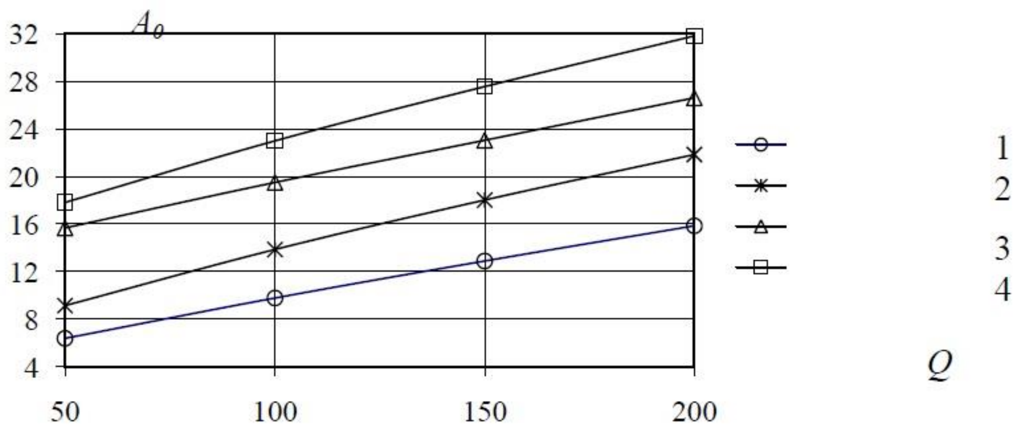

Fig. 1 Variation of cross-sectional area A_0 with load Q

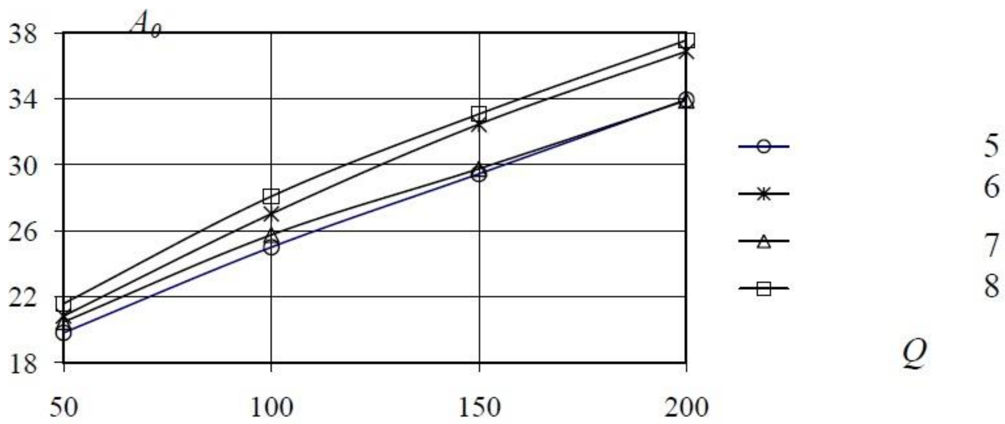

Fig. 2 Variation of cross-sectional area A_0 with load$Q$

8. Conclusions

As can be seen from Table 1,in the case of a weak dependence of corrosion on the stress state the optimal cross section for member in tension is circular. This trend is changing with increase of the parameter *m*. This can be deduced from Table 2, where the inner radius $\underline{R}$ is not equal to zero. Optimization results for thin annular cross section are listed in Tables 3 and 4 indicating that the initial cross-sectional area A_0 increases as compared with the corresponding results of Table 1, by about 1.45 to 2.45 times (see also Fig. 1).As for the optimization results obtained for compression members it is obvious that the compression case the best cross-section is a thin ring.

In this paper an analytical expression of durability is derived for the bar under tensile or compressive load in the corrosive environment. Design problem is solved to determine the size of the elements of annular cross-section for a given period of operation, utilizing the linear model.

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