

Vibration analysis of heterogeneous nonlocal beams in thermal environment

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Abstract. In this paper, the thermo-mechanical vibration characteristics of functionally graded (FG) nanobeams subjected to three types of thermal loading including uniform, linear and non-linear temperature change are investigated in the framework of third-order shear deformation beam theory which captures both the microstructural and shear deformation effects without the need for any shear correction factors. Material properties of FG nanobeam are assumed to be temperature-dependent and vary gradually along the thickness according to the power-law form. Hence, applying a third-order shear deformation beam theory (TSDBT) with more rigorous kinetics of displacements to anticipate the behaviors of FG nanobeams is more appropriate than using other theories. The small scale effect is taken into consideration based on nonlocal elasticity theory of Eringen. The nonlocal equations of motion are derived through Hamilton's principle and they are solved applying analytical solution. The obtained results are compared with those predicted by the nonlocal Euler-Bernoulli beam theory and nonlocal Timoshenko beam theory and it is revealed that the proposed modeling can accurately predict the vibration responses of FG nanobeams. The obtained results are presented for the thermo-mechanical vibration analysis of the FG nanobeams such as the effects of material graduation, nonlocal parameter, mode number, slenderness ratio and thermal loading in detail. The present study is associated to aerospace, mechanical and nuclear engineering structures which are under thermal loads.

Keywords: thermal vibration; functionally graded nanobeam; nonlocal elasticity theory

1. Introduction

Functionally graded materials (FGMs) are known as microscopically inhomogeneous spatial composite materials which provide wide potential applications for various machineries which are subjected to vigorous thermo-mechanical loadings, such as spacecraft heat shields, plasma coatings for fusion reactors, jet fighter structures, and heat engine components. According to this point that FG materials have been placed in the classification of composite materials, the volume fractions of two or more material constituents such as a pair of ceramic-metal are assumed to vary smoothly and continuously throughout the gradient directions. The FGM materials are made to

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take advantage of desirable features of its constituent phases, for example, in a thermal protection system; the ceramic constituents are capable to withstand extreme temperature environments due to their better thermal resistance characteristics, while the metal constituents provide stronger mechanical performance and diminish the possibility of catastrophic fracture. Hence, presenting novel mechanical properties, FGMs have gained its applicability in several engineering fields, such as biomedical engineering, nuclear engineering and mechanical engineering. In view of these advantages, a number of investigations, dealing with static, buckling, dynamic characteristics of functionally graded (FG) structures, had been published in the scientific literature (Ebrahimi and Rastgoo 2008a, b, c, Ebrahimi, 2013, Ebrahimi *et al.* 2008, 2009a, b, 2016a, Ebrahimi and Zia 2015, Ebrahimi and Mokhtari 2015).

Moreover, considerable progression in the utilization of structural elements such as beams and plates with micro or nanolength scale in micro/nano electro-mechanical systems (MEMS/NEMS), due to their outstanding mechanical, chemical, and electronic properties, led to a sudden momentum in modeling of micro and nano scale structures. In these applications, size effects become prominent. Since the invention of carbon nanotubes (CNTs) by Iijima (1999), nanoscale engineering materials have exposed to considerable attention in modern science and technology. These structures possess extraordinary mechanical, thermal, electrical and chemical performances that are superior to the conventional structural materials. Therefore nanostructures attract great interest by researchers based on molecular dynamics and continuum mechanics. The problem in using the classical theory is that the classical continuum mechanics theory does not take into account the size effects in micro/nano scale structures. The classical continuum mechanics over predicts the responses of micro/nano structures. Another way to capture the size effects is to rely on molecular dynamic simulations (MD) which is considered as a powerful and accurate implement to study of structural components at nanoscale. But even the molecular dynamic simulation at nano scale is computationally exorbitant for modeling the nanostructures with large numbers of atoms. So a conventional form of continuum mechanics that can capture the small scale effect is required. Eringen's nonlocal elasticity theory (Eringen 1983) is the most commonly used continuum mechanics theory that includes small scale effects with good accuracy to model micro/nano scale devices and systems. The nonlocal elasticity theory assumes that the stress state at a reference point is a function of the strain at all neighbor points of the body. Hence, this theory could take into consideration the effects of small scales. Lots of studies have been performed to investigate the size-dependent response of structural systems based on Eringen's nonlocal elasticity theory (Ebrahimi and Salari 2015a, b, 2016, Ebrahimi *et al.* 2015a, 2016c, Ebrahimi and Nasirzadeh 2015, Ebrahimi and Barati 2016 a, b, c, d, e, f, Ebrahimi and Hosseini 2016 a, b, c).

Concerning taking into account the size-effect for FG beam structures based on the nonlocal constitutive relation of Eringen, a large number of studies have been conducted attempting to develop nonlocal beam models for predicting the mechanical responses of nanobeams. The potential of application of nonlocal Euler-Bernoulli beam theory to materials in micro and nano scale proposed by Peddieson *et al.* (2003) as the first researchers to propose nonlocal elasticity theory to nano structures. Then, the nonlocal elasticity theory attracted considerable attention among the nanotechnology community and application of this theory generalized in various mechanical analyses. Reddy (2007) formulated various available beam theories, including the Euler-Bernoulli, Timoshenko, Reddy, and Levinson beam theories through nonlocal differential relations of Eringen. In other scientific work, Wang and Liew (2007) carried out the static analysis of micro and nano scale structures based on nonlocal continuum mechanics using Euler-Bernoulli and Timoshenko beam theory. Aydogdu (2009) presented a general nonlocal beam model for

analysis bending, buckling, and vibration of nanobeams using different beam theories. Pradhan and Murmu (2010) investigated the flapwise bending-vibration of rotating nano-cantilevers by using Differential quadrature method (DQM). They noticed that size effects have a main role in the vibration behavior of rotating nanostructures. Civalek *et al.* (2010) proposed formulation of the governing equations of nonlocal Euler-Bernoulli beams to investigate bending of cantilever microtubules via the differential quadrature method. Thai (2012) suggested a nonlocal higher order beam theory to study mechanical responses of nanobeams. Simsek (2014) proposed a non-classical beam model based on the Eringen's nonlocal elasticity theory for nonlinear vibration of nanobeams with various boundary conditions. Zenkour *et al.* (2014) presented the nonlocal beam model based on Eringen's theory for the vibration of FG and composite nanobeams. Most recently Ebrahimi and Barati (2016g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, 2017a, b) and Ebrahimi *et al.* (2017) explored thermal and hygro-thermal effects on nonlocal behavior of FG nanobeams and nanoplates.

World wide applications of FGMs in micro/nano structures are increasingly being recognized in recent years. To properly apply this class of novel micro/nano materials in micro/nano electromechanical systems (MEMS/NEMS), their mechanical behavior needs to be investigated. Recently, Eltahir *et al.* (2012) presented a finite element analysis for free vibration of FG nanobeams using nonlocal EBT. Rahmani and Pedram (2014) analyzed the size effects on vibration of FG nanobeams based on nonlocal TBT. Also, recently Hosseini-Hashemi *et al.* (2014) investigated free vibration of FG nanobeams with consideration surface effects and piezoelectric field using nonlocal elasticity theory. Most recently Ebrahimi *et al.* (2015b) and Ebrahimi and Salari (2015c) examined the applicability of differential transformation method in investigations on vibrational characteristics of FG size-dependent nanobeams. In another work, Ebrahimi and Salari (2015d) presented a semi-analytical method for vibrational and buckling analysis of FG nanobeams considering the physical neutral axis position.

Although the size-dependent FG beam models have been developed in the aforementioned studies, most of them ignore the effects of thermal environment. Also, the common use of FGMs in high temperature environment leads to considerable changes in material properties. For example, Young's modulus usually reduces when temperature increases in FGMs. To predict the behavior of FGMs subjected to extreme temperatures more accurately, it is necessary to consider the temperature dependency on material constituent's properties. Furthermore, due to the expansion of new industries and technologies, many systems and structures experience severe thermal environments, resulting in various types of thermal loads. This situation has created a need for a text that is focused on the analysis of thermal vibration. Thermal vibration is a phenomenon in many structures that should be checked to ensure the safety of structures. Consequently, thermal vibration analysis of beam structures is common in structural mechanics. Hence, presenting an accurate model of FG nanobeams is very important for successful NEMS design. Considering great application of beams in different engineering fields such as aerospace and mechanical engineering, and due to the fact that making temperatures and working temperature of structures are not equal, for more accurate design, it is useful to study their thermos-mechanical behavior. Several studies have been performed to investigate the thermal effect on mechanical responses of FGM beams. Wattanasakulpong *et al.* (2011) investigated thermal buckling and elastic vibration of third-order shear deformable FG beams. Thermo-mechanical buckling and nonlinear free vibration analysis of FG beams on nonlinear elastic foundation is investigated by Fallah and Aghdam (2012). Small amplitude vibrations of a FG material beam under in-plane thermal loading in the prebuckling and postbuckling regimes is studied by Esfahani *et al.* (2014). It should be cited that,

in these work the beams are in macro scale and small scale effect is not taken into consideration. For the FG nanobeam problems, Ebrahimi and Salari (2015e) investigated the thermal effects on buckling and free vibration characteristics of FG size-dependent Timoshenko nanobeams subjected to an in-plane thermal loading. In another study, Thermo-mechanical vibration analysis of nonlocal temperature-dependent FG nanobeams with various boundary conditions is investigated by Ebrahimi and Salari (2015f). It is noticed that most of the previous studies on mechanical analysis of FG nanobeams have been carried out based on Euler-Bernoulli and Timoshenko beam theories. It should be noted that the EBT fails to consider the influences of shear deformations. This theory is only applicable for slender beams and should not be applied for thick beams, and also it suppose that the transverse perpendicular to the neutral surface stays normal during and after bending, which indicates that the transversal shear strain is equal to zero Hence, the buckling loads and natural frequencies of thick beams are overestimated in which shear deformation effects are prominent. Timoshenko Beam Theory can enumerate the influences of shear deformations for thick beams with presumption of a constant shear strain state in the direction of beam thickness. So, as a disadvantage of this theory, a shear correction factor is required to properly demonstration of the deformation strain energy. To prevent using the shear correction factors, many higher-order shear deformation theories have been developed such as the third-order shear deformation theory proposed by Reddy (2007), the generalized beam theory proposed by Aydogdu (2009) and sinusoidal shear deformation theory of Touratier (1991). Reddy's third order beam theory (RBT) can be used with supposing the higher order longitudinal displacement variations of beam along the thickness. By verifying zero transverse shear stresses at the upper and lower surfaces of the beam, this theory captures both the microstructural and shear deformation effects. Therefore, The Reddy beam theory is more exact and provides better representation of the physics of the problem, which does not need any shear correction factors. This theory relaxes the limitation on the warping of the cross sections and allows cubic variations in the longitudinal direction of the beam, so it can produce adequate accuracy when applying for beam analysis. Therefore, a few numbers of studies have been conducted to investigate the mechanical responses of FG micro/nano beams by using higher shear deformation beam theories. Zhang *et al.* (2005) developed a size-dependent FG beam model resting on Winkler-Pasternak elastic foundation based on an improved third-order shear deformation theory and provided the analytical solutions for the bending, buckling and free vibration problems. By searching the literature, it is found that a work analyzing the thermal vibration of FG nanobeams using the third-order shear deformation beam theory hasn't been yet published.

In the present work, the thermal vibration analysis of third order shear deformable simply supported FG nanobeams is presented and effects of three types of thermal loading namely, uniform, linear and nonlinear temperature rise on vibration behavior of FG nanobeams are investigated. Material properties of FG nanobeam are assumed to change continuously along the thickness according to the power-law form and are assumed to be temperature-dependent. By using the Hamilton's principle the governing equations of motion are derived and Navier's type solution method is used to solve the equations. The obtained results based on third order shear deformation beam theory are compared with those predicted by the previous works to verify the accuracy of the present model. Selected numerical results are presented to indicate the effects of the power-law index, nonlocal parameter, slenderness ratio and thermal load on the vibration characteristics of FG nanobeams.

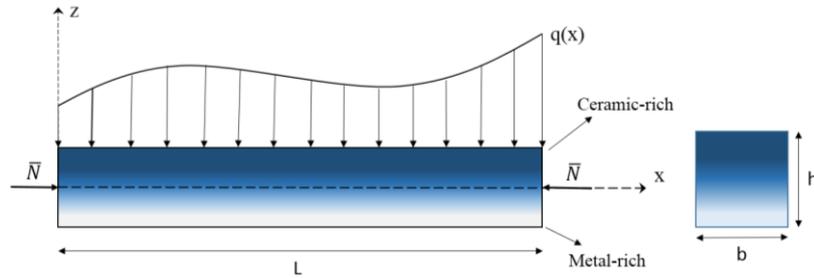


Fig. 1 Geometry and coordinates of FG nanobeam

2. Governing equations

2.1 Power-law functionally graded material (P-FGM) beam

One of the most favorable models for FGMs is the power-law model, in which material properties of FGMs are supposed to change according to a power law about spatial coordinates. The coordinate system for FG nano beam is shown in Fig. 1. The FG nanobeam is assumed to be combination of ceramic and metal and effective material properties (P_f) of the FG beam such as Young's modulus E_f and mass density ρ are supposed to change continuously in the direction of z -axis (thickness direction) according to an power function of the volume fractions of the material constituents. So, the effective material properties, P_f can be stated as

$$P_f = P_c V_c + P_m V_m, \tag{1}$$

where subscripts m and c denote metal and ceramic, respectively and the volume fraction of the ceramic is associated to that of the metal in the following relation

$$V_c + V_m = 1. \tag{2a}$$

The volume fraction of the ceramic constituent of the beam is assumed to be given by

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^p, \tag{2b}$$

Here p is the power-law exponent which determines the material distribution through the thickness of the beam and z is the distance from the mid-plane of the FG nanobeam

Therefore, from Eqs. (1) and (2), the effective material properties of the FG nanobeam such as Young's modulus (E), mass density (ρ), Poisson's ratio (ν), coefficient of thermal expansion (α_t) and thermal conductivity (κ) can be expressed as follows

$$\begin{Bmatrix} E(z) \\ \rho(z) \\ \nu(z) \\ \alpha_t(z) \\ \kappa(z) \end{Bmatrix} = \begin{Bmatrix} E_c - E_m \\ \rho_c - \rho_m \\ \nu_c - \nu_m \\ \alpha_c - \alpha_m \\ \kappa_c - \kappa_m \end{Bmatrix} \left(\frac{z}{h} + \frac{1}{2}\right)^p + \begin{Bmatrix} E_m \\ \rho_m \\ \nu_m \\ \alpha_m \\ \kappa_m \end{Bmatrix}. \tag{3}$$

The material composition of FG nanobeam at the upper surface ($z = +h/2$) is supposed to be the pure ceramic and it changes continuously to the opposite side surface ($z = -h/2$) which is

Table 1 Temperature dependent coefficients for Si3N4 and SUS304

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si3N4	$E(\text{Pa})$	348.43e+9	0	-3.070e-4	2.160e-7	-8.946e-11
	$\alpha(\text{K}^{-1})$	5.8723e-6	0	9.095e-4	0	0
	$\rho(\text{Kg/m}^3)$	2370	0	0	0	0
	$\kappa(\text{W/mK})$	13.723	0	-1.032e-3	5.466e-7	-7.876e-11
	ν	0.24	0	0	0	0
SUS304	$E(\text{Pa})$	201.04e+9	0	3.079e-4	-6.534e-7	0
	$\alpha(\text{K}^{-1})$	12.330e-6	0	8.086e-4	0	0
	$\rho(\text{Kg/m}^3)$	8166	0	0	0	0
	$\kappa(\text{W/mK})$	15.379	0	-1.264e-3	2.092e-6	-7.223e-10
	ν	0.3262	0	-2.002e-4	3.797e-7	0

pure metal. To more accurate prediction of FGMs behavior under high temperature, it is necessary to consider the temperature dependency on material properties. The nonlinear equation of thermo-elastic material properties in function of temperature $T(\text{K})$ can be expressed as Ebrahimi and Salari (2015e)

$$P = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3), \quad (4)$$

where P_0, P_{-1}, P_1, P_2 and P_3 are the temperature dependent coefficients of temperature, T (K) which can be seen in the Table 1 that contains material properties of Si_3N_4 and SUB304.

2.2 Kinematic relations

Based on the third order shear deformation (Reddy) beam theory, the displacement field at any point of the beam can be written as

$$\begin{aligned} u_x(x, z) &= u(x) + z\varphi(x) - \alpha z^3 \left(\varphi + \frac{\partial w}{\partial x} \right), \\ u_z(x, z) &= w(x), \end{aligned} \quad (5)$$

where $\alpha = \frac{4}{3h^2}$ and u and w are the longitudinal and the transverse displacements, φ is the rotation of the cross section at each point of the neutral axis. Nonzero strains of the Reddy beam model are expressed as follows

$$\varepsilon_{xx} = \varepsilon_{xx}^{(0)} + z\varepsilon_{xx}^{(1)} + z^3\varepsilon_{xx}^{(3)}, \quad \gamma_{xz} = \gamma_{xz}^{(0)} + z^2\gamma_{xz}^{(2)}. \quad (6)$$

where

$$\begin{aligned} \varepsilon_{xx}^{(0)} &= \frac{\partial u}{\partial x}, \quad \varepsilon_{xx}^{(1)} = \frac{\partial \varphi}{\partial x}, \quad \varepsilon_{xx}^{(3)} = -\alpha \left(\frac{\partial \varphi}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right), \\ \gamma_{xz}^{(0)} &= \varphi + \frac{\partial w}{\partial x}, \quad \gamma_{xz}^{(2)} = -\beta \left(\varphi + \frac{\partial w}{\partial x} \right), \end{aligned} \quad (7)$$

and $\beta = \frac{4}{h^2}$. By using the Hamilton's principle, in which the motion of an elastic structure in the time interval $t_1 < t < t_2$ is so that the integral with respect to time of the total potential energy is extremum

$$\int_0^t \delta(K - U - V)dt = 0, \quad (8)$$

where K is the kinetic energy, U is the total strain energy and V is the work done by external forces. The virtual strain energy can be calculated as

$$\delta U = \iiint_V \sigma_{ij} \delta \varepsilon_{ij} dv = \iiint_V (\sigma_{xx} \delta \varepsilon_{xx} + \sigma_{xz} \delta \gamma_{xz}) dv. \quad (9)$$

Substituting Eq. (6) into Eq. (9) yields

$$\delta U = \int_0^L \left(N \delta \varepsilon_{xx}^{(0)} + M \delta \varepsilon_{xx}^{(1)} + P \delta \varepsilon_{xx}^{(3)} + Q \delta \gamma_{xz}^{(0)} + R \delta \gamma_{xz}^{(2)} \right) dx. \quad (10)$$

in which the variables introduced in arriving at the last expression are defined as follows

$$\begin{aligned} \{N, M, P\} &= \iint_A \sigma_{xx} \{1, z, z^3\} dA, \\ \{Q, R\} &= \iint_A \sigma_{xz} \{1, z^2\} dA, \end{aligned} \quad (11)$$

The first variation of the work done by applied forces can be written in the form

$$\delta V = \int_0^L \left[\left(N^T \frac{\partial w}{\partial x} \frac{\partial}{\partial x} + q + \alpha P \frac{\partial^2}{\partial x^2} \right) \delta w + f \delta u - N \delta \varepsilon_{xx}^{(0)} - \bar{M} \frac{\partial \delta \varphi}{\partial x} - \bar{Q} \delta \gamma_{xz}^{(0)} \right] dx, \quad (12)$$

where N^T is thermal resultant, $\bar{M} = M - \alpha P$, $\bar{Q} = Q - \beta R$ and N is the applied axial compressive load and $q(x)$ and $f(x)$ are the transverse and axial distributed loads and k_w and k_p are linear and shear coefficient of elastic foundation. The thermal resultant can be expressed as

$$N^T = \int_{-h/2}^{h/2} E(z, T) \alpha_t(z, T) (T - T_0) dz. \quad (13)$$

The first variation of the virtual kinetic energy can be written in the form

$$\begin{aligned} \delta K = \int_0^L \left\{ I_0 \left(\frac{\partial u}{\partial t} \frac{\partial \delta u}{\partial t} + \frac{\partial w}{\partial t} \frac{\partial \delta w}{\partial t} \right) + I_1 \left(\frac{\partial u}{\partial t} \frac{\partial \delta \varphi}{\partial t} + \frac{\partial \varphi}{\partial t} \frac{\partial \delta u}{\partial t} \right) + I_2 \frac{\partial \varphi}{\partial t} \frac{\partial \delta \varphi}{\partial t} - \alpha \left[I_3 \frac{\partial u}{\partial t} \left(\frac{\partial^2 \delta w}{\partial x \partial t} + \frac{\partial \delta \varphi}{\partial t} \right) + I_3 \frac{\partial \delta u}{\partial t} \left(\frac{\partial^2 w}{\partial x \partial t} + \frac{\partial \varphi}{\partial t} \right) + I_4 \frac{\partial \varphi}{\partial t} \left(\frac{\partial^2 \delta w}{\partial x \partial t} + \frac{\partial \delta \varphi}{\partial t} \right) + I_4 \frac{\partial \delta \varphi}{\partial t} \left(\frac{\partial^2 w}{\partial x \partial t} + \frac{\partial \varphi}{\partial t} \right) - \alpha I_6 \left(\frac{\partial^2 w}{\partial x \partial t} + \frac{\partial \varphi}{\partial t} \right) \left(\frac{\partial^2 \delta w}{\partial x \partial t} + \frac{\partial \delta \varphi}{\partial t} \right) \right] \right\} dx, \end{aligned} \quad (14)$$

where I_j represent the mass inertia

$$I_j = \iint_A \rho z^j dA. \quad (15)$$

It is to be noted that for homogeneous nanobeams we have $I_1 = I_3 = 0$.

By substituting Eqs. (10), (12) and (14) into Eq. (8) and setting the coefficients of δu , δw and $\delta \varphi$ to zero, the following Euler-Lagrange equation can be obtained

$$\begin{aligned} \frac{\partial N}{\partial x} + f &= I_0 \frac{\partial^2 u}{\partial t^2} + \hat{I}_1 \frac{\partial^2 \varphi}{\partial t^2} - \alpha I_3 \frac{\partial^3 w}{\partial x \partial t^2}, \\ \frac{\partial \bar{M}}{\partial x} - \bar{Q} &= \hat{I}_1 \frac{\partial^2 u}{\partial t^2} + \hat{I}_2 \frac{\partial^2 \varphi}{\partial t^2} - \alpha \hat{I}_4 \left(\frac{\partial^3 w}{\partial x \partial t^2} + \frac{\partial^2 \varphi}{\partial t^2} \right), \\ \frac{\partial \bar{Q}}{\partial x} - \frac{\partial}{\partial x} \left(N^T \frac{\partial w}{\partial x} \right) + \alpha \frac{\partial^2 P}{\partial x^2} + q &= I_0 \frac{\partial^2 w}{\partial t^2} + \alpha I_3 \frac{\partial^3 u}{\partial x \partial t^2} + \alpha I_4 \frac{\partial^3 \varphi}{\partial x \partial t^2} - \alpha^2 I_6 \left(\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^3 \varphi}{\partial x \partial t^2} \right) \end{aligned} \quad (16)$$

where $\hat{I}_j = I_j - \alpha I_{j+2}$.

2.3 The nonlocal elasticity model for FG nanobeam

According to Eringen nonlocal elasticity model, the stress state at a point inside a body is regarded to be function of strains of all points in the neighbor regions. For homogeneous elastic solids the nonlocal stress-tensor components σ_{ij} at each point x in the solid can be defined as

$$\sigma_{ij}(x) = \iiint_{\mathcal{V}} \psi(|x - x'|, \tau) t_{ij}(x') dv(x'), \quad (17)$$

where $t_{ij}(x')$ are the components available in local stress tensor at point x which are associated to the strain tensor components ε_{kl} as

$$t_{ij} = C_{ijkl} \varepsilon_{kl}. \quad (18)$$

The concept of Eq. (17) is that the nonlocal stress at any point is weighting average of local stress of all points in the near region that point, the size that is related to the nonlocal kernel $\psi(|x - x'|, \tau)$. Also $|x - x'|$ is Euclidean distance and τ is a constant given by

$$\tau = \frac{e_0 a}{l}. \quad (19)$$

which indicates the relation of a characteristic internal length, (for instance lattice parameter, C-C bond length and granular distance) and a characteristic external length, l (for instance crack length and wavelength) using a constant, e_0 , dependent on each material. The value of e_0 is experimentally estimated by comparing the scattering curves of plane waves and atomistic dynamics. According to (Eringen 1983) for a class of physically admissible kernel $\psi(|x - x'|, \tau)$. It is possible to represent the integral constitutive relations given by Eq. (17) in an equivalent differential form as

$$[1 - (e_0 a)^2 \nabla^2] \sigma_{kl} = t_{kl}, \quad (20)$$

where ∇^2 is the Laplacian operator. Thus, the scale length $e_0 a$ consider the influences of small scales on the response of nano-structures. The magnitude of the small scale parameter relies on several parameters including mode shapes, boundary conditions, chirality and the essence of motion. The parameter $e_0 = (\pi^2 - 4)^{1/2} / 2\pi \cong 0.39$ was given by Eringen (1983). Also, Zhang *et al.* (2005) found the value of 0.82 nm for nonlocal parameter when they compared the vibrational results of simply supported single-walled carbon nanotubes with molecular dynamics simulations. The nonlocal parameter, μ , is experimentally obtained for various materials; for instance, a conservative estimate of $\mu < 4 \text{ (nm)}^2$ for a single-walled carbon nanotube is proposed (Wang and Hu 2005). It is worth mentioning that this magnitude is dependent of size and chirality, because the properties of carbon nanotubes are extensively confirmed to be dependent of chirality. There is no serious study conducted to determining the value of small scale to simulate mechanical behavior of

FG micro/nanobeams. Hence all researchers who worked on size-dependent mechanical behavior of FG nanobeams on the basis the nonlocal elasticity method investigated the influence of small scale parameter on mechanical behavior of FG nanobeams by changing the value of the small scale parameter. In the present work, the nonlocal parameter is assumed to be in the range of 0-5 (nm)² (Eltaher *et al.* 2012). So, for a material in the one-dimension case, the constitutive relations of nonlocal theory can be expressed as

$$\begin{aligned} \sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} &= E \varepsilon_{xx}, \\ \sigma_{xz} - (e_0 a)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} &= G \gamma_{xz}, \end{aligned} \tag{21}$$

where σ and ε are the nonlocal stress and strain, respectively. E is the Young's modulus, $G(z) = \frac{E(z)}{2[1+\nu(z)]}$ is the shear modulus (where ν is the Poisson's ratio). For a nonlocal FG beam, Eqs. (21) can be written as

$$\begin{aligned} \sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} &= E(z) \varepsilon_{xx}, \\ \sigma_{xz} - \mu \frac{\partial^2 \sigma_{xz}}{\partial x^2} &= G(z) \gamma_{xz}, \end{aligned} \tag{22}$$

where $(\mu = (e_0 a)^2)$. Integrating Eq. (22) over the beam's cross-section area, we obtain the force-strain and the moment-strain of the nonlocal Reddy FG beam theory can be obtained as follows

$$N - \mu \frac{\partial^2 N}{\partial x^2} = A_{xx} \frac{\partial u}{\partial x} + (B_{xx} - \alpha E_{xx}) \frac{\partial \varphi}{\partial x} - \alpha E_{xx} \frac{\partial^2 w}{\partial x^2}, \tag{23}$$

$$M - \mu \frac{\partial^2 M}{\partial x^2} = B_{xx} \frac{\partial u}{\partial x} + (D_{xx} - \alpha F_{xx}) \frac{\partial \varphi}{\partial x} - \alpha F_{xx} \frac{\partial^2 w}{\partial x^2}, \tag{24}$$

$$P - \mu \frac{\partial^2 P}{\partial x^2} = E_{xx} \frac{\partial u}{\partial x} + (F_{xx} - \alpha H_{xx}) \frac{\partial \varphi}{\partial x} - \alpha H_{xx} \frac{\partial^2 w}{\partial x^2}, \tag{25}$$

$$Q - \mu \frac{\partial^2 Q}{\partial x^2} = (A_{xz} - \beta D_{xz}) \left(\frac{\partial w}{\partial x} + \varphi \right), \tag{26}$$

$$R - \mu \frac{\partial^2 R}{\partial x^2} = (D_{xz} - \beta F_{xz}) \left(\frac{\partial w}{\partial x} + \varphi \right), \tag{27}$$

in which the cross-sectional rigidities are defined as follows

$$\{A_{xx}, B_{xx}, D_{xx}, E_{xx}, F_{xx}, H_{xx}\} = \int_{-h/2}^{h/2} E(z) \{1, z, z^2, z^3, z^4, z^6\} dz, \tag{28}$$

$$\{A_{xz}, D_{xz}, F_{xz}\} = \int_{-h/2}^{h/2} G(z) \{1, z^2, z^4\} dz. \tag{29}$$

The explicit relation of the nonlocal normal force can be derived by substituting for the second

derivative of N from Eq. (16)₁ into Eq. (23) as follows

$$N = A_{xx} \frac{\partial u}{\partial x} + K_{xx} \frac{\partial \varphi}{\partial x} - \alpha E_{xx} \frac{\partial^2 w}{\partial x^2} + \mu \left(I_0 \frac{\partial^3 u}{\partial x \partial t^2} + \hat{I}_1 \frac{\partial^3 \varphi}{\partial x \partial t^2} - \alpha I_3 \frac{\partial^4 w}{\partial t^2 \partial t^2} - \frac{\partial f}{\partial x} \right). \quad (30)$$

Eliminating \bar{Q} from Eqs. (16)₂ and (16)₃, we obtain the following equation

$$\begin{aligned} \frac{\partial^2 \bar{M}}{\partial x^2} = \frac{\partial}{\partial x} \left(N^T \frac{\partial w}{\partial x} \right) - \alpha \frac{\partial^2 P}{\partial x^2} - q + I_0 \frac{\partial^2 w}{\partial t^2} + I_1 \frac{\partial^3 u}{\partial x \partial t^2} + I_2 \frac{\partial^3 \varphi}{\partial x \partial t^2} \\ - \alpha I_4 \left(\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^3 \varphi}{\partial x \partial t^2} \right). \end{aligned} \quad (31)$$

Also the explicit relation of the nonlocal bending moment can be derived by substituting the above equations into Eqs. (24) and (25) as follows

$$\begin{aligned} \bar{M} = K_{xx} \frac{\partial u}{\partial x} + \bar{I}_{xx} \frac{\partial \varphi}{\partial x} - \alpha J_{xx} \frac{\partial^2 w}{\partial x^2} + \mu \left[\frac{\partial}{\partial x} \left(N^T \frac{\partial w}{\partial x} \right) - \alpha \frac{\partial^2 P}{\partial x^2} - q \right. \\ \left. + I_0 \frac{\partial^2 w}{\partial t^2} + I_1 \frac{\partial^3 u}{\partial x \partial t^2} + I_2 \frac{\partial^3 \varphi}{\partial x \partial t^2} - \alpha I_4 \left(\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^3 \varphi}{\partial x \partial t^2} \right) \right], \end{aligned} \quad (32)$$

where

$$K_{xx} = B_{xx} - \alpha E_{xx}, \quad I_{xx} = D_{xx} - \alpha F_{xx}, \quad J_{xx} = F_{xx} - \alpha H_{xx}, \quad \bar{I}_{xx} = I_{xx} - \alpha J_{xx}. \quad (33)$$

By substituting for the second derivative of \bar{Q} from Eq. (16)₃ into Eq. (26) with the aid of Eq. (27) the following expression for the nonlocal shear force will be derived

$$\begin{aligned} \bar{Q} = A_{xz}^* \left(\frac{\partial w}{\partial x} + \varphi \right) + \mu \left[\frac{\partial^2}{\partial x^2} \left(N^T \frac{\partial w}{\partial x} \right) - \alpha \frac{\partial^3 P}{\partial x^3} - \frac{\partial q}{\partial x} + I_0 \frac{\partial^3 w}{\partial x \partial t^2} + \alpha I_3 \frac{\partial^4 u}{\partial x^2 \partial t^2} \right. \\ \left. + \alpha I_4 \frac{\partial^4 \varphi}{\partial x^2 \partial t^2} - \alpha^2 I_6 \left(\frac{\partial^5 w}{\partial x^3 \partial t^2} + \frac{\partial^4 \varphi}{\partial x^2 \partial t^2} \right) \right], \end{aligned} \quad (34)$$

where

$$A_{xz}^* = \bar{A}_{xz} - \beta \bar{D}_{xz}, \quad \bar{A}_{xz} = A_{xz} - \beta D_{xz}, \quad \bar{D}_{xz} = D_{xz} - \beta F_{xz}. \quad (35)$$

Now we use \bar{M} and \bar{Q} from Eqs. (32) and (34) and the identity that given from Eq. (25) to get

$$\alpha \frac{\partial^2}{\partial x^2} \left(P - \mu \frac{\partial^2 P}{\partial x^2} \right) = \alpha E_{xx} \frac{\partial^3 u}{\partial x^3} + \alpha J_{xx} \frac{\partial^3 \varphi}{\partial x^3} - \alpha^2 H_{xx} \frac{\partial^4 w}{\partial x^4}. \quad (36)$$

The nonlocal governing equations of third-order shear deformation FG nanobeam in terms of the displacement can be derived by substituting for N , \bar{M} and \bar{Q} from Eqs. (30), (32) and (34), respectively, and using Eq. (36) into Eqs. (16) as follows

$$\begin{aligned} A_{xx} \frac{\partial^2 u}{\partial x^2} + K_{xx} \frac{\partial^2 \varphi}{\partial x^2} - \alpha E_{xx} \frac{\partial^3 w}{\partial x^3} + f - I_0 \frac{\partial^2 u}{\partial t^2} - \hat{I}_1 \frac{\partial^2 \varphi}{\partial t^2} + \alpha I_3 \frac{\partial^3 w}{\partial x \partial t^2} \\ - \mu \left(\frac{\partial^2 f}{\partial x^2} - I_0 \frac{\partial^4 u}{\partial x^2 \partial t^2} - \hat{I}_1 \frac{\partial^4 \varphi}{\partial x^2 \partial t^2} + \alpha I_3 \frac{\partial^5 w}{\partial x^3 \partial t^2} \right) = 0, \end{aligned} \quad (37)$$

$$\begin{aligned} K_{xx} \frac{\partial^2 u}{\partial x^2} + \bar{I}_{xx} \frac{\partial^2 \varphi}{\partial x^2} - \alpha J_{xx} \frac{\partial^3 w}{\partial x^3} - A_{xz}^* \left(\frac{\partial w}{\partial x} + \varphi \right) - \hat{I}_1 \frac{\partial^2 u}{\partial t^2} - \hat{I}_2 \frac{\partial^2 \varphi}{\partial t^2} + \alpha \hat{I}_4 \left(\frac{\partial^3 w}{\partial x \partial t^2} + \frac{\partial^2 \varphi}{\partial t^2} \right) \\ + \mu \left[\hat{I}_1 \frac{\partial^4 u}{\partial x^2 \partial t^2} + \hat{I}_2 \frac{\partial^4 \varphi}{\partial x^2 \partial t^2} - \alpha \hat{I}_4 \left(\frac{\partial^5 w}{\partial x^3 \partial t^2} + \frac{\partial^4 \varphi}{\partial x^2 \partial t^2} \right) \right] = 0, \end{aligned} \quad (38)$$

$$\begin{aligned}
 A_{xz}^* \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \varphi}{\partial x} \right) + \alpha E_{xx} \frac{\partial^3 u}{\partial x^3} + \alpha J_{xx} \frac{\partial^3 \varphi}{\partial x^3} - \alpha^2 H_{xx} \frac{\partial^4 w}{\partial x^4} - \frac{\partial}{\partial x} \left(N^T \frac{\partial w}{\partial x} \right) + q - I_0 \frac{\partial^2 w}{\partial t^2} \\
 - \alpha I_3 \frac{\partial^3 u}{\partial x \partial t^2} - \alpha I_4 \frac{\partial^3 \varphi}{\partial x \partial t^2} + \alpha^2 I_6 \left(\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^3 \varphi}{\partial x \partial t^2} \right) + \mu \left[\frac{\partial^3}{\partial x^3} \left(N^T \frac{\partial w}{\partial x} \right) - \frac{\partial^2 q}{\partial x^2} + I_0 \frac{\partial^4 w}{\partial x^2 \partial t^2} \right. \\
 \left. + \alpha I_3 \frac{\partial^5 u}{\partial x^3 \partial t^2} + \alpha I_4 \frac{\partial^5 \varphi}{\partial x^3 \partial t^2} - \alpha^2 I_6 \left(\frac{\partial^6 w}{\partial x^4 \partial t^2} + \frac{\partial^5 \varphi}{\partial x^3 \partial t^2} \right) \right] = 0.
 \end{aligned} \tag{39}$$

3. Solution procedures

Here, on the basis the Navier method, an analytical solution of the governing equations for free vibration of a simply-supported FG nanobeam is presented. To satisfy governing equations of motion and the simply supported boundary condition, the displacement variables are adopted to be of the form

$$\begin{Bmatrix} u(x, t) \\ w(x, t) \\ \varphi(x, t) \end{Bmatrix} = \sum_{n=1}^{\infty} \begin{Bmatrix} U_n \cos\left(\frac{n\pi}{L}x\right) \\ W_n \sin\left(\frac{n\pi}{L}x\right) \\ \Phi_n \cos\left(\frac{n\pi}{L}x\right) \end{Bmatrix} e^{i\omega_n t}, \tag{40}$$

where (U_n, W_n, Φ_n) are the unknown Fourier coefficients to be determined for each n value. The boundary conditions for simply-supported beam are given by

$$u(0, t) = 0, \quad \left. \frac{\partial u}{\partial x} \right|_{x=L} = 0, \quad w(0, t) = w(L, t) = 0, \quad \left. \frac{\partial \varphi}{\partial x} \right|_{x=0} = \left. \frac{\partial \varphi}{\partial x} \right|_{x=L} = 0. \tag{41}$$

Substituting Eq. (40) into Eqs. (37)-(39), respectively, leads to

$$\begin{aligned}
 \left\{ I_0 \left[1 + \mu \left(\frac{n\pi}{L} \right)^2 \right] \omega_n^2 - A_{xx} \left(\frac{n\pi}{L} \right)^2 \right\} U_n + \left\{ \hat{I}_1 \left[1 + \mu \left(\frac{n\pi}{L} \right)^2 \right] \omega_n^2 - K_{xx} \left(\frac{n\pi}{L} \right)^2 \right\} \Phi_n \\
 + \frac{n\pi}{L} \alpha \left\{ E_{xx} \left(\frac{n\pi}{L} \right)^2 - I_3 \left[1 + \mu \left(\frac{n\pi}{L} \right)^2 \right] \omega_n^2 \right\} W_n = 0,
 \end{aligned} \tag{42}$$

$$\begin{aligned}
 \left\{ \hat{I}_1 \left[1 + \mu \left(\frac{n\pi}{L} \right)^2 \right] \omega_n^2 - K_{xx} \left(\frac{n\pi}{L} \right)^2 \right\} U_n - \left\{ \bar{I}_{xx} \left(\frac{n\pi}{L} \right)^2 + A_{xz}^* - \hat{I}_2 \left[1 + \mu \left(\frac{n\pi}{L} \right)^2 \right] \omega_n^2 \right\} \Phi_n \\
 + \frac{n\pi}{L} \alpha \left\{ J_{xx} \left(\frac{n\pi}{L} \right)^2 - A_{xz}^* - \alpha \hat{I}_4 \left[1 + \mu \left(\frac{n\pi}{L} \right)^2 \right] \omega_n^2 \right\} W_n = 0,
 \end{aligned} \tag{43}$$

$$\begin{aligned}
 \frac{n\pi}{L} \alpha \left\{ E_{xx} \left(\frac{n\pi}{L} \right)^2 - I_3 \left[1 + \mu \left(\frac{n\pi}{L} \right)^2 \right] \omega_n^2 \right\} U_n \\
 + \frac{n\pi}{L} \alpha \left\{ J_{xx} \left(\frac{n\pi}{L} \right)^2 - A_{xz}^* - \alpha \hat{I}_4 \left[1 + \mu \left(\frac{n\pi}{L} \right)^2 \right] \omega_n^2 \right\} \Phi_n \\
 \left\{ - \left(\frac{n\pi}{L} \right)^2 \left[A_{xz}^* + \alpha^2 H_{xx} \left(\frac{n\pi}{L} \right)^2 \right] + \left[1 + \mu \left(\frac{n\pi}{L} \right)^2 \right] \left[N^T \left(\frac{n\pi}{L} \right)^2 + \left(I_0 + \alpha^2 I_6 \left(\frac{n\pi}{L} \right)^2 \right) \omega_n^2 \right] \right\} W_n \\
 = 0.
 \end{aligned} \tag{44}$$

By setting the determinant of the coefficient matrix of the above equations, the analytical solutions can be obtained from the following equations

$$([K] + N^T[K^T] - \omega_n^2[M])\{\Delta\} = \{0\}, \quad (45)$$

where $\{\Delta\} = \{U_n, W_n, \Phi_n\}^T$, $[K]$ is the stiffness matrix, $[K^T]$ is the coefficient matrix of temperature change, and $[M]$ is the mass matrix. By setting this polynomial to zero, we can find natural frequencies ω_n .

4. Types of thermal loading

4.1 Uniform temperature rise (UTR)

For a FG nanobeam at reference temperature T_0 the temperature is uniformly raised to a final value T which the temperature change is $\Delta T = T - T_0$.

4.2 Linear temperature rise (LTR)

For a FG nanobeam for which the beam thickness is thin enough, the temperature distribution is assumed to be varied linearly through the thickness as follows

$$T = T_m + \Delta T \left(\frac{z}{h} + \frac{1}{2} \right), \quad (46)$$

where the buckling temperature difference is $\Delta T = T_c - T_m$ in which T_c and T_m are the temperature of the top surface which is ceramic-rich and the bottom surface which is metal-rich, respectively.

4.3 Nonlinear temperature rise (NLTR)

The one-dimensional temperature distribution through-the-thickness can be obtained by solving the steady-state heat conduction equation with the boundary conditions on bottom and top surfaces of the beam across the thickness

$$-\frac{d}{dz} \left(\kappa(z, T) \frac{dT}{dz} \right) = 0, \quad T|_{z=h/2} = T_c, \quad T|_{z=-h/2} = T_m \quad (47)$$

The solution of above equation is

$$T = T_m + (T_c - T_m) \frac{\int_{-h/2}^z \frac{1}{\kappa(z, T)} dz}{\int_{-h/2}^{h/2} \frac{1}{\kappa(z, T)} dz}. \quad (48)$$

5. Numerical results and discussions

The thermal vibration analysis of FG nanobeams are investigated using the Navier method based upon third order shear deformation theory and nonlocal elasticity theory. The effective material properties, that elasticity modulus and mass density of the FG nanobeam vary through the thickness direction according to power law distribution. Simply supported boundary condition is considered for which required the Navier method. The effects of FG material gradation, nonlocality effect, slenderness ratio and thermal load on the non-dimensional natural frequencies

Table 2 Comparison of the non-dimensional fundamental frequency for a FG nanobeam with various power-law indexes at $L/h = 20$ and $\Delta T = 0$ K

μ	$p = 0$			$p = 0.5$		
	EBT (Eltaher <i>et al.</i> 2012)	TBT (Rahmani and Pedram (24))	Present RBT	EBT (Eltaher <i>et al.</i> 2012)	TBT (Rahmani and Pedram (24))	Present RBT
0	9.8797	9.8296	9.82957	7.8061	7.7149	7.71546
1	9.4238	9.3777	9.377686	7.4458	7.3602	7.36078
2	9.0257	8.9829	8.982894	7.1312	7.0504	7.0509
3	8.6741	8.6341	8.634103	6.8533	6.7766	6.77714
4	8.3607	8.323	8.323021	6.6057	6.5325	6.53296
5	8.0789	8.0433	8.043309	6.383	6.3129	6.31342

μ	$p = 1$			$p = 5$		
	EBT (Eltaher <i>et al.</i> 2012)	TBT (Rahmani and Pedram (24))	Present RBT	EBT (Eltaher <i>et al.</i> 2012)	TBT (Rahmani and Pedram (24))	Present RBT
0	7.0904	6.9676	6.967613	6.0025	5.9172	5.916152
1	6.7631	6.6473	6.6473	5.7256	5.6452	5.644175
2	6.4774	6.3674	6.367454	5.4837	5.4075	5.406561
3	6.2251	6.1202	6.120217	5.2702	5.1975	5.196632
4	6.0001	5.8997	5.899708	5.0797	5.0103	5.0094
5	5.7979	5.7014	5.701436	4.9086	4.8419	4.841049

of the FG nanobeam will be figured out. FG nanobeam is composed of steel (SUS304) and alumina (Al_2O_3) where its properties are given in Table 1. The bottom surface of the beam is pure Steel, whereas the top surface of the beam is pure Alumina. The beam geometry has the following dimensions: L (length) = 10000 nm, b (width) = 1000 nm. A 5K increase in metal surface to reference temperature T_0 of FG nanobeam is considered, i.e., $T_m - T_0 = 5K$. The following dimensionless relation is defined in order to calculate the non-dimensional natural frequencies

$$\hat{\omega} = \omega_n L^2 \sqrt{\frac{\rho_c A}{E_c I}} \tag{49}$$

where $I = bh^3/12$ is the moment of inertia of the cross section of the nanobeam. For the verification purpose, the non-dimensional natural frequency of simply supported FG nanobeam with various nonlocal parameters and power-law exponents are compared with the results presented by Eltaher *et al.* (2012) For Euler-Bernoulli FG nanobeams and Rahmani and Pedram (2014) which has been obtained by analytical method for FG Timoshenko nanobeam. In these works, the material properties are selected as: $E_m = 210$ GPa, $E_c = 390$ GPa, $\rho_m = 7800$ kg/ m^3 , $\rho_c = 3900$ kg/ m^3 , $\nu_m = 0.3$ and $\nu_c = 0.24$.

The reliability of the presented method and procedure for FG nanobeam may be concluded from Table 2; where the results are in an excellent agreement as values of non-dimensional frequency are consistent with presented analytical solution. It can be observed from Table 2 that the results of nonlocal Reddy beam theory are smaller than those of nonlocal Euler beam theory.

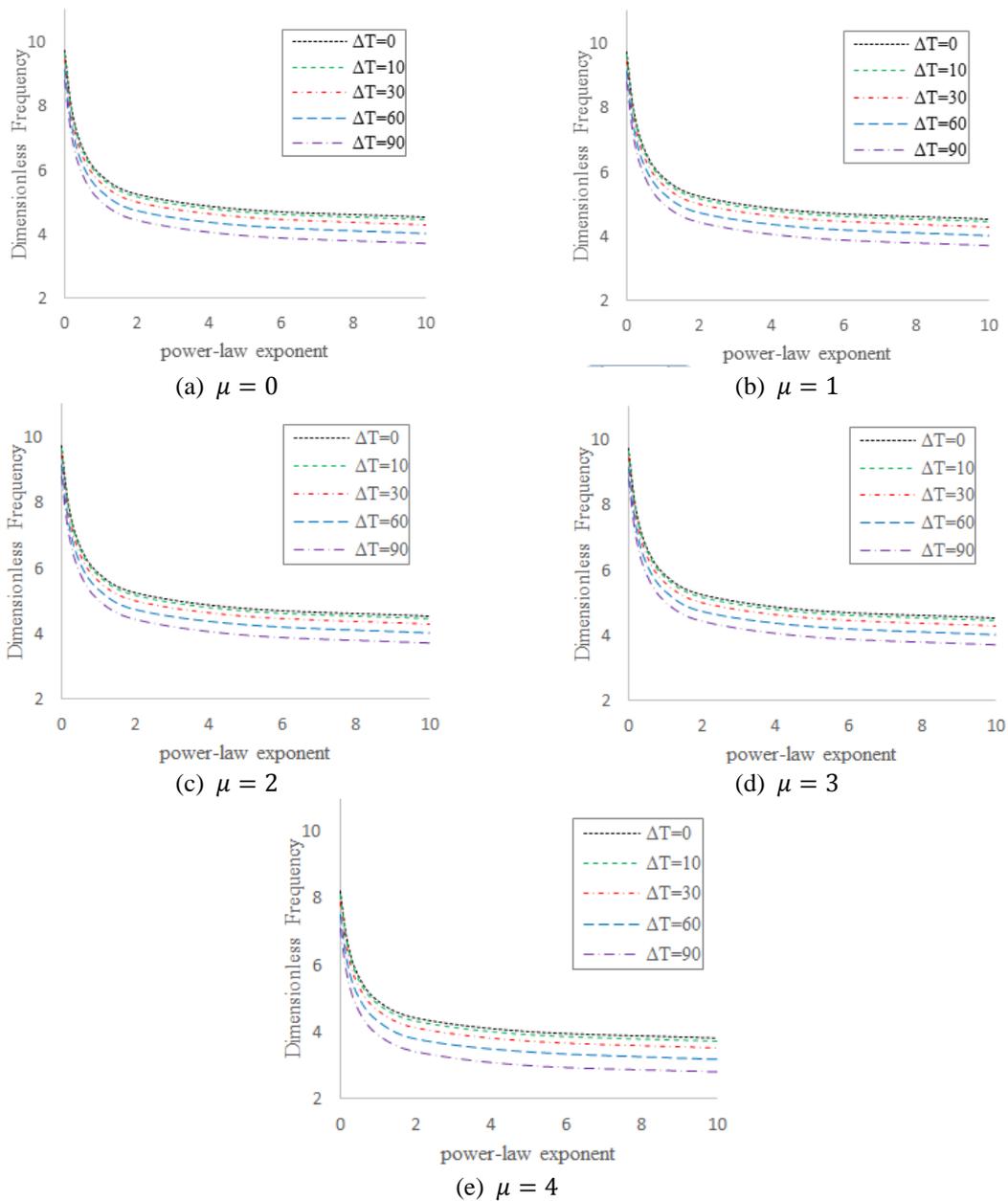


Fig. 2 The variation of the first dimensionless frequency of FG nanobeam under non-linear temperature change for different nonlocal parameters ($L/h = 20$)

This is due to the fact that Euler-Bernoulli beam model cannot capture shear deformation effect.

The dimensionless frequencies of FG nanobeam versus the power-law exponent under non-linear temperature rise through-the-thickness at $L/h = 20$ are depicted in Fig. 2, respectively. In this figure, regardless of the thermal loading types, the dimensionless natural frequency decreases suddenly as the power-law exponent increases from 0 to 2, then decreases monotonically as the

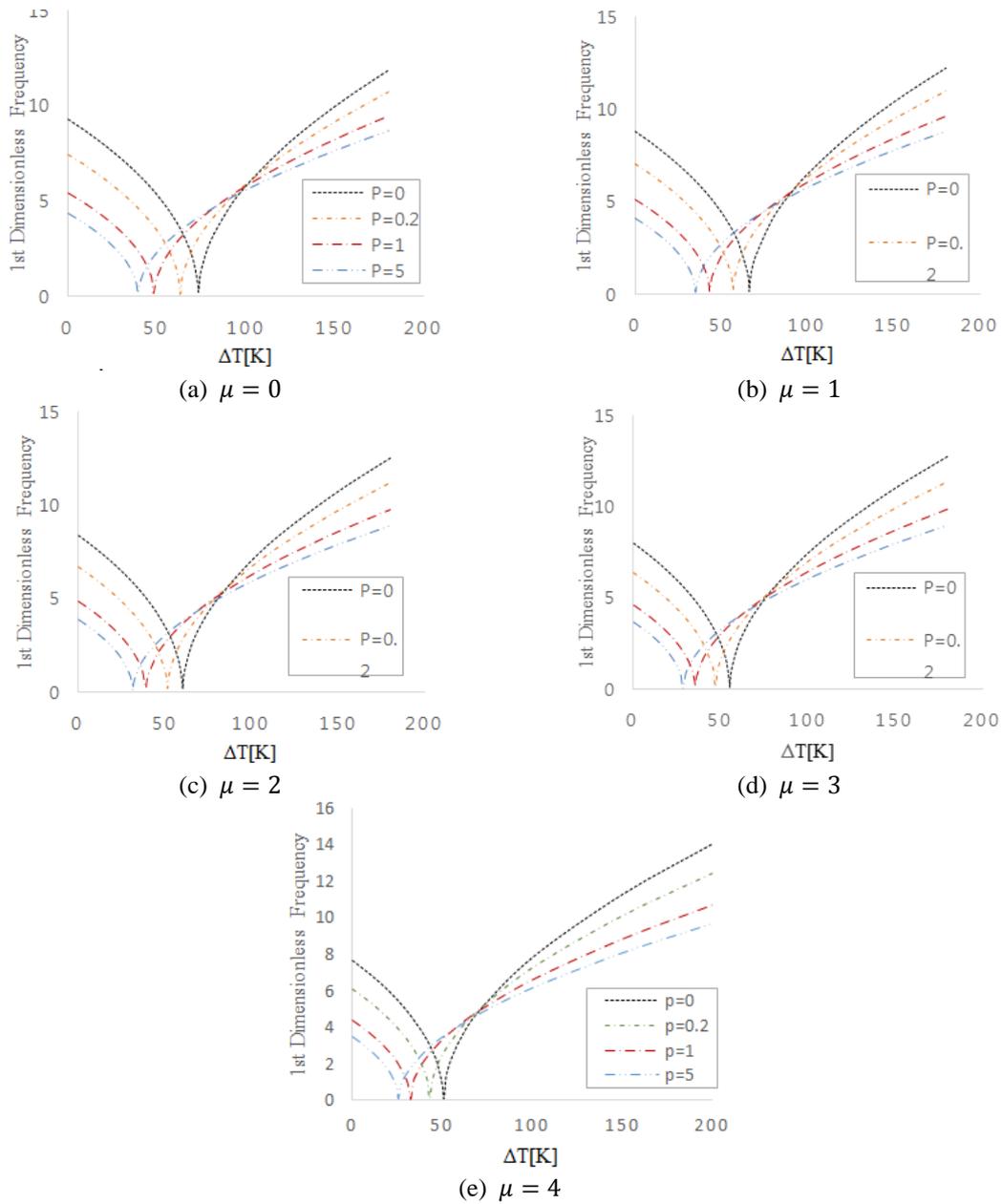


Fig. 3 Variations of the first dimensionless natural frequency of the FG nanobeam with respect to linear temperature change for different values of power-law exponent and nonlocal parameters ($L/h = 25$)

power-law exponent increases from 2 to 10. It can be observed from the results of this figure that by increasing the nonlocal parameter the first dimensionless frequency reduce for every power-law exponent and temperature change, which indicates the notability of the nonlocal effect.

Also, it must be noted that the dimensionless natural frequency of the FG nanobeam under non-linear temperature rise is greater than that of the FG nanobeam under linear temperature rise and

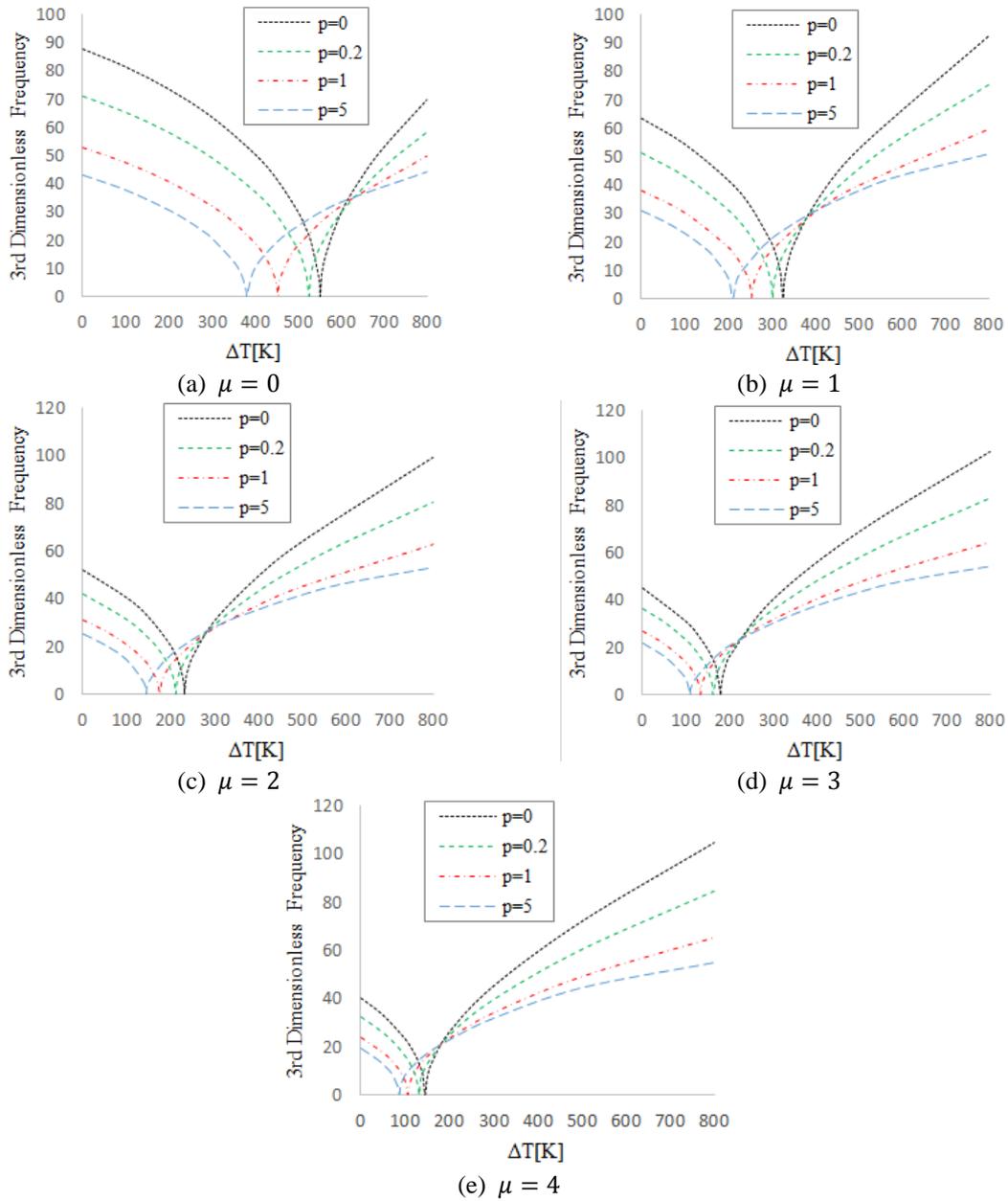


Fig. 4 Variations of the third dimensionless natural frequency of the FG nanobeam with respect to non-linear temperature change for different values of gradient indexes and nonlocal parameters ($L/h = 50$)

the latter is greater than that of the FG nanobeam under uniform temperature rise. Fig. 3 illustrates the variation of first dimensionless natural frequency with changing of the power-law exponent for different nonlocal parameter at slenderness ratio $L/h = 50$ of FG nanobeam under linear temperature change. Also, the variation of third dimensionless natural frequency with temperature rise is presented in Fig. 4. It is seen that before a prescribed temperature, i.e. the critical buckling

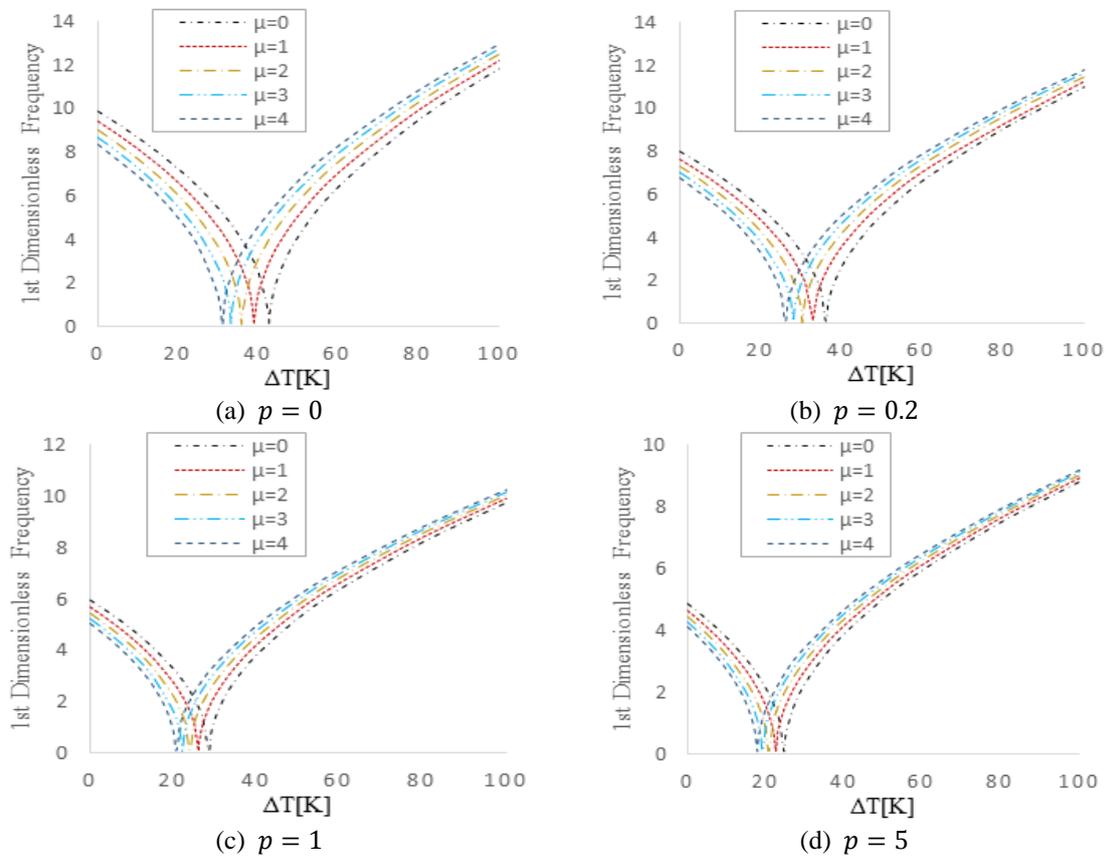


Fig. 5 Variations of the first dimensionless natural frequency of the FG nanobeam with respect to uniform temperature change for different values of nonlocal parameters and power-law exponents ($L/h = 50$)

temperature, as temperature increases the first dimensionless frequency reduces. This is associated to the reduction in total stiffness of the beam, since geometrical stiffness of FG nanobeam diminishes as temperature rises. Near the critical buckling temperature, dimensionless frequency trends to zero. It is observed that temperature dependency of the material constituents leads to more accurate conclusions, whereas with supposing temperature independent material properties, critical buckling temperature point is exaggerated. Moreover, in the pre-buckling region, with the temperature dependent assumption, predicted frequencies are smaller than those frequencies obtained with the assumption of temperature independent material. This is due to the less Young's modulus of the material constituents in the case of temperature dependent material. Also, it should be stated that, by increasing the nonlocal parameter the critical temperature point shifts to the left. Fig. 5 show the variations of the first dimensionless natural frequency of the FG nanobeam under uniform temperature change with respect to temperature change, respectively for different values of nonlocal parameters and power-law indexes ($L/h = 50$). It should be noted that compressive axial forces as resultants of thermal stresses arising from the temperature rise in beams with micro/nano scales, can lead to buckling the beams if its value passes the critical value. By imposing a high external pressure to the FG nanobeam structure, the high stresses induced in the structure will influence its integrity and the structure is talented and disposed to failure. Therefore,

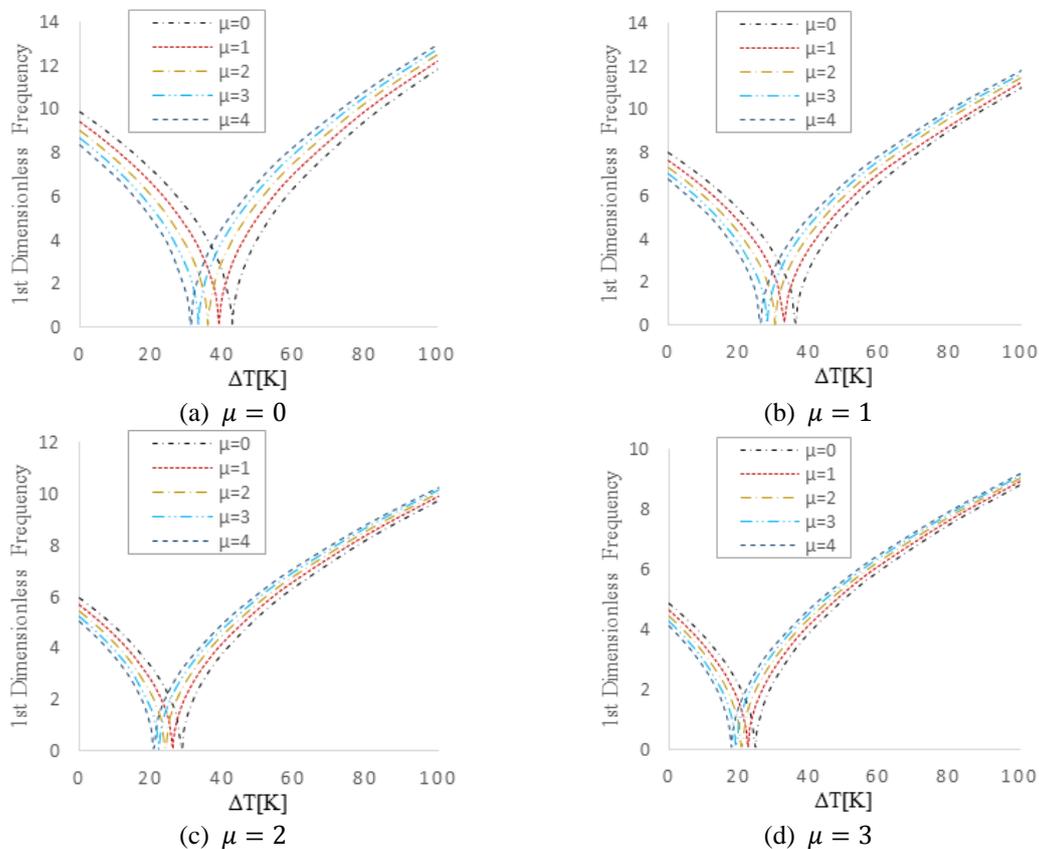


Fig. 6 Variations of the first dimensionless natural frequency of the FG nanobeam with respect to non-linear temperature change for different values of slenderness ratios and nonlocal parameters

it is observed from these figures that dimensionless frequencies of the FG nanobeam approaches to zero around a prescribed temperature which is the critical buckling temperature. Before the critical buckling point, as temperature rises the dimensionless frequency reduces, but after that point, as temperature grows the dimensionless frequency increases. Moreover, in the pre-buckling domain, by increasing the nonlocal parameter, the dimensionless natural frequency diminishes at a constant power-law exponent, while in the post-buckling domain, as the nonlocal parameter increases the dimensionless frequency raises. Another notable observation is that, the critical point is postponed with the assumption of the smaller power-law indexes, related to the fact that the lower power-law indexes result in the increase of stiffness of the beam. The variations of the first dimensionless natural frequency of the FG nanobeam under non-linear temperature rise with respect to temperature change for different values of slenderness ratios ($L/h = 40, 50, 60$) and nonlocal parameters at $p = 0.2$ is presented in Fig. 6. It is revealed that for FG nanobeams in the pre-buckling domain, increasing slenderness ratio leads to decrease in natural frequency. But in the post-buckling domain increase of slenderness ratio leads to increment in natural frequency. Also, it is seen that when nonlocal parameter increases the critical buckling point continuously moves to the left at a fixed material power-law index.

It is seen from the results of these figures that in all cases of thermal loading increasing

nonlocal scale parameter leads to decreasing in the non-dimensional frequencies at a constant power-law exponent. So it is worth noting that nonlocal parameter has a remarkable effect on the natural frequencies of FG nanobeams. Also, it is observed that, by fixing nonlocal parameter and increasing power-law exponent the non-dimensional frequencies reduces, especially for lower values of power-law exponent. In addition, it is concluded that the values of non-dimensional frequencies temperature in the case of nonlinear temperature change are bigger than those of uniform and linear temperature change at a constant power-law exponent and slenderness ratio.

6. Conclusions

Thermal vibration characteristics of the temperature-dependent FG nanobeams in thermal environment are investigated based on nonlocal third order shear deformation beam theory in conjunction with Navier analytical method. The effects of three types of thermal loads namely uniform, linear and nonlinear temperature changes on vibrational responses of FG nanobeams are studied. Thermo-mechanical properties of the FG nanobeams are both temperature-dependent and position-dependent. The nonlocal governing differential equations in thermal environment are derived by implementing Hamilton's principle and using nonlocal constitutive equations of Eringen. Accuracy of the results is examined using available data in the literature. The effects of small scale parameter, material gradation, thermal loading and slenderness ratio on thermal vibration of FG nanobeams are investigated. It is observed that the fundamental frequency decreases with the increase in temperature and trends to zero at the critical temperature point. Diminution of frequency with thermal load before the critical point is attributed to the weakening effect of thermally induced compressive stress on the beam stiffness. Moreover, after passing the critical buckling temperature, the fundamental frequency increases with the increment of temperature. Also, it is concluded that under all types of temperature rises, as the power-law exponent grows the natural frequencies diminish, whereas, a reverse trend is observed in the post-buckling domain. In addition it is revealed that for the FG nanobeams subjected to nonlinear temperature changes through the thickness, the obtained frequencies are higher than that for the FG nanobeams subjected to the uniform and linear temperature changes.

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