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Reliability of an elastic bar under tension in a corrosive environment

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Abstract. In this study we investigate the reliability of a bar subjected to a random tensile load in presence of corrosion. We consider linear, quadratic and exponential models that connect the stress in the bar with the corrosion rate. Two probability densities are considered for the load, with attendant derivation of the time-dependant reliability. The design time of operation is determined utilizing the requirement that the reliability must not be less than the required value.

Keywords: reliability; corrosion; bar; time to failure

1. Introduction

Dolinskii (1967) apparently pioneered analytical corrosion study postulating the linear relationship between corrosion rate and stress. Elishakoff, Ghyselinck and Miglis (2011) generalized Dolinskii's (1967) work by dealing with the durability of an elastic bar under tension to include non-linear relationships between corrosion rate and stress. The goal of this study is to supplement the previous deterministic analysis incorporating the notion of reliability.

Several reliability studies have been conducted in the context of corrosion. Sarveswaran, Smith and Blockley (1998) dealt with interval probability concept, Karimi and Ramachandran (2000) considered the chloride-induced corrosion of reinforcement in concrete structures. Duprat and Sellier (2006) studied corrosion risk due to carbonation using an adaptive response surface method. Stewart and Mullard (2007) studied reliability of reinforced concrete structures exposed to chloride ion attack.

Sarveswaran, Smith and Blockley (1998) stressed that "The remaining capacity of corroded steel structures provides a good example of different aspects of uncertainly. These include: an unknown or partially known extent of damage; variability in loading and an uncertain reserve of structural capacity depending on mode of failure". A careful review of some recent works for reliability assessment of ageing structures was prepared by Val and Stewart (2009).

This work deals with a single aspect of uncertainly, namely the variability in loading. Time-dependant reliability analysis is conducted. Analytical expressions for reliability are derived for

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linear or quadratic relationships between stress and corrosion rate; numerical analysis is shown to be need when the above dependence is of exponential nature.

2. Posing of the problem

Consider a bar whose cross sectional area represents a thin tube, with inner radius $\underline{R}$ and outer radius $\overline{R}$, The bar is subjected to a tensile load Q. The normal stress $\sigma(t)$ in the cross-section reads

$$\sigma(t) = \frac{Q}{\pi[\overline{R}^2(t)-\underline{R}^2]} \tag{1}$$

Outer surface of the bar is in contact with corrosive environment. Therefore, through the corrosion process, the outer radius decreases whereas the inner radius remains constant. We express the outer radius $\overline{R}(t)$ from Eq. (1) as

$$\overline{R}(t) = \sqrt{Q/\pi\sigma(t)+\underline{R}^2} \tag{2}$$

Note that since $\overline{R}$ is a function of time, so is the resulting stress $\sigma(t)$. We assume that the outer radius varies as a function of the corrosion velocity $v(t)$ also referred as the corrosion rate as follows

$$\overline{R}(t) = \overline{R}_0 - \int_0^t v(x)dx \tag{3}$$

where $\overline{R}_0$ is the initial value of the outer radius and t is the time instant when the outer radius is recorded. The problem consists in evaluating the durability of the structure and the associated time-dependant reliability.

3. Linear relationship between corrosion rate and stress

We first consider the simplest possible relationship between corrosion rate and stress.

According to Dolinskii (1967), the corrosion velocity v(t) linearly depends on stress value $\sigma(t)$

$$v(t) = v_0 + m\sigma(t) \tag{4}$$

where m is a coefficient dependant on the material and the corrosive environment. The expression in the right hand side must be equated with value of outer radius given in Eq. (2). The following equation is obtained

$$\overline{R}_0 - v_0 t - \int^t m\sigma(x)dx = \sqrt{Q/\pi\sigma(t)+\underline{R}^2} \tag{5}$$

Differentiation with respect to t yields the first order differential equation with variable coefficients

$$-v_0 - m\sigma(t) = -\frac{Q}{2\sigma^2(t)\pi\sqrt{Q/\pi\sigma(t)+\underline{R}^2}}\frac{d\sigma}{dt} \tag{6}$$

We integrate Eq. (6) to get

$$\frac{Q}{\pi}\int_{\sigma_0}^{\sigma}\frac{d\sigma(t)}{2\sigma^2(t)[v_0+m\sigma(t)]\sqrt{Q/\pi\sigma(t)+\underline{R}^2}} = \int_0^t dt \tag{7}$$

Its solution reads

$$t = \left[\left(-\frac{mQ\sigma(t)\cdot\ln\left(\dfrac{Qv_0 - Qm\sigma(t) + 2v_0\underline{R}_0^2\pi\sigma(t) - 2\sqrt{v_0(v_0\underline{R}_0^2\pi - Qm)}\sqrt{\sigma(t)(Q+\underline{R}_0^2\pi\sigma(t))}}{v_0+m\pi\sigma(t)}\right)}{2v_0\sqrt{v_0\pi(v_0\underline{R}_0^2\pi - Qm)}\sqrt{\sigma(t)(Q+\underline{R}_0^2\pi\sigma(t))}} - 1\right)\sqrt{\frac{Q+\underline{R}_0^2\pi\sigma(t)}{\sigma(t)}}\right]_0^t \tag{8}$$

The durability is obtained by replacing in Eq. (6) t by T and $\sigma(T)$ by σ_y, the yield stress. Thus we postulate that the stress $\sigma(t)$ reaching the yield stress level constitutes the failure of the system

$$\begin{aligned} T = {} & \left(\frac{mQ\sigma_y\cdot\ln\left(-\dfrac{Qv_0 - Qm\sigma_y + 2v_0\underline{R}_0^2\pi\sigma_y - 2\sqrt{v_0(v_0\underline{R}_0^2\pi - Qm)}\sqrt{\sigma_y(Q+\underline{R}_0^2\pi\sigma_y)}}{v_0+m\pi\sigma_y}\right)}{2v_0\sqrt{v_0\pi(v_0\underline{R}_0^2\pi - Qm)}\sqrt{\sigma_y(Q+\underline{R}_0^2\pi\sigma_y)}} - 1\right)\sqrt{\frac{Q+\underline{R}_0^2\pi\sigma_y}{\sigma_y}} \\ & - \left(\frac{mQ\sigma_y\cdot\ln\left(-\dfrac{Qv_0 - Qm\sigma_y + 2v_0\underline{R}_0^2\pi\sigma_0 - 2\sqrt{v_0(v_0\underline{R}_0^2\pi - Qm)}\sqrt{\sigma_0(Q+\underline{R}_0^2\pi\sigma_0)}}{v_0+m\pi\sigma_0}\right)}{2v_0\sqrt{v_0\pi(v_0\underline{R}_0^2\pi - Qm)}\sqrt{\sigma_y(Q+\underline{R}_0^2\pi\sigma_0)}} - 1\right)\sqrt{\frac{Q+\underline{R}_0^2\pi\sigma_0}{\sigma_0}} \end{aligned} \tag{9}$$

The initial stress σ_0 in Eqs. (8) and (9), is a function of Q

$$\sigma_0 = \frac{Q}{\pi(\bar{R}_0^2 - \underline{R}^2)} \tag{10}$$

The substitution of Eq. (8) into Eq. (7) yields to T as a function of load Q and other constant parameters v_0, σ_y, $\bar{R}_0$, $\underline{R}_0$, m.

Fig. 1 represents a plot of durability as a function of Q, other parameters being set at v_0 =

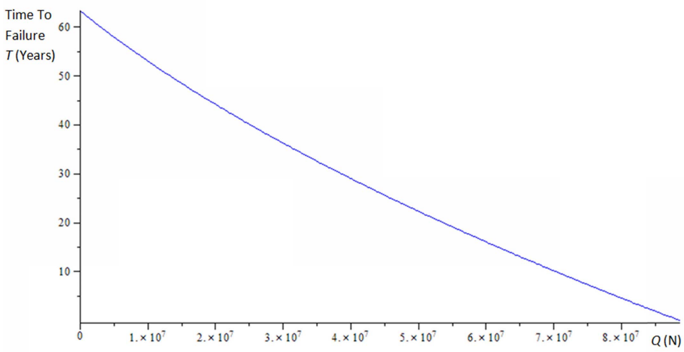

Fig. 1 Durability of the bar in years versus load Q

$10^{-10}\mathrm{m}^2 \cdot \mathrm{N} \cdot$ s-1. $\sigma_y = 2.35 \cdot 103\, P_\alpha$, $\underline{R} = 0.2$ m, $\overline{R}_0 = 0.4$ m, $m = 10^{-20}\,\mathrm{m}^2 \cdot N^{-1} \cdot s^{-1}$.

We observe that $T = 0$ is obtained for a critical value of the load Q_{cr}

$$Q_{cr} = \sigma_y \cdot \pi \cdot (\overline{R}_0^2 - \underline{R}^2) \tag{11}$$

This means that if the initial stress σ_0 equals the yield stress σ_y the structure fails at delivery. In this case, for the set parameters, we have $Q_{cr} \approx 8.86 \cdot 10^7 N$.

4. Reliability of the bar

Let us consider a realistic situation in which the load Q constitutes a random variable with given probability density $f_Q(q)$. We consider two different cases for the random variable Q. First one is the simplest possible case in which Q has a continuous uniform density, namely

$$f_Q(q) = \begin{cases} \dfrac{1}{q_{\max} - q_{\min}} & \text{for } q_{\min} \le q \le q_{\max} \\ 0 & \text{for } q < q_{\min} \text{ or } q > q_{\max} \end{cases} \tag{12}$$

The cumulative distribution function $F_Q(q)$ of the load is

$$F_Q(q) = \begin{cases} 0 & \text{for } q < q_{\min} \\ \dfrac{q - q_{\min}}{q_{\max} - q_{\min}} & \text{for } q_{\min} \le q < q_{\max} \\ 1 & \text{for } q \ge q_{\max} \end{cases} \tag{13}$$

Fig. 2 reproduces for the time to failure T versus the load q superimposed with the probability density function $f_Q(q)$ of Q. We recall that our goal is to determine the reliability of a structure, namely the probability that its time to failure T is not less than a specified value t_α.

Second case is when the random variable Q has a triangular density, namely

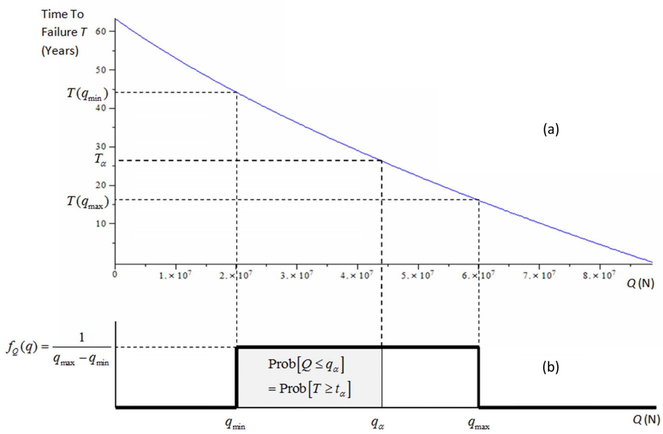

Fig. 2 (a) Time to failure t versus load Q and (b) probability density function of Q

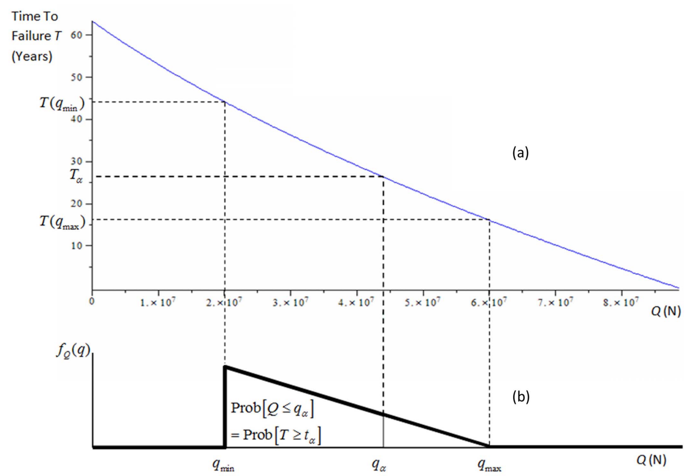

Fig. 3 (a) Time to failure T versus load Q and (b) probability density function of Q

$$f_Q(q) = \begin{cases} -\dfrac{2}{(q_{\max}-q_{\min})}Q + \dfrac{2q_{\max}}{(q_{\max}-q_{\min})} & \text{for } q_{\min} \le q < q_{\max} \\ 0 & \text{for } q \le q_{\min} \text{ or } q \ge q_{min} \end{cases} \tag{14}$$

The cumulative distribution function $F_Q(q)$ of the load is then

$$F_Q(q) = \begin{cases} 0 & \text{for } q < q_{\min} \\ -\dfrac{Q - q_{\max}Q + 2q_{\max}q_{\min} - q_{\min}}{(q_{\max}-q_{\min})} & \text{for } q_{\min} \le q < q_{\max}, \\ 1 & \text{for } q \ge q_{\max} \end{cases} \tag{15}$$

Fig. 3 reproduces for the time to failure T versus the load Q superimposed with the probability density function $f_Q(q)$ of Q.

We observe from these figures that in the range $q_{\min} \leq q \leq q_{max}$, in order inequality $T \geq t_{\alpha}$ to hold, the load q should not be greater than value q_{α} found from equation

$$T(q_{\alpha}) = t_{\alpha} \tag{16}$$

Thus, we get

$$Prob[Q \leq q_{\alpha}] = Prob[T \geq t_{\alpha}] = R(t_{\alpha}) \tag{17}$$

Moreover, two particular values for the reliability are obtained

$$\begin{aligned} Prob[T \geq t(q_{\min})] &= R[t(q_{\min})] = 0 \\ Prob[T \geq t(q_{\max})] &= R[t(q_{\max})] = 1 \end{aligned} \tag{18}$$

where $t(q_{\min})$ is obtained from Eq. (9) by substituting $q = q_{\min}$. Likewise $t(q_{\max})$ is derived from Eq. (9) by substituting $q = q_{\max}$.

Eq. (18) establishes limiting values of reliability. To find the analytical expression of $R(t) \equiv R$ for load q in the range $q_{\min} \leq q \leq q_{\max}$, we act as follows: We pick a specified value q in this range. The reliability is determined as the shaded area in Fig. 2 and equals the probability distribution function $F_Q(q)$

$$R = Prob[Q \leq q] = F_Q(q) = \frac{q - q_{\min}}{q_{\max} - q_{\min}} \tag{19}$$

The load value q can be thus related with the reliability as follows

$$q = (q_{\max} - q_{\min})R + q_{\min} \tag{20}$$

We do the same for the triangular density and get the following relationship between q and R

$$q = -\sqrt{1-R}(q_{\max} - q_{\min}) + q_{\max} \tag{21}$$

Now our goal is to establish the function $R = R(t)$. Since it appears unfeasible to express the inverse function $Q = Q(t)$ from Eq. (10), we will find t as a function of R rather than R as a function of t. Since R equals the probability that $Q \leq q_r$, we substitute Eq. (20) in view of Eq. (10) into Eq. (9) and derive $T = T(q)$. Thus, the reliability level R is achieved at t equal to T. Then T becomes a function of R, where R is the probability that $T \geq t$.

$$\begin{aligned} t = {} & \sqrt{\frac{[(q_{\max}-q_{\min})R+q_{\min}]+\underline{R}_0^2\pi\sigma_y}{\sigma_y}}\left[-1+\frac{m[(q_{\max}-q_{\min})R+q_{\min}]\sigma_y}{2v_0\sqrt{v_0\pi(v_0\underline{R}_0^2\pi-[(q_{\max}-q_{\min})R+q_{\min}]m)}\sqrt{\sigma_y([(q_{\max}-q_{\min})R+q_{\min}]+\underline{R}_0^2\pi\sigma_y)}}\right. \cdot \\ & -\frac{[(q_{\max}-q_{\min})R+q_{\min}]v_0-[(q_{\max}-q_{\min})R+q_{\min}]m\sigma_y+2v_0\underline{R}_0^2\pi\sigma_y-2\sqrt{v_0(v_0\underline{R}_0^2\pi-[(q_{\max}-q_{\min})R+q_{\min}]m)}\sqrt{\sigma_y([(q_{\max}-q_{\min})R+q_{\min}]+\underline{R}_0^2\pi\sigma_y)}}{v_0+m\sigma_y} \\ & -\sqrt{\frac{[(q_{\max}-q_{\min})R+q_{\min}]+\underline{R}_0^2\pi\sigma_y}{\sigma_0}}\left[-1+\frac{m[(q_{\max}-q_{\min})R+q_{\min}]\sigma_0}{2v_0\sqrt{v_0\pi-((q_{\max}-q_{\min})R+q_{\min})m}\sqrt{\sigma_0([(q_{\max}-q_{\min})R+q_{\min}]+\underline{R}_0^2\pi\sigma_0)}}\right. \cdot \\ & \left.\ln\left(-\frac{[(q_{\max}-q_{\min})R+q_{\min}]v_0-[(q_{\max}-q_{\min})R+q_{\min}]m\sigma_0+2v_0\underline{R}_0^2\pi\sigma_0-2\sqrt{v_0(v_0\underline{R}_0^2\pi-[(q_{\max}-q_{\min})R+q_{\min}]m)}\sqrt{\sigma_0([(q_{\max}-q_{\min})R+q_{\min}]+\underline{R}_0^2\pi\sigma_0)}}{v_0+m\sigma_0}\right)\right] \end{aligned} \tag{22}$$

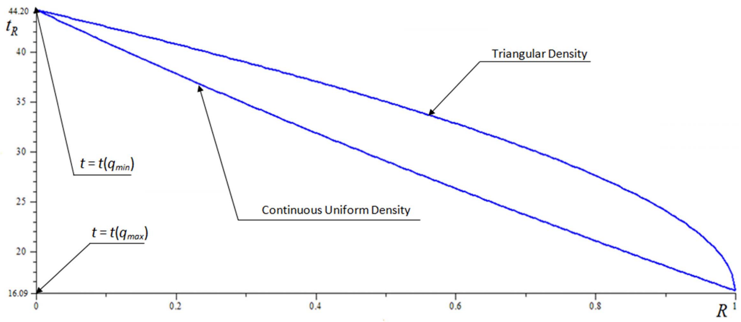

Fig. 4 Variation of time instant t as a function of reliability R in the range $q_{min} \leq q \leq q_{max}$

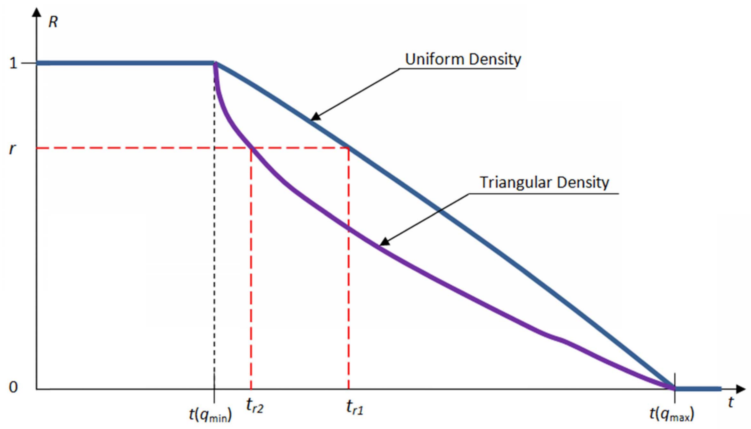

Fig. 5 Reliability of the structure versus time

Fig. 4 represents a plot of time instant t as a function of R, with the parameters set as in the Fig. 1.

Once reliability R is derived (Fig. 5) we are interested in designing the structure. Design requirement consists in demanding that reliability is not less than a required, specified value r. The condition

$$R(t) \geq r \tag{23}$$

results in the design operation time t_r such that

$$R((t_r) \equiv r) \tag{24}$$

This means that if the structure is exploited not above time interval $[0, t_r]$, reliability value will not be less than required reliability r.

5. Quadratic relationship between corrosion rate and stress

An additional relationship between the corrosion velocity $v(t)$ and the stress $\sigma(t)$ can be considered. Indeed, the linear relationship is the simplest one; it cannot exhaust all possible relationships between the stress and the corrosion rate. Now we consider a quadratic relationship. Thus, we express the corrosion rate as follows

$$v(t) = v_0 + m\sigma(t) + n\sigma^2(t) \tag{25}$$

Introducing Eq. (25) into Eq. (3) yields

$$\overline{R}_0 - v_0 t - \int_0^t (m\sigma(x) + n\sigma^2(x))dx = \sqrt{Q/\pi\sigma(t) + \underline{R}^2} \tag{26}$$

Differentiating the resulting expression leads to

$$\frac{Q}{\pi}\int^t \frac{d\sigma(t)}{[v_0 + m\sigma(t) + n\sigma^2(t)]2\sigma^2(t)\sqrt{Q/\pi\sigma(t) + \underline{R}^2}} = \int_0^t dt \tag{27}$$

We convert $v_0 + m\sigma(t) + n\sigma^2(t)$ into a partial fraction

$$\frac{1}{v_0 + m\sigma(t) + m\sigma^2(t)} = \frac{1}{n[\sigma(t) - s_1](s_1 - s_2))} + \frac{1}{n[\sigma(t) - s_2](s_2 - s_1))} \tag{28}$$

with s_1 and s_2

$$s_1 = \frac{-m + \sqrt{m^2 - 4v_0 n}}{2n}, \quad s_2 = \frac{-m - \sqrt{m^2 - 4v_0 n}}{2n} \tag{29}$$

Then we rewrite Eq. (27) using Eq. (29)

$$\frac{Q}{\pi}\int_{\sigma_0}^{\sigma} \frac{d\sigma(t)}{2\sigma^2(t)\sqrt{Q/\pi\sigma(t) + \underline{R}^2}}\left[\frac{1}{n[\sigma(t) - s_1](s_1 - s_2)} + \frac{1}{n[\sigma(t) - s_2](s_2 - s_1)}\right] = \int_0^t dt \tag{30}$$

The dependence between the time and the stress reads

$$\begin{aligned} t = \sum_{i=1}^{2} \frac{1}{Qs_i^3 n\pi\underline{R}\left(\frac{{}^{S}\omega(i+1)}{{}^{S}\omega(1)} - 1\right)\left(1 + \frac{Q}{\pi\underline{R}^2 S_j}\right)} & \Bigg[-Q\pi\underline{R}^2\left[\sigma\left(1 + \frac{Q}{\pi\underline{R}^2\sigma}\right)^{3/2} - \sigma_0\left(1 + \frac{Q}{\pi\underline{R}^2\sigma}\right)^{3/2} - \pi^2\underline{R}^4 S_i\left[\sigma\left(1 + \frac{Q}{\pi\underline{R}^2\sigma}\right)^{3/2} - \sigma_0\left(1 + \frac{Q}{\pi\underline{R}^2\sigma}\right)^{3/2}\right]\right. \\ & + Q\underline{R}^2\pi\left[\sigma\sqrt{1 + \frac{Q}{\pi\underline{R}^2\sigma}} - \sigma_0\sqrt{1 + \frac{Q}{\pi\underline{R}^2\sigma_0}}\right] + \pi^2\underline{R}^4 s_i\left[\sigma\sqrt{1 + \frac{Q}{\pi\underline{R}^2\sigma}} - \sigma_0\sqrt{1 + \frac{Q}{\pi\underline{R}^2\sigma_0}}\right] \\ & + \frac{1}{2}Q^2\sqrt{1 + \frac{Q}{s_j\pi\underline{R}^2}}\ln\left(\frac{\left(\frac{\sigma_0}{s_i} - 1\right)\left[1 + \frac{\sigma_y}{s_j} + \frac{2\underline{R}^2 y}{Q} + 2\pi\underline{R}^2 - \frac{y}{Q}\sqrt{1 + \frac{Q}{s_j\pi\underline{R}^2}}\sqrt{1 + \frac{Q}{\pi\underline{R}^2\sigma}}\right]}{\left(\frac{\sigma_y}{s_i} - 1\right)\left[1 + \frac{\sigma_0}{s_i} + \frac{2\underline{R}^2 o}{Q} + 2\pi\underline{R}^2 - \frac{y}{Q}\sqrt{1 + \frac{Q}{s_j\pi\underline{R}^2}}\sqrt{1 + \frac{Q}{\pi\underline{R}^2\sigma_0}}\right]}\right)\Bigg] \end{aligned} \tag{31}$$

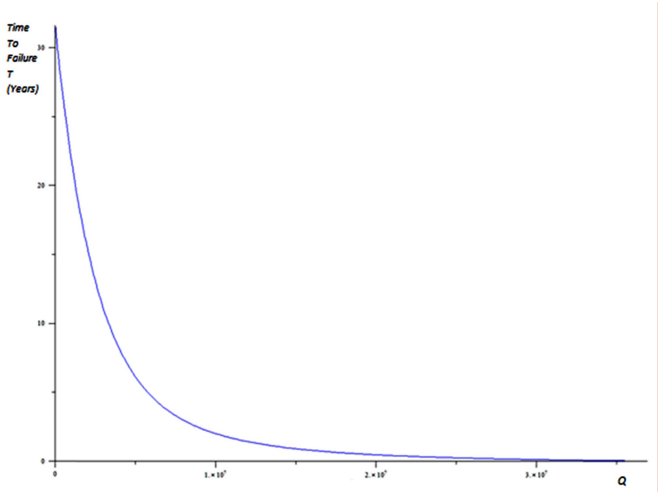

Fig. 6 Time to failure T versus load Q

with the permutation function ω defined as

$$\omega = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$$

The permutation function is function which associates the values of the front row line to his image in the second row line, so that we have $s_{\omega(1)} = s_1$, $s_{\omega(2)} = s_1$, $s_{\omega(3)} = s_1$.

The durability is then obtained by replacing in Eq. (28) t by T and $\sigma(T)$ by σ_y, the yield stress

$$\begin{aligned} T = \sum_{i=1}^{2} & \frac{1}{Q s_i^3 n \pi \underline{R}\left(\frac{s_{\omega(i+1)}}{s_{\omega(i)}} - 1\right)\left(1 + \frac{Q}{\pi \underline{R}^2 s_i}\right)} \Bigg[-Q\pi\underline{R}^2 \left[\sigma\left(1+\frac{Q}{\pi\underline{R}^2\sigma_y}\right)^{3/2} - \sigma_0\left(1+\frac{Q}{\pi\underline{R}^2\sigma_0}\right)^{3/2} \right] - \pi^2\underline{R}^4 s_i \left[\sigma\left(1+\frac{Q}{\pi\underline{R}^2\sigma_y}\right)^{3/2} - \sigma_0\left(1+\frac{Q}{\pi\underline{R}^2\sigma_0}\right)^{3/2} \right] \\ & + Q\underline{R}^2\pi\left[\sigma_y\sqrt{1+\frac{Q}{\pi\underline{R}^2\sigma_y}} - \sigma_0\sqrt{1+\frac{Q}{\pi\underline{R}^2\sigma_0}}\right] + \pi^2\underline{R}^4 s_j\left[\sigma_y\sqrt{1+\frac{Q}{\pi\underline{R}^2\sigma_y}} - \sigma_0\sqrt{1+\frac{Q}{\pi\underline{R}^2\sigma_0}}\right] \\ & + \frac{1}{2}Q^2\sqrt{1+\frac{Q}{s_j\pi\underline{R}^2}}\ln\left(\frac{\left(\frac{\sigma_0}{s_j}-1\right)\left[1+\frac{\sigma_y}{s_j}+\frac{2\pi\underline{R}^2\sigma_y}{Q}+2\pi\underline{R}^2\frac{\sigma_y}{Q}\sqrt{1+\frac{Q}{s_j\pi\underline{R}^2}}\sqrt{1+\frac{Q}{s_j\pi\underline{R}^2\sigma_y}}\right]}{\left(\frac{\sigma_y}{s_j}-1\right)\left[1+\frac{\sigma_0}{s_j}+\frac{\underline{R}^2\sigma_0}{Q}+2\pi\underline{R}^2\frac{\sigma_0}{Q}\sqrt{1+\frac{Q}{s_j\pi\underline{R}^2}}\sqrt{1+\frac{Q}{s_j\pi\underline{R}^2\sigma_0}}\right]} \right) \end{aligned} \tag{32}$$

Fig. 6 represents a plot of durability as a function of Q, using parameters adopted for the linear model, and taking for the coefficient n in Eq. (29) value $n = 10^{-25} \cdot m^5 \cdot N^{-2} \cdot s^{-1}$.

6. Reliability in nonlinear σ(t) – v(t) Setting

As in chapter 3, Q constitutes a random variable with given probability density. Eqs. (14), (15) and (18) remain the same. Figs. 7, 8 and 9 are plots of t versus R for three different values of n.

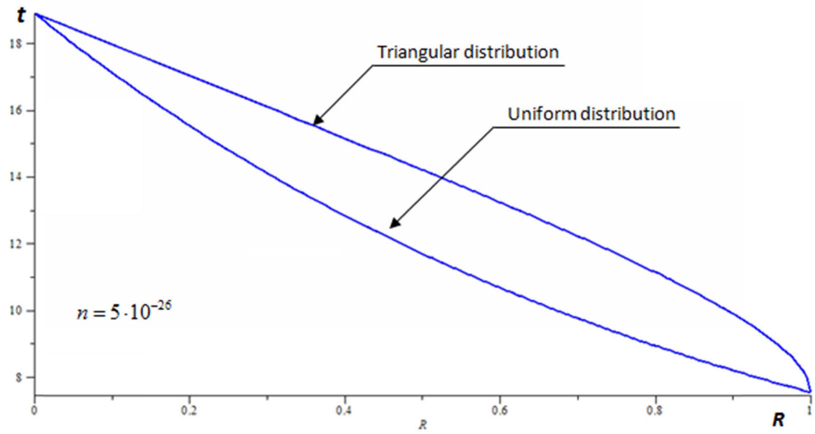

Fig. 7 ariation of time instant t as a function of reliability R in the range $q_{min} \le q \le q_{max}$ for $n = 5\cdot10^{-26}$

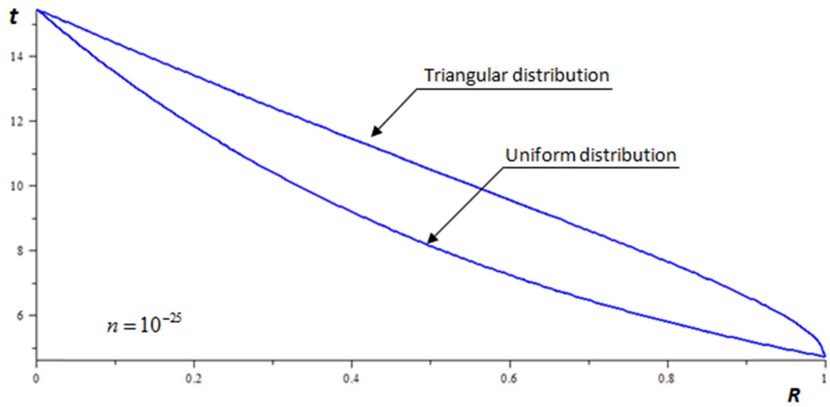

Fig. 8 Variation of time instant t as a function of reliability R in the Range $q_{min} \le q \le q_{max}$ for $n = 10^{-56}$

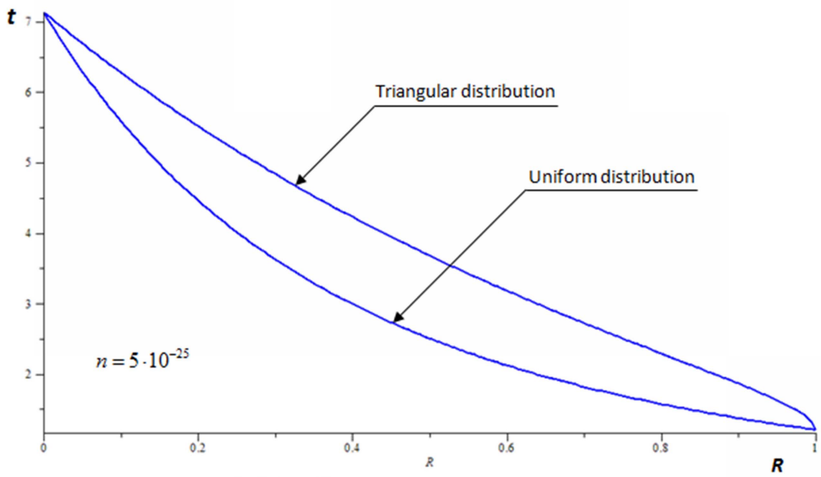

Fig. 9 Variation of time instant t as a function of reliability R in the range $q_{min} \le q \le q_{max}$ for $n = 5\cdot10^{-55}$

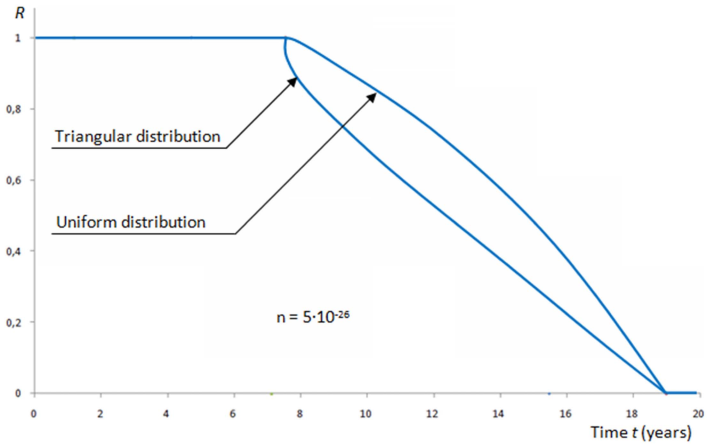

Fig. 10 Reliability of the structure versus time for $n = 5{\cdot}10^{-26}$

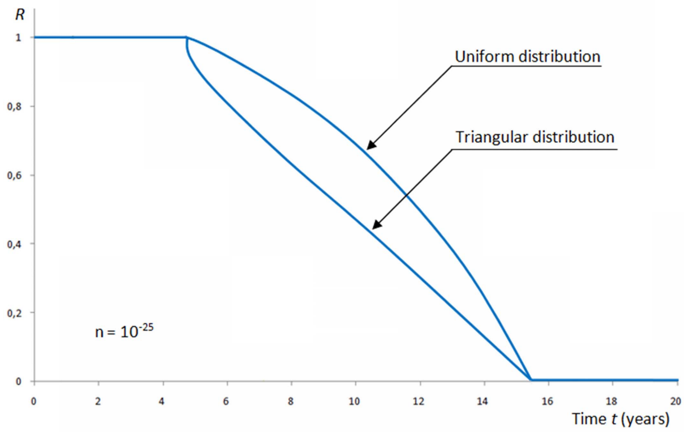

Fig. 11 Reliability of the structure versus time for $n = 5{\cdot}10^{-25}$

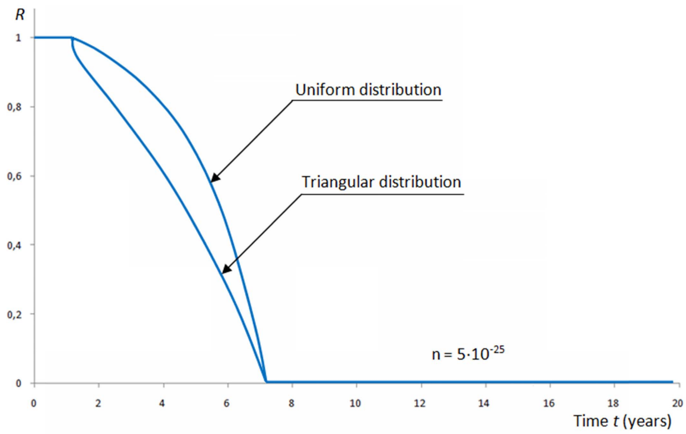

Fig. 12 Reliability of the structure versus time for $n = 5{\cdot}10^{-25}$

7. Exponential relationship between corrosion rate and stress

We now consider an exponential stress corrosion model for the tube as suggested by Gutman, Zainullin and Zaripov (1984) in deterministic setting

We recall that the tensile stress reads

$$\sigma(t) = \frac{Q}{\pi(\overline{R}(t) - \underline{R})} \tag{33}$$

We differentiate stress σ with respect to time t in Eq. (33) and get

$$\frac{d\sigma}{dt} = \frac{-2Q\overline{R}t}{\pi(\overline{R}(t) - \underline{R})} \cdot \frac{d\overline{R}}{dt} \tag{34}$$

At the initial time the outer radius equals $\overline{R}_0$. The stress level at $t = 0$ equals

$$\sigma_0 = \frac{Q}{\pi(\overline{R}_0^2 - \underline{R}^2)} \tag{35}$$

In view of the fact that $(\overline{R}_0 - \underline{R})\sigma_0 = Q/\pi$, Eq. (35) can be rewritten as

$$\frac{d\sigma}{dt} = \frac{-2\overline{R}(t) \cdot (\overline{R}_0 - \underline{R})}{(\overline{R}_0^2 - \underline{R}^2)^2} \cdot \frac{d\overline{R}}{dt} \tag{36}$$

From Eq. (1) we express $(\overline{R}(t)^2 - \underline{R}^2) = Q^2/\pi^2 \sigma(t)^2$. Hence the derivation of stress becomes

$$\frac{d\sigma}{dt} = -\left(\frac{2\overline{R}(t)\sigma_0(\overline{R}_0^2 - \underline{R}^2)\sigma^2\pi^2}{Q}\frac{d\overline{R}}{dt}\right) \tag{37}$$

Eq. (1) gives the following equality $(\overline{R}_0 - \underline{R})\sigma_0\pi/Q = 1/\sigma_0(\overline{R}_0 - \underline{R})$ that we substitute in Eq. (36) to get

$$\frac{d\sigma}{dt} = \frac{-2\sigma\overline{R}t}{\sigma_0(\overline{R}_0^2 - \underline{R}^2)} \cdot \frac{d\overline{R}}{dt} \tag{38}$$

According to Gutman, Zainullin and Zaripov (1984) the variation of the outer radius can be described as a sum

$$\frac{d\overline{R}}{dt} = \left(\frac{d\overline{R}}{dt}\right)_1 + \left(\frac{d\overline{R}}{dt}\right)_2 \tag{39}$$

The term $(d\overline{R}/dt)_1$ is associated with the Poisson's effect, and the deformation in transverse direction reads

$$\varepsilon_y = \frac{\overline{R}(t) - \overline{R}_0}{\overline{R}_0 - \underline{R}} \tag{40}$$

Using Poisson's equation $\varepsilon_y = -\mu \cdot \varepsilon_x$ with $\varepsilon_x = \sigma(t)/E$ we get

$$\frac{\overline{R}(t)-\overline{R}_0}{\overline{R}_0-\underline{R}} = -\mu\frac{\sigma(t)}{E} \tag{41}$$

where μ is the Poisson's ration. Differentiating Eq. (41) with respect to the time yields

$$\left(\frac{d\overline{R}}{dt}\right)_1 = -\frac{\mu}{E}(\overline{R}_0-\underline{R})\frac{d\sigma}{dt} \tag{42}$$

The second term is postulated by Gutman, Zainullin and Zuripov as

$$\left(\frac{d\overline{R}}{dt}\right)_2 = -v_0 exp\left(\frac{\sigma(t)V_m}{3RT}\right) \tag{43}$$

where V_{m} is the molar volume of the metal, $R \cong 8.31 J\cdot(mol\cdot K)^{-1}$ the universal gas constant, T the absolute temperature in K. Introducing Eqs. (42) and (43) into Eq. (38) yields

$$\frac{d\sigma}{dt} = \frac{2\sigma(t)\overline{R}t(\overline{R}_0-\underline{R})}{\sigma_0(\overline{R}_0^2-\underline{R}^2)}\cdot\frac{\mu}{E}\frac{d\sigma}{dt}+\frac{2\sigma(t)\overline{R}t}{\sigma_0(\overline{R}_0^2-\underline{R}^2)}\cdot v_0 exp\left(\frac{\sigma(t)V_m}{3RT}\right) \tag{44}$$

In view of the identity $\overline{R} = \sqrt{Q/\pi\sigma(t)+\underline{R}^2}$ we can write Eq. (44) as follows

$$\frac{d\sigma}{dt} = \frac{2\sigma(t)\sqrt{Q\pi\sigma(t)+\underline{R}^2}}{\sigma_0(\overline{R}_0^2-\underline{R}^2)}\left[\frac{\mu(\overline{R}_0-\underline{R})}{E}\frac{d\sigma}{dt}+v_0 exp\left(\frac{\sigma(t)V_m}{3RT}\right)\right] \tag{45}$$

or after some algebra

$$\frac{d\sigma}{dt}\left[1-\frac{2\sigma(t)\sqrt{Q\pi\sigma t+\underline{R}^2}}{\sigma_0(\overline{R}_0^2-\underline{R}^2)}\frac{\mu(\overline{R}_0-\underline{R})}{E}\right] = \frac{2\sigma(t)\sqrt{Q\ \pi\sigma(t)+\underline{R}^2}}{\sigma_0(\overline{R}_0^2-\underline{R}^2)}v_0 exp\left(\frac{\sigma(t)V_m}{3RT}\right) \tag{46}$$

Eq. (46) is then rewritten as follows

$$d\sigma\left[\frac{\sigma_0\sqrt{\pi}(\overline{R}_0^2-\underline{R}^2)\sigma(t)^{-\frac{3}{2}}}{2v_0\sqrt{Q+\underline{R}^2\pi\sigma(t)}}exp\left(-\frac{\sigma(t)V_m}{3RT}\right)-\frac{\mu(\overline{R}_0-\underline{R})}{v_0E}exp\left(-\frac{\sigma(t)V_m}{3RT}\right)\right] = dt \tag{47}$$

We integrate this function between σ_0 to σ_{y} for the stress and from 0 to t for the time

$$\int_{\sigma_0}^{\sigma}\frac{\sigma_0\sqrt{\pi}(\overline{R}_0^2-\underline{R}^2)\sigma(t)^{-\frac{3}{2}}}{2v_0\sqrt{Q+\underline{R}^2\pi\sigma(t)}}exp\left(-\frac{\sigma(t)V_m}{3RT}\right)d\sigma-\int_{\sigma_0}^{\sigma}\frac{\mu(\overline{R}_0-\underline{R})}{v_0E}\exp\left(-\frac{\sigma(t)V_m}{3RT}\right) = \int_0^t dt \tag{48}$$

We substitute σ by σ_y and t by T_f in Eq. (48) to get the durability of the thin tube

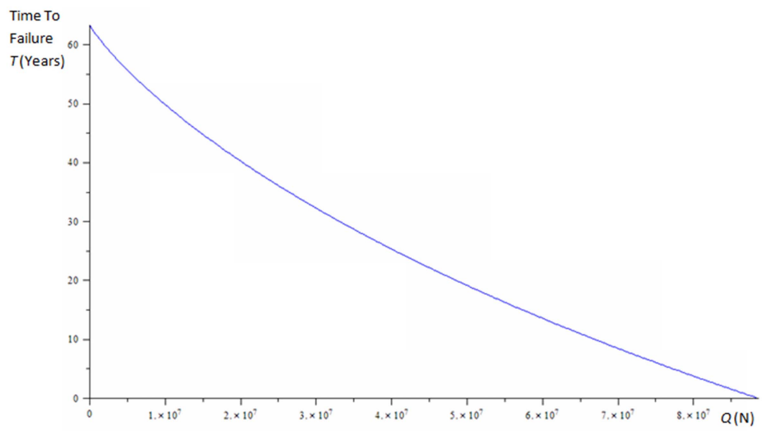

Fig. 13 Durability of the bar in years versus load Q

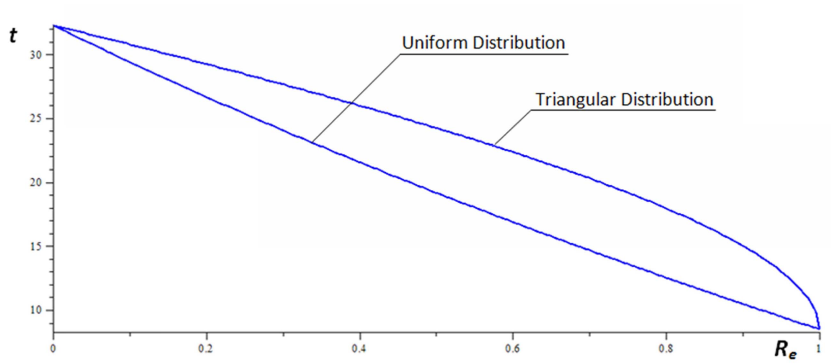

Fig. 14 Time t as a function of the reliability R_e

$$\int_{\sigma_0}^{\sigma_y} \frac{\sigma_0\sqrt{\pi}(\overline{R}_0^2-\underline{R}^2)\sigma(t)^{-\frac{3}{2}}}{2v_0\sqrt{Q+\underline{R}^2\pi\sigma(t)}} exp\left(-\frac{\sigma(t)V_m}{3RT}\right)d\sigma - \int_{\sigma_0}^{\sigma_y} \frac{\mu(\overline{R}_0-\underline{R})}{v_0 E} exp\left(-\frac{\sigma(t)V_m}{3RT}\right) = T_f \tag{49}$$

Fig. 13 represents a plot of the durability T_f as a function of the load Q, using parameters adopted for the linear and quadratic models.

8. Reliability for exponential dependence σ(t) – v(t) setting

As in sections 4 and 6, the load Q is now treating as constituting a random variable with given probability density. Thus, we plot the time as a function of the reliability R_e using Eqs. (20) and (21).

Fig. 14 is a plot of t versus R_e for the parameters used to plot Fig. 13.

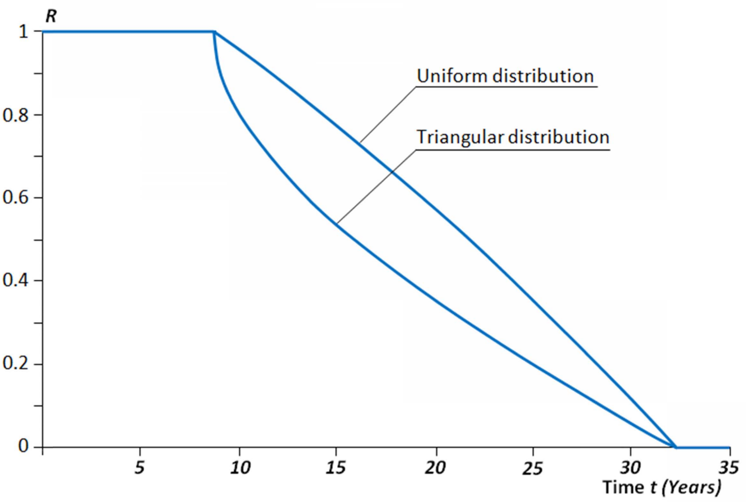

Fig. 15 Reliability R of the Structure versus Time t

9. Conclusions

Apparently for the first time in literature the time-dependant reliability was derived for structures in corrosive environment. Both theoretical and numerical analyses were conducted. Time of safe operation was derived as the time interval in which reliability equals or in excess the codified value.

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