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Computer modeling of elastoplastic stress state of fibrous composites with hole

Askhad M. Polatov[*1], Akhmat M. Ikramov[1a] and Abduvali A.Khaldjigitov[2b]

[1]Department of Mathematics, National University of Uzbekistan, 4 University Str., Tashkent 100174, Republic of Uzbekistan
[2]Samarkand branch of Tashkent University of Information Technologies, 100140, Sh. Mirzo str. 47-a Samarkand, Uzbekistan

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Abstract. The paper represents computer modeling of the deformed state of physically nonlinear transversally isotropic bodies with hole. In order to describe the anisotropy of the mechanical properties of transversally-isotropic materials a structurally phenomenological model has been used. This model allows representing the initial material in the form of the coupled isotropic materials: the basic material (binder) considered from the positions of continuum mechanics and the fiber material oriented along the anisotropy direction of the original material. It is assumed that the fibers perceive only the axial tensile-compression forces and are deformed together with the base material.
To solve the problems of the theory of plasticity, simplified theories of small elastoplastic deformation have been used for a transversely-isotropic body, developed by B.E. Pobedrya. A simplified theory allows applying the theory of small elastoplastic deformations to solve specific applied problems, since in this case the fibrous medium is replaced by an equivalent transversely isotropic medium with effective mechanical parameters. The essence of simplification is that with simple stretching of composite in direction of the transversal isotropy axis and in direction perpendicular to it, plastic deformations do not arise. As a result, the intensity of stresses and deformations both along the principal axis of the transversal isotropy and along the perpendicular plane of isotropy is determined separately. The representation of the fibrous composite in the form of a homogeneous anisotropic material with effective mechanical parameters allows for a sufficiently accurate calculation of stresses and strains. The calculation is carried out under different loading conditions, keeping in mind that both sizes characterizing the fibrous material fiber thickness and the gap between the fibers-are several orders smaller than the radius of the hole. Based on the simplified theory and the finite element method, a computer model of nonlinear deformation of fibrous composites is constructed. For carrying out computational experiments, a specialized software package was developed. The effect of hole configuration on the distribution of deformation and stress fields in the vicinity of concentrators was investigated.

Keywords: FEM; transversally-isotropic medium; computational experiment; fibrous composite; elasticity; plasticity; hole; strain; stress

∗Corresponding author, Professor, E-mail: asad3@yandex.ru
[a]Associate Professor, E-mail: axmat3@yandex.ru
[b]Professor, E-mail: akhald@mail.ru

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1. Introduction

Recently, the development of science and computer technologies allows us to create computer models that could give us a clear reflection of the stress state of composite structures. Special attention has been given to the study of the influence of the structural features of fibrous materials and stress concentrators.

The development of models and solving problems of physically nonlinear deformation of fibrous composite materials are devoted to the work of many authors, a list of which is cited in (Pobedrya 1997). Where sets out the basic provisions (postulates) of continuum mechanics. Along with the classical models, new models are considered, where the material is a composite and the connectedness of mechanical fields has been taken into account. An elastoplastic strain analysis is carried out for fibrous composites by using numerical modeling. (Polatov 2013) Application of homogeneous transversely-isotropic model was chosen based on a problem solution of a square plate with a circular hole under uniaxial tension. The results obtained in this study matches the solution of fiber model trial problem, as well as to analytical solution. Further, numerical algorithm and software has been developed, based on simplified theory of small elastic strains for transversely-isotropic bodies and FEM. According to a research (Yang and Chow 1998) the results of the experimental and numerical finite element determination of the indices of the anisotropic stress-strain state. The problem of uniaxial stretching of graphite-epoxy layered composite rectangular plates with unidirectional reinforcing carbon fibers that contain a central circular hole has been solved. In monograph (Jain and Mittal 2008) was given, an analysis of the concentration and distribution of stresses in isotropic, orthotropic and layered composite plates with a central round hole subjected to transverse static load. Moreover, research paper (Abdul and Ishrat 2016), represents the effect of a stress concentrator in a rectangular plate. Where the concentrator is a round hole. The values of stress concentration factors in the vicinity of the hole, are given and obtained by the finite element method. In a research (Tomashevskiy 2011), an algorithm and a solution to the problem were considered by taking into account the physical nonlinearity of bodies based on the theory of small elastoplastic deformations. It is noted that the solution process using the deformation theory is performed much faster than within the framework of the flow theory. It is known that the presence in the bodies of structural holes significantly affects the deformation in their vicinity. In the paper (Yazici 2007) the elastoplastic analysis of the stresses of an isotropic plate in the vicinity of a square hole is carried out. The boundaries of the field of plastic stresses around conformally displayed square holes are searched using the elastic equations of G.N. Savin. A finite element approach is used to find numerical solutions. Theoretical and finite-element elastoplastic solutions for isotropic plates with square holes with corners rounded are compared. For the description of elastic-plastic strain process of fibrous composites based on averaging method different versions of the plasticity theory are proposed, in which the composite material is replaced by a homogeneous anisotropic medium (Bravo-Castillero *et al.* 2005). The simplified theory of small elastoplastic deformations opens up possibilities for solving specific applied problems. In the case of bodies with holes, it allows for fairly accurate calculation of stresses and strains under various types of loading. Provided that both sizes characterizing the fibrous material - the fiber thickness and the width of the gap between the fibers - are several orders of magnitude smaller than the radius of the hole (Karpov 2002). To calculate the values of the effective mechanical parameters of fibrous materials uses relations derived from asymptotic methods. Where the radial interaction of the components (matrices and fibers) is also taken into account, due to the difference in their Poisson ratios (Andrianov *et al.* 2007). Since the deformation of the matrix ensures the loading of high-strength fibers, the consideration of its plastic deformations is an important task as well. This makes the study of the stress-strain state of fibrous composites most complete (Vasil'yev *et al.* 1990).

Composite materials mechanics is one of very rapidly developing research areas, which obtained significant theoretical and experimental results. However, non-linear strain processes of composite materials with concentrators are not well investigated yet. Modern developments in mathematical modeling of transversely-isotropic materials' elastic-plastic strain process cannot be considered as complete. Wide implementation of composite materials has led to the emergence of new fields in science related to the study of elastic-plastic materials strain (Lee *et al.* 2012). In connection with this, in order to have reliable composite materials strength evaluation it is relevant to use the modern computer technologies (Meer and Sluys 2009).

In this paper is carried out computer modeling of elastoplastic stress state of fibrous composites with hole. A computer model of deforming physically nonlinear transversely isotropic bodies has been developed. Solutions of elastic and elastoplastic problems of deforming structures made of fibrous composites are obtained. The influence of the configuration of elliptical holes and cracks on the stress-strain state of the bonding matrix of fibrous composites in the vicinity of the stress concentrator was studied and discussed.

Structural-phenomenological model is used to describe the anisotropy of the mechanical properties of transversely isotropic materials. The starting material is represented as a complex of two isotropic materials that work together. The main material (binder) and the material of the fibers, which are oriented along the anisotropy direction of the source material. In this case, the binder is considered from the standpoint of continuum mechanics. The material of the fibers is based on the assumption that the fibers perceive only axial tensile – compression forces and are deformed together with the binder. To solve the problem of the theory of plasticity, a simplified theory of small elastoplastic deformations is used for a transversely isotropic body (Pobedrya 1984). Simplification consists in replacing the original fibrous medium with an equivalent transversely isotropic medium with effective mechanical parameters. As a result, with simple stretching of the composite in the direction of the axis of transversal isotropy and in the direction perpendicular to it, plastic deformation does not occur. The values of stress intensity and deformations are determined separately both along the main axis of the transverse isotropy $Oz-(Q_u, q_u)$, and in the perpendicularly located isotropy plane $Oxy-(P_u, p_u)$.

2. Problem statement and solution method

The elastoplastic medium of inhomogeneous solid material is investigated. The medium consists of two components: fibers and a matrix (binder) material. The matrix material ensures the joint operation of reinforcing elements. To solve the problem, the theory of small elastoplastic deformations is used for a transversely isotropic medium (Pobedrya 1984).

The general formulation of the boundary value problem of the theory of elasticity for anisotropic bodies includes:

- equilibrium equations

$$\sigma_{ij,j} + X_i = 0, \quad x_i \in V \tag{1}$$

- generalized Hooke's law

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{2}$$

- Cauchy relations

$$\varepsilon_{kl} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right) \tag{3}$$

- boundary conditions

$$u_i\big|_{\Sigma_1} = u_i^0, \qquad x_i \in \Sigma_1, \quad \sum_{j=1}^{3}\sigma_{ij}\, n_j\big|_{\Sigma_2} = S_i^0, \qquad x_i \in \Sigma_2, \tag{4}$$

where u_i- is the component of the displacement vector; X_i, S_i^0- bulk and surface forces; Σ_1, Σ_2- part of the volume Σ bounding surface V; n_j- external normal to the surface Σ_2; C_{ijkl}- tensor of elastic constants.

In the simplified theory of small elastoplastic deformations of a transversely isotropic medium, the generalized Hooke law (2) takes the following form

$$\tilde{\sigma} = (\lambda_2 + \lambda_4)\tilde{\theta} + \lambda_3\varepsilon_{33}, \qquad \sigma_{33} = \lambda_3\tilde{\theta} + \lambda_1\varepsilon_{33}, \qquad P_{ij} = \frac{P_u}{p_u}p_{ij}, \qquad Q_{ij} = \frac{Q_u}{q_u}q_{ij}, \tag{5}$$

where

$$P_u = 2\lambda_4(1-\pi(p_u))p_u, \quad Q_u = 2\lambda_5(1-\chi(q_u))q_u \tag{6}$$

$\pi(p) = \bar{\lambda}_1(1-p_s/p)$ and $\chi(q) = \bar{\lambda}_2(1-q_s/q)$ - A. Ilyushin's plasticity functions, whose values in the elastic zone are equal to zero.

$\bar{\lambda}_1$, p_s - hardening coefficients and elastic deformation limits in the isotropy plane Oxy.

$\bar{\lambda}_2$, q_s - hardening coefficients and elastic deformation limits along the isotropy axis Oz.

In the elastic area, the parameters σ_{ij} are determined from Hooke's law. In the area of plastic deformation, the parameters σ_{ij} are determined on the basis of the A. Ilyushin's deformation theory;

λ_i - elastic constants of a transversally-isotropic medium;

(P_{ij}, p_{ij}) - components of the deviator parts of the transversely-isotropic stress and strain tensors in the isotropy plane Oxy;

(Q_{ij}, q_{ij}) - components of the deviator parts of the transversely-isotropic stress and strain tensors along the isotropy axis Oz (Pobedrya 1984)

$$\begin{aligned} P_u &= \sqrt{\frac{1}{2}P_{ij}P_{ij}} = \frac{\sqrt{2}}{2}\sqrt{(\sigma_{11}\text{-}\sigma_{22})^2 + 4\sigma_{12}^2}, \\ p_u &= \sqrt{\frac{1}{2}p_{ij}p_{ij}} = \frac{\sqrt{2}}{2}\sqrt{(\varepsilon_{11}\text{-}\varepsilon_{22})^2 + 4\varepsilon_{12}^2}, \end{aligned} \tag{7}$$

$$\begin{aligned} Q_u &= \sqrt{\frac{1}{2}Q_{ij,}Q_{ij}} = \sqrt{\sigma_{13}^2 + \sigma_{23}^2}, \\ q_u &= \sqrt{\frac{1}{2}q_{ij}q_{ij}} = \sqrt{\varepsilon_{13}^2 + \varepsilon_{23}^2}, \end{aligned} \tag{8}$$

where

$$p_{ij}=\varepsilon_{ij}+\frac{\tilde{\theta}}{2}\left(\delta_{i3}\delta_{j3}-\delta_{ij}\right)+\varepsilon_{33}\delta_{i3}\delta_{j3}-\left(\varepsilon_{i3}\delta_{j3}+\varepsilon_{3j}\delta_{i3}\right) \tag{9}$$

$$q_{ij}=\varepsilon_{i3}\delta_{j3}+\varepsilon_{3j}\delta_{i3}-2\varepsilon_{33}\delta_{i3}\delta_{j3}, \qquad \tilde{\theta}=\varepsilon_{11}+\varepsilon_{22}, \tag{10}$$

$$P_{ij}=\sigma_{ij}+\tilde{\sigma}\left(\delta_{i3}\delta_{j3}-\delta_{ij}\right)+\sigma_{33}\delta_{i3}\delta_{j3}-\left(\sigma_{i3}\delta_{j3}+\sigma_{3j}\delta_{i3}\right) \tag{11}$$

$$Q_{ij}=\sigma_{i3}\delta_{j3}+\sigma_{3j}\delta_{i3}-2\sigma_{33}\delta_{i3}\delta_{j3}, \qquad \tilde{\sigma}=\left(\sigma_{11}+\sigma_{22}\right)/2. \tag{12}$$

The mechanical parameters of the transversely isotropic material are related to the modules λ_i by the following relations

$$\lambda_1=E'_{ef}\left(1-\mu_{tf}\right)/l, \quad \lambda_2=E_{ef}(\mu_{ef}+k\mu_{ef}{}^2)/\left[(1+\mu_{ef})/l\right], \quad \lambda_3=E_{ef}\mu'_{ef}/l,$$
$$\lambda_4=G_{ef}=E_{ef}/\left[2(1+\mu_{ef})\right], \quad \lambda_5=G'_{ef}, \quad l=1-\mu-2\mu'^2_{ef}k, \quad k=E_{ef}/E'_{ef}.$$

where μ_{ef}- effective Poisson's ratio and E_{ef}- effective elastic moduli in the isotropy plane of the transversely isotropic material; μ'_{ef}- effective Poisson ratios and E'_{ef} - effective elastic moduli along the isotropy axis of the transversely isotropic material.

It is assumed that the transversal isotropy plane coincides with the plane *Oxy*, and the isotropy axis with the axis *Oz*. The studied medium is homogeneous with effective mechanical parameters both along the isotropy axis and along the isotropy plane. Based on this, the iterative process of the initial stress method is used to solve the elastoplastic problem (Brovko *et al.* 2011).

3. Verification of the proposed approach

Software provides users with the universal tools for preparation of the process routine tasks automation, data processing, and storage (Polatov and Nodirjanova 2014). In the software structure's finite element model formation is considered, finite elements in the type of quadrangular parallelepiped has been used. Computational experiments were performed on the basis of FEM. The reliability and correctness of the proposed homogeneous model is confirmed by the coincidence of the results obtained on its basis, in the case of an elastic problem, with the results of solving a test problem of fibrous structure stretching in the form of a square plate with a central circular hole along the fiber (Karpov 2002). And also, on the model of a homogeneous transversely isotropic material with effective elastic mechanical characteristics of boron/aluminum (Polatov 2013). In the case of an elastoplastic problem, the calculation results coincide with the results of solving the problem of stretching a square plate of fibrous material based on the variation-difference method (Khaldjigitov 2003). The geometrical dimensions of structures, stress concentrators and nodes displacement are dimensionless relative to the length side of a square plate.

First test. To test the algorithm, the results of the calculation of the problem of two-sided compression of a transversely isotropic elastoplastic single cube from a magmatic with uniformly distributed loads $P_{xx}=\pm 10^4$ MPa along the axis OX are considered. Construction material has a linear hardening. The main axis of transversal isotropy is directed along the OZ axis. For a given

Table 1 Comparison of calculation results

Model	u	σ_{xx} [MPa]	p_u	P_u[MPa]
Transversely-isotropic	-10.20*10^{-2}	-0.997*10^{4}	0.1494	0.705*10^{4}
(Khaldjigitov 2003)	-09.57*10^{-2}	-1.003*10^{4}	0.1500	0.707*10^{4}

Table 2 Comparison of calculation results

Model	σ_{xx} [MPa]	σ_{zz} [MPa]	τ_{zx} [MPa]	τ_{zx} [MPa]
Fibrous (Karpov 2002)	5.91	63.16	-9.12	1.81
Transversely-isotropic	5.65	60.32	-9.83	1.64

load, a uniaxial stress state is observed throughout the cube. The material is completely in plastic state. In the first line of Table 1 presents the results of solving the above elastoplastic problem. To substantiate the reliability of the obtained results of the calculation, the second line contains the solutions of a similar problem based on the variation-difference method (Khaldjigitov 2003). Comparison of results confirms the correctness of the results obtained and it should be noted that with uniaxial stress, there is a steady convergence of the iterative process.

Second test. To confirm correctness transversely isotropic model-elastic problem of stretching a plate with a central circular notch was solved.

The plate is stretched along the axis OZ by a uniformly distributed load $P_{zz}=\pm 10$ MPa. The studied material is boron/aluminum, which is a fibrous composite. The bonding material is D16 aluminium alloy, and as an armature - boron fibers directed along the axis OZ are used. The volume fraction of boron in the material v=60%.

For aluminium alloy: Young's modulus E = 7*10^4 MPa, Poisson's ratio μ = 0,32.

For boron fiber: E' = 3,9*10^5 MPa, Poisson's ratio μ' = 0,21.

For boron aluminum, the effective elastic characteristics are as follows:

E = 16*10^4 MPa, E' = 2,6*10^5 MPa, G' = 5,1*10^4 MPa (Karpov 2002).

For comparison, Table 2 shows the results of solving the problem using fibrous and transversely isotropic models. The proximity of the solutions suggests that the developed model is correct.

4. Finite element outcomes and discussion

4.1 Elastic calculation

To study the effect of an isolated hole on the body's stress-strain state of unidirectional composite, a three-dimensional elastic problem of deforming a rectangular plate (its height is 1, width – 0,5 and thickness – 0,1) is considered with a uniform uniaxial tension along the axis with a distributed load P_{zz} =100 MPa, applied on the lower and upper edges (Fig. 1). The parameters of σ_{ij} are determined from Hooke's law, and in relation (6) of the values of the functions of plasticity $\pi(p)$ and $\chi(q)$ are equal to zero. As the matrix material is used aluminium alloy D16 with the parameters: E=7,1·10^4 MPa (modulus of elasticity), μ=0,32 (Poisson's ratio), p_s=0,003. Boron fibers with characteristics are used as reinforcing elements: E'=39,7·10^4 MPa, μ' =0,21 (Vasil'yev *et al.* 1990). The fibers of the material are oriented along the axis Oz, the volume content of fibers

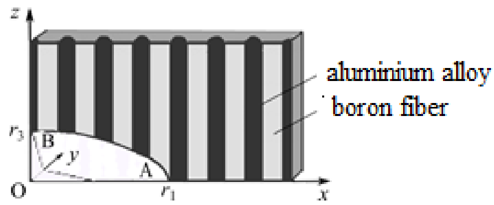

Fig. 1 The fourth part of the plate section on plane *Oxz*

Table 3 Parameters of stress-strain state at point *A* of a cracked plate

r_3/r_1	$u\cdot10^5$	σ_{xx}, MPa	σ_{zz}, MPa	τ_{zx}, MPa	p_u	P_u, MPa
4	–0,5710	4,215	5,5695	–0,6092	0,00151	1,972
2	–1,2372	8,684	11,427	–1,0252	0,00266	3,489
4/3	–1,9507	14,250	18,058	–1,3152	0,00351	4,592
1	–2,7033	20,406	25,219	–1,5375	0,00421	5,5225
3/4	–2,6036	32,746	37,874	–1,0085	0,00352	4,6143
1/2	–2,4438	53,186	59,049	–0,1149	0,00317	4,1593
1/4	–1,9935	85,801	94,233	0,9611	0,00500	6,5544
0	–2,060	74,888	30, 862	2,2877	0,00244	32,0307

Table 4 Parameters of stress-strain state at point *B* of a plate with an elliptical hole (τ_{zy}=0)

r_3/r_1	$w\cdot10^5$	σ_{xx}, MPa	σ_{zz}, MPa	p_u	P_u, MPa
4	3,0528	–46,102	–12, 586	0,00181	23,705
2	4,3270	–69,287	–13,306	0,00302	39, 595
4/3	5,6192	–78,653	–10,791	0,00366	47,997
1	6,9720	–83,022	–7,9411	0,00405	53,104
3/4	6,4114	–81,179	–7,4131	0,00398	52,174
1/2	5,8184	–80,714	–7,0060	0,00398	52,132
1/4	5,0181	–85,329	–7,6285	0,00419	54,956
0	4,7642	–77,201	–4,4764	0,00393	51,437

in the composite is v=60%, the corresponding effective mechanical parameters of boron/aluminum are as follows: E=1,3992·10^5 MPa, E'=2,6682·10^5 MPa MPa, G=0,6551·10^5 MPa, G'=0,5396·10^5 MPa, μ=0,0682, μ'=0,2480 (Karpov 2002).

By computational experiment, the influence of the shape of the hole on the distribution of deformation fields and stresses in the plane *Oxy* of transversal isotropy is investigated. The sizes of the large (r_1) and small (r_3) axes of the concentrator in the form of an ellipse vary. Logically, the process is completed by solving an elastic problem of fibrous structure stretching with a horizontal rectilinear crack in the center. On the border of a crack, a front is marked out - a plane in which the cracks join. From the point of view of the formulation and solution of the problem, the crack banks play the role of an additional body boundary. Due to the smallness of the distance between the crack tips, it can be considered a mathematical cut, that is, a cavity of zero volume, which is

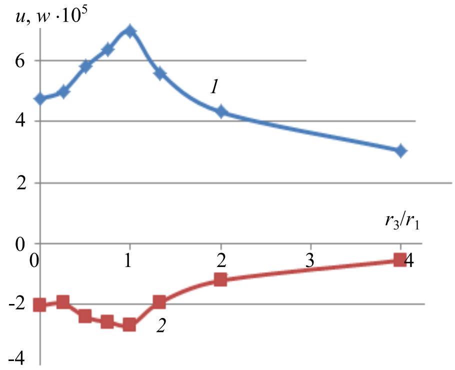

Fig. 2 Changes in the displacements of *u*- at point *A* (curve 1) and *w*- at point *B* (curve 2)

bounded by two geometrically coinciding surfaces—the cut edges. Obviously, the greatest stress concentration will be observed near the front. Therefore, in the calculations, the vicinity of the crack is divided into smaller finite elements.

As a stress concentrators are considered:

- perforating hole in the shape of an ellipse (r_1=0,05 and r_3=0,01);
- perforating horizontal rectilinear crack (l=0,1).

Characteristic points *A* and *B* are located in zones of stress concentration:
for an ellipse (Fig. 1), this is point *A* at the intersection of the hole contour with the axis and point *B* at the intersection with the axis; for a crack: *A*- point of the crack tip, *B* - point in the middle of the crack length).

Tables 3 and 4 show the values of the components of the stress-strain state at characteristic points in the vicinity of the hole in the cross section (*u*, *w* are the projections of the displacement of points, respectively, on the axis *Ox* and *Oz*) in the isotropy plane. When r_3/r_1=0 (the case of a crack) the intensity of stresses P_u in the isotropy plane increases. This is due to the increase in the difference of values σ_{xx} and σ_{zz}. At the same time, the value of the component of the shear stress τ_{zx} increases and its sign changes to the opposite (Table 3).

However, the value of the intensity of deformations in the isotropy plane remains half as much as if there is an opening in the shape of an ellipse (r_3/r_1=1/4) (Table 4). Since the transversely isotropic medium is a model of a fiber composite, the zone of the last fiber cut by a crack becomes the place of the greatest concentration of breaking stress (Fig. 1). The first fiber, adjacent to the broken, takes the main load and reduces the intensity of stresses at the points of fiber break (Karpov 2002).

Fig. 2 shows the behavior of displacements *u* at point *A* (curve 1) and *w* at point *B* (curve 2).

Each of them reaches its maximum value at a ratio r_3/r_1=1; in this case, the hole takes the form of a circle and has the largest area.

Fig.3 shows the distributions of the intensity of deformations p_u in the isotropy plane in the cross section *Oxz* depending on the ratio r_3/r_1. Elevated values p_u are localized and concentrated in the vicinity of the hole. Under the action of tensile load, the central part of the structure, together with the hole, is compressed axially *Ox*. This is also confirmed by the presence of a compressive stress component σ_{xx} (Table 4). As a result, the hole along the axis *Oz* is stretched.

Increased levels of strain intensities and stresses in the isotropy plane are observed in the vicinity of the upper and lower parts of the hole. The highest values p_u=0,00419 and P_u=54,96

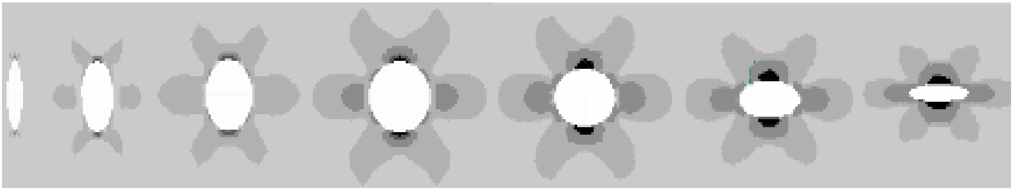

Fig. 3 Patterns of the distribution of strain intensity p_u in the *Oxz* section

Fig. 4 Distribution of strain intensity values -p_u in case of crack

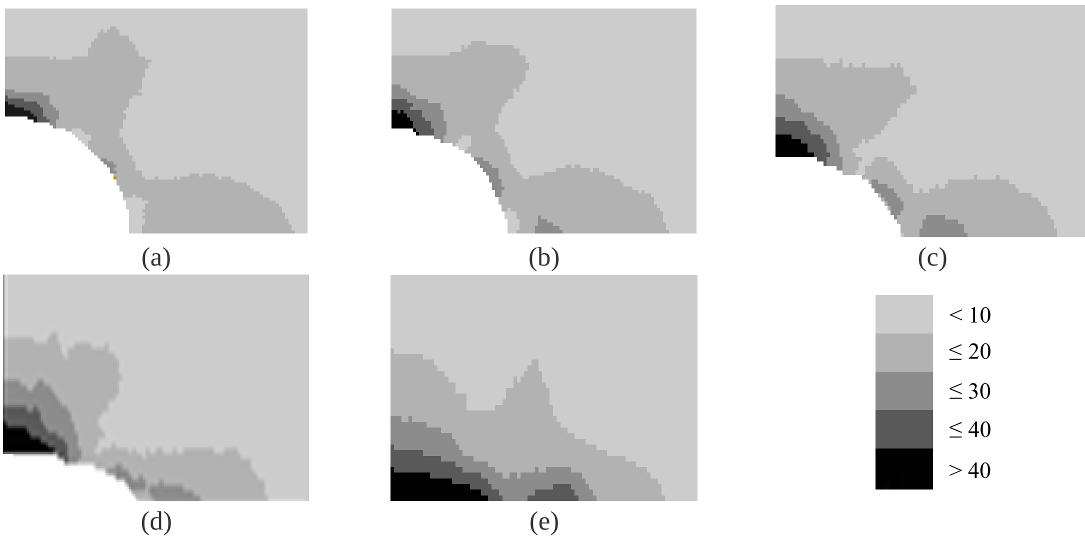

Fig. 5 The stress intensity distribution P_u [MPa] (1/4 of the plate)

MPa are achieved at r_3=0,0125. The distributions of the strain intensity in three computational experiments (with ratios r_3/r_1=4, 2 and 4/3) confirm the locality of the effect of the hole on the change in the strain intensity field. According to Fig. 3, it is limited to areas located in the vicinity of the upper and lower parts of the hole's contour.

In the presence of an isolated straight-line crack (Fig. 4), the maximum values of the strain intensity in the isotropy plane are concentrated along the crack brinks, and in the vicinity of the top of its value are somewhat lower.

Zones of stress intensity P_u in the isotropy plane in the vicinity of the stress concentrator with a ratio r_3/r_1 from 1 to 1/8 (r_1=0,005) are shown in Fig. 5(a)-5(d). The maximum of intensity values

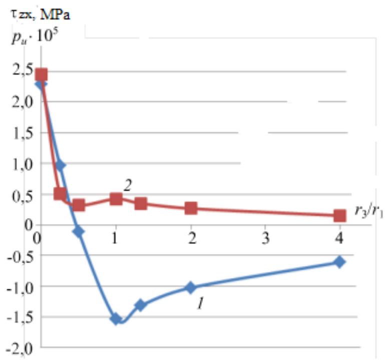

Fig. 6 Graph of tension changes τ_{zx} (curve 1) and p_u at point *A* (curve 2)

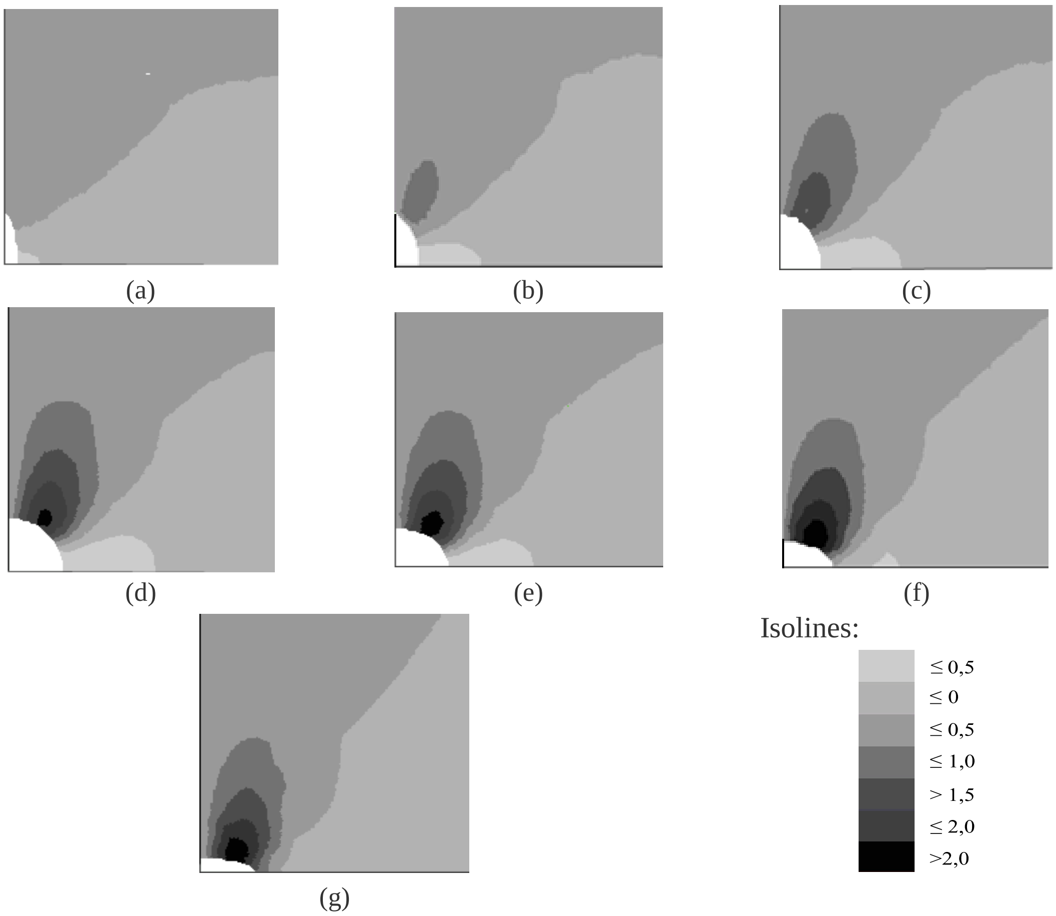

Fig. 7 Distribution of tangential stresses τ_{zx} [MPa] in 1/4 of the plate

P_u are concentrated near the top (and, at the bottom accordingly, not shown here) of the hole. However, as the vertical size of the hole decreases near its lateral parts, zones of elevated stress intensity are also formed, which is associated with an increase in the curvature coefficient of the

Fig. 8 Distribution of values of tangential stresses τ_{zx} [MPa] in case of crack

ellipse. It should be noted that in the vicinity of point *A*, the intensity values of the stresses are insignificant. This zone can be clearly seen in Fig. 5(e), which corresponds to the case of an isolated straight-line crack.

Graphs of changes in the tangential component of the stress τ_{zx} and strain rate in the isotropy plane p_u in the vicinity of point *A* are shown in Fig. 6. Analysis of the results shows that in the presence of a crack and a flattened (r_3/r_1 =1/8) in the direction of the axis of the ellipse, the values of the tangential stresses are positive.

With an increase in the value of the ratio of semiaxes r_3/r_1 in the range from 1/4 and higher, the tangential stresses change sign to negative and reach the maximum value at r_3/r_1 =1 (circle). The presence of tangential stresses usually causes a change in shape. The change in sign with a decrease in the value of the vertical axis of the elliptical hole means that the region of maximum tangential stresses is formed in the vicinity of point *A*. This leads to cracks. They propagate along the fibers over the entire length of the sample and can lead to a general destruction of the structure. To determine the zones of maximum tangential stresses τ_{zx} with different ratios of the hole parameters, computational experiments were carried out with the values r_3/r_1 given in Table 1. The results of the experiments are presented in Fig. 7.

At values of r_3/r_1 =1 (circle) and smaller in the vicinity of the hole, a stable region of maximum values τ_{zx} is formed. With a decrease in the size of the semi-axis r_3 of the elliptical hole (at a constant r_1 =0,05), the maximum value area encompasses the hole contour (Fig. 7(a)-7(g). In a problem with a crack, tangential stresses τ_{zx} form in the vicinity of its tips (Fig. 8). These areas are the most vulnerable in terms of strength in fiber structures with a hole or crack. Since in this place there is a probability of separation of the matrix from high-strength fibers.

4.2 Elastoplastic calculation

The three-dimensional elastoplastic problem of uniformly distributed tension of the plate along the axis of the load (P_{zz}=950 MPa) is considered. The load is applied to the lower and upper edges of the plate. The plate has an isolated hole in the shape of an ellipse (r_3/r_1 =1/8) or a rectilinear horizontal crack. The geometric and mechanical parameters of the problem are identical to the parameters of the problem in the case of elastic calculation. In the first approximation, the effective material constants of the plasticity function of an equivalent transversely isotropic medium are equal to the material constants of the duralumin matrix. For comparison, the results in Table 5 and 6 show the values of the components of the elastic and elastoplastic state of the plate in the transverse isotropy *Oxy* plane for the section *Oxz*. They characterize of the matrix material

Table 5 Stress-strain state parameters at point *B* of a plate with an elliptical hole (τ_{zx} = 0)

Type of calculation	p_u	σ_{xx}, MPa	P_u, MPa
elastic	0,00398	–810,63	522,09
elastoplastic	0,00327	–622,23	410,95
difference, in %	17,84	23,24	21,29

Table 6 Comparison of parameters calculated differently for a plate with a crack

	at the top of the crack (*A*)		difference, in %	at the midpoint (*B*)		difference, in %
	elastic	elastoplastic		elastic	elastoplastic	
p_u	0,00232	0,00155	33,19	0,00373	0,00338	9,38
σ_{xx}, MPa	711,44	492,22	30,81	–733,41	–634,13	13,54
P_u, MPa	304,29	203,10	33,25	488,65	418,18	14,42

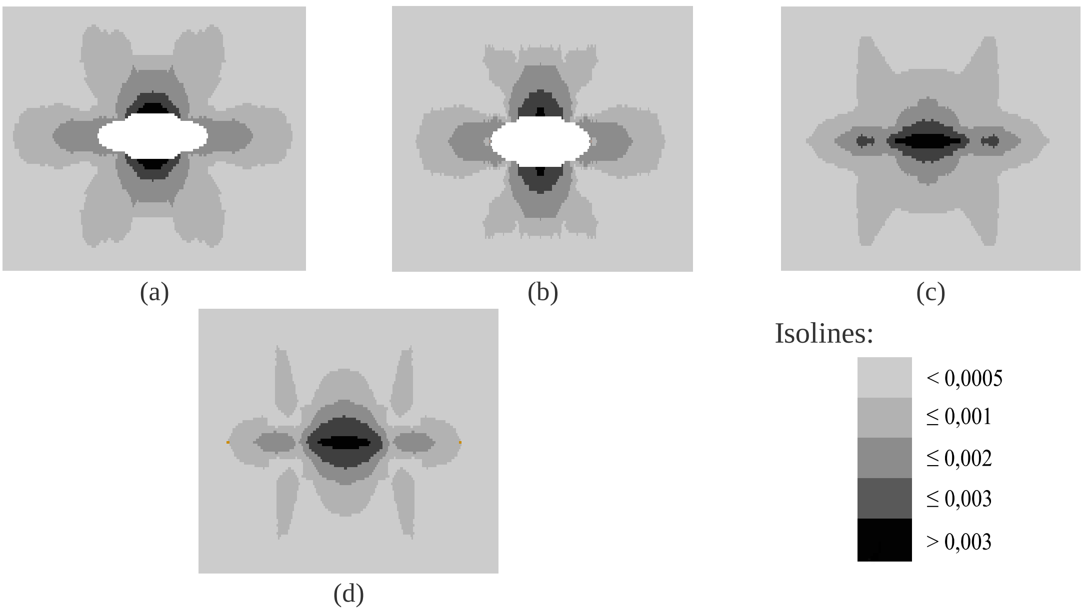

Fig. 9 Distribution of strain intensity p_u in a plate with an elliptical hole (a, b) and a crack (c, d) in an elastic (a, c) and elastoplastic (b, d) cases

behavior of fibrous composites. Table 5 contains the values of the parameters of the stress-strain state in the isotropy plane in the vicinity of point *B* of the elliptical hole. Data analysis indicates a significant decrease in the values of the intensity of deformations p_u and stresses P_u in the isotropy plane due to plastic deformations.

Next, the results of the calculation of the stress-strain state of a plate with a notch in the form of a horizontal straight crack (Table 6) are analyzed.

In the vicinity of the midpoint on the crack tips, the strain intensity values $p_u>p_s$ in the isotropy plane determine the plastic zone. However, in the vicinity of the crack tips, the strain intensity values $p_u<p_s$ correspond to the elastic zone. This confirms that when the fibrous composite is stretched along the axis of transverse isotropy, the region of plastic deformations along the

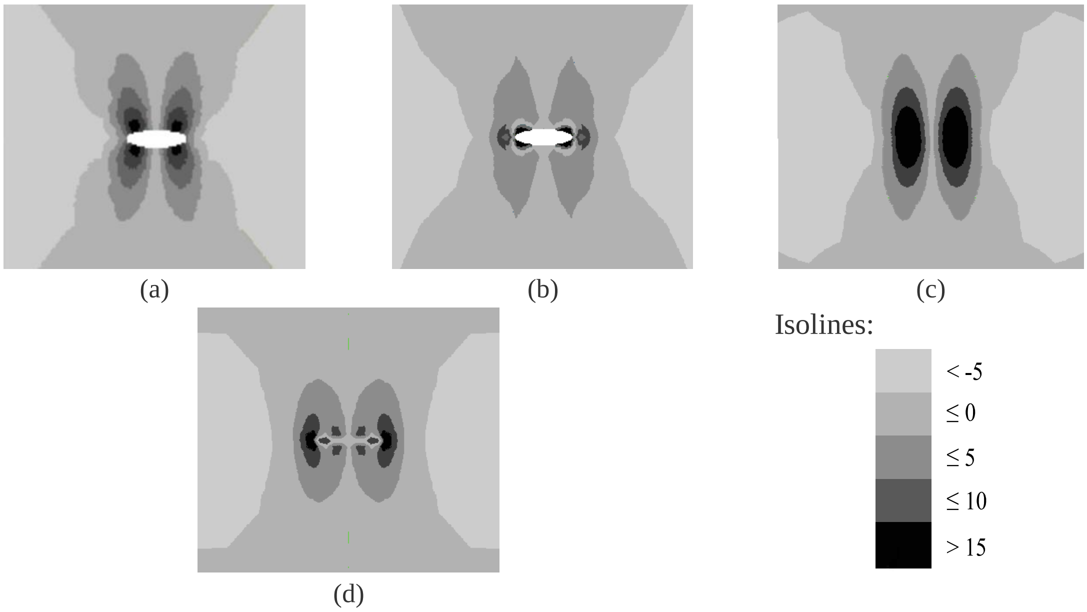

Fig. 10 Distribution of tangential stresses τ_{zx} [MPa] in a plate with an elliptical hole (a, b) and a crack (c, d) in an elastic (a, c) and elastoplastic (b, d) cases

isotropy plane concentrates in the vicinity of the middle of the crack tips. And in the vicinity of the tips of the crack - the area is elastic.

Fig. 9 shows the distributions of the intensity of deformations in the isotropy plane for a plate with a hole in the form of an ellipse at r_3/r_1=1/8, plates with a crack obtained from elastic (Fig. 9(a), 9(c)) and elastoplastic calculations (Fig. 9(b), 9(d)). The areas of plastic deformations are concentrated in the vicinity of the upper and lower parts of the ellipse (Fig. 9(b)) and in the vicinity of the middle of the crack tips (Fig. 9(d)).

The same redistribution can be observed even in the presence of a central horizontal rectilinear crack (Fig. 10(c), 10(d)). The maximum values of the tangential stress component are formed in the vicinity of the crack tips. In the elastoplastic case, the maximum values of this component are concentrated directly at the crack tips (Fig. 10(c)).

Thus, the computational experiment makes it possible to investigate the influence of the shape of the holes on the stress-strain state of structures made of fibrous composites, to determine the zones of formation of plastic deformations and the localization of areas with maximum tangential stresses causing fiber separation from the matrix. The results of the elastoplastic calculation make it possible to specify the stress-strain state of the structures and evaluate the true behavior of fibrous composites.

5. Conclusions

As a result of theoretical studies and computational experiments made, the following were performed:

• A computer model has been developed for solving three-dimensional problems of elastic and elastoplastic deformation of fibrous composites on the basis of the simplified theory of small elastoplastic deformations of transversely isotropic media and the finite element method.

• The effect of holes' shape on the stress-strain state and the distribution of the strain intensity in the isotropy plane in the vicinity of the fibrous composites' stress concentrator was studied.

• The plastic deformations areas in the isotropy plane and the redistribution of the parameters of the fibrous composites' stress-strain state in the vicinity of stress concentrators were determined.

• By means of computational experiment, for different configurations of holes in fibrous composites, the location of areas with maximum values of tangential stresses, where high strength fibers can detach from the matrix, is defined.

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