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Stiffness of hybrid systems with and without pre-stressing

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Abstract. Constructive merging of "basic" systems of different behavior creates hybrid systems. In doing so, the structural elements are grouped according to the behavior in carrying the load into a geometric order that provides sufficient load and structure functionality and optimization of the material consumption. Applicable in all materializations and logical geometric forms is a transparent system suitable for the optimization of load-bearing structures. Research by individual authors gave insight into suitable system constellations from the aspect of load capacity and the approximate method of estimating the participation of partial stiffness within the rigidity of the hybrid system. The obtained terms will continue to be the basis for our own research of the influence of variable parameters on the behavior of hybrid systems formed of glued laminated girder and cable of different geometric shapes. Previous research has shown that by applying the strut-type hybrid systems can increase the load capacity and reduce the deformability of the free girder. The implemented parametric analysis points to the basic parameter in the behavior of these systems - the rigidity of individual elements and the overall stiffness of the system. The basic idea of pre-stressing is that, in the load system or individual load-bearing element, prior to application of the exploitation load, artificially challenge the forces that should optimize the final system behavior in the overall load. Pre-stressing is possible only if the supporting system or system's element possess sufficient strength or stiffness, or reaction to the imposed forces of pre-stressing. In this paper will be presented own research of the relationship of partial stiffness of strut-type hybrid systems of different geometric forms. Conducted parametric analysis of hybrid systems with and without pre-stressing, and on the example of the glulam-steel strut-type hybrid system under realistic conditions of change in the moisture content of the wooden girder, resulted in accurate expressions and diagrams suitable for application in practice.

Keywords: optimization of load-bearing structures; hybrid system; pre-stressing of hybrid system; glued laminated timber; cable; effective force of pre-stressing; geometrical stiffness/rigidity, self- stiffness/rigidity

1. Introduction

The rigidity of the load-bearing system represents ratio between the load and the resulting deformations on the system, where there are commonly known interdependencies, where high rigidity causes small deformations and large forces in the system, and vice versa. Today, using high-quality materials in civil engineering, mobile bearing systems are increasingly being used, which are usable with additional stiffening.

Comparing geometric changes in load-bearing structures in the transfer of loads it is possible to

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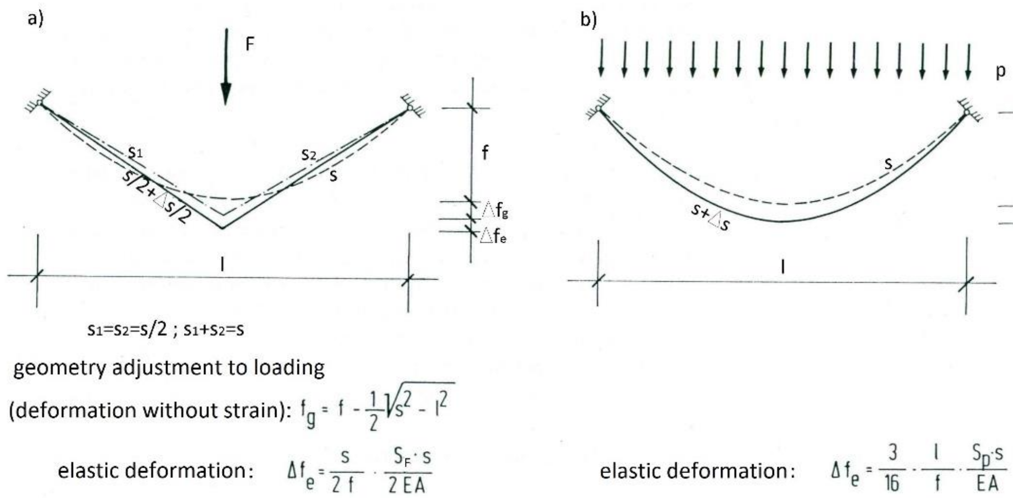

Fig. 1 Components of deformation of the system with stiffness on a stretch (a) geometrically non-inverse load, (b) geometrically inclined load (Wagner, Schl 172/13-1)

notice that certain systems are geometrically prone to load, while other systems with their rigidity provide resistance to deformation.

Thus, load bearing structures according to the behavior in load transfer can be divided into two groups: structures with geometric changes and without the stress of the basic material and structure with geometric changes due to the stress of the base material.

The first group of structures is rod or wrinkled systems with joint joints, and in which geometry is adapted to the load[1] (affine systems). Such systems have a delicate, but stable balance when the rods are solely tightened (Fig. 1). In the opposite case, the structure is unusable.

These load-bearing systems do not initially have geometric stiffness[2], so the system “adjusts” in the direction of load action, which is a geometric deformation of the system without the stress of the base material. The second kind of deformation is perceived by the system after “adjusting” according to the applied load, after which the geometric changes due to the deformation (stretching) of the base material occur.

The geometric stiffness of this system occurs only when the system achieves a stable balance under load and when deformations of the base material occur, and in the deformed state, the total stiffness of the system is both geometric and mechanical stiffness.

The second group of structures consists of elements that have stiffness, and thus resistance to the change in the geometry of the structure, which causes deformations and stresses in the basic material.

These systems are not adjustable to the load, and are capable of achieving an indifferent balance.

Hybrid systems are created by unifying these two “basic” systems, whose advantages come true with the correct geometric combination.

2. Hybrid systems

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[1]Additional loads, which are not prone to geometry of the system, cause changes in geometry.
[2]Geometry does not contribute to system's stiffness.

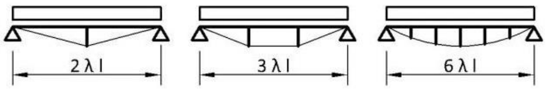

Fig. 2 Geometric constellation of strut-type hybrid systems

2.1 Strut type hybrid systems

Strut type hybrid system is created by constructively combining the two basic systems, that is, a girder that has bending rigidity and a cable with stretching rigidity, with common points at the ends of the girder (Fig. 2). Their constructive unification provides verticals of infinite stiffness, articulated to a rigid girder. They are favorable in cases of large spans or loads where elements with bending rigidity would require a disproportionately large material consumption to increase the span.

2.2 Geometric cable shape and its behaviour under the load

The cable within the hybrid system represents the affine structure, which, in addition to the task of tightening the force, has the function of geometric adjustment of the system to a given load. The cable's geometry varies, assuming a lack of bending rigidity[3], adapting to the loading of the hand over the rigid struts. Combining a rigid bending element creates a mixed (hybrid) system. In this way, the mobile system receives rigid components, which ensures limiting the deformation of the hybrid system.

In order to perform the rigidity of these affine systems in the general form, according to the theory of cables, a system is considered in the plane of the suspended cables with a geometrically affine load arranged normally on the direction between the articular supports (Figure 3).

From the equilibrium conditions of the vertical components of the system, the differential equation for the cable loaded with its own weight on the length dL

$$dF_S^V = -q \cdot dL = -q \cdot \sqrt{1-y'^2} = F_S^H \cdot y'' \tag{1}$$

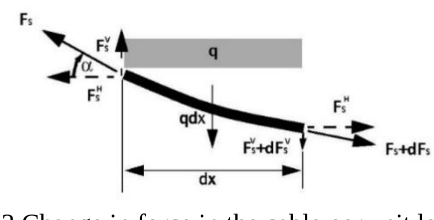

Fig. 3 Change in force in the cable per unit length

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[3]The line of cable, supported on the end supports and loaded with its own weight, differs from the line of the chain, because the cable has a certain bending rigidity, and the cable line is heavily curved in the area of the crown from the line of the chain. It means that there is a difference in the horizontal component of the force component in the support of the cable and the chain, and therefore the arrow cables f_s and the chain f_L, all depending on the cable length and support range (*L/l*). For practice, the budget of the line of the cable with its own weight can be taken simplified by the equation of the second-order parabola for $L/l \leq 1,20$.

Solution of the definition of equation is

$$y(x)=\frac{F_S^H}{q}\cdot\cosh\left(\frac{q}{F_S^H}\cdot\left(x-C_1\right)\right)+C_2 \tag{2}$$

For the cable, under the assumption of a parabolic curve, which possesses stiffness on stretching, burdened by uniform load q, simple relationships of force in the cable, arrow and span are obtained

$$f=\sqrt{\frac{3}{8}\cdot l\cdot\left(L-l\right)} \tag{3}$$

$$F_S^H=\frac{q\cdot l^2}{8}\cdot\frac{1}{\sqrt{\frac{3}{8}\cdot l\cdot\left(L-l\right)}} \tag{4}$$

where:

F_S^H - horizontal component of the force in the cable constant over the entire length of the cable,

f - arrow of the cable.

Deformable cables, relatively low weight, have very low bending rigidity and are very adaptive to transverse loads. For the determination of the total length of the stretched cable and the force in the cable, the terms obtained from the geometry and the conditions of the balance of the deformed cable are applied.

2.3 Cross-sectional forces and deformations of cables with irreplaceable supports according to the Theory of the second order

According to (Kleinhanss 1973), by the Theory of second order, it is possible to obtain approximate formulas for the dependence of force in the cable and displacement depending on the relevant parameters (geometry, stiffness, load and method of support). The approximate solution is obtained on the flat system of suspended, evenly curved, cables with fixed supports, used for the performance of general, valid claims for the transmission of load on the cable. Approximate terms, according to Theory of second order, can also be applied to finding a change in the geometric stiffness of the hybrid system.

From the analogue of the deformation line of the cable and the bending moment line of the replacement girder of the same span and load, the general expression for the length of the curve, and the effect of the elastic elongation of the cable due to the effect of force according to the Theory

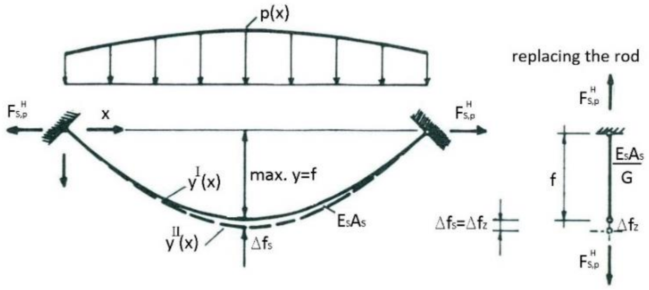


Fig. 4 Replacement rod for determining the characteristic geometric value of *G* (Kleinhanss 1973)

of the first order, is as follows

$$F_S^{H\,I} \cdot f = F_S^{H\,II} \cdot (f + \Delta f) = \left(F_S^{H\,I} + \Delta F_S^{H}\right) \cdot (f + \Delta f)$$
$$\Delta F_S^{H} \cdot \Delta f \cong 0 \quad \Rightarrow \Delta F_S^{H} = -\frac{\Delta f}{f} \cdot F_S^{H\,I} \tag{5}$$

where, for relatively small changes in geometry, the iteration process can be reduced to the introduction of an unknown force according to Theory of the second order. The unknown values in relation to the output values according to the Theory of the first order are:

ΔF_S^H- change of the force in the cable from the external load,

ΔL - elastic elongation of the cable with horizontal force according to the Theory of the second order (assuming $y(x) = y'(x)$),

Δf - follows from the elongation of the cable as a change in the characteristic ordinates in l/2.

The relation between Δf and ΔL for impact of the force in the cable according to the Theory of the second order is given as:

$$\Delta f = \frac{F_S^{H\,II}}{E_S \cdot A_S} \cdot f \cdot G$$
$$G = \frac{1}{f} \cdot \frac{\Delta f}{\Delta L} \cdot \int_o^l \left(1 + y'(x)^2\right) dx \tag{6}$$

where:

G - geometric characteristic value (represents the linear relation between the change of the arrow and the elongation of the cable).

On the basis of the preceding one can be expressed directly the force value in the cable according to the Theory of the second order, without iteration

$$\Delta F_S^{H} = F_{S,p}^{H\,II} - F_{S,p}^{H\,I} = -\frac{F_{S,p}^{H\,II}}{E_S \cdot A_S} \cdot G \cdot F_S^{H\,I}$$
$$F_{S,p}^{H\,II} = \frac{F_{S,p}^{H\,I}}{1 + \dfrac{G \cdot F_S^{H\,I}}{E_S \cdot A_S}} = \frac{F_{S,p}^{H\,I}}{1+K} = \beta \cdot F_{S,p}^{H\,I} \tag{7}$$

Where

$$K = \frac{F_S^{H\,I} \cdot G}{E_S \cdot A_S}$$
$$\beta = \frac{1}{1+K} \tag{8}$$

The characteristic geometric value of G according to the expressions (6) and (7) is the influence of the geometry of the cable in a closed form on the rigidity of the system. This size can be represented as the “geometric stiffness” E_SA_S/G of the true replacement rod of length f (Fig. 5), whose change in length Δf_z due to normal tensile force $F_{S,p}{}^H$ represents the change of the arrow of the cable Δf_S, with the corresponding rigidity E_SA_S under by the action of additional force $F_{S,p}{}^H$.

On the basis of the preceding one, it is concluded that the influence of Theorem II of the order β, for the known line $y^{I}(x)$ for the imaginary not deformed cable, can be determined through the total horizontal force F_S^H, as well as the geometric characteristics of G, and the stiffness of the stretching cable $E_S A_S$. K and G are directly proportional values, and this applies to all cable forms as characteristic parameters depending on f^2/l^2.

2.4 The stiffness of the strut type hybrid system for external load q

The rigidity of the hybrid system consists of partial stiffness, as follows: stiffness on elastic deformations of individual hybrid elements and rigidity due to the geometric distribution of system elements in relation to load reception. Based on the previously presented simplification of the problem of the influence of the Theory of second order on the force changes in the deformable cable, it is possible to derive the terms for the "geometric stiffness" of the cables within the strut systems of different geometric forms through the geometric characteristic G. Each achieved cable shape under a unique image of the load corresponds to one geometric characteristic value G which depends on the ratio f/l and not the absolute dimensions of the cable. According to expression (6) it is possible to calculate G for different forms of curved cable[4], where:

$\int_o^l \left(1+y'(x)^2\right)dx$ - elongation of the cable ΔL due to the effect of force $F_{S,p}^H = E_S \cdot A_S$,

$\frac{\Delta f}{\Delta L}$ - ratio by which is geometrically shown the shift of the arrow dependent on the change in the length of the cable,

$\frac{1}{f}$ - gives a non-dimensional value G.

If it is assumed that the normal tensile force is on the replacement rod $F_{S,p}^H = E_S \cdot A_S$, then

$$\Delta f = \frac{F_{S,p}^H \cdot f}{\frac{E_S \cdot A_S}{G}} = \frac{E_S \cdot A_S \cdot f}{\frac{E_S \cdot A_S}{G}} \Rightarrow \Delta f = G \cdot f \tag{9}$$

Using the geometric characteristic values G, the "geometric" rigidity of the hybrid system can be determined with respect to the characteristic shape of the cable, that is, the unique image of the transmission of the evenly distributed load q on the cable[5] (Table1).

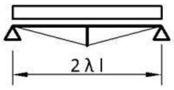
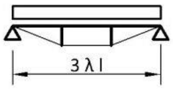
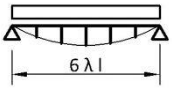
Reducing the deformability of the hybrid system in relation to the bending girder is caused by the activation of the rigidity of the cable, which consists of a geometric stiffness and stiffness on stretching.

The deformability of the hybrid system can be analyzed by looking at the hybrid system in the boundary cases of the deformability of the individual systems from which it is made. The characteristic observed cross sections are those in which the relationship between the individual basic systems is realized, i.e., the cross sections where the compatibility of the deformations of the

[4]The term is usually determined by the development of order and neglect of higher order members.

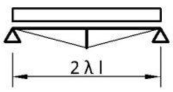
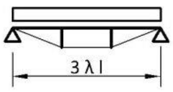
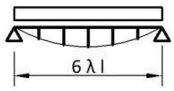
[5]The derived terms represent the rigidity of the cable within a hybrid system of different geometric forms and can be applied to test the behavior of the hybrid system under load. The geometric characteristic G was used besides the possibility of describing the geometric parameters of stiffness, also due to the calculation of transient quantities and deformations according to the theory of higher order.

Table 1 Geometric stiffness of the characteristic forms of cable of the hybrid system (Miljanović and Zlatar 2016)

Type of hybrid	Δf	$K_{G,S}$
 *	$\frac{F^{H\ II}_{S,p}}{E_S\cdot A_S}\cdot f\cdot G=\frac{q\cdot l^2}{8\cdot f\cdot E_S\cdot A_S}\cdot f\cdot\frac{l^2}{4\cdot f^2\cdot\cos^3\alpha}$	$32\cdot f^2\cdot\cos^3\alpha\cdot E_S\cdot A_S$
 **	$\frac{F^{H\ II}_{S,p}}{E_S\cdot A_S}\cdot f\cdot G=\frac{q\cdot l^2}{8\cdot f\cdot E_S\cdot A_S}\cdot f\cdot\frac{l^2\cdot\left(1+\cos^3\alpha\right)}{9\cdot f^2\cdot\cos^3\alpha}$	$\frac{72\cdot E_S\cdot A_S\cdot f^2\cdot\cos^3\alpha}{(1+\cos^3\alpha)}$
 ***	$\frac{F^{H\ II}_{S,p}}{E_S\cdot A_S}\cdot f\cdot G=\frac{q\cdot l^2}{8\cdot f\cdot E_S\cdot A_S}\cdot f\cdot\left(1+\frac{3}{16}\cdot\frac{l^2}{f^2}\right)$	$\frac{128}{3}\cdot f^2\cdot E_S\cdot A_S$

Assumptions:
∗A new deformed cable shape is approximately parallel to the initial default shape;
∗∗ The horizontal component of the cable elongation at the load point is neglected;
∗∗∗An approximate expression can be used for forms where $f/l\leq 0{,}25$, where the effects by the Theory of the second order are taken with sufficient accuracy.

Table 2 The value of the real force in the strut, and real deflection in the characteristic section

Type of hybrid	$V_{hyb,stv}\ (q_{uk})$	$\omega_{hyb}\ (q_{uk})=\omega_n(q_{uk})-\omega_n(V)+\Delta f$
	$\frac{q_{uk}\cdot l^2\cdot\sin\alpha}{4\cdot f}\left(1-\frac{48\cdot E_n\cdot I_n}{32\cdot f^2\cdot\cos^3\alpha\cdot E_S\cdot A_S}\right)$	$\frac{5}{384}\cdot\frac{q_{uk}\cdot l^4}{E_n\cdot I_n}-\frac{V_{hyb,stv}\cdot l^3}{48\cdot E_n\cdot I_n}$
	$\frac{q_{uk}\cdot l^2\cdot\sin\alpha}{8\cdot f}\cdot\left(1-\frac{9\cdot\left(1+\cos^3\alpha\right)\cdot E_n\cdot I_n}{40\cdot f^2\cdot\cos^3\alpha\cdot E_S\cdot A_S}\right)$	$\frac{11}{972}\cdot\frac{q_{uk}\cdot l^4}{E_n\cdot I_n}-\frac{5\cdot V_{hyb,stv}\cdot l^3}{162\cdot E_n\cdot I_n}$
	$\frac{q_{uk}\cdot l^2}{4\cdot f}\cdot\sin\alpha_3\cdot\left(1-\frac{270\cdot E_n\cdot I_n}{176\cdot f^2\cdot E_S\cdot A_S}\right)$	$\frac{5}{384}\cdot\frac{q_{uk}\cdot l^4}{E_n\cdot I_n}-\frac{176}{2304}\cdot\frac{V_{hyb,stv}\cdot l^3}{E_n\cdot I_n}$

ω_{hyb} (q_{uk})-the deflection of the hybrid system, where another member of the expression gives a qualitative effect of strut a particular hybrid system.
ω_n (q_{uk})-the deflection of the girder from total load with a yielding cable,
ω_n (V)-the deflection of the girder in the middle of the span of the total activated force in the strut for an unyielding cable,
Δf -vertical deformation of the cable due to the effect of the full force of pressure in strut.

individual systems within the hybrid system can be established (Table 2).

According to the given expressions (Table 2), it can be concluded that the total stiffness of the system cannot be expressed only by adding the individual stiffness of the system elements. The expressions for geometric stiffness within the total stiffness of the cable (Table 1) can be used with sufficient accuracy, with an approach that takes into account any geometric "adjustment" of the system to a given load.

3. Pre-stressed hybrid systems

The strut type hybrid system has been supported by various geometric shapes, which are influenced by: the number and strut schedule, the tension of the tension in the cable the stream of tension in the cable, or the shape and position of the cable in relation to the girder and the cross-section of the girder. When selecting a girder all of the listed parameters can vary geometrically, which has consequences on the behavior of the load system when they are directly affecting the system's stiffness. In addition to geometric parameters, the rigidity of the hybrid system is influenced by the variation of the parameters of the properties of the embedded materials and their mutual relations.

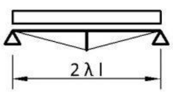
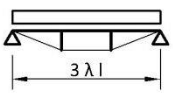
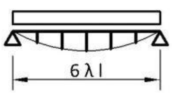
3.1 Participation of basic systems in load transfer

For further analysis of the stiffness ratio of hybrid systems, as well as constellation suitable for pre-stressing, it is necessary to consider the participation of basic systems in load transfer. The behavior of the hybrid system can be expressed by the effect of strut, i.e., by the force created in strut. For the analysis of the influence of the basic parameters of the hybrid system in the transmission of the total load, the relative value of the force in the struts of the considered hybrid systems is introduced (Table 3).

The condition $V_{hyb,stv}=0$, from the expression for the total deformation of the hybrid system (Table 2), means that the strut has no effect on the deformability of the system. In this case, according to the terms for the actual force in strut (Table 2), this condition satisfies the equilibrium of the partial stiffness, the bending of girder and the stretching cable in its geometric form. According to the previous one, it is concluded that for the equal redistribution of loads between the girder and the cable, the hybrid system should be pre-stressed to reduce deformability.

Constant parameters E_n/E_S were taken to analyze the impact of the form, and f/l, h/l and A_n/A_S vary. On the ordinate of the diagram shown in Fig. 5 are given the relative values of the force in support of the total system load, and for different system forms, which is shown at abscissa and expressed by the values of the mutual strut distance of the system.

Table 3 Relative value of the force in support to the total system load (Miljanović and Zlatar 2016)

Type of hybrid	$V_{hyb,stv}/q_{uk}l$
	$\frac{l}{4\cdot f}\cdot\frac{1}{\sqrt{1+\frac{l^2}{4\cdot f^2}}}\cdot\left(1-\frac{1}{2}\cdot\frac{E_n}{E_S}\cdot\frac{A_n}{A_S}\cdot\frac{h^2}{l^2}\cdot\frac{l^2}{4\cdot f^2}\cdot\frac{1}{\sqrt{\left(\frac{4f^2}{l^2}+1\right)^3}}\right)$
	$\frac{l}{8\cdot f}\cdot\frac{1}{\sqrt{1+\frac{l^2}{9\cdot f^2}}}\cdot\left[1-\frac{3}{20}\cdot\frac{E_n}{E_S}\cdot\frac{A_n}{A_S}\cdot\frac{l^2}{8\cdot f^2}\cdot\frac{h^2}{l^2}\cdot\left(\sqrt{\left(\frac{9\cdot f^2}{l^2}+1\right)^3}+1\right)\right]$
	$\frac{l}{4\cdot f}\cdot\frac{1}{\sqrt{1+9\frac{l^2}{4\cdot f^2}}}\cdot\left(1-\frac{90}{176}\cdot\frac{E_n}{E_S}\cdot\frac{A_n}{A_S}\cdot\frac{h^2}{l^2}\cdot\frac{l^2}{4\cdot f^2}\right)$

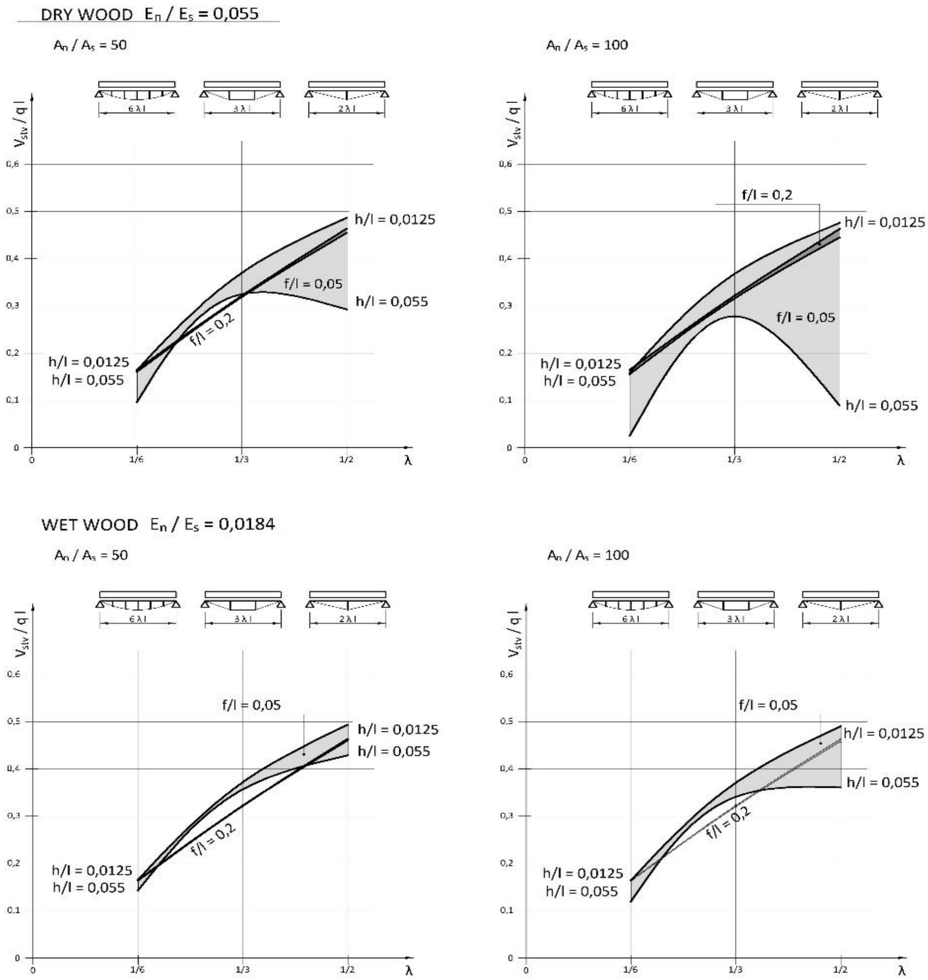


Fig. 5 The size of the relative force of the pressure in the strut at the place of maximum deflection depending on the form

The change in the relative force of the pressure in strut is shown in Figure 5., whereby for the suitable constelations of the hybrid system the value of the strut force is between the two envelopes, as follows:

-which from the Table 2 means that it is the equalization of the individual stiffness of the bending girder and the stretching cable, and for $V_{hyb,stv}/q_{uk}\cdot l \leq 0$, the rigidity of the bending girder is larger and the girder transmits most of the load (which practically allows a bit of *f/l*),

- $V_{hyb,stv}/q_{uk}\cdot l=1$, meaning that the rigidity of the cable is considerably greater than the rigidity of the bending girder, and the load of the system is largely redistributed to the cable.

Based on Fig. 5 and previous considerations it is concluded that:

- for the small geometric heights of the hybrid system of the considered forms, the greatest effect of strut is achieved with the single strut type system,
- the single strut type system shows the highest sensitivity to the change in the geometric height of the hybrid and the rigidity of the bending girder.

3.2 Influence of the cross-sectional shape of the girder

In further analysis, the change in the force of pressure in the strut, or the effect of strut from the point of deformability of the system, was considered, and depending on the shape of the cross-section of the wooden girder. For the purposes of this analysis, two final cases of the cross-sectional shape of the girder were taken into account, with regard to the realization of the bending rigidity, namely: a square and a split cross section, and expressed as a relationship of the positional and own moment of inertia. The size of the relative force of pressure in strut were observed, and for the boundary values of the height of the girder and the geometric height of the hybrid system, for certain relations of the surfaces of the cross-section of the girder and cable, and the modifications of the elasticity of the wooden girder due to the increase of the percentage of moisture of the wood were considered in particular (Fig. 6).

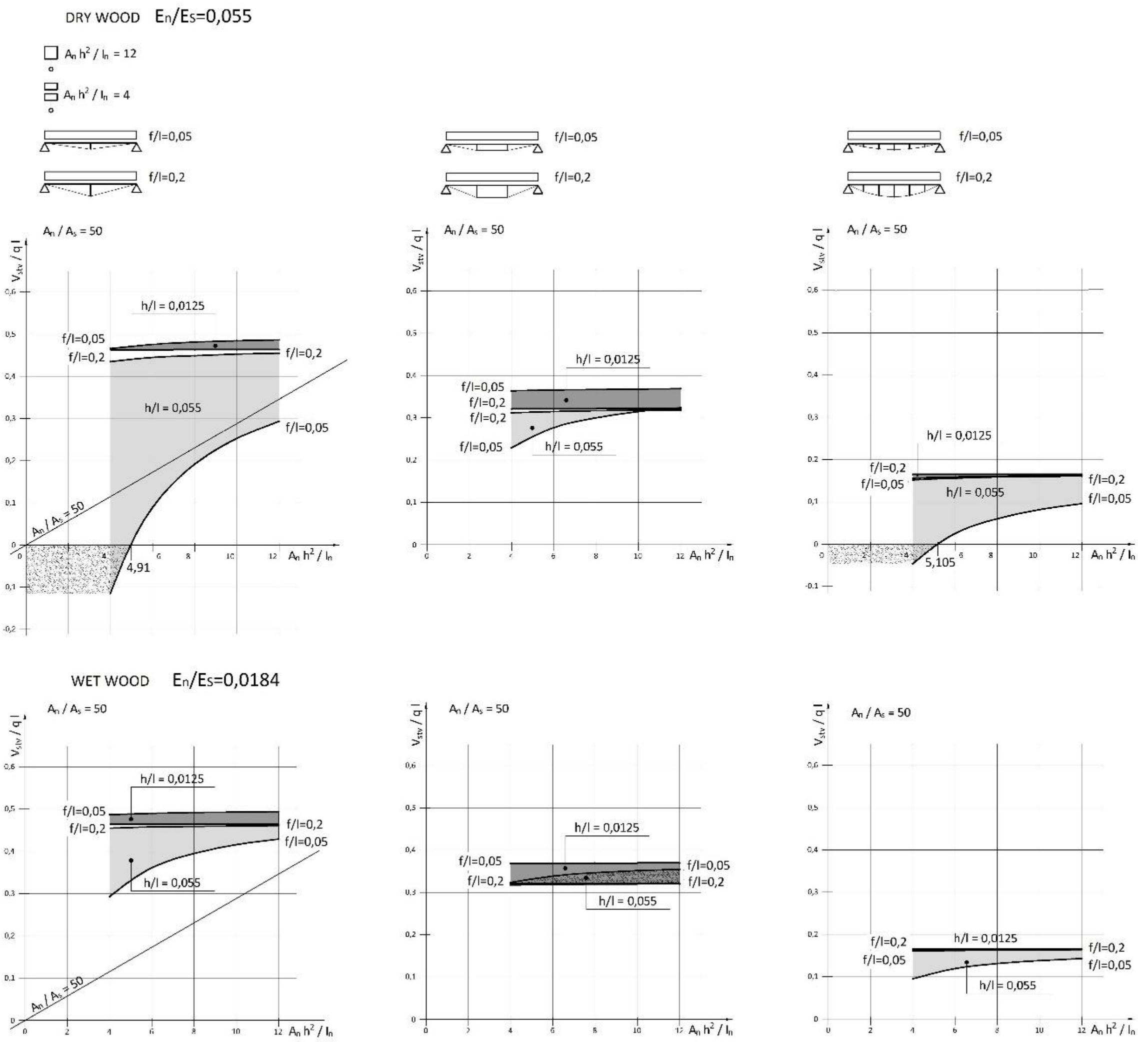
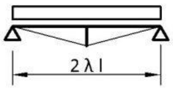
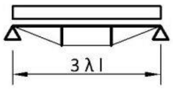
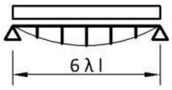


Fig. 6 Relative force in strut of changing the rigidity of the wooden girder

Table 4 Necessary stiffness relationships to activate the hybrid system (Miljanović and Zlatar 2016)

Type of hybrid	$k_n(V)/k_S(V)$
	$\frac{3}{2}\cdot\frac{E_n}{E_S}\cdot\frac{I_n}{A_n\cdot h^2}\cdot\frac{A_n}{A_S}\cdot\frac{h^2}{l^2}\cdot\frac{l^2}{f^2}\cdot\frac{1}{\sqrt{\left(\frac{4\cdot f^2}{l^2}+1\right)^3}}\le 1$
	$\frac{9}{40}\cdot\frac{E_n}{E_S}\cdot\frac{I_n}{A_n\cdot h^2}\cdot\frac{A_n}{A_S}\cdot\frac{h^2}{l^2}\cdot\frac{l^2}{f^2}\cdot\left(\sqrt{\left(\frac{9\cdot f^2}{l^2}+1\right)^3}+1\right)\le 1$
	$\frac{270\cdot E_n\cdot I_n}{176\cdot f^2\cdot E_S\cdot A_S}=\frac{270}{176}\cdot\frac{E_n}{E_S}\cdot\frac{I_n}{A_n\cdot h^2}\cdot\frac{A_n}{A_S}\cdot\frac{h^2}{l^2}\cdot\frac{l^2}{f^2}\le 1$

$k_n(V)$-the rigidity of the girder for the action of the activated force in strut,
$k_S(V)$- stiffness of the cable for the effect of the force in strut,
$k_n(V)/k_S(V)\le 1$-a condition for given system burdened by external load acts as a hybrid system, that is, it performs the redistribution of load on the girder and cable in certain relationships.

Hybrid systems with a rigid girder and shallow arrow can achieve robust stiffness relations when behavior is reduced to the behavior of the “basic” girder system, so that the cable does not make sense (Fig. 6).

Thus, for example, for the system of single shallow strut f/l=0,05 and high girder height h/l=0,055, and for the ratio of cross-sectional surfaces A_n/A_S=50 and to $A_n\cdot h^2/I_n$=4,91, the cable in the system does not have sufficient role of carrying the load. Thus, for the same type of hybrid system with a split cross-section of the girder, where it is $A_n\cdot h^2/I_n$=4, the cable makes sense only for the geometric height $f/l\geq$0,05559.

In Table 4 are given expressions for finding the ratio of the individual stiffness of the girder to the bending and cable to the stretching in the characteristic form of the hybrid system, and for the action of the activated force in strut of the total load.

Finally, for all the analysis carried out, the changes in the behavior of different forms of the strut type hybrid system with a glued laminated wooden girder and a steel cable, the same characteristics of the cross-section of the girder, the geometric heights and the appropriate relationship of the cross-sectional surfaces of the hybrid elements can be summarized as follows:

- For hybrids of small geometric heights with increasing rigidity of the girder, increasing the cross-sectional height or breaking the cross-section of the girder, the cable does not have a sufficient role in carrying the load, that is, for the described limit values there is no role in reducing the deformability of the system. It means that a strut type hybrid system makes sense if the rearrangement of the loads between the girders and cables is done so that the cable gets a greater role in carrying the load;
- With the increase in the ratio of the cross-sectional area of the individual elements, the girder becomes even more expressed in the load transfer, and it can be concluded that the ratio of the stiffness of the “basic” systems is crucial for the activation of the hybrid system;
- For examined forms of hybrid systems in which the small geometric height, the highest sensitivity in the change in the stiffness ratio and the greatest deformability shows the single-strut system, and given the proportion of the geometric stiffness of the cable in the rigidity of the system and the number of strut, i.e., the possibility of occurrence of major deformations of non-

supported cross- increase of percentage of moisture of wood;
- For all forms of hybrid systems, the high rigidity of the girder to the bending is disadvantageous when the system has a small geometric height, and for boundary cases when the cable does not have a sufficient role of carrying the load in the system, that is, it does not make sense without pre-stressing;
- For all forms of hybrid systems it is possible to apply pre-stressing in order to increase the rigidity of the cable in the system and more correctly redistribute the load to the elements of the system;
- By introducing the pre-stressing force, the own frequency of the cable increases, provided that it is not sub-dimensioned. If, however, the rigidity of the bending girder is greater than the rigidity of the cable of a particular form, the activation of the hybrid as a system is achieved by pre-pressing.
- "Total" pre-stressing can be applied to systems where, due to an unfavorable constellation, it is not, or is not sufficiently activated, or "tapping" in systems in which we want to increase the load capacity or reduce the deformability of the system.

3.3 Change in the stiffness of the hybrid system in the case of the pre-stressing force

For the use of pre-stressing, the most favorable hybrid system constellation is determined according to the share of the cable in carrying the load. If the rigidity of the girder is greater than the total stiffness of the cable in the system, the pre-stressing is further activated by the cable and the stiffness ratio of individual basic systems is changed.

From the point of view of increasing the load capacity of hybrid systems, pre-stressing is considered in the order of failure of the characteristic cross-section. For hybrid systems with higher girder rigidity and low geometric heights, due to the larger partaking of load bearing girder, the influence of the bending moment on normal force is more dominant. The pre-stressing of this system is obtained by increasing the normal force, and the bending moments are equalized by the length of the girder. Then the system gets more favorable picture of the stressing, because the limit state of the load in the characteristic sections is achieved, first of all by the plasticization of the pressed zone, and then by rapidly canceling the tightened zone, which represents a delayed break in relation to the system without pre-stressing.

In order to find the size of the effective pre-stressing force, from the aspect of reducing the deformability of the hybrid system, the boundary cases of deformation of the basic systems from which it is made are observed, and due to the effect of the force of pre-stress. If the hybrid system is loaded only by the force of pre-stressing introduced into the narrow stretch, the pressure force in the strut is activated which causes vertical deformation of the girder and cable. If these deformations are observed for the extreme cases of behavior of these two basic systems, depending on the stiffness, a direct dependence of the length of the cable elongation and the hybrid system deflection is obtained.

If this approach is applied to different geometric forms of hybrid systems, it is possible to obtain accurate expressions for finding the effective size of the cable elongation depending on the required girders deflection at the point of strut (Miljanović and Zlatar 2015, Miljanović and Zlatar 2017). The stretching size of the cable is pre-stressed without loss due to elastic and plastic deformations of elements and geometric changes in the system.

In this paper, the deformability of hybrid systems of various forms and the relationship of partial stiffness to the effect of even load is analyzed, and with the setting of the conditions that the

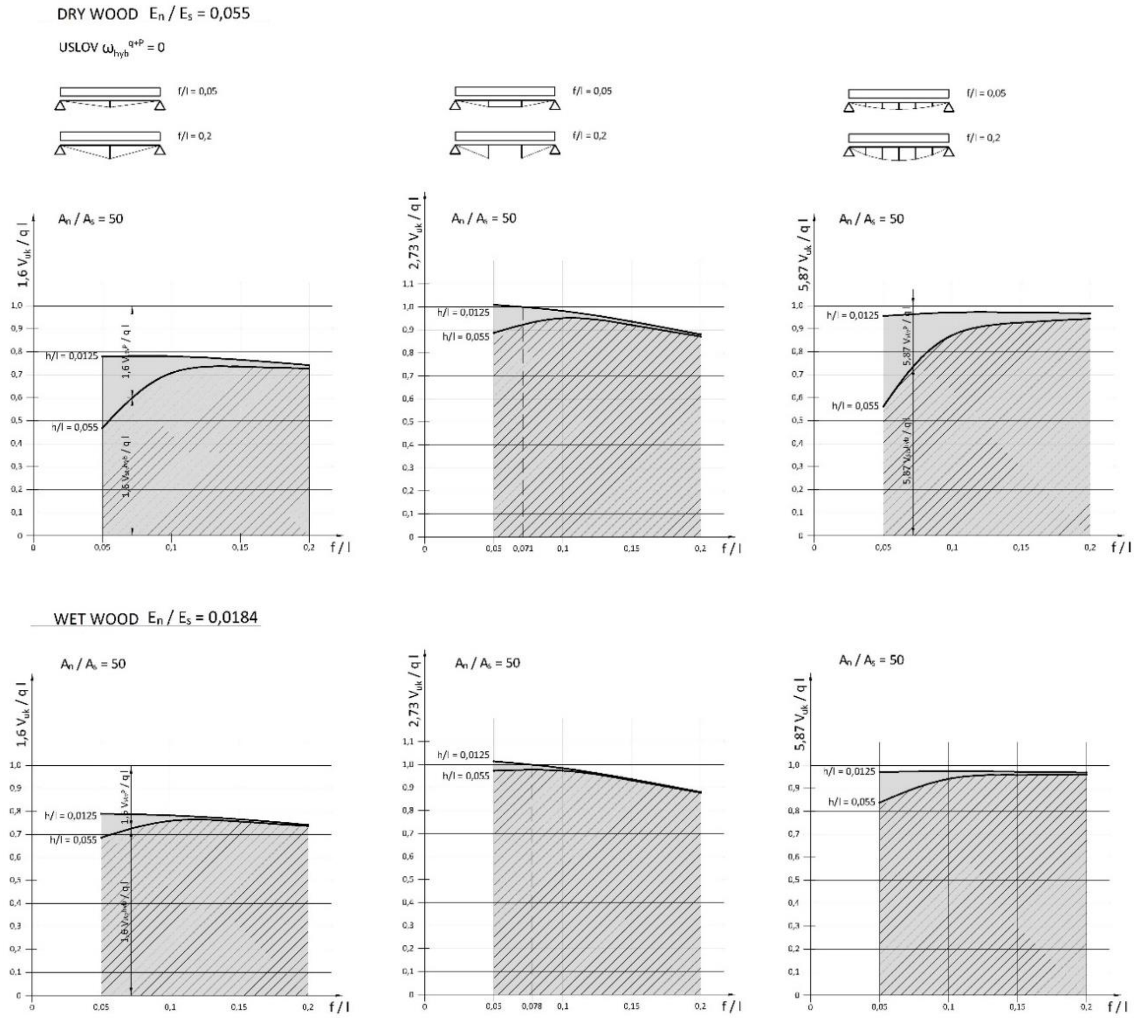


Fig. 7 Relative values of the actual force of the pressure in strut from the external load q and the necessary effective force of pre-stressing, provided that the deflection of the pre-stressed system ω (x) = 0 at the point of strut

characteristic strut cross sections of the girder due to the strut effect do not have vertical displacements, that is, the deformation in the characteristic cross sections of the load girder *q* is equal to zero. Practically, the obtained expressions represent the values of the full force in the strut needed to achieve the conditions $\omega(x)$=0, and the actual value of the force in strut depends on the relationship of the partial stiffness of the system.

In Fig. 7, the actual values of the force achieved in strut of a pre-stressed hybrid system loaded with even load *q* are given, so that the values of the actual force in strut for external load are only determined by the conditions of the vertical incompatibility of the cross section and the ratio of the partial stiffness, and the value of the force in strut that needs to be achieved by pre-stressing makes the difference to full value 1. Based on these diagrams it is possible to determine the necessary effective pre-stressing force for hybrid systems of different constellations (with the condition that the deflection is at the characteristic cross section 0), that is, the necessary elongation of the cable according to (Miljanović and Zlatar 2015, Miljanović and Zlatar 2017).

4. Conclusions

If hybrid systems are observed, from the aspect of increasing the load capacity, with different rigidity girders it can be concluded that:

• The uneven distribution of momentum is expressed in very rigid cross-sectional girders, as well as for the use of wooden girders, especially when changing the percentage of moisture of the wood, within the hybrid system, with the expressed rigidity of cable support,

• Pre-stressing more effectively affects the system's stiffness and the uniformity of the bending moment under the different load schemes, which is the better mutual compliance of the partial stiffness of the hybrid system, i.e. bending girder and stretching cable.

From the aspect of reducing the deformability:

• For all forms of hybrid systems, the strong stiffness of the bending girders is disadvantageous when the system has a small geometric height, and for boundary cases when the cable does not have a sufficient role of carrying the load in the system, that is, it does not make sense without pre-stressing;

• For all forms of hybrid systems it is possible to apply pre-stressing in order to increase the rigidity of the cable in the system and more correctly redistribute the load to the elements of the system;

• By increasing the percentage of moisture, the rigidity of the girder is reduced, and therefore the load of the cable is increased. It means that for hybrid systems of low geometric height and high rigidity of the bending girders, the increase in the percentage of moisture has a positive effect, that is, triggers the activation of the system as a hybrid.

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