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Analysis of interfacial stresses of the reinforced concrete foundation beams repairing with composite materials plate

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Abstract. This paper presents a careful theoretical investigation into interfacial stresses in reinforced concrete foundation beam repairing with composite plate. The essential issue in the analysis of reinforced structures with composite materials is to understand the individual behaviour of each material and its interaction with the remaining ones. The present model is based on equilibrium and deformations compatibility requirements in and all parts of the repaired RC foundation beam, i.e., the reinforced concrete foundation beam, the composite plate and the adhesive layer. The theoretical predictions are compared with other existing solutions, By comparisons between the existing solutions and the present new solution enable a clear appreciation of the effects of various parameters such as the geometric characteristics and mechanical properties of the components of the repaired beam, as well as the geotechnical stresses of the soil are considered. This research is helpful for the understanding on mechanical behaviour of the interface and design of the composite-concrete hybrid structures.

Keywords: RC foundations beam; interfacial stresses; repair of structures; composite plates

1. Introduction

The structural plating of reinforced concrete beams with fibre reinforced polymer (FRP) laminates represents a new technology in the civil engineering field. In fact, until today, the most common method in structural strengthening has been the Hermitian technique using the application of steel plates. The present paper is devoted to understand the mechanism of debonding failure mode in reinforced concrete foundation beam repairing with composite plate. This brittle mode of failure is a result of the high shear and vertical normal (peeling) stress concentrations arising at the edges of the bonded FRP strip. Accurate predictions of such stresses are thus important for designing against debonding failures.

In the foundation construction of structures the most used solution are isolated bases under columns of structures. Nevertheless, based on surface foundation calculation codes, when two bases are relatively close one another, or when reducing differential settlement for them is needed, the beam foundations which are essential and are commonly used to transmit the stress to the ground. In this context, the civil engineering calculator is required to comply with the calculation codes based on the following three aspects, namely resistance conditions, stability phenomena and the economic aspect which is very important. In that case, the studying of the foundation beams is

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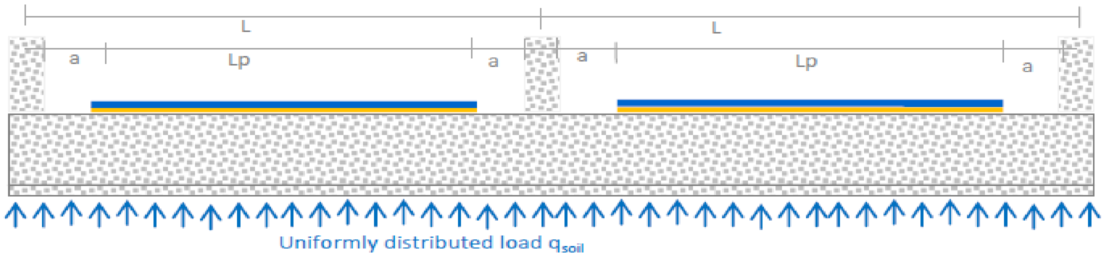
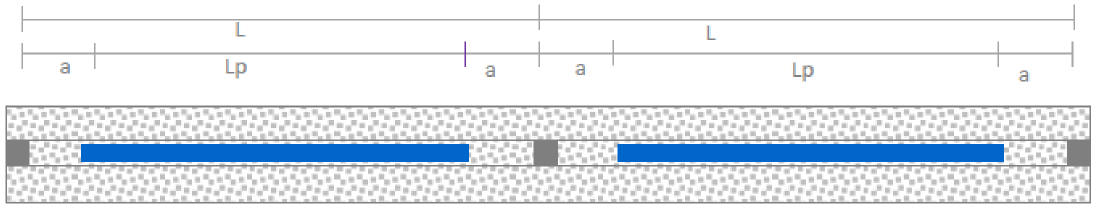
important from different points of view. In the first place, from the engineering point of view, is important because the task of foundations is to transmit the loads of structures to the ground, so that any failure in the foundations affects the rest of the structure directly. Secondly, from the economic standpoint, it is important to achieve an efficient design with the minimum cost. In the third place, from the structural safety point of view, as any failure represents a risk to people and property.

Composite material made from two or more immiscible components together produce material with properties that exceed the individual components. Experts and researchers determined various solutions to deal with the crucial problem of structures aging caused by different loads, and they have been using new composite materials, fiber-reinforced polymers (FRPs), which have several practical applications. The materials currently used for both repair and enhancement of reinforced concrete structures in civil engineering, are composite materials. This type of external reinforcement, in the form of plate, has successfully replaced the steel sheets since fiber composites have good tensile strength and light weight, are easy to handle and have good resistance to external agents and fatigue. In addition no special tools are required for their placement so that the work can be done quickly and easily. The FRP composites have been mainly used to reinforce concrete beams; extensive research has shown that the bonding between an FRP plate to the face of concrete beam can effectively develop its capability force and ultimate strength (Yang *et al*. 2007, Yeghnem *et al*. 2019, Benhenni *et al*. 2019, Benferhat *et al*. 2016, Adim *et al*. 2018, Rabhi *et al*. 2020, Rahmani *et al*. 2020, Refrafi *et al*. 2020, Sahla *et al*. 2019, Ammar *et al*. 2020, Tahar *et al*. 2008, Belkacem *et al*. 2016b, Bensattalah *et al*. 2018, Matouk *et al*. 2020, Rabahi *et al*. 2019, Menasria *et al*. 2020, Shariati *et al*. 2020, Zine *et al*. 2020, Bekki *et al*. 2019); these types of reinforced models have been able to reduce the influence of the interfacial stresses effect, which are affected by the phenomenon of solicitations, and the success of this technique is due to the effect of the transferred interfacial stresses (Tounsi 2006, Guenaneche *et al*. 2014, Al-Furjan *et al*. 2020b, Alimirzaei *et al*. 2019, Hamrat *et al*. 2020,Balubaid *et al*. 2019, Belbachir *et al*. 2020, Hassaine Daouadji *et al*. 2016, Benhenni *et al*. 2018, Daouadji *et al*. 2016b, Belkacem *et al*. 2016c, Panjehpour *et al*. 2014, Smith and Teng 2002) from the beam to the externally bonding FRP.

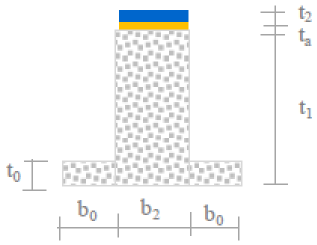
The present paper is devoted to understanding the mechanism of debonding failure mode and the development of sound design rules. This brittle mode of failure is a result of the high shear and vertical normal (peeling) stress concentrations arising at the edges of the bonded composite strip (Benyoucef *et al*. 2007, Tahar *et al*. 2016, Tayeb *et al*. 2020, Abualnour *et al*. 2019, Addou *et al*. 2019, Khiloun *et al*. 2020, Belkacem *et al*. 2016a, Adim *et al*. 2016, Daouadji *et al*. 2016a, Medani *et al*. 2019, Al-Furjan *et al*. 2020a, Hassaine Daouadji *et al*. 2019, Benferhat *et al*. 2018, Tounsi *et al*. 2020, Zidour *et al*. 2020, Tounsi *et al*. 2008, Rabahi 2016, El Mahi *et al*. 2014). Hence, this limited area in the close vicinity of the bonded strip edge, subjected to high peeling and interfacial shear stresses, proves to be among the most critical parts of the strengthened beams. Consequently, the determination of interfacial stresses has been researched for the last decade for beams bonded with either steel or advanced composite materials. In particular, several closed-form analytical solutions have been developed (Ait Atmane *et al*. 2015, Alfarabi *et al*. 2020, Amara *et al*. 2019, Hassaine Daouadji 2017, Bourada *et al*. 2020, Chaabane *et al*. 2019, Chikr *et al*. 2020, Draiche *et al*. 2019, Draoui *et al*. 2019, Hussain *et al*. 2020, Kaddari *et al*. 2020, Krour *et al*. 2014, Rabia *et al*. 2016, Chaded *et al*. 2018, Abdederak *et al*. 2018, Abdelhak *et al*. 2016, Mohammadimehr *et al*. 2020, Panjehpour *et al*. 2016, Hassaine Daouadji 2013, Belbachir *et al*. 2019, Boutaleb *et al*. 2019, Bousahla *et al*. 2020, Boussoula *et al*. 2020, Bensattalah *et al*. 2016, Chergui *et al*. 2019, Mohamed Amine *et al*. 2019, Rabia *et al*. 2019, Bourada *et al* 2019). All these solutions are for linear elastic materials and employ the same key assumption that the adhesive is subject to normal and shear



(a) Longitudinal section



(b) Plan view



(c) Cross section

Fig. 1 Reinforced concrete foundation beam repairing with composite plate

stresses that are constant across the thickness of the adhesive layer. It is this key assumption that enables relatively simple closed-form solutions to be obtained. Recently, Tounsi (2006) have developed some other methods based also on the deformation compatibility approach to predict the interfacial stresses in FRP-RC hybrid foundation beams. The main objective of the present study is to analyze the interfacial stresses in reinforced concrete foundation beam repaired with composite plate. The simple approximate closed-form solutions discussed in this paper provide a useful but simple tool for understanding the interfacial behaviour of an externally repaired with composite plate. This paper is focused on the possibility of repairing in reinforced concrete foundation beam with composite plate. The foundation beam has been modeled and analyzed analytically, where the present model used has been validated by comparison with other existing solutions for the case of a reinforced concrete foundation beam with composite plate.

2. Theoretical analysis and solutions procedure

2.1 Research significance

The most common failure modes for composite repaired reinforced concrete foundation beam are debonding of the composite plate or ripping of the concrete cover. These types of failures prevent the repaired RC foundation beam for reaching its ultimate flexural capacity, and therefore they must be included in design considerations. Both of these premature failure modes are caused by shear and normal stress concentrations in adhesive layer. Closed form solutions of stress concentrations are required in developing design guidelines for repaired reinforced concrete foundation beam with composite plates.

2.2 Theoretical formulation

2.2.1 Basic assumptions:

To simplify the theoretical derivations of interfacial stresses in FRP-strengthened reinforced concrete foundation beam (Fig. 1), the following assumptions are adopted in this article.

- The composite materials, including concrete foundation beam, adhesive layer and FRP laminates, are all linear elastic.
- The average cross-sectional strain in bending conforms to the plane-section assumption before and after strengthening.
- There is no slip at the FRP to adhesive inter-face or at the adhesive to concrete foundation beam interface before debonding failure.
- The shear and normal stresses in the adhesive layer are constant through the thickness, and the in plane bending stiffness of the adhesive layer is neglected.
- In the derivation of interfacial shear stress, the moment curvatures of both the concrete foundation beam and the externally bonded FRP laminates are equal.

In the present analysis, a linear elastic behavior is assumed for the three materials (concrete foundation beam, adhesive and composite plate). The adhesive layer is supposed to play a role in transferring the stresses from the concrete to the composite plate with constant stresses throughout its thickness (Fig. 2).

2.2.2 Elasticity equations:

The deformation in reinforced concrete foundation beam in the vicinity of the adhesive layer can be expressed by (Fig. 2)

$$\varepsilon_1(x) = \frac{du_1(x)}{dx} = \varepsilon_1^M(x) + \varepsilon_1^N(x) \tag{1a}$$

with

$$\varepsilon_1^M(x) = \frac{y_1}{E_1 I_1} M_1(x) \quad \text{and} \quad \varepsilon_1^N(x) = \frac{du_1^N(x)}{dx} = \frac{N_1}{E_1 A_1} \tag{1b}$$

$$\varepsilon_1(x) = \frac{du_1(x)}{dx} = \frac{y_1}{E_1 I_1} M_1(x) + \frac{N_1}{E_1 A_1} \tag{1c}$$

Based on the theory of laminated sheets, the deformation of the composite sheet in the vicinity of the adhesive layer is given by

$$\varepsilon_2(x) = \frac{du_2(x)}{dx} = \varepsilon_2^M(x) + \varepsilon_2^N(x) \tag{2a}$$

with

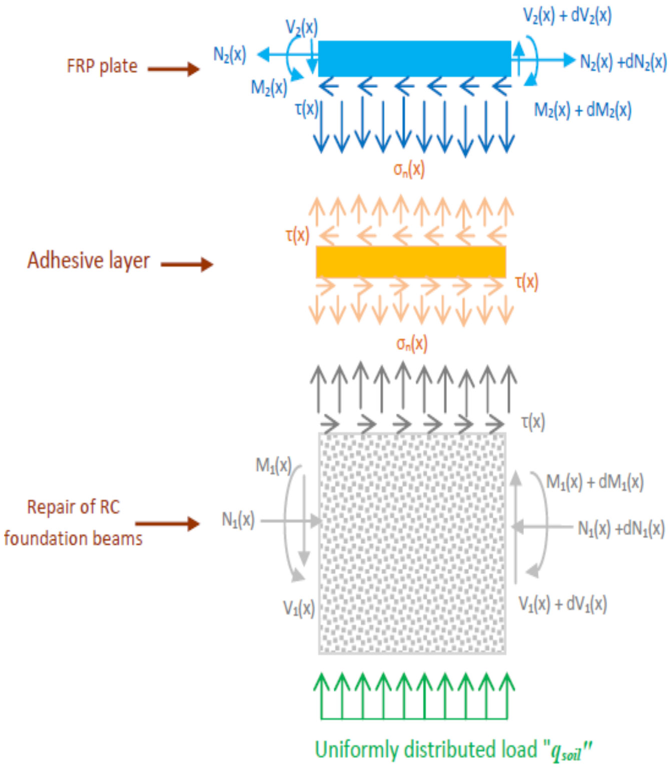


Fig. 2 Forces in an infinitesimal element of an FRP- repairing RC foundation beam

$$\varepsilon_2^M(x) = \frac{-y_2}{E_2 I_2} M_2(x) \quad \text{and} \quad \varepsilon_2^N(x) = \frac{du_2^N(x)}{dx} = A'_{11}\frac{N_2(x)}{b_2} \tag{2a}$$

$$\varepsilon_2(x) = \frac{du_2(x)}{dx} = -D'_{11}\frac{y_2}{b_2} M_2(x) + A'_{11}\frac{N_2(x)}{b_2} \tag{2c}$$

Where $u_1(x)$ and $u_2(x)$ are the horizontal displacements of the reinforced concrete foundation beam and the composite plate respectively. $M_1(x)$ and $M_2(x)$ are respectively the bending moments applied to the concrete beam and the composite plate; E_1 is the Young's modulus of reinforced concrete foundation beam; I_1 the moment of inertia, N_1 and N_2 are the axial forces applied to the reinforced concrete foundation beam and the composite plate respectively, b_1 and t_1 are the width and thickness of the reinforcement plate, $[A']=[A^{-1}]$ is the inverse of the membrane matrix $[A]$, $[D']=[D^1]$ is the inverse of the bending matrix.

By writing the conditions of equilibrium of the member 1 (reinforced concrete foundation beam), we will have:

In the *x* direction

$$\frac{dN_1(x)}{dx} = -b_1\tau(x) \tag{3a}$$

Where $\tau(x)$ is the shear stress in the adhesive layer.

In the y direction

$$\frac{dV_1(x)}{dx} = -(\sigma_n(x)b_1 + q_{soil}b_1) \tag{3b}$$

Where $V_1(x)$ the sheer force of the concrete beam is, $\sigma(x)$ is the normal stress at the adhesive layer, q_{soil} is the distributed load and b_1 the width of the concrete foundation beam.

The moment of balance

$$\frac{dM_1(x)}{dx} = V_1(x) - b_1 y_1 \tau(x) \tag{4}$$

The balance of the FRP repaired plate in the x and y directions, as well as the moment of equilibrium are written as follows:

In the x direction

$$\frac{dN_2(x)}{dx} = b_2 \tau(x) \tag{5a}$$

In the y direction

$$\frac{dV_2(x)}{dx} = \sigma_n(x) b_2 \tag{5b}$$

The moment of balance

$$\frac{dM_2(x)}{dx} = V_2(x) - b_2 y_2 \tau(x) \tag{5c}$$

Where $V_2(x)$ is the shear force of the reinforcement plate.

In what follows, the stiffness of the reinforcement plate is significantly lower than that of the reinforced concrete foundation beam to be repaired. The bending moment in the composite plate can be neglected to simplify the shear stress derivation operations. On the other hand, the laminate theory is used to determine the stress and strain of the externally bonded composite plate in order to investigate the whole mechanical performance of the composite strengthened structure. The effective modules of the composite laminate are varied by the orientation of the fibre directions and arrangements of the laminate patterns. The classical laminate theory is used to estimate the strain of the composite plate, i.e.

$$\begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \tag{6}$$

$$\begin{aligned} \left[A'\right] &= [A]^{-1} + [A]^{-1}[B]\left[D^*\right]^{-1}[B][A]^{-1} \\ \left[B'\right] &= -[A]^{-1}[B]\left[D^*\right]^{-1} \\ \left[C'\right] &= \left[B'\right]^T \\ \left[D'\right] &= \left[D^*\right]^{-1} \\ \left[D^*\right] &= [D] - [B][A]^{-1}[B] \end{aligned} \tag{7}$$

The terms of the matrices $[A]$, $[B]$ and $[D]$ are written as:

Extensional matrix $A_{ij} = \sum_{k=1}^{NN} \overline{Q}_{ij}^{k} ((y_2)_k - (y_2)_{k-1})$ (8)

Extensional -bending coupled matrix: $B_{ij} = \frac{1}{2} \sum_{k=1}^{NN} \overline{Q}_{ij}^{k} ((y_2^2)_k - (y_2^2)_{k-1})$ (9)

Flexural matrix: $D_{ij} = \frac{1}{3} \sum_{k=1}^{NN} \overline{Q}_{ij}^{k} ((y_2^3)_k - (y_2^3)_{k-1})$ (10)

The subscript *NN* represents the number of laminate layers of the FRP plate, $\bar{Q}_{ij}$ can be estimated by using the off-axis orthotropic plate theory, where

$$\overline{Q}_{11} = Q_{11} m^4 + 2(Q_{12} + 2Q_{33}) m^2 n^2 + Q_{22} n^4 \quad (11a)$$

$$\overline{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{33}) m^2 n^2 + Q_{12}(n^4 + m^4) \quad (11b)$$

$$\overline{Q}_{22} = Q_{11} n^4 + 2(Q_{12} + 2Q_{33}) m^2 n^2 + Q_{22} m^4 \quad (11c)$$

$$\overline{Q}_{33} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{33}) m^2 n^2 + Q_{33}(n^4 + m^4) \quad (11d)$$

And

$$Q_{11} = \frac{E_1}{1 - \upsilon_{12}\upsilon_{21}} \quad (12a)$$

$$Q_{22} = \frac{E_2}{1 - \upsilon_{12}\upsilon_{21}} \quad (12b)$$

$$Q_{12} = \frac{\upsilon_{12} E_2}{1 - \upsilon_{12}\upsilon_{21}} = \frac{\upsilon_{21} E_1}{1 - \upsilon_{12}\upsilon_{21}} \quad (12c)$$

$$Q_{33} = G_{12} \quad (12d)$$

$$m = \cos(\theta_j) \quad n = \sin(\theta_j) \quad (12e)$$

Where *j* is number of the layer; *h*, $\bar{Q}_{ij}$ and θ_j are respectively the thickness, the Hooke's elastic tensor and the fibers orientation of each layer.

Assume that the ply arrangement of the plate is symmetrical with respect to the mid-plane axis y_2=0. A great simplification in laminate analysis then occurs by assuming that the coupling matrix *B* is identically zero. Therefore Eqs. (6)-(10) can be simplified to the following matrix form for a plate with a width of b_2

$$\{\varepsilon^0\} = [A']\{N\}_2 \quad \text{and} \quad \{k\} = [D']\{M\}_2 \quad (13)$$

Where

$$\{\varepsilon^0\}_2 = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix}, \text{ and } \{k\}_2 = \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \quad (14)$$

$$\{N\}_2 = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}_2 \text{, and } \{M\}_2 = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}_2 \tag{15}$$

In the present study, only an axial load N_x and the bending moment M_x in the beam's longitudinal axis are considered, i.e., N_y=N_{xy}=0 and M_y=M_{xy}=0. Therefore, Eqs. (14) and (15) can be simplified to

$$\varepsilon_x^0 = \frac{A'_{11} N_x}{b_2} \text{ and } k_x = \frac{D'_{11} M_x}{b_2} \tag{16}$$

Using CLT, the strain at the top of the FRP plate 2 is given as

$$\varepsilon_2(x) = \varepsilon_x^0 - k_x \frac{t_2}{2} \tag{17}$$

Substituting Eq. (12b) in (12c) gives the following equation

$$\varepsilon_2(x) = \frac{du_2(x)}{dx} = -D'_{11} \frac{t_2}{2b_2} M(x) + A'_{11} \frac{N_2(x)}{b_2} \tag{18}$$

Where: $N_2(x) = N_x$ and $M_2(x) = M_x$

The subscripts 1 and 2 denote adherends 1 and 2, respectively. $M(x)$, $N(x)$ and $V(x)$ are the bending moment, axial and shear forces in each adherend.

2.3 Shear stress distribution along the FRP-concrete foundation beam interface:

The shear stress at the adhesive layer can be expressed as follows

$$\tau_a = \tau(x) = K_s \Delta u(x) = K_s \left[u_2(x) - u_1(x) \right] \tag{19}$$

Where K_s is the shear stiffness of the adhesive layer per unit length. From Eq. (19) we can deduce the expression of K_s which is given by

$$K_s = \frac{\tau(x)}{\Delta u(x)} = \frac{\tau(x)}{\Delta u(x)/t_a} \frac{1}{t_a} = \frac{G_a}{t_a} \tag{20}$$

$\Delta u(x)$ is the displacement relative to the adhesive interface, G_a et t_a are the modulus and thickness of the adhesive layer, respectively.

By differentiating the Eqs. (19), (1) and (2) with respect to x, and neglecting the bending moment of the composite plate we will have

$$\frac{d\tau(x)}{dx} = K_s \left[\frac{du_2(x)}{dx} - \frac{du_1(x)}{dx} \right] \tag{21a}$$

$$\frac{d\tau(x)}{dx} = K_s [[-D'_{11} \frac{y_2}{b_2} M_2(x) + A'_{11} \frac{N_2(x)}{b_2}] - [\frac{y_1}{E_1 I_1} M_1(x) + \frac{N_1}{E_1 A_1}]] \tag{21b}$$

Assuming that the curvatures in the member 1 and 2 are equal, the relationship between the moments in the two members can be written as follows

$$M_1(x) = RM_2(x) \tag{22}$$

With

$$R = \frac{E_1 I_1 D_{11}^{'}}{b_2} \tag{23}$$

The total moment of equilibrium of the differential element of Fig. 2 is given by

$$M_T(x) = M_1(x) + M_2(x) + N(x)(y_1 + t_a + y_2) \tag{24a}$$

$$M_T(x) = M_1(x) + M_2(x) + N(x)(y_1 + t_a + \frac{t_2}{2}) \tag{24b}$$

$M_T(x)$ is the total moment and $N(x)$ is given by the following expression

$$N(x) = N_2(x) = N_1(x) = b_2 \int_0^x \tau(x) \tag{25}$$

The bending moment of the reinforced beam expressed as a function of the total moment applied and the interface shear stresses is given by

$$M_1(x) = \frac{R}{R+1}\left[M_T(x) - b_2 \int_0^x \tau(x)(y_1 + t_a + \frac{t_2}{2})dx \right] \tag{26}$$

$$M_2(x) = \frac{1}{R+1}\left[M_T(x) - b_2 \int_0^x \tau(x)(y_1 + t_a + \frac{t_2}{2})dx \right] \tag{27}$$

By deriving expressions (16) and (17) we will have

$$\frac{dM_1(x)}{dx} = \frac{R}{R+1}\left[V_T(x) - b_2 \tau(x)(y_1 + t_a + \frac{t_2}{2}) \right] \tag{28}$$

$$\frac{dM_2(x)}{dx} = \frac{1}{R+1}\left[V_T(x) - b_2 \tau(x)(y_1 + t_a + \frac{t_2}{2}) \right] \tag{29}$$

By differentiating Eq. (31) we will have

$$\frac{d^2\tau(x)}{dx^2} = K_s \left[\frac{A_{11}^{'}}{b_2} \frac{dN_2(x)}{dx} - D_{11}^{'} \frac{t_2}{2b_2} \frac{dM_2(x)}{dx} - \frac{y_1}{E_1 I_1} \frac{dM_1(x)}{dx} + \frac{1}{E_1 A_1} \frac{dN_1(x)}{dx} \right] \tag{30}$$

The substitution of shear force (Eqs. (28) and (29)) and normal force (Eq. (25)) in Eq. (30) allows us to obtain the differential equation of the shear interface stresses

$$\frac{d^2\tau(x)}{dx^2} - K_s \left(A_{11}^{'} + \frac{b_2}{E_1 A_1} + \frac{(y_1 + \frac{t_2}{2})(y_1 + t_a + \frac{t_2}{2})}{E_1 I_1 D_{11}^{'} + b_2} b_2 D_{11}^{'} \right) \tau(x) + K_s \left(\frac{(y_1 + \frac{t_2}{2})}{E_1 I_1 D_{11}^{'} + b_2} D_{11}^{'} \right) V_T(x) = 0 \tag{31}$$

The solution to the differential equation (Eq. (31)) above is given by:

$$\tau(x)=\beta_1\cosh(\eta x)+\beta_2\sinh(\eta x)+\left(\frac{\dfrac{(y_1+\frac{t_2}{2})}{E_1I_1D_{11}^{'}+b_2}D_{11}^{'}}{A_{11}^{'}+\dfrac{b_2}{E_1A_1}+\dfrac{(y_1+\frac{t_2}{2})(y_1+t_a+\frac{t_2}{2})}{E_1I_1D_{11}^{'}+b_2}b_2D_{11}^{'}}\right)V_T(x) \tag{32}$$

With

$$\eta=\left[K_s\left(A_{11}^{'}+\frac{b_2}{E_1A_1}+\frac{(y_1+\frac{t_2}{2})(y_1+t_a+\frac{t_2}{2})}{E_1I_1D_{11}^{'}+b_2}b_2D_{11}^{'}\right)\right]^{\frac{1}{2}} \tag{33}$$

β_1 and β_2 are constant coefficients determined from the boundary conditions. For our case of a uniformly distributed load (q_{soil}), the formulation of the shear stress is given by the following equation

$$\tau(x)=\left(\frac{K_s}{\eta^2}\left(\frac{2(y_1+\frac{t_2}{2})}{E_1I_1D_{11}^{'}+b_2}D_{11}^{'}\right)+\frac{K_sy_1}{E_1I_1}a^2\right)\frac{q_{soil}}{2\eta}e^{-\eta x}+\frac{K_s}{\eta^2}\left(\frac{(y_1+\frac{t_2}{2})}{E_1I_1D_{11}^{'}+b_2}D_{11}^{'}\right)q_{soil}(a+x) \tag{34}$$

2.4 Normal stress distribution along the FRP- concrete foundation beam interface

The normal stress in the adhesive can be expressed as follows

$$\sigma_n(x)=K_n\Delta w(x)=K_n\left[w_2(x)-w_1(x)\right] \tag{35}$$

Where K_n the normal stiffness of the adhesive is layer per unit of length and can be deduced as follows

$$K_n=\frac{\sigma_n(x)}{\Delta w(x)}=\frac{\sigma_n(x)}{\Delta w(x)/t_a}\left(\frac{1}{t_a}\right)=\frac{E_a}{t_a} \tag{36}$$

$w_c(x)$ and $w_p(x)$ are the vertical displacements of the member 1 and 2 respectively. By deriving Eq. (27) twice it results

$$\frac{d^2\sigma_n(x)}{dx^2}=K_n\left[\frac{d^2w_2(x)}{dx^2}-\frac{d^2w_1(x)}{dx^2}\right] \tag{37}$$

By considering the relations moment curvature of the concrete beam and the reinforcement plate respectively we can write

$$\frac{d^2w_1(x)}{dx^2}=-\frac{M_1(x)}{E_1I_1}\text{, and }\frac{d^2w_2(x)}{dx^2}=-\frac{D_{11}^{'}M_2(x)}{E_2I_2} \tag{38}$$

Based on the equations of balance (3) and (8), the differential equations of equilibrium of the members 1 and 2 expressed as a function of the shear stresses and the normal stresses are given as follows:

The balance of members 1 and 2 leads to the following relationships:

Adherent1: $$\frac{d^4 w_1(x)}{dx^4} = \frac{1}{E_1 I_1} b_2 \sigma_n(x) + \frac{y_1}{E_1 I_1} b_2 \frac{d\tau(x)}{dx} + \frac{q_{soil}}{E_1 I_1} \quad (39)$$

Adherent 2: $$\frac{d^4 w_2(x)}{dx^4} = -D'_{11}\sigma_n(x) + D'_{11}\frac{t_2}{2}\frac{d\tau(x)}{dx} \quad (40)$$

The substitution of Eqs. (39) and (40) in the fourth derivative of the normal stress obtained from Eq. (35) gives the differential equation governing the normal interface stress

$$\frac{d^4\sigma_n(x)}{dx^4} + K_n\left(D'_{11} + \frac{b_2}{E_1 I_1}\right)\sigma_n(x) - K_n\left(D'_{11}\frac{t_2}{2} - \frac{y_1 b_2}{E_1 I_1}\right)\frac{d\tau(x)}{dx} + \frac{q_{soil}K_n}{E_1 I_1} = 0 \quad (41)$$

The general solution of the differential equation of order 4 is:

$$\begin{aligned}\sigma_n(x) &= e^{-\lambda x}\left[\beta_3\cos(\lambda x) + \beta_4\sin(\lambda x)\right] + e^{\lambda x}\left[\beta_5\cos(\lambda x) + \beta_6\sin(\lambda x)\right] \\ &\quad - \frac{y_1 b_2 - \dfrac{D'_{11}E_1 I_1 t_2}{2}}{D'_{11}E_1 I_1 + b_2}\frac{d\tau(x)}{dx} - \frac{1}{D'_{11}E_1 I_1 + b_2} q_{soil}\end{aligned} \quad (42)$$

For large values of “*x*” we assume that the normal stress tends to zero, and it results $\beta_5=\beta_6=0$. The general solution becomes

$$\sigma_n(x) = e^{-\lambda x}\left[\beta_3\cos(\lambda x) + \beta_4\sin(\lambda x)\right] - \frac{y_1 b_2 - \dfrac{D'_{11}E_1 I_1 t_2}{2}}{D'_{11}E_1 I_1 + b_2}\frac{d\tau(x)}{dx} - \frac{1}{D'_{11}E_1 I_1 + b_2} q_{soil} \quad (43)$$

Where

$$\lambda = \left[\frac{K_n}{4}\left(D'_{11} + \frac{b_2}{E_1 I_1}\right)\right]^{\frac{1}{4}} \quad (44)$$

The integration constants β_3 and β_4 are given by

$$\begin{aligned}\beta_3 &= \frac{K_n}{2\lambda^3 E_1 I_1}\left[V_T(0) + \lambda M_T(0)\right] - b_2 K_n\left(\frac{y_1}{E_1 I_1} - \frac{D'_{11}t_2}{2b_2}\right)\frac{\tau(0)}{2\lambda^3} \\ &\quad + \frac{y_1 b_2 - \dfrac{D'_{11}E_1 I_1 t_2}{2}}{2\lambda^3(D'_{11}E_1 I_1 + b_2)}\left(\frac{d^4\tau(0)}{dx^4} + \lambda\frac{d^3\tau(0)}{dx^3}\right)\end{aligned} \quad (45)$$

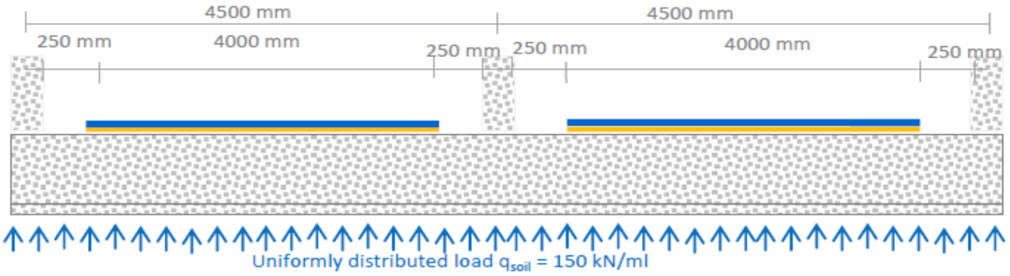
$$\beta_4 = -\frac{K_n}{2\lambda^2 E_1 I_1} M_T(0) - \frac{y_1 b_2 - \dfrac{D'_{11}E_1 I_1 t_2}{2}}{2\lambda^2(D'_{11}E_1 I_1 + b_2)}\frac{d^3\tau(0)}{dx^3} \quad (46)$$

The above expressions for the constants β_3 and β_4 has been left in terms of the bending moment $M_T(0)$ and shear force $V_T(0)$ at the end of the soffit plate. With the constants β_3 and β_4 determined, the interfacial normal stress can then be found using Eq. (43).

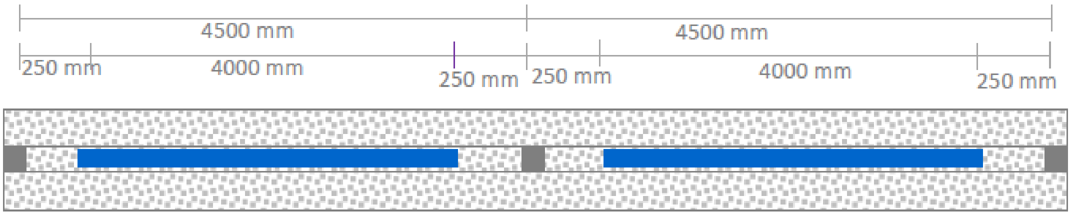
3. Results: Discussion and analysis

Table 1 Mechanical properties of the materials used

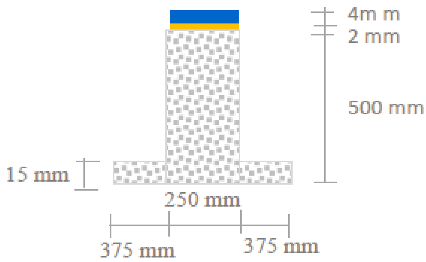
Component	Young's modulus (MPa)	Poisson's ratio
Reinforced concrete foundation beam	E_1=30000	0.18
Adhesive layer	E_a=6700	0.4
Sika Carbodur repaired plate	E=165 000	0.3
Sika Wrap repaired plate	E_2=230 000	0.3
CFRP repaired plate	E_2=140 000	0.28
GFRP repaired plate	E_2=50 000	0.28



(a) Longitudinal section



(b) Plan view



(c) Cross section

Fig. 3 Geometric characteristic of a reinforced concrete foundation beam repaired with composite plate

3.1 Geometric and material properties

The material used for the present studies is an reinforced concrete foundation beam repaired with different type of composite materials plate (GFRP, CFRP, Sika Carbodur and Sika Wrap). A summary of the geometric and material properties is given in Table 1 and Fig. 3. The span of the reinforced concrete foundation beam is 9000 mm (2×4500 mm), the distance from the support to the end of the plate is 250 mm and the uniformly distributed load (UDL) is q_{soil}=150 kN/ml.

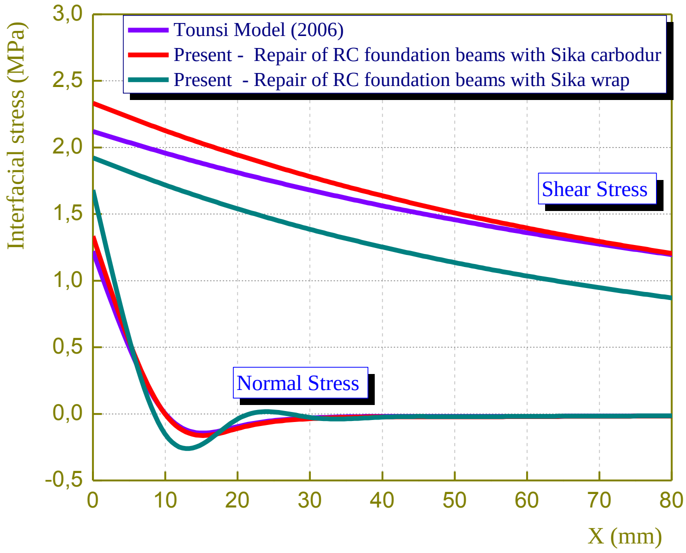


Fig 4 Comparison analytical results of interfacial stresses for composite plate repaired RC foundation beam

3.2 Comparison with analytical solutions

A comparison of the interfacial shear and normal stresses from the different existing closed-form solutions and the present new solution is undertaken in this section. A reinforced concrete foundation beam repaired with composite plate is considered. The continuous beam is simply supported and subjected to a uniformly distributed load. A summary of the geometric and material properties is given in Table 1 and Fig. 3. The span of The RC beam is 9000 mm (2×4500 mm), the distance from the support to the end of the plate is 250 mm and the uniformly distributed load is q_{soil}=150 kN/ml. Fig. 4 plots the interfacial shear and normal stresses near the plate end for the example reinforced concrete foundation beam repaired with composite plate (sika wrap plate and sika carbodur plate) for the uniformly distributed load case "q_{soil}". Overall, the predictions of the different solutions agree closely with each other. The interfacial normal stress is seen to change sign at a short distance away from the plate end. Hence, it is apparent that the adherend shear deformation reduces the interfacial stresses concentration and thus renders the adhesive shear distribution more uniform. The interfacial normal stress is seen to change sign at a short distance away from the plate end.

3.3 Effect of composite plate stiffness on interfacial stress

Fig. 5 gives interfacial normal and shear stresses for the RC beam bonded with a sika carbodur, sika wrap, CFRP plate and GFRP plate, respectively, which demonstrates the effect of plate material properties on interfacial stresses. The length of the plate is L_p=4000 mm, and the thickness of the plate and the adhesive layer are both 4 mm. The results show that, as the plate material becomes softer (from sika carbodur, CFRP, sika wrap and then GFRP), the interfacial stresses become smaller, as expected. This is because, under the same load, the tensile force developed in the plate is smaller, which leads to reduced interfacial stresses. The position of the peak interfacial shear stress moves closer to the free edge as the plate becomes less stiff.

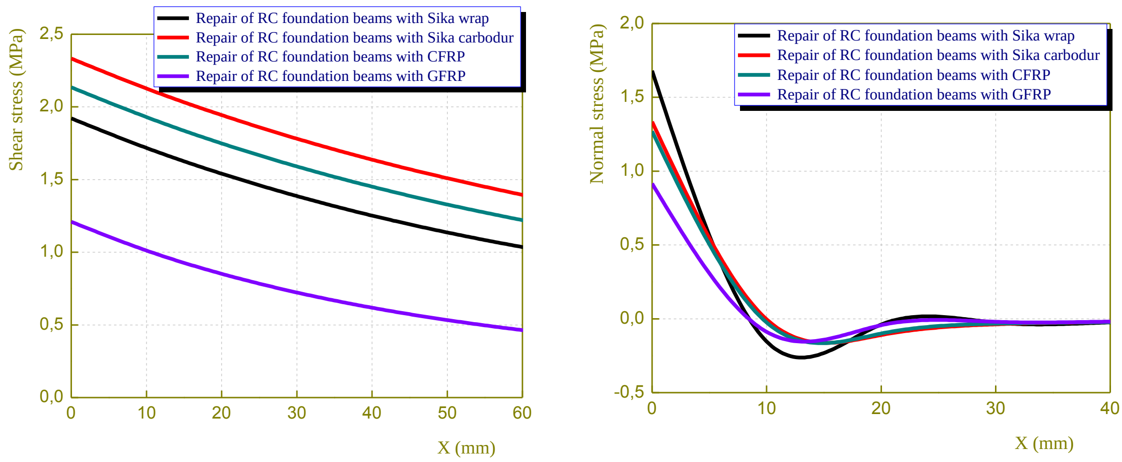


Fig. 5 Effect of plate stiffness on interfacial stresses in reinforced concrete foundation beam repaired with composite plate

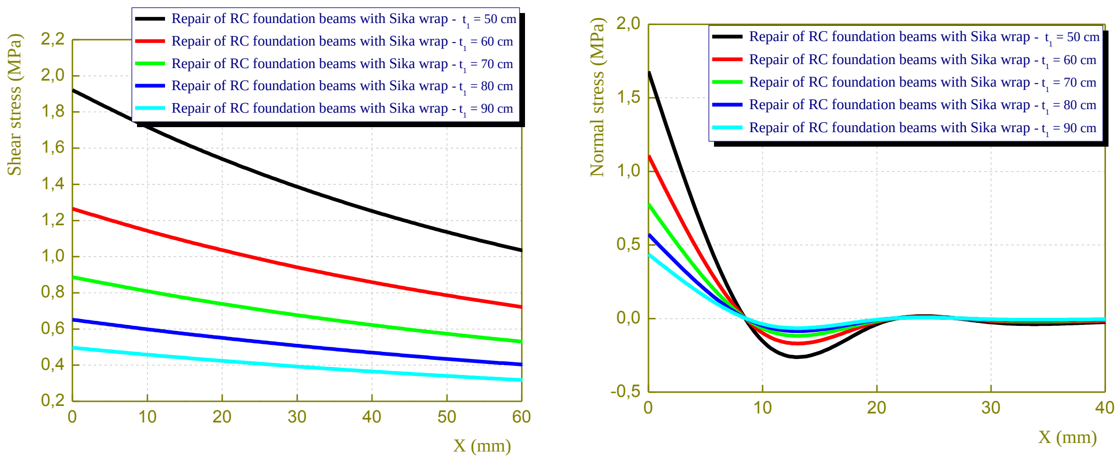


Fig. 6 Effect of rigidity of the RC beam on interfacial stresses in reinforced concrete foundation beam repaired with Sika Wrap composite plate

3.4 Effect of RC foundation beam stiffness on interfacial stress

Figs. 6, 7 and 8 show the effect of rigidity of the RC beam on interfacial stresses in reinforced concrete foundation beam repaired with composite plate (Sika Carbodur plate and Sika Wrap plate), respectively, for different dimensions of the RC beam, namely: 25×50 cm^2, 25×60 cm^2, 25×70 cm^2, 25×80 cm^2 and 25×90 cm^2, are considered here. It is shown that the rigidity of the RC influences the interfacial shear stress considerably and the interfacial normal stress slightly. The position of the peak interfacial shear stress moves closer to the free edge as the plate becomes less stiff.

3.5 Effect of soil stress characteristics on interfacial stress

The soil stress characteristics characteristics is an important design variable in practice. Figs. 9,

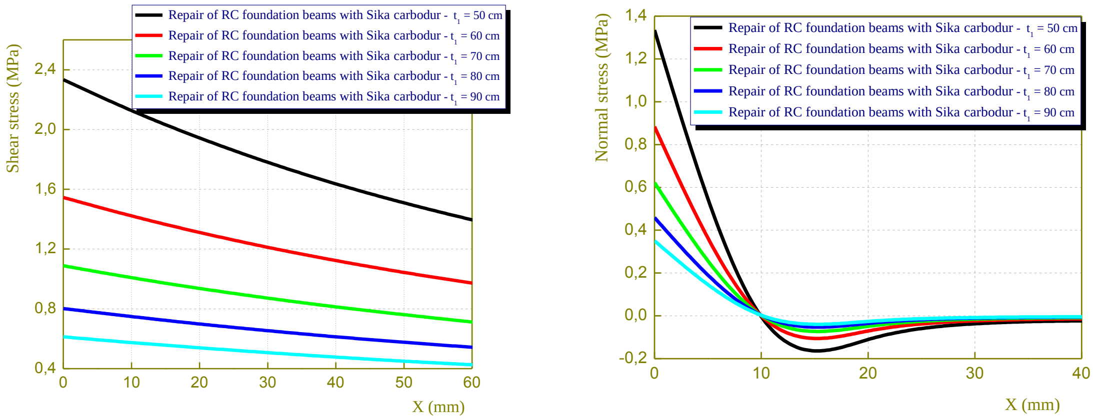


Fig. 7 Effect of rigidity of the RC beam on interfacial stresses in reinforced concrete foundation beam repaired with Sika Carbodur composite plate

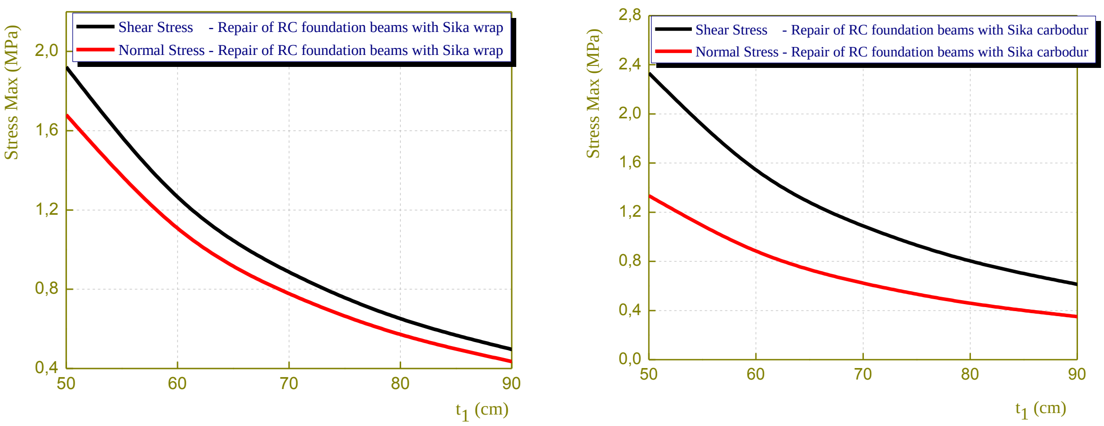


Fig. 8 Effect of height of the RC beam on interfacial stresses in reinforced concrete foundation beam repaired with composite plate

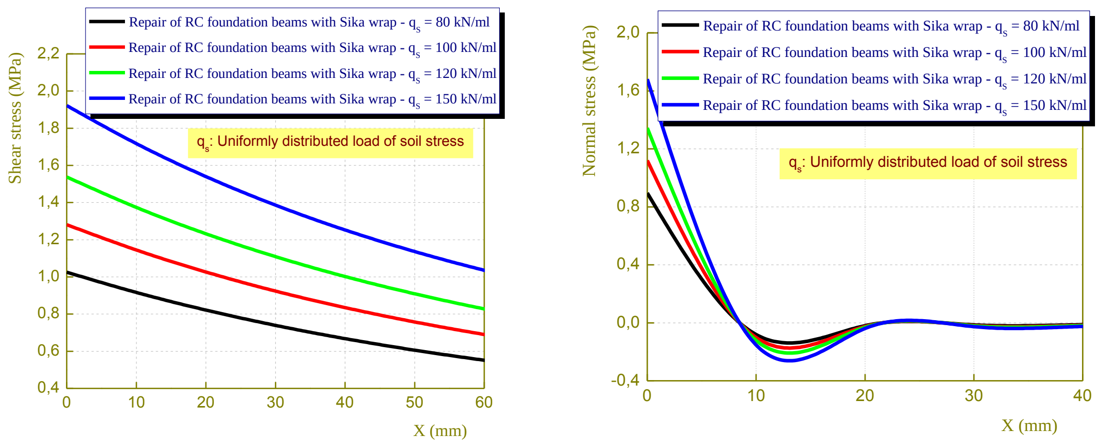


Fig. 9 Effect soil stress characteristics on interfacial stresses in reinforced concrete foundation beam repaired with Sika Wrap composite plate

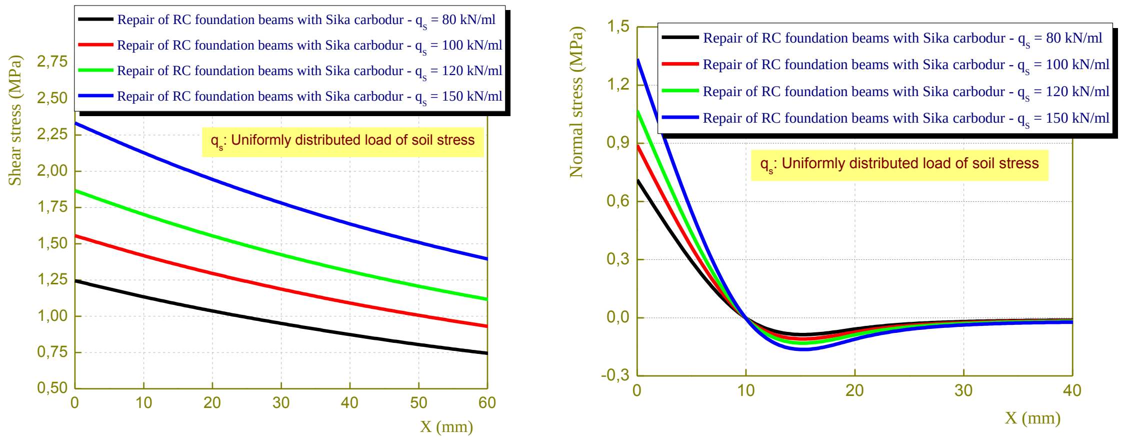


Fig. 10 Effect soil stress characteristics on interfacial stresses in reinforced concrete foundation beam repaired with Sika Carbodur composite plate

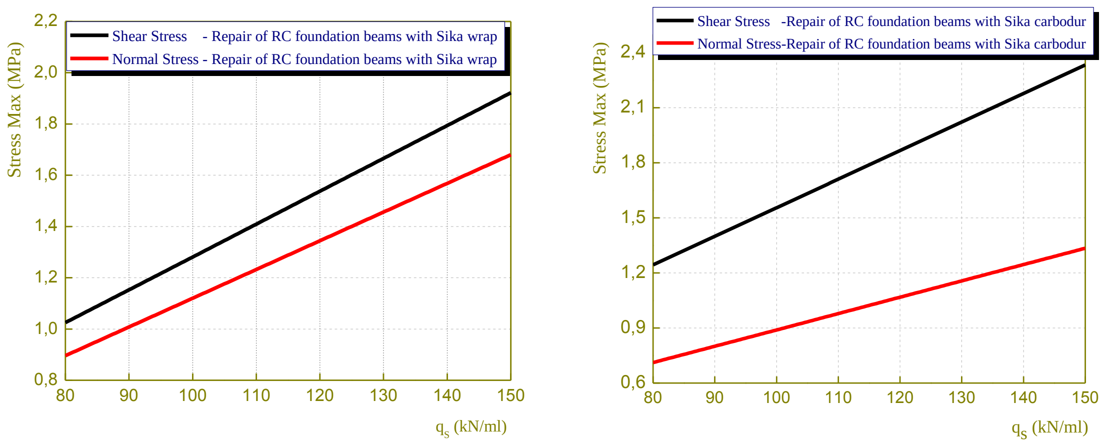


Fig. 11 Effect soil stress characteristics on interfacial stresses in reinforced concrete foundation beam repaired with composite plate

10 and 11 shows the effect soil stress characteristics on interfacial stresses in reinforced concrete foundation beam repaired with Sika Wrap and Sika Carbodur composite plate. Here, four values of the soil stress characteristics, take in our case the uniformly distributed load of soil stress: q_s=80 kN/ml, q_s=100 kN/ml, q_s=120 kN/ml and q_s=150 kN/ml, are considered. It is shown that the level and concentration of interfacial stress are influenced considerably by the uniformly distributed load of soil stress. The interfacial stresses increase as the uniformly distributed load of soil stress increases.

3.6 Effect on the length of the repair plate L_p

The influence of the length of the ordinary-beam region (the region between the support and the end of the composite strip on the edge stresses) appears in Figs. 12 and 13 (in the case of a repair plate in Sika Wrap and in Sika Carbodur). It is seen that, as the plate terminates further away from

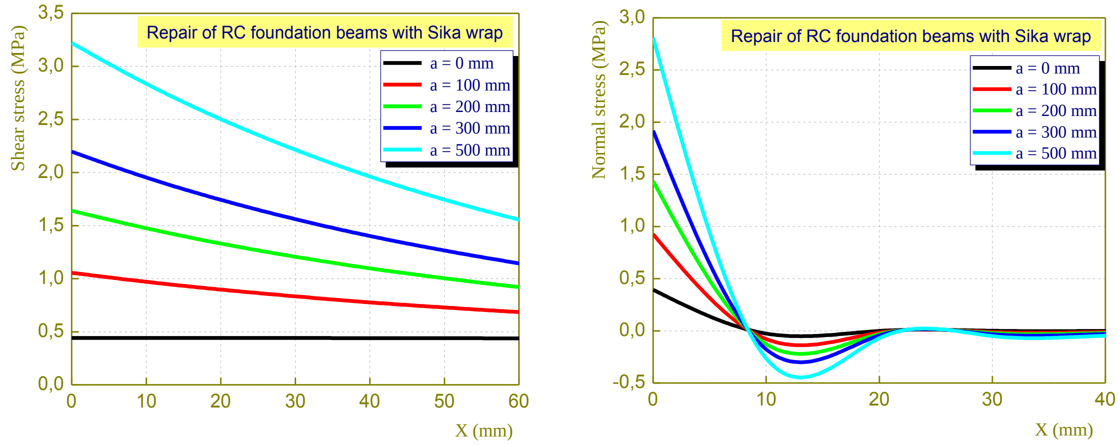


Fig. 12 Effect of the length of the repair plate L_p on interfacial stresses in reinforced concrete foundation beam repaired with Sika Wrap composite plate

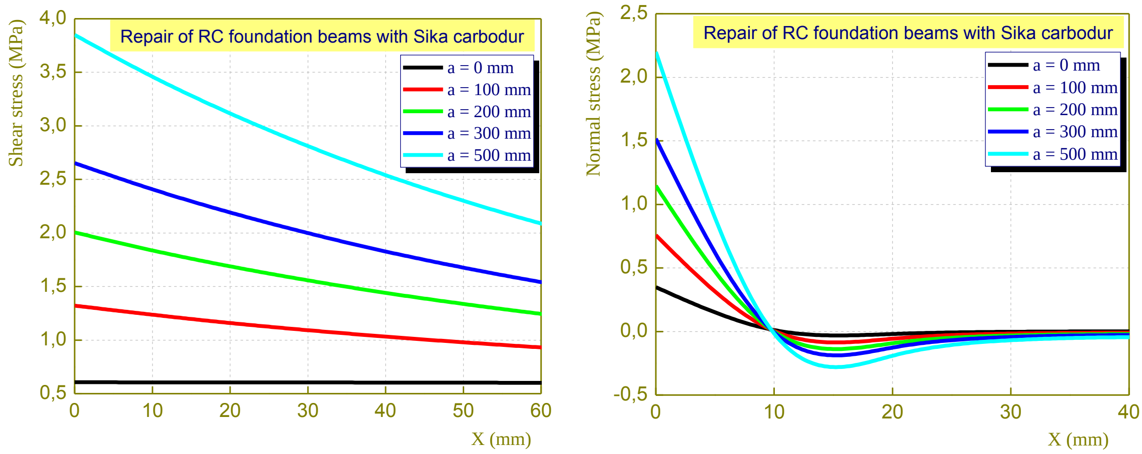


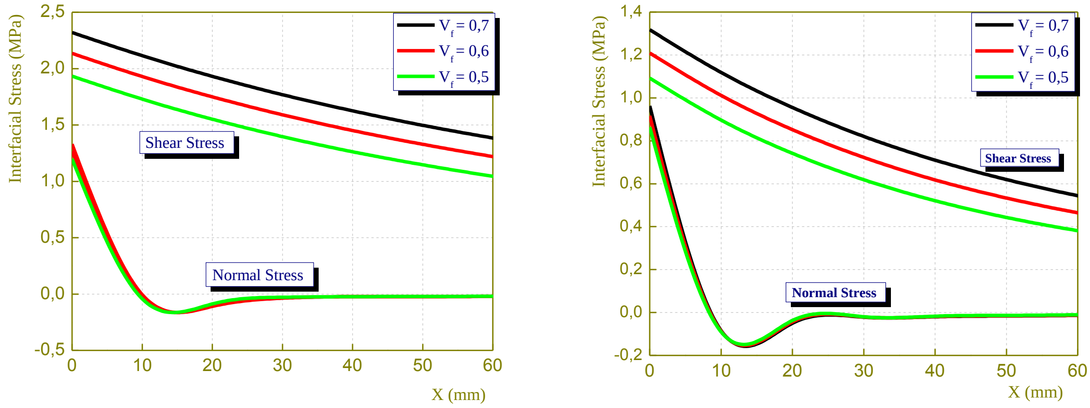
Fig. 13 Effect of the length of the repair plate L_p on interfacial stresses in reinforced concrete foundation beam repaired with Sika Carbodur composite plate

the supports, the interfacial stresses increase significantly. This result reveals that in any case of strengthening, including cases where retrofitting is required in a limited zone of maximum bending moments at midspan, it is recommended to extend the strengthening strip as possible to the lines.

3.7 Effect of fiber volume fraction on interfacial stress

Fig. 14 shows, Effect of fiber volume fraction V_f (=0.5, 0.6 and 0.7) on the variation of shear and normal adhesive stresses in reinforced concrete foundation beam repaired with composite plate. It can be seen that the interfacial shear stresses are reduced with decreases in fiber volume fraction. However, almost no effect is observed on the variation of interfacial normal stresses.

3.8 Effect of fiber orientation on interfacial stress



(a) CFRP plate (b) GFRP plate

Fig. 14 Effect of fiber volume fraction on interfacial stresses in reinforced concrete foundation beam repaired with composite plate (a) CFRP plate and (b) GFRP plate

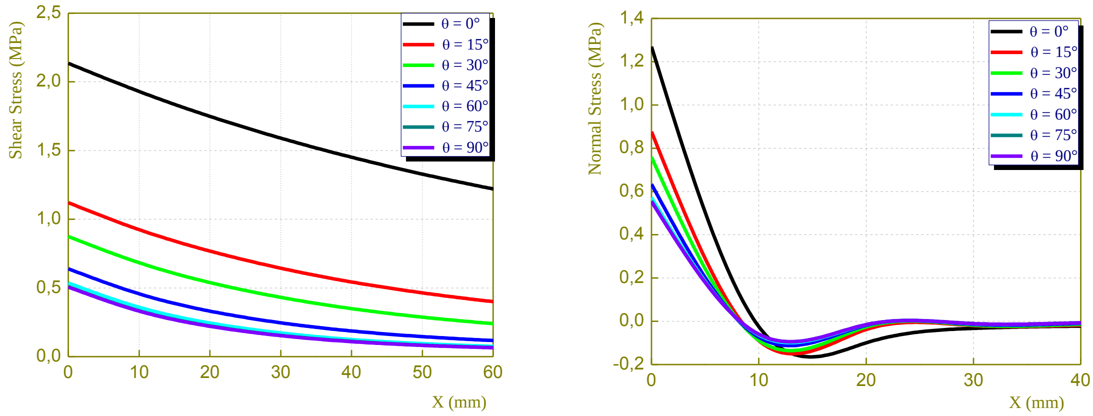


Fig. 15 Effect of fiber orientation on interfacial stresses in reinforced concrete foundation beam repaired with CFRP composite plate

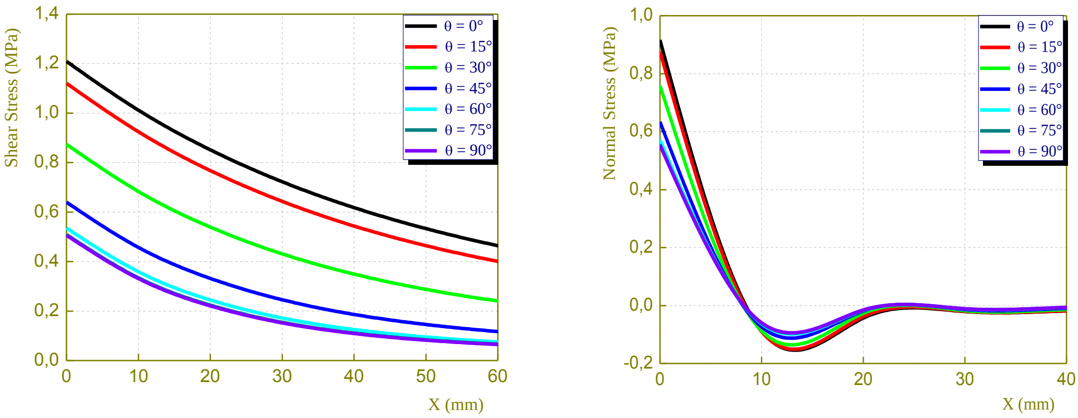
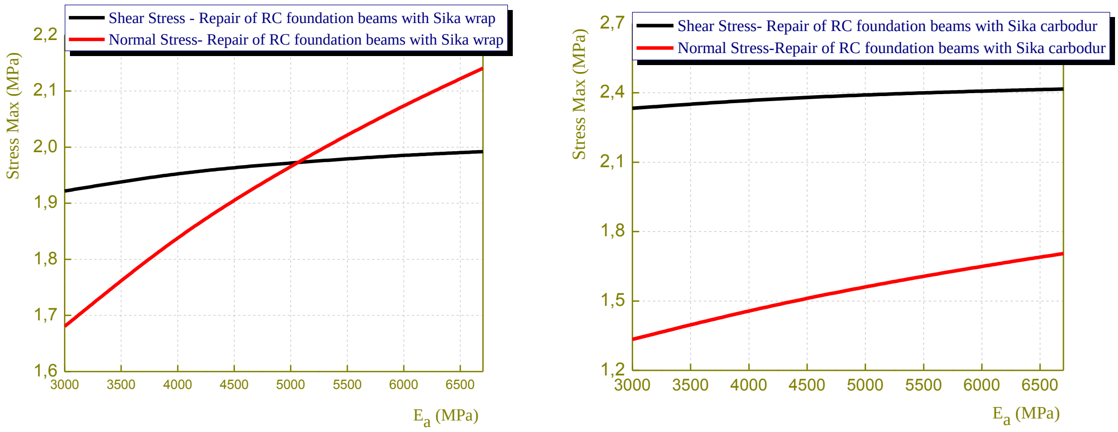


Fig. 16 Effect of fiber orientation on interfacial stresses in reinforced concrete foundation beam repaired with GFRP composite plate



(a) Sika Wrap plate (b) Sika Carbodur plate

Fig. 17 Effect of elasticity modulus of adhesive layer on interfacial stresses in reinforced concrete foundation beam repaired with composite plate

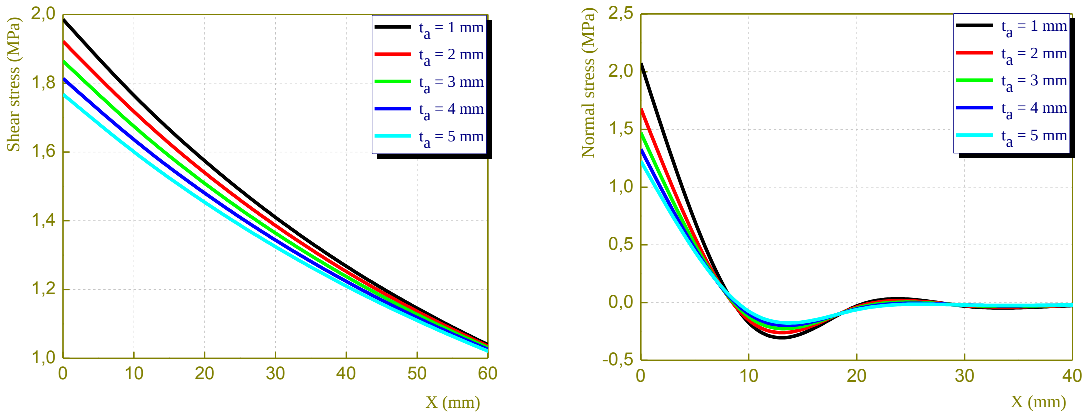


Fig. 18 Effect of adhesive layer thickness on interfacial stresses in reinforced concrete foundation beam repaired with Sika Wrap composite plate

The effect of fiber orientation on adhesive stresses is show in Figs. 15 and 16 (in the case of a repair plate in CFRP and in GFRP), the maximum interfacial stresses increase with increasing alignment of all high strength fibers in the composite plate in beam's longitudinal direction *x*.

3.9 Effect of elasticity modulus of adhesive layer

The adhesive layer is a relatively soft, isotropic material and has a smaller stiffness. The four sets of Young's moduli are considered here, which are 3, 4, 5 and 6.7 GPa. The Poisson's ratio of the adhesive is kept constant. The numerical results in Fig. 17 show that the property of the adhesive hardly influences the level of the interfacial stresses in reinforced concrete foundation beam repaired with composite plate (Sika Wrap plate and Sika Carbodur plate), whether normal or shear stress, but the stress concentrations at the end of the plate increase as the Young's modulus of the adhesive increases.

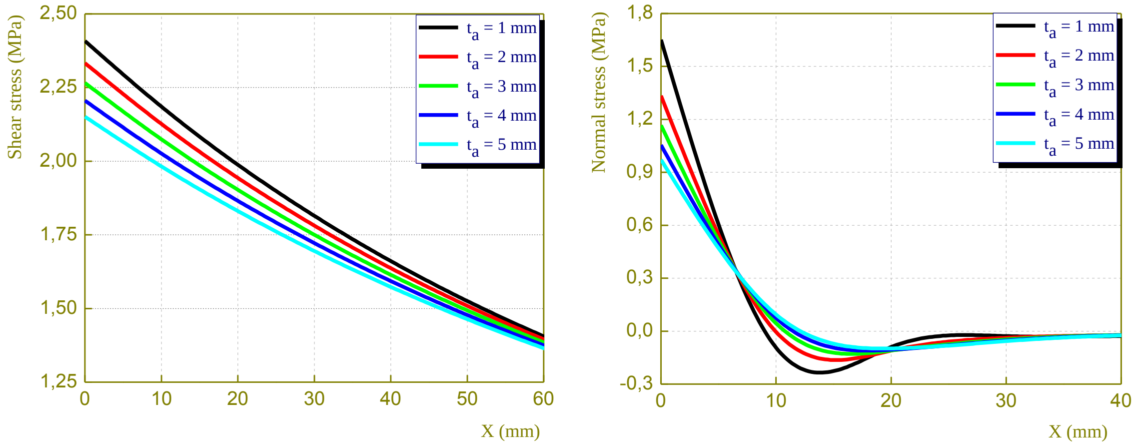


Fig. 19 Effect of adhesive layer thickness on interfacial stresses in reinforced concrete foundation beam repaired with Sika Carbodur composite plate

3.10 Effect of the adhesive layer thickness

Figs. 18 and 19 shows the effects of the thickness of the adhesive layer on the interfacial stresses in reinforced concrete foundation beam repaired with Sika wrap (Fig. 18) and sika Carbodur (Fig.19) composite plate. It is seen that increasing the thickness of the adhesive layer leads to significant reduction in the peak interfacial stresses. In order to solve the problem, thus using thick adhesive layer, especially in the vicinity of the edge, is recommended.

4. Conclusions

In order to evaluate the improvement in the flexion and strength in reinforced concrete foundation beams with composite external repair various cases were analyzed. The composite plate have been used to repair reinforced concrete foundation beams and to increase the strength of uncracked structures. Different locations and amounts of composite plate were used to study their influence of various parameters, in particular the geometric characteristics and mechanical properties of the components of the repaired beam, as well as the geotechnical stresses of the soil of the reinforced concrete foundation beam. With the studies presented and the results found the following conclusions can be taken, several of which may be useful recommendations for designers:

• The use of composite materials as bending reinforcement provides increases in strength and rigidity for the reinforced concrete foundation beam.

• There are stress concentrations at the end of the composite plate. The normal stress concentration is a tensile stress but quickly towards to zero through a small oscillation. The initial delamination of the composite plate from the reinforced concrete foundation beam results from joints effects of the shear and normal stress at the end of the composite plate.

• The maximum edge interfacial stresses increase with increasing alignment of all high strength fibers in composite plate longitudinal direction.

• The interfacial shear stresses are reduced with decreases in fiber volume fraction. However, almost no effect is observed on the variation of interfacial normal stresses.

• The interfacial stresses are influenced by the geometry parameters such as thickness of the adhesive layer and composite plate in range of the different degrees. It is shown that the edge stresses and levels increase obviously with the increase of the thickness of the composite plate. However, it is seen that increasing the thickness of the adhesive layer leads to significant reduction in the peak interfacial stresses.
• Another outcome based on the parametric study indicates that extending the composite strip as close as possible to the support reduces the stresses at the edge.
• Several factors are involved in the mechanical behaviour of the reinforced concrete foundation beams that need to be taken into account when evaluating the use of composite materials in repair and/or external reinforcement. The analysis of the structural behaviour is more complex than the other bending elements reinforced.

Considering these conclusions and taking into account the fact that the composite materials are simple and easily applied, this type of repair is a good option to repair foundation beams, with taking into account of course the recommendations cited well before.

Acknowledgments

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