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Seismic response of foundation-mat structure subjected to local uplift

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Abstract. The effects of large rotations and p-delta on the dynamic response of a structure subjected to seismic loading and local uplift of its foundation were analyzed in this work. The structure was modeled by an equivalent flexible mat mounted on a rigid foundation that is supported either by a Winkler soil type or a rigid soil. The equations of motion of the system were derived by taking into account the equilibrium of the coupled foundation-mat system where the structure was idealized as a single-degree-of-freedom. The obtained nonlinear coupled system of ordinary differential equations was integrated by using an adequate numerical scheme. A parametric study was performed then in order to evaluate the maximum response of the system as function of the intensity of the earthquake, the slenderness of the structure, the ratio of the mass of the foundation to the mass of the structure. Three cases were considered: (i) local uplift of foundation under large rotation with the p-delta effect, (ii) local uplift of foundation under large rotation without including the p-delta effect, (iii) local uplift of foundation under small rotation. It was found that, in the considered ranges of parameters and for moderate earthquakes, assuming small rotation of foundation under seismic loading can yield more adverse structural response, while the p-delta effect has almost no effect.

Keywords: seismic response; Winkler foundation; rigid soil; local uplift; large rotations; p-delta effect

1. Introduction

A common feature of civil engineering structures is that they involve contact with ground. When the external forces, such as those generated by earthquakes, act on these systems, the structural displacements and the ground displacements are coupled to each other. The process in which the response of the soil influences the motion of the structure and vice versa is termed as soil-structure interaction (SSI).

Under some conditions such the case for light structures consisting of low rise buildings laying on a relatively stiff soil, neglecting the effects of SSI may be reasonable. These effects can however be important for relatively weighty structures founded on soft soils such as high-rise buildings and elevated-highways on soft soil.

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The most general framework for analyzing SSI response appearing in foundation-mat structure subjected to seismic action is to use fully coupled transient dynamic soil-structure equations. In dealing with this complete formulation approach, one major difficulty to overcome is the modeling of unbounded media. Various methods have been introduced for this purpose. Among them, one finds continuum based approaches, Wolf (1985). Methods related to this approach are numerous. They can be classified in several families such as: finite-infinite elements, Lysmer and Kuhlemeyer (1969), thin layer transmitting boundaries, Lysmer (1970), boundary elements, Brebbia *et al.* (1984), absorbing layers, Song and Wolf (1994) and finite soil domain with continued fraction absorbing boundaries, Guddati and Lim (2006). A common characteristic of these continuum based methods is that they need elevated numerical effort to obtain an approximation of the coupled problem solution.

Considering earthquake response evaluation of large linear structural systems with local non-linearities, an efficient dynamic substructuring technique was presented by Ibrahimbegovic and Wilson (1990a). The authors have provided a rational approach to the SSI problem with predetermined non-linearities occurring along the structure-foundation interface. In particular, they have addressed uplifting of the structure as a natural base isolation concept and have illustrated that through a numerical example. To enhance computational efficiency and accuracy in SSI problems consisting of large linear systems with non-proportional damping, Ibrahimbegovic *et al.* (1990b) have compared real and complex Ritz vector bases with conventional eigenvector bases. The authors have shown that Ritz vector bases were superior in terms of numerical efficiency. In another work Ibrahimbegovic and Wilson (1992) have presented several methods for enhancing computational effectiveness in both static and dynamic analysis of structural systems with localized non-linear behavior. A significant reduction of computational effort with respect to brute-force non-linear analysis was achieved with no significant loss of accuracy.

Using continuum based approaches in the presence of structural geometric nonlinearities or for the case where the displacements are not small like for analyzing superstructure rocking problem is still a real challenge. The computational effort needed by the global procedures increases considerably due to the restriction imposed on the time step size for the convergence of the numerical solution to occur. To reduce the computational effort which is required by continuum based methods, alternative approaches were introduced such as those based on Winkler foundation concept, Houlsby *et al.* (2005). In this way of modeling, a series of discrete elements like for example springs are used to represent the sub-grade soil behavior.

In the following, focus is on the Winkler based foundation approach and the soil will be modeled by equivalent discrete supports. Meanwhile, the elastic superstructure will be represented by a limited number of degrees-of-freedom with lumped masses. These simplifications enable to take into account more straightforwardly localized nonlinearities such as foundation base uplifting, soil-footing friction or structural large displacements.

The effect of foundation uplifting on the dynamical response of a structure has been largely investigated by many researchers. Housner (1963) was the first to study in detail the problem of structures with uplift and to observe some favorable effect of uplift on structural response magnitude. Meek (1975) studied the effect of tipping-uplift on the response of a single-degree-of-freedom (SDOF) system and reported that allowing the SDOF system to uplift alters its natural frequency and lead to significant reductions in base reactions and in transverse deformations. Meek (1978) performed analysis of a core stiffened buildings and concluded that in comparison with a fixed-base core-braced structures, tipping greatly reduces the base shear and moment when subjected to seismic excitation.

Recent investigation on foundation uplifting effect on the structural dynamical response include the work by Oliveto *et al.* (2003) who improved the analytical model utilized by previous researchers (Meek 1975, Psycharis 1991, Chopra and Yim 1984, Chopra and Yim 1985, Apostolou *et al.* 2007) and have derived the equations of motion for a SDOF structure with uplift under the assumption of large displacements. The transition conditions between successive phases of motion were derived and enabled to interpret some behaviors that occur in the structural response that have been reported in the literature. The minimum horizontal acceleration impulses for the uplift and the overturning of the system were determined in closed analytical form. The authors have concluded that for earthquake ground motions, the linearized models generally underestimate the structural response.

Using the assumption of small rotations, Anastasopoulos *et al.* (2012) have assumed inelastic behavior of the soil-structure interaction problem. The authors compared the performance of two approaches to design against earthquakes: classical over-designing and new under-designing. They have concluded that for large intensity earthquakes the performance of the new design scheme is more advantageous in avoiding collapse, as it limits excessive inelastic structural deformations while increasing residual settlement and rotations of the foundation.

Using the finite element method, Faramarz *et al.* (2012) have analyzed the effects of p-delta and uplift phenomena on the response of a structure which is founded on an elastic-plastic soil. Their main conclusions are that foundation uplift reduces the lateral stiffness of system, yielding a decrease of story shear and base shear. The same observation was made regarding the p-delta effect as it was found to be favorable in decreasing base shear.

Acikgoz and DeJong (2012) derived the equations of motion for a foundation-mat structure by using a Lagrangian formulation for large rotations. They investigated the interaction of elasticity and rocking in an attempt to clarify the fundamental dynamics of flexible rocking systems. A parametric analysis was performed to measure the effect of elasticity on uplift, overturning instability and resonance occurring under harmonic excitation.

Acikgoz and DeJong (2014) characterized and predict the maximum rocking response of large and flexible structures to earthquakes using an idealized structural model. To achieve this, the maximum rocking demand caused by different earthquake records was evaluated using several ground motion intensity measures.

Calio and Greco (2014) investigated some typical aspects of the nonlinear behavior of a flexible structure subjected to harmonic base excitation and foundation uplift. Its response was evaluated with reference to large rigid rotations and small elastic deformations.

Sinan *et al.* (2016) presented experimental results of free vibration and earthquake excitation tests to investigate the dynamic behavior of freely rocking flexible structures with different geometric and vibration characteristics. The tests were conducted to identify the dynamic characteristics of the structure, and to determine how these characteristics influence displacement and acceleration demands.

In the present work, the effect on the seismic response of a foundation-mat structure undergoing foundation local uplift is analyzed by considering both small and large rotations. The modeling considers also p-delta effect resulting from second order projection terms. The system is represented by a SDOF system which is considered to be bonded on two types of foundations: Winkler like foundation and rigid soil. Based on some reference models already existing in the literature (Yim and Chopra 1979, Sinan *et al.* 2016), the equations of motion are derived in a new synthesized hierarchical form enabling to make comprehensible comparison between the various models. The degrees of freedom are related to mat tip lateral displacement, base vertical

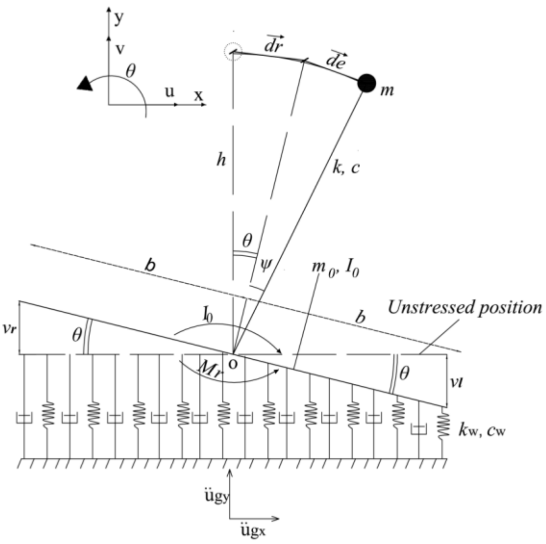


Fig. 1 Flexible structure on Winkler foundation

displacement and base rotation.

In all cases, the obtained mathematical model takes the form of a coupled system of second order ordinary differential equations that are strongly nonlinear. These systems are integrated by using a special numerical scheme.

A parametric analysis is performed in order to evaluate the maximum response for these various models as function of key parameters under the action of given scaled seismic accelerations that are applied at the bottom of the base mat. Two different ground motions are considered: El Centro 1940 and Kobe NS earthquakes.

2. Materials and methods

2.1 Analytical model of flexible structure on elastic foundation

The structure is assumed to behave like a beam mat which can be further characterized by its first natural mode of vibration. The structure is like this represented by a SDOF system of mass m, lateral stiffness k and lateral damping c. The mat is supposed to be mounted on a rigid foundation basis that is assumed to react as a rigid rectangular plate of negligible thickness. The foundation mass denoted by m_0 is taken to be uniformly distributed; the total moment of inertia of the foundation about its center is designated by I_0.

The soil-structure interaction takes place at the interface separating the rigid footing and the foundation soil. This interaction can be described by springs and damping elements that are distributed over the entire width of the foundation.

Fig. 1 gives a schematic representation of the coupled foundation-mat system. The horizontal slippage between the mat and supporting elements is assumed to be negligible. For the foundation model, the stiffness per unit length is designated by k_w and damping per unit length coefficient is c_w.

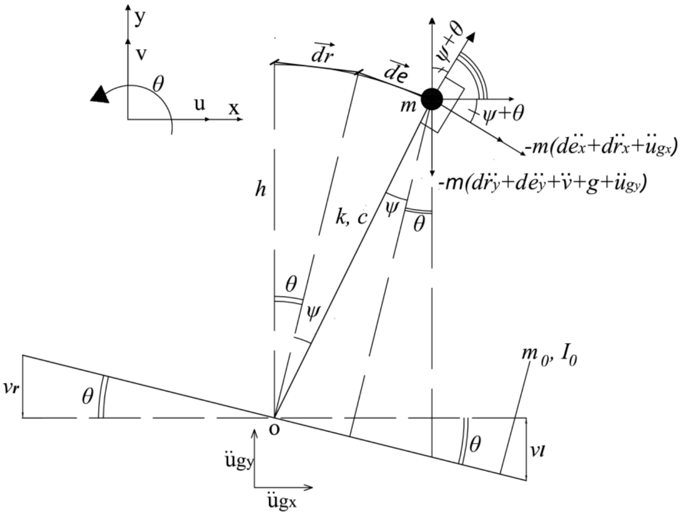


Fig. 2 Free body diagram of the system with uplift showing the considered dependent and independent degrees of freedom

They are assumed constant and independent from displacement amplitude or excitation frequency. The seismic excitation acting on the base foundation is specified as horizontal and vertical accelerations. Under the influence of this excitation, the foundation mat undergoes a rigid body motion described by the rotation of angle θ and global vertical displacement of its centre of gravity v which are both defined with regards to the unstressed position. Uplifting can occur when for a given soil foundation contact point the local vertical displacement is ascending.

In Fig. 1, h designates the height of the structure from the base, M_r the total moment acting on the base mat, $\vec{d}_r$ the rigid horizontal displacement, $\vec{d}_e$ the elastic horizontal displacement of the mat tip relative to the base, $\ddot{u}_g$ the seismic acceleration, v the vertical displacement of the centre of gravity of the base mat, θ the angle of rotation of the mat base, Ψ the angle rotation due to the deformation of the structure which is assumed to remain small so that sin(Ψ) and cos(Ψ) can be approximated respectively by Ψ and 1. Finally, b is the half width of the foundation mat.

In the following the other notations used are given next.

$\omega=\sqrt{k/m}$ natural frequency of the rigidly supported structure; $\omega_v=\sqrt{2k_w b/\left(m+m_0\right)}$ vertical vibration frequency of the system with its foundation mat bonded to the supporting elements; $\alpha=h/b$ slenderness ratio; $\beta=\omega_v/\omega$ frequency ratio; $\gamma=m_0/m$ foundation mass to superstructure mass ratio; $\xi=c/(2m\omega)$ damping ratio of the rigidly supported structure; $\xi_v=c_w b/[(m+m_0)\omega_v]$ damping ratio in vertical vibration of the system with its foundation mat bonded to the supporting elements.

The equations of motion of the entire system are derived by taking into account the equilibrium of the coupled foundation-mat system. The free body diagram of the system with inertial forces is shown in Fig. 2. The three equilibrium equations are:

-Equilibrium of horizontal forces acting on the coupled foundation-mat system: $\Sigma F_x=0$

-Equilibrium of vertical forces acting on the coupled foundation-mat system: $\Sigma F_y=0$

-Equilibrium of moments acting on the coupled foundation-mat system written at the center of the foundation: $\Sigma M_z=0$

2.1.1 Equations of motion for large rotations with including p-delta effect

The equations of motion are coupled equations that can be written in terms of the reduced parameters: d_{ex}, v and θ.

From geometrical considerations one obtains the following relations

$$d_{rx} = 2h\sin\left(\frac{\theta}{2}\right)\cos(\theta) \tag{1}$$

$$d_{ry} = -2h\sin\left(\frac{\theta}{2}\right)\sin(\theta) \tag{2}$$

$$d_{ex} = 2h\sin\left(\frac{\psi}{2}\right)\cos(\psi+\theta) \tag{3}$$

$$d_{ey} = -2h\sin\left(\frac{\psi}{2}\right)\sin(\psi+\theta) \tag{4}$$

As the elastic rotation Ψ is assumed to be small, the following simplifications hold

$$\cos(\theta+\psi) = \cos(\theta) - \psi\sin(\theta) \tag{5}$$

$$\sin(\theta+\psi) = \psi\cos(\theta) + \sin(\theta) \tag{6}$$

Substituting Eqs. (5)-(6) in Eqs. (3)-(4) yields

$$\psi = \frac{1}{h\cos(\theta)} d_{ex} \tag{7}$$

$$d_{ey} = -h\psi\sin(\theta) \tag{8}$$

Considering the equilibrium of forces in the lateral direction x, the equation of motion in terms of the mat tip lateral displacement $u=d_{ex}$ writes

$$m\left(\ddot{u}+\ddot{d}_{rx}+\ddot{u}_{gx}\right)\cos(\theta+\psi)+c\dot{u}+ku = -m\left(\ddot{v}+\ddot{d}_{ey}+\ddot{d}_{ry}+\ddot{u}_{gy}+\mathrm{g}\right)\cos(\theta+\psi)\sin(\theta+\psi) \tag{9}$$

Which reduces by using Eqs. (5)-(9) to the following coupled nonlinear second order differential equation

$$\begin{aligned} &\ddot{u}+\frac{ch\cos(\theta)}{m\left(h\cos^2(\theta)-\sin(\theta)u\right)}\dot{u}+\frac{kh\cos(\theta)}{m\left(h\cos^2(\theta)-\sin(\theta)u\right)}u = -\ddot{u}_{gx}-\ddot{d}_{rx} \\ &-\left(\ddot{v}+\ddot{d}_{ey}+\ddot{d}_{ry}+\ddot{u}_{gy}+\mathrm{g}\right)\left(\frac{1}{h}u+\sin(\theta)\right) \end{aligned} \tag{10}$$

With

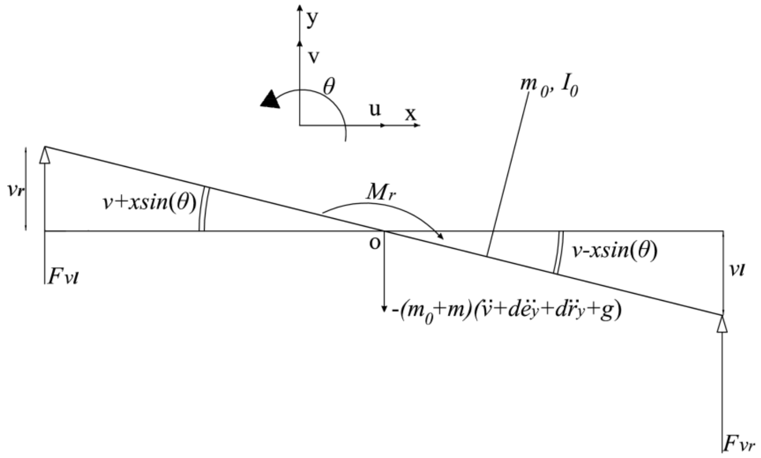


Fig. 3 Free body diagram for the base

$$\ddot{d}_{rx} = \frac{h\ddot{\theta}}{2}\left[3\cos\left(\frac{3\theta}{2}\right)-\cos\left(\frac{\theta}{2}\right)\right]+\frac{h\dot{\theta}^2}{4}\left[-9\sin\left(\frac{3\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)\right] \tag{11}$$

$$\ddot{d}_{ry} = \frac{h\ddot{\theta}}{2}\left[\sin\left(\frac{\theta}{2}\right)-3\sin\left(\frac{3\theta}{2}\right)\right]+\frac{h\dot{\theta}^2}{4}\left[\cos\left(\frac{\theta}{2}\right)-9\cos\left(\frac{3\theta}{2}\right)\right] \tag{12}$$

$$\ddot{d}_{ey} = -\tan(\theta)\ddot{d}_{ex} - 2\left(1+\tan^2(\theta)\right)\dot{\theta}\dot{d}_{ex} - \left(1+\tan^2(\theta)\right)\left(\ddot{\theta}+2\dot{\theta}^2\tan(\theta)\right)de_x \tag{13}$$

The equilibrium of forces in the vertical direction can be written as

$$(m+m_0)\left(\ddot{v}+\ddot{d}_{ey}+\ddot{d}_{ry}\right)+F_v = -(m+m_0)g+(m+m_0)\ddot{u}_{gy} \tag{14}$$

With

$$F_v = \int_{-b}^{b}\left(k_w v_{(\ell,r)}+c_w\dot{v}_{(\ell,r)}\right)dx \tag{15}$$

The vertical displacements at the edges of the foundation mat, see Fig. 3, measured from the initial unstressed positions are given by

$$v_i = v \pm x\sin(\theta),\ i = l, r \tag{16}$$

In Eq. (15), F_v is the total vertical force acting on the base mat. This force is obtained as

$$F_v = k_w\int\left(v \pm x\sin(\theta)\right)dx + c_w\int\left(\dot{v} \pm x\dot{\theta}\cos(\theta)\right)dx \tag{17}$$

Because the Winkler foundation cannot extend above its initial unstressed position, an edge of the foundation mat would uplift at the time instant for which the following condition is fulfilled, Housner (1963),

$$v_i(t) > 0, \quad i = l, r \tag{18}$$

Integrating Eq. (17) by taking into account the uplifting condition given by Eq. (18), one arrives after simplifications at the following expression

$$F_v = \left(1-\varepsilon_1^2\right)\varepsilon_2 \frac{b^2}{2}\left[c_w\dot{\theta}\cos(\theta)+k_w\sin(\theta)\right]-\left(1+\varepsilon_1\right)b\left(c_w\dot{v}+k_w v\right) \tag{19}$$

With

$$\varepsilon_1 = \begin{cases} 1 & \text{contact at both edges} \\ \dfrac{\varepsilon_2 v}{b\sin(\theta)} & \text{one edge is uplifted} \end{cases} \tag{20}$$

$$\varepsilon_2 = \begin{cases} -1 & \text{left edge uplifted} \\ 0 & \text{contact at both edges} \\ 1 & \text{right edge uplifted} \end{cases} \tag{21}$$

Substituting Eq. (19) in Eq. (14) one obtains the following equation

$$\ddot{v} = -\left(1-\varepsilon_1^2\right)\varepsilon_2\xi_v\beta\omega\frac{b}{2}\left[\dot{\theta}\cos(\theta)+\frac{k_w}{c_w}\sin(\theta)\right]-\left(1+\varepsilon_1\right)\xi_v\beta\omega\left(\dot{v}+\frac{k_w}{c_w}v\right)-\left(\ddot{d}_{ry}+\ddot{d}_{ey}\right)-g+\ddot{u}_{gy} \tag{22}$$

Taking the resultant moment about the centre of the foundation at the base of the mat, the following equation is readily obtained

$$I_0\ddot{\theta}+hm\left(\ddot{d}_{rx}+\ddot{u}+\ddot{u}_{gx}\right)\cos(\theta+\psi)+M_r = 0 \tag{23}$$

With $I_0 = m_0 b^2/3$ and where M_r is the resistant moment which represents the global action of spring and dashpot system acting on the foundation base. It is derived by considering the forces applied on the free body diagram of the base mat as

$$M_r = k_w \int_{-b}^{b}\left[\pm vx \pm x^2\sin(\theta)\right]dx + c_w\int_{-b}^{b}\left[\pm xv \pm x^2\dot{\theta}\cos(\theta)\right]dx \tag{24}$$

Integrating Eq. (24) and taking into account Eq. (18), one obtains

$$M_r = \left(1-\varepsilon_1^2\right)\varepsilon_2 c_w \frac{b^2}{2}\left(k_w v + c_w\dot{v}\right)+\left(1+\varepsilon_1^3\right)c_w\frac{b^3}{3}\left(k_w\sin(\theta)+c_w\dot{\theta}\cos(\theta)\right) \tag{25}$$

Using Eq. (6), calculating the integral in Eq. (23) and substituting the obtained result in Eq. (22) yields the following equation

$$\ddot{\theta} = \frac{3\alpha}{\gamma b}\left(2\xi\omega\dot{u}+\omega^2 u\right)-\left(1-\varepsilon_1^2\right)\varepsilon_2\frac{3c_w}{2m\gamma}\left(\dot{v}+\frac{k_w}{c_w}v\right)-\left(1+\varepsilon_1^3\right)\frac{c_w b}{m\gamma}\left(\dot{\theta}\cos(\theta)+\frac{k_w}{c_w}\sin(\theta)\right) \tag{26}$$

The equations of motion of the system under the assumption of large rotation are formed by the system of Eq. (10), Eqs. (20)-(22) and Eq. (26) where Eqs. (11)-(13) are also be used. Substituting Eqs. (11)-(13) in Eq. (10), and Eqs. (22)-(26) one arrives at the following nonlinear second order differential system of equations

$$H(Y)\ddot{Y} = G(Y,\dot{Y}) \tag{27}$$

With

$$Y = \begin{bmatrix} u \\ v \\ \theta \end{bmatrix} \tag{28}$$

$$H = \begin{bmatrix} 1-\tan(\theta)\left(\frac{1}{h}u+\sin(\theta)\right) & \frac{1}{h}u+\sin(\theta) & \chi+\frac{h}{2}\left[3\cos\left(\frac{3\theta}{2}\right)-\cos\left(\frac{\theta}{2}\right)\right] \\ -\tan(\theta) & 1 & \frac{h}{2}\left[\sin\left(\frac{\theta}{2}\right)-3\sin\left(\frac{3\theta}{2}\right)\right]-\left(1+\tan^2(\theta)\right)u \\ 0 & 0 & 1 \end{bmatrix} \tag{29}$$

$$\chi = \left(\frac{1}{h}u+\sin(\theta)\right)\left\{\frac{h}{2}\left[\sin\left(\frac{\theta}{2}\right)-3\sin\left(\frac{3\theta}{2}\right)\right]-\left(1+\tan^2(\theta)\right)u\right\} \tag{30}$$

$$G(Y,\dot{Y}) = \begin{bmatrix} G_u \\ G_v \\ G_\theta \end{bmatrix} \tag{31}$$

$$\begin{aligned} G_u = {} & \dot{\theta}^2\left\{\frac{h}{4}\left[9\sin\left(\frac{3\theta}{2}\right)-\sin\left(\frac{\theta}{2}\right)\right]-\left(\frac{1}{h}u+\sin(\theta)\right)\left\{\frac{h}{4}\left[\cos\left(\frac{\theta}{2}\right)-9\cos\left(\frac{3\theta}{2}\right)\right]\right.\right. \\ & \left.\left.-2\tan(\theta)\left(1+\tan^2(\theta)\right)u\right\}\right\}+\left(\frac{1}{h}u+\sin(\theta)\right)\left[2\left(1+\tan^2(\theta)\right)\dot{\theta}\dot{u}-g-\ddot{u}_{gy}\right] \\ & -\frac{2\xi h\omega\cos(\theta)}{h\cos^2(\theta)-u\sin(\theta)}\dot{u}-\frac{\omega^2 h\cos(\theta)}{h\cos^2(\theta)-u\sin(\theta)}u-\ddot{u}_{gx} \end{aligned} \tag{32}$$

$$\begin{aligned} G_v = {} & -\left\{\frac{h\dot{\theta}^2}{4}\left[\cos\left(\frac{\theta}{2}\right)-9\cos\left(\frac{3\theta}{2}\right)\right]\right\}+2\left(1+\tan^2(\theta)\right)\dot{\theta}\dot{u}+2\tan(\theta)\left(1+\tan^2(\theta)\right)\dot{\theta}^2 u \\ & -\left(1-\varepsilon_1^2\right)\varepsilon_2\xi_v\beta\omega\frac{b}{2}\left[\dot{\theta}\cos(\theta)+\frac{k_w}{c_w}\sin(\theta)\right]-\left(1+\varepsilon_1\right)\xi_v\beta\omega\left(\dot{v}+\frac{k_w}{c_w}v\right)-\mathrm{g}+\ddot{u}_{gy} \end{aligned} \tag{33}$$

$$G_\theta = \frac{3\alpha}{\gamma b}\left(2\xi\omega\dot{u} + \omega^2 u\right) - \left(1-\varepsilon_1^2\right)\varepsilon_2 \frac{3c_w}{2m\gamma}\left(\dot{v} + \frac{k_w}{c_w}v\right) - \left(1+\varepsilon_1^3\right)\frac{c_w b}{m\gamma}\left(\dot{\theta}\cos(\theta) + \frac{k_w}{c_w}\sin(\theta)\right) \quad (34)$$

2.1.2 Equations of motion for large rotations without p-delta effect

The p-delta effect corresponds to the third term in the right hand side of Eq. (10). In the model which does not take into account p-delta effect, this term is neglected, while the other Eqs. (22)-(26) remain the same. The equations of motion under this condition are now given by

$$H_w(Y)\ddot{Y} = G_w(Y,\dot{Y}) \quad (35)$$

With

$$H_w = \begin{bmatrix} 1 & 0 & \frac{h}{2}\left[3\cos\left(\frac{3\theta}{2}\right) - \cos\left(\frac{\theta}{2}\right)\right] \\ -\tan(\theta) & 1 & \frac{h}{2}\left[\sin\left(\frac{\theta}{2}\right) - 3\sin\left(\frac{3\theta}{2}\right)\right] - \left(1+\tan^2(\theta)\right)u \\ 0 & 0 & 1 \end{bmatrix} \quad (36)$$

$$G_w(Y,\dot{Y}) = \begin{bmatrix} G_{wu} \\ G_{wv} \\ G_{w\theta} \end{bmatrix} \quad (37)$$

$$G_{wd_{ex}} = \frac{h}{4}\left[9\sin\left(\frac{3\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right]\dot{\theta}^2 - \frac{2\xi h\omega\cos(\theta)}{h\cos^2(\theta) - d_{ex}\sin(\theta)}\dot{u} - \frac{\omega^2 h\cos(\theta)}{h\cos^2(\theta) - d_{ex}\sin(\theta)}u - \ddot{u}_{gx} \quad (38)$$

And $G_{wv}=G_v$ and $G_{w\theta}=G_\theta$.

2.1.3 Equations of motion for small rotations

The equations of motion of the system under the hypothesis of small rotation of the foundation are derived by taking $\theta \ll 1$. They are obtained from Eqs. (10)-(13), Eqs. (20)-(22) and Eq. (26) by letting the following approximations

$$\cos(\theta) \approx 1 \quad (39)$$

$$\sin(\theta) \approx \theta \quad (40)$$

And by neglecting all the higher order terms in the displacements and their time derivatives in

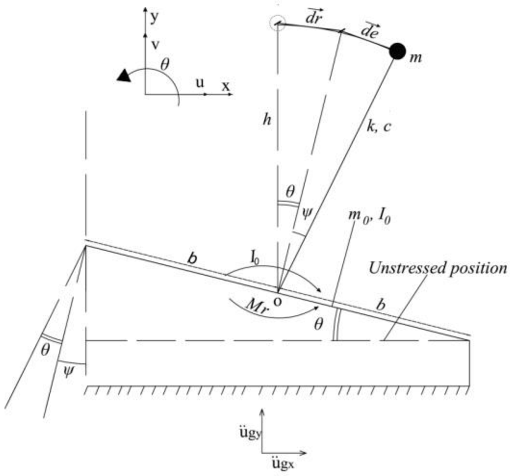


Fig. 4 Flexible structure on rigid soil

the system defined by Eqs. (20)-(21) and Eqs. (27)-(34). One obtains then after performing all the simplifications

$$H_s(Y)\ddot{Y} = G_s(Y,\dot{Y}) \tag{41}$$

With

$$H_s = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{42}$$

$$G_s(Y,\dot{Y}) = \begin{bmatrix} G_{su} \\ G_{sv} \\ G_{s\theta} \end{bmatrix} \tag{43}$$

$$G_{su} = -2\xi\omega\dot{u} - \omega^2 u - \ddot{u}_{gx} \tag{44}$$

$$G_{sv} = -\left(1-\varepsilon_1^2\right)\varepsilon_2\xi_v\beta\omega\frac{b}{2}\left(\dot{\theta}+\frac{k_w}{c_w}\theta\right)-\left(1+\varepsilon_1\right)\xi_v\beta\omega\left(\dot{v}+\frac{k_w}{c_w}v\right)-\mathrm{g}+\ddot{u}_{gy} \tag{45}$$

$$G_{s\theta} = \frac{3\alpha}{\gamma b}\left(2\xi\omega\dot{u}+\omega^2 u\right)-\left(1-\varepsilon_1^2\right)\frac{3\varepsilon_2}{2m_0 h}\left(c_w\dot{v}+k_w v\right)-\left(1+\varepsilon_1^3\right)\frac{b}{m_0 h}\left(c_w\dot{\theta}+k_w\theta\right) \tag{46}$$

$$\varepsilon_1 = \begin{cases} 1 & \text{contact at both edges} \\ \dfrac{\varepsilon_2 v}{b\theta} & \text{one edge is uplifted} \end{cases} \tag{47}$$

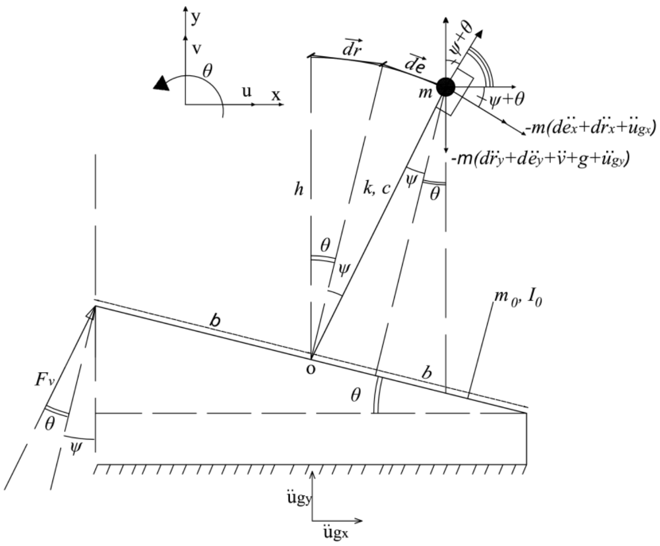


Fig. 5 Free body diagram of the system with uplift showing the considered dependent and independent degrees of freedom

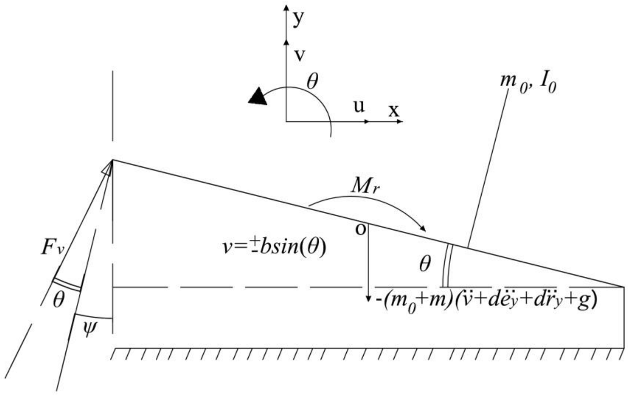


Fig. 6 Free body diagram of the system with uplift showing the considered dependent and independent degrees of freedom

And ε_2 having the same definition as in Eq. (21).

2.2 Equations of motion for flexible structure on rigid soil

The model of foundation-mat on a rigid soil assumes that the rigid foundation is supported by a soil which is infinitely rigid. This corresponds for the foundation-mat on Winkler foundation to the limit case where the foundation springs are very stiff $k_w \to \infty$. The motion of the foundation is then a rocking taking place on the two foundation edges, see Fig. 4.

The equations of motion of the entire system under the hypothesis of rigid soil are obtained by using the same approach than that used in the case of flexible structure on Winkler foundation. Fig. 5 gives the free body diagram corresponding to this case.

2.2.1 Equations of motion for flexible structure on rigid soil with including p-delta effect

The vertical displacements at the edges of foundation mat, see Fig. 6, measured from the initial unstressed position are given by

$$v_i = \varepsilon_2 \,\mathrm{b}\sin(\theta), \quad i = l, r \tag{48}$$

With ε_2 having the same definition as in Eq. (21). So, v and θ are not independent variables here and only two equations of motion are needed.

Eq. (48) replaces Eq. (16) obtained in the case of a Winkler foundation.

The contact of the foundation with the rigid soil is assumed to be frictionless on either of its edges. Thus, the reaction remains orthogonal to the actual direction of the foundation. Assuming the elastic rotation to remain small, the equations of motion are obtained from Eqs. (9)-(10), Eq. (14) and Eq. (48) by substituting Eq. (22) and Eq. (26) by the following equations

$$\begin{aligned} F_v = \frac{m(1+\gamma)h\cos(\theta)}{h\cos^2(\theta) - u\sin(\theta)} \Bigg\{ & \ddot{u}_{gy} - g - \varepsilon_2 b\left(\ddot{\theta}\cos(\theta) - \dot{\theta}^2\sin(\theta)\right) + \tan(\theta)\ddot{u} + 2\left(1+\tan^2(\theta)\right)\dot{\theta}\dot{u} \\ & + \left(1+\tan^2(\theta)\right)\left(\ddot{\theta} + 2\dot{\theta}^2\tan(\theta)\right)u - \frac{h\ddot{\theta}}{2}\left[\sin\left(\frac{\theta}{2}\right) - 3\sin\left(\frac{3\theta}{2}\right)\right] - \frac{h\dot{\theta}^2}{4}\left[\cos\left(\frac{\theta}{2}\right) - 9\cos\left(\frac{3\theta}{2}\right)\right]\Bigg\} \end{aligned} \tag{49}$$

$$\begin{aligned} M_r = \frac{mb(1+\gamma)h\cos(\theta)}{h\cos^2(\theta) - u\sin(\theta)} \Bigg\{ & \ddot{u}_{gy} - g - \varepsilon_2 b\left(\ddot{\theta}\cos(\theta) - \dot{\theta}^2\sin(\theta)\right) + \tan(\theta)\ddot{u} + 2\left(1+\tan^2(\theta)\right)\dot{\theta}\dot{u} \\ & + \left(1+\tan^2(\theta)\right)\left(\ddot{\theta} + 2\dot{\theta}^2\tan(\theta)\right)u - \frac{h\ddot{\theta}}{2}\left[\sin\left(\frac{\theta}{2}\right) - 3\sin\left(\frac{3\theta}{2}\right)\right] - \frac{h\dot{\theta}^2}{4}\left[\cos\left(\frac{\theta}{2}\right) - 9\cos\left(\frac{3\theta}{2}\right)\right]\Bigg\} \end{aligned} \tag{50}$$

Substituting Eq. (50) in Eq. (23) and using Eqs. (27)-(34) and Eq. (48) one obtains after performing some easy algebra the following coupled second order system of ordinary differential equations

$$H_r(Y)\ddot{Y}_r = G_r(Y_r, \dot{Y}_r) \tag{51}$$

With χ given by Eq. (30) and

$$Y_r = \begin{bmatrix} u \\ \theta \end{bmatrix} \tag{52}$$

$$H_r = \begin{bmatrix} 1 - \tan(\theta)\left(\frac{1}{h}u + \sin(\theta)\right) & \chi + \frac{h}{2}\left[3\cos\left(\frac{3\theta}{2}\right) - \cos\left(\frac{\theta}{2}\right)\right] + \varepsilon_2 b\left(\frac{1}{h}u + \sin(\theta)\right)\cos(\theta) \\ \frac{3\alpha(1+\gamma)\tan(\theta)}{\gamma\left(h\cos(\theta) + d_{ex}\tan(\theta)\right)} & 1 - \frac{3\alpha(1+\gamma)}{\gamma\left(h\cos(\theta) + d_{ex}\tan(\theta)\right)}k \end{bmatrix} \tag{53}$$

$$\kappa = \left\{\frac{h}{2}\left[\sin\left(\frac{\theta}{2}\right) - 3\sin\left(\frac{3\theta}{2}\right)\right] - \left(1+\tan^2(\theta)\right)d_{ex} + \varepsilon_2 b\cos(\theta)\right\} \tag{54}$$

$$G_{ru} = \dot{\theta}^2 \left\{ \frac{h}{4}\left[9\sin\left(\frac{3\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right] - \left(\frac{1}{h}u + \sin(\theta)\right)\left\{ \frac{h}{4}\left[\cos\left(\frac{\theta}{2}\right) - 9\cos\left(\frac{3\theta}{2}\right)\right] - \varepsilon_2 b\sin(\theta) \right.\right.$$
$$\left.\left. - 2\tan(\theta)\left(1+\tan^2(\theta)\right)u \right\}\right\} + \left(\frac{1}{h}u + \sin(\theta)\right)\left[2\left(1+\tan^2(\theta)\right)\dot{\theta}\dot{u} - g - \ddot{u}_{gy}\right] \tag{55}$$
$$- \frac{2\xi h\omega\cos(\theta)}{h\cos^2(\theta) - u\sin(\theta)}\dot{u} - \frac{\omega^2 h\cos(\theta)}{h\cos^2(\theta) - u\sin(\theta)}u - \ddot{u}_{gx}$$

$$G_{r\theta} = \frac{3\alpha(1+\gamma)}{\gamma\left(h\cos(\theta) + d_{ex}\tan(\theta)\right)}\left\{ -\ddot{u}_{gy} + g - 2\left(1+\tan^2(\theta)\right)\dot{\theta}\dot{u} \right.$$
$$\left. -\dot{\theta}^2\left\{ \frac{h}{4}\left[\cos\left(\frac{\theta}{2}\right) - 9\cos\left(\frac{3\theta}{2}\right)\right] - 2\tan(\theta)\left(1+\tan^2(\theta)\right)u - \varepsilon_2 b\sin(\theta)\right\}\right\} - \frac{3\alpha}{\gamma b}\left(2\xi\omega\dot{u} + \omega^2 u\right) \tag{56}$$

2.2.2 Equations of motion for flexible structure on rigid soil without p-delta effect

The equations of motion of the system under the assumption of large rotation but without including p-delta effect can be obtained form Eqs. (51)-(56) and by suppressing all the terms resulting from the right hand side of Eq. (9). The obtained equations of motions are then

$$H_{rw}(Y)\ddot{Y}_r = G_{rw}(Y_r, \dot{Y}_r) \tag{57}$$

$$H_{rw} = \begin{bmatrix} 1 & \frac{h}{2}\left[3\cos\left(\frac{3\theta}{2}\right) - \cos\left(\frac{\theta}{2}\right)\right] \\ \frac{3\alpha(1+\gamma)\tan(\theta)}{\gamma\left(h\cos(\theta) + d_{ex}\tan(\theta)\right)} & 1 - \frac{3\alpha(1+\gamma)}{\gamma\left(h\cos(\theta) + d_{ex}\tan(\theta)\right)}\kappa \end{bmatrix} \tag{58}$$

With k by Eq. (54)

$$G_{ru} = -\frac{2\xi h\omega\cos(\theta)}{h\cos^2(\theta) - d_{ex}\sin(\theta)}\dot{u} - \frac{\omega^2 h\cos(\theta)}{h\cos^2(\theta) - d_{ex}\sin(\theta)}u - \frac{h\dot{\theta}^2}{4}\left[-9\sin\left(\frac{3\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right] - \ddot{u}_{gx} \tag{59}$$

And $G_{rw\theta}=G_{r\theta}$.

2.2.3 Equations of motion for flexible structure on rigid soil for small rotation

The equations of motion of the system under the hypothesis of small rotation of the foundation are obtained by neglecting all the displacements terms exceeding the first order in Eqs. (51)-(56).

This yields the following second order ordinary differential equations

$$H_{rs}(Y)\ddot{Y}_r = G_{rs}(Y_r, \dot{Y}_r) \tag{60}$$

With

Table 1 Considered combinations in the parametric study for the foundation-mat on a Winkler foundation type

Combination	α	β	γ
1	2	2	0.2
2	2	2	0.25
3	2	2	0.3
4	2	4	0.2
5	2	4	0.25
6	2	4	0.3
7	2	6	0.2
8	2	6	0.25
9	2	6	0.3
10	4	2	0.2
11	4	2	0.25
12	4	2	0.3
13	4	4	0.2
14	4	4	0.25
15	4	4	0.3
16	4	6	0.2
17	4	6	0.25
18	4	6	0.3
19	8	2	0.2
20	8	2	0.25
21	8	2	0.3
22	8	4	0.2
23	8	4	0.25
24	8	4	0.3
25	8	6	0.2
26	8	6	0.25
27	8	6	0.3

$$H_{rs}=\begin{bmatrix}1 & h\\ 0 & 1-\dfrac{3(1+\gamma)\varepsilon_2 b}{\gamma b}\end{bmatrix} \tag{61}$$

$$G_{rsu}=-2\xi\omega\dot{u}-\omega^2 u-\ddot{u}_{gx} \tag{62}$$

$$G_{rs\theta}=\frac{3\alpha}{\gamma b}(2\xi\omega d\,\dot{e}_y+\omega^2 de_y)-\frac{3(1+\gamma)}{yb}(\ddot{u}_{gy}-g) \tag{63}$$

Table 2 Considered combinations in the parametric study for the foundation-mat on rigid soil

Combination	α	γ
1	2	0.2
2	2	0.25
3	2	0.3
4	4	0.2
5	4	0.25
6	4	0.3
7	8	0.2
8	8	0.25
9	8	0.3

3. Results and discussion

The systems of second order ordinary differential Eqs. (27), (35), (41), (51), (57) and (60) corresponding either to a Winkler foundation type or to a rigid soil are multi-form. This is because the conditions corresponding to Eqs. (20)-(21) or Eq. (47) are depending on the actual solution. Consequently, the numerical integration can be achieved only iteratively by guessing after each calculated step for the uplifting criterion as given by Eqs. (20)-(21) or Eq. (47) and actualizing the contact interface interval between the foundation and the ground.

As the system is stiff, the integration is performed in the following by using a proper numerical scheme based on the Matlab command ode15 s. The proposed numerical scheme was found to be robust and fast. This enabled to calculate the response of the foundation-mat structure under various conditions of loading and for different values of parameters.

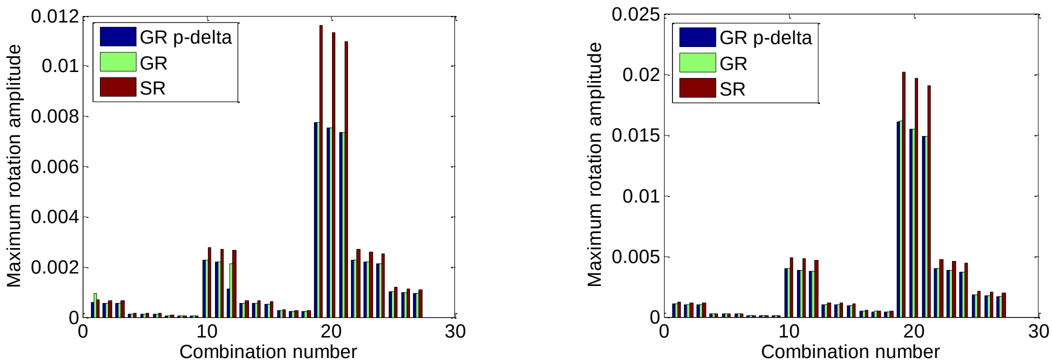
A parametric study was conducted in order to analyze the effect of the key parameters α, β and γ on the maximum amplitudes of base rotation and of the elastic horizontal displacement of the mat tip. The following parameters were fixed during simulations: ξ=0.05, ξ_v=0.4, m=227 kg, h=1.89 m, b=0.305 m and ω=5.3 Hz. The foundation-mat system response is investigated in both cases of large and small rotation of the base, and with or without including the p-delta effect.

Two different earthquake records were chosen for the excitation of the foundation base: the El Centro 1940 and Kobe NS. These records were scaled to have the same peak ground acceleration which was fixed at 0.32 g.

A total number of 3^3=27 combinations were considered for the Winker foundation type while only 3^2=9 combinations were considered for the rigid soil as in this last case the parameter β does not intervene. Table 1 and Table 2 recall the combinations that have been taken into account by choosing three levels for each parameter. The calculations were performed for both scaled seismic ground acceleration in order to investigate the effects of spectral content and earthquake duration on the coupled soil-structure system response for both large and small rotations, and by taking into account p-delta effect or discarding it.

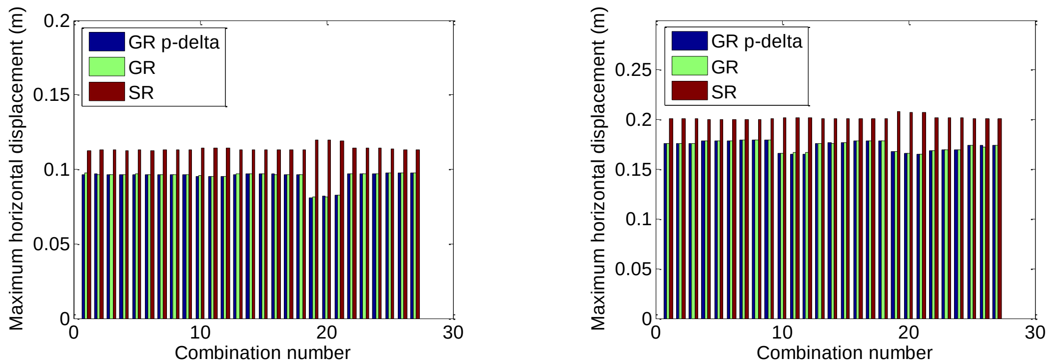
Fig. 7 gives variation of maximum rotation as function of the combination number in the case of flexible foundation-mat on Winkler foundation type for the three models: great rotation with p-delta effect (GR p-delta); great rotation without p-delta effect (GR) and small (SR). Both El Centro and Kobe NS earthquakes were taken into account.

Fig. 8 gives variation of maximum horizontal displacement as function of the combination



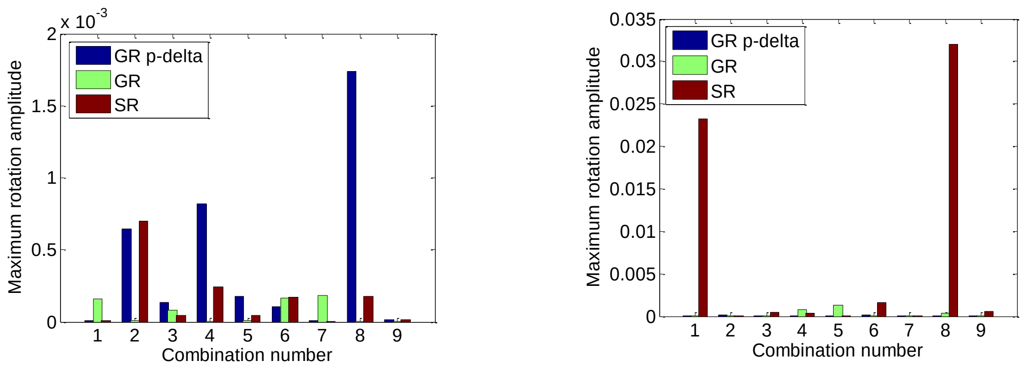
El Centro Kobe NS

Fig. 7 Variation of maximum rotation as function of the combination number in the case of flexible foundation-mat on Winkler foundation type; (GR p-delta) designates great rotation with p-delta effect; (GR) great rotation without p-delta effect and (SR) small rotation



El Centro Kobe NS

Fig. 8 Variation of maximum horizontal displacement as function of the combination number in the case of flexible foundation-mat on Winkler foundation type; (GR p-delta) designates great rotation with p-delta effect; (GR) great rotation without p-delta effect and (SR) small rotation



El Centro Kobe NS

Fig. 9 Variation of maximum rotation as function of the combination number in the case of flexible foundation-mat on rigid soil; (GR p-delta) designates great rotation with p-delta effect; (GR) great rotation without p-delta effect and (SR) small rotation

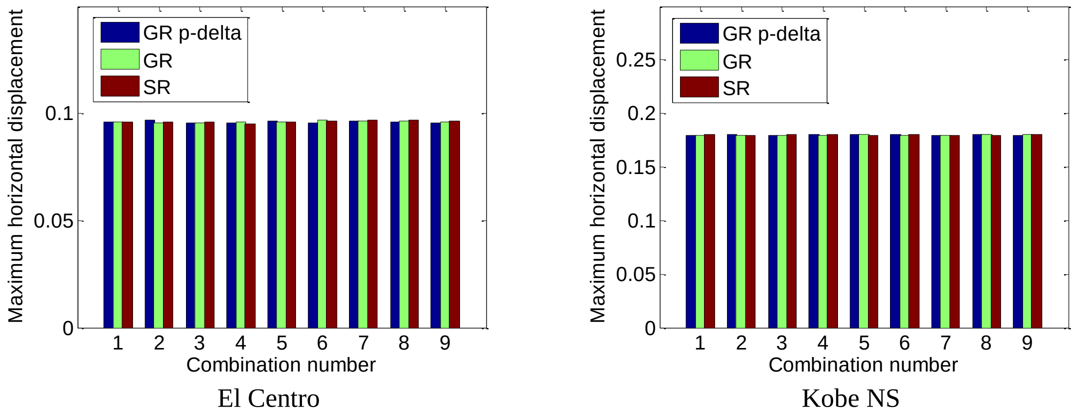


Fig. 10 Variation of maximum horizontal displacement as function of the combination number in the case of flexible foundation-mat on rigid soil; (GR p-delta) designates great rotation with p-delta effect; (GR) great rotation without p-delta effect and (SR) small rotation

number in the case of flexible foundation-mat on Winkler foundation type for the three considered models. The same designation of models and both earthquakes records were considered.

Fig. 9 gives variation of maximum rotation as function of the combination number in the case where the structure in founded on a rigid soil.

Fig. 10 gives variation of maximum horizontal displacement as function of the combination number in the case where the structure in founded on a rigid soil.

Fig. 1 shows that the slenderness ratio α, frequency ratio β have huge influence on the maximum rotation of the base, while the mass ratio γ in the considered range of variation has almost no effect of the maximum rotation.

Fig. 2 shows that for the maximum horizontal displacement, the frequency ratio has the most effect, while the slenderness ration and the ratio of masses have almost no influence.

Figs. 1-2 show that the small rotation approximation of the foundation-mat problem yields a large overestimation of rotation and mat tip horizontal displacement. These figures show also that the p-delta effect appears to have only a small influence on the maximum values of the rotation and elastic horizontal displacement, for the considered range of parameters.

Fig. 7 shows that the maximum values of base rotation increase when α increases. They decrease when β or γ increases.

Fig. 9 shows that for the case of rigid soil, the behavior of the foundation-mat system is irregular, that is to say no general tendencies like those obtained for the case of Winkler like foundation are recognized. There is an erratic behavior as for the Kobe NS earthquake the small rotation model has yielded the most severe rotations while for the El Centro earthquake the largest rotation was for the great rotation model including p-delta effect.

Fig. 4 shows that for the case of rigid soil, the maximum horizontal displacement is almost insensitive to the considered parameters α and γ. It is not also sensitive to the considered earthquake.

Figs. 1 and 3 show that even when fixing at the same level the maximum acceleration of the earthquakes, the obtained foundation-mat structure response is quiet different for the two considered seismic records. The responses under Kobe NS earthquake are almost the double of those obtained under the action of El Centro earthquake.

4. Conclusions

Analysis of the dynamic effects caused by local uplift of foundation base on the maximum response of a flexible structure was performed in this work as function of key dimensionless parameters. These included the slenderness of the structure, the frequency ratio and the ratio of the mass of the foundation to the mass of the structure. The foundation-mat structure was considered to set up either on a Winkler like foundation or a rigid soil. The modeling considered also the p-delta effect appearing in large rotation regime. Three cases for each kind of foundation soil type were investigated depending on whether large rotation and p-delta effect is considered or not.

The obtained results have shown that in the considered ranges of parameters the assumption of small rotation yields a large overestimation of maximum rotation in the case of foundation-mat on Winkler like soil. Assuming small rotations of the foundation could noticeably increase the maximum response of the structure under seismic loading. The obtained results have shown also that the p-delta effect has only a small influence on the results in the considered ranges of parameters.

When considering the foundation-mat to be on a rigid soil, the obtained results have shown that the maximum horizontal displacements are likely the same than those obtained for the Winkler like foundation. However, the base rotation does not compare easily with the Winkler like foundation. An erratic behavior is observed where the maximum rotation may be smaller or greater than that obtained for the Winkler like foundation. Also, the small rotation model may yield smaller or greater rotation in comparison with than the model assuming great rotation.

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