

# The effects of thermo-mechanical behavior of living tissues under thermal loading without energy dissipation

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**Abstract.** This study seeks to develop analytical solutions for the biothermoelastic model without accounting for energy dissipation. These solutions are then applied to estimate the temperature changes induced by external heating sources by integrating relevant empirical data characterizing the biological tissue of interest. The distributions of temperature, displacement, and strain were obtained by utilizing the eigenvalues approach with the Laplace transforms and numerical inverse transforms method. The impacts of the rate of blood perfusion and the metabolic activity parameter on thermoelastic behaviors were discussed specifically. The temperature, displacement, and thermal strain results are visually represented through graphical representations.

**Keywords:** eigenvalues approach; Laplace transformations; thermal and mechanical interactions; tissue; without energy dissipation

## 1. Introduction

There are several methods available to measure the thermal properties of living organisms, but the outcomes can differ. Achieving accurate measurements of the temperature characteristics of tissues within a living organism is still challenging. The complete understanding of temperature properties in living tissues remains incomplete due to the complexities involved in measuring them in vivo. This complexity arises from the potential alterations to tissue temperature caused by postmortem conditions, as well as the absence of the perfusion effect when examining tissue outside of the body. In recent research, the complexity of heat transfer in skin tissues has been recognized as a significant challenge. Therapeutic procedures such as laser tissue welding (Gabay

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*et al.* 2011), hyperthermia (Mahjoob and Vafai 2009), and laser surgeries (Zhou *et al.* 2009) have become commonplace in modern medicine.

The impact of diseases is influenced by the blood flow reaching the affected region. Accurate monitoring of thermal properties in damaged tissue can aid in the timely and effective application of appropriate therapies. Analyzing heat transfer within living tissues is a complex and demanding task due to the diverse internal structures they possess. To describe heat transfer in living tissues, Pennes' bioheat model (Pennes 1948) is employed, which is based on Fourier's law of thermal conduction. Phase change phenomena are observed across a wide range of biological tissues. To address this, researchers have developed adjusted versions of Penne's bioheat models using various numerical techniques found in the literature. Three models, namely GN-III, GN-II, and GN-I, were introduced by Green and Naghdi (Green and Naghdi 1991, Green and Naghdi 1992, Green and Naghdi 1993). The constitutive formulations of the G-N theories have been linearized. GN-I closely resembles the classical coupled thermoelastic theory, while GN-II exhibits the propagation of thermal signals at a finite velocity without energy dissipations. On the other hand, GN-III suggests finite speed propagation with energy dissipations.

The temperature distribution within skin tissues is subject to intricate phenomena, such as blood circulation and metabolic heating generation. As a result, researchers have extended fundamental relationships to incorporate these complexities. These relationships encompass diverse phenomenological mechanisms, including metabolic heat production, thermal conduction, blood perfusion, radiation, and phase changes. The transformation stages of biological tissue can manifest in various forms. The relevant literature offers modified versions of Penne's bioheat models that employ a variety of numerical approaches. Multiple numerical methods are utilized to solve these models, which include the homotopy perturbation technique (Gupta *et al.* 2010), Legendre wavelets Galerkin approaches (Yadav *et al.* 2014, Kumar *et al.* 2015), and the finite element methods (Gupta *et al.* 2013). In their study, Esneault and Dillenseger (2010) utilized finite difference methods to examine the progression of temperature improvement over time, specifically focusing on cases with abnormally low body temperatures. Ghanmi and Abbas (Ghanmi and Abbas 2019) conducted an analytical investigation into the fractional time derivative within skin tissues through thermal therapy. Marin *et al.* (2021) employed the finite element method to analyze the non-linear bio-heat model in skin tissues induced by an external heating source. Hobiny and Abbas (2021) performed an analytical study on the fractional bioheat model in spherical tissues.

To understand the interactions between heat and mechanical effects in anisotropic laser-induced tissue hyperthermia, Fahmy (2019) proposed a novel boundary element model. Diaz *et al.* (2002) utilized the finite element approach to address the thermo-diffusion problem in biological tissues, aiming to model thermal damage. The thermo-elastic behaviors of tissues are governed by generalized thermo-elastic models, such as the G-N model, G-NII model, DPL, and fractional model. Li *et al.* (2018, 2018, 2019) further studied the impact of heat-induced mechanical responses in skin tissues, taking into account temperature-dependent properties. Youssef and Alghamdi (2020) focused on modeling the thermoelastic dual-phase-lag behavior of living tissues exposed to various thermal loads in a one-dimensional setting. Shen *et al.* (2005) employed a thermo-mechanical model to investigate the interactions between skin tissues and high temperatures, examining both thermal and mechanical effects. Kim *et al.* (2016) focused on studying the mechanical and thermal consequences resulting from the absorption of pulsed laser in human skin. In a recent study, (Lata 2019) examined thermomechanical interactions in a transversely isotropic magneto-thermoelastic solid where two different temperatures were considered without factoring in energy dissipation. (Lata and Kaur 2022) examined the effects of

two temperatures and energy dissipation within an axisymmetric isotropic thermoelastic solid modeled using modified couple stress theory. In one study, (Singh and Lata 2023) examined the impact of two temperatures and nonlocal effects on an isotropic thermoelastic thick circular plate, without considering energy dissipation. Xu *et al.* (2008a, 2008b, 2008c, Marin 2010, Othman 2020, Marin 2021, Marin *et al.* 2022) developed a theoretical framework for understanding the interconnected thermomechanical behaviors of the skin, considering it as a layered material. They emphasized that heat-induced stress can contribute to thermal discomfort and adopted a sequentially coupled approach for ease of solution. Zhu *et al.* (2002) explored rate process models of thermal damage and light energy deposition in tissue using diffusion theory. Numerous researchers have attempted to find numerical or analytical solutions to address the challenges posed by linear and nonlinear models of heat transfer when investigating thermal phenomena in finite media (Zenkour and Abbas 2014, Abbas and Kumar 2016, Li *et al.* 2019, Naik and Sayyad 2020, Mohammed and Ismael 2022, Sobhy and Zenkour 2022).

The objective of this study is to develop an analytical methodology to investigate the thermo-mechanical interactions in living tissue without energy dissipation when subjected to rapid heating and exhibiting varying thermal and mechanical properties. By employing the eigenvalues approach with Laplace transform, precise solutions can be obtained for each physical field, enabling the calculation of thermo-elastic responses in living tissues that experience instantaneous heating. The temperature, displacement, and strain changes are depicted through graphical representations.

## 2. Statement of the problem

In this study, we assume that the skin tissues are uniform, exhibiting linear, isotropic and homogeneous thermoelastic properties. Consequently, the thermoelastic equations governing the behavior of the skin tissue, considering varying thermal properties according to the bioheat conduction model, can be represented as follows (Li *et al.* 2018), assuming the absence of anybody force

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \gamma T_{,i} = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (1)$$

$$k^* \nabla^2 T = \frac{\partial}{\partial t} \left( \rho c_e \frac{\partial T}{\partial t} + \rho_b \omega_b c_b (T - T_b) + \gamma T_o \frac{\partial u_{i,i}}{\partial x} - Q_m \right), \quad (2)$$

$$\sigma_{ij} = \left( \lambda u_{k,k} - \gamma (T - T_o) \right) \delta_{ij} + \mu (u_{i,j} + u_{j,i}), \quad (3)$$

where  $T_o$  the initial temperature of the local tissues,  $T$  is the tissues temperature,  $\rho$  is the tissue mass density,  $\lambda, \mu$  refer to the Lamé's constants,  $T_b$  is the blood temperature,  $\omega_b$  is the blood perfusion rate,  $\rho_b$  is the blood mass density,  $c_b$  is the blood specific heat,  $c_e$  refer to the specific heat at constant strain,  $u_i$  are the displacement components,  $\gamma = (3\lambda + 2\mu)\alpha_t$ ,  $\alpha_t$  refer to the linear thermal expansion coefficient,  $k^*$  is the rate of thermal conductivity in these models,  $e_{ij}$  are the strain components,  $t$  is the time,  $\sigma_{ij}$  are the components of the stress,  $\delta_{ij}$  is the Kronecker symbol and  $Q_m$  is the metabolic heat generation in skin tissue. According to Mitchell *et al.* (1970), it was observed that the production of metabolic heat relies on the temperature of nearby tissues and can be represented as follows

$$Q_m = Q_{m0} \times 2^{\alpha \left( \frac{T - T_o}{10} \right)}, \quad (4)$$

where  $Q_{mo}$  denotes the reference metabolic heat source and  $\alpha$  is a constant that pertains to metabolic activity. In practical situations, it is commonly acceptable to approximate the generation of metabolic heat as a linear function of the temperature of the surrounding tissues. This can be expressed as

$$Q_m = Q_{mo} \left( 1 + \alpha \left( \frac{T - T_o}{10} \right) \right). \quad (5)$$

Within this context, we assume that the skin tissue in a confined domain, with a thickness of  $h$ , possesses surface and bottom boundaries. As a result, the displacement components and strain can be given by

$$u_x = u(x, t), u_y = 0, u_z = 0, e = \frac{\partial u}{\partial x}. \quad (6)$$

Hence, the model can be presented as follows

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \quad (7)$$

$$k^* \frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial^2 T}{\partial t^2} + \rho_b \omega_b c_b \frac{\partial T}{\partial t} + \gamma T_o \frac{\partial^3 u}{\partial t^2 \partial x} - \frac{\alpha}{10} Q_{mo} \frac{\partial T}{\partial t}, \quad (8)$$

$$\sigma = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma(T - T_o). \quad (9)$$

### 3. Initial and boundary conditions

To obtain solutions to the equations, it is essential to establish two sets of initial and boundary conditions that correspond to the specifications of the physical model

$$\sigma(x, 0) = 0, \left. \frac{\partial u(x, t)}{\partial t} \right|_{t=0} = 0, T(x, 0) = T_b, \left. \frac{\partial T(x, t)}{\partial t} \right|_{t=0} = 0, \quad (10)$$

$$\sigma(0, t) = 0, k^* \left. \frac{\partial T(x, t)}{\partial x} \right|_{x=0} = -q_o \frac{t(2t_p - t)e^{-\frac{t}{t_p}}}{16t_p^3}, u(h, t) = 0, T(h, t) = 0, \quad (11)$$

where  $q_o$  is constant and the parameter  $t_p$  represents the characteristic time associated with the pulsating heat flux. To streamline the governing equations, we will employ the subsequent non-dimensional variables to facilitate the computations

$$(x', u') = \frac{(x, u)}{L}, T' = \frac{T - T_o}{T_o}, \sigma' = \frac{\sigma}{\lambda + 2\mu}, (t', t'_p) = \frac{v}{L} (t, t_p), \quad (12)$$

where  $v = \sqrt{\frac{k^*}{\rho c}}$ . The governing equations can be expressed using non-dimensional parameters (12) in their dimensionless form after removing the dashes

$$\frac{\partial^2 u}{\partial x'^2} = \epsilon_1 \frac{\partial^2 u}{\partial t'^2} + \epsilon_2 \frac{\partial T}{\partial x'}, \quad (13)$$

$$\frac{\partial^2 T}{\partial x'^2} = \frac{\partial^2 T}{\partial t'^2} + \epsilon_3 \frac{\partial T}{\partial t'} + \epsilon_4 \frac{\partial^3 u}{\partial t'^2 \partial x'}, \quad (14)$$

$$\sigma = \frac{\partial u}{\partial x'} - b_2 T, \quad (15)$$

$$u(x, 0) = 0, \left. \frac{\partial u(x, t)}{\partial t} \right|_{t=0} = 0, T(x, 0) = 0, \left. \frac{\partial T(x, t)}{\partial t} \right|_{t=0} = 0, \quad (16)$$

$$\sigma(0, t) = 0, \left. \frac{\partial T(x, t)}{\partial x} \right|_{x=0} = -\frac{q_0 v}{T_0 k^*} \frac{t(2t_p - t)e^{-\frac{t}{t_p}}}{16t_p^3}, u(h, t) = 0, T(h, t) = 0, \quad (17)$$

where  $\epsilon_1 = \frac{\rho v^2}{\lambda + 2\mu}$ ,  $\epsilon_2 = \frac{T_0 \gamma_e}{\lambda + 2\mu}$ ,  $\epsilon_3 = \frac{Lv}{k^*} \left( \rho_b \omega_b c_b - \frac{\alpha}{10} Q_{mo} \right)$ ,  $\epsilon_4 = \frac{\gamma_e}{\rho c_e}$ .

Through the utilization of Laplace transforms, Eqs. (13) to (17) can be transformed

$$\bar{g}(x, s) = L[g(x, t)] = \int_0^\infty g(x, t) e^{-st} dt. \quad (18)$$

Hence, we can obtain the following equations

$$\frac{d^2 \bar{u}}{dx^2} = s^2 \epsilon_1 \bar{u} + \epsilon_2 \frac{d\bar{T}}{dx}, \quad (19)$$

$$\frac{d^2 \bar{T}}{dx^2} = s(s + \epsilon_3) \bar{T} + s^2 \epsilon_4 \frac{d\bar{u}}{dx}, \quad (20)$$

$$\bar{\sigma} = \frac{d\bar{u}}{dx} - \epsilon_2 \bar{T}, \quad (21)$$

$$\bar{\sigma}(0, s) = 0, \left. \frac{d\bar{T}(x, s)}{dx} \right|_{x=0} = -\frac{q_0 v s t_p}{8T_0 k^* (1 + s t_p)^3}, \bar{u}(L, s) = 0, \bar{T}(L, s) = 0. \quad (22)$$

Using Eqs. (19) and (20), we can express the vector-matrix differential equation by

$$\frac{dV}{dx} = BV, \quad (23)$$

$$\text{where } V = \begin{pmatrix} \bar{u} \\ \bar{T} \\ \frac{d\bar{u}}{dx} \\ \frac{d\bar{T}}{dx} \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ s^2 \epsilon_1 & 0 & 0 & \epsilon_2 \\ 0 & s(s + \epsilon_3) & s^2 \epsilon_4 & 0 \end{pmatrix}.$$

To solve Eq. (23), we can utilize the eigenvalue techniques outlined in (Das *et al.* 1997, Baksi *et al.* 2006, Santra *et al.* 2014, Abbas *et al.* 2016, Gupta and Das 2016, Kumar *et al.* 2016, Kumar *et al.* 2017, Hobiny and Abbas 2019, Abbas *et al.* 2020) to obtain the characteristic relation of matrix  $B$ .

$$\omega^4 - (s^2 \epsilon_1 + s(s + \epsilon_3) + s^2 \epsilon_2 \epsilon_4) \omega^2 + s^3 \epsilon_1 (s + \epsilon_3) = 0, \quad (24)$$

The roots of Eq. (24), denoted as  $\pm\omega_1$  and  $\pm\omega_2$ , correspond to the eigenvalues of matrix  $B$ . It is worth noting that the terms in Eq. (24) consist of function of the Laplace parameters  $s$ . Consequently, the general solution can be written as follows

$$V(x, s) = A_1 X_1 e^{-\omega_1 x} + A_2 X_2 e^{\omega_1 x} + A_3 X_3 e^{-\omega_2 x} + A_4 X_4 e^{\omega_2 x}. \quad (25)$$

Therefore, within the Laplace domain, the overall solutions for displacement, temperature, and strain can be described as follows

$$\bar{u}(x, s) = B_1 U_1 e^{-\omega_1 x} + B_2 U_2 e^{\omega_1 x} + B_3 U_3 e^{-\omega_2 x} + B_4 U_4 e^{\omega_2 x}. \quad (26)$$

$$\bar{T}(x, s) = B_1 T_1 e^{-\omega_1 x} + B_2 T_2 e^{\omega_1 x} + B_3 T_3 e^{-\omega_2 x} + B_4 T_4 e^{\omega_2 x}. \quad (27)$$

$$\bar{e} = -\omega_1 U_1 B_1 e^{-\omega_1 x} + \omega_1 U_2 B_2 e^{\omega_1 x} - \omega_2 U_3 B_3 e^{-\omega_2 x} + \omega_2 U_4 B_4 e^{\omega_2 x}, \quad (28)$$

here,  $T_i$  and  $U_i$  represent the eigenvector of temperature and displacement, respectively. The values

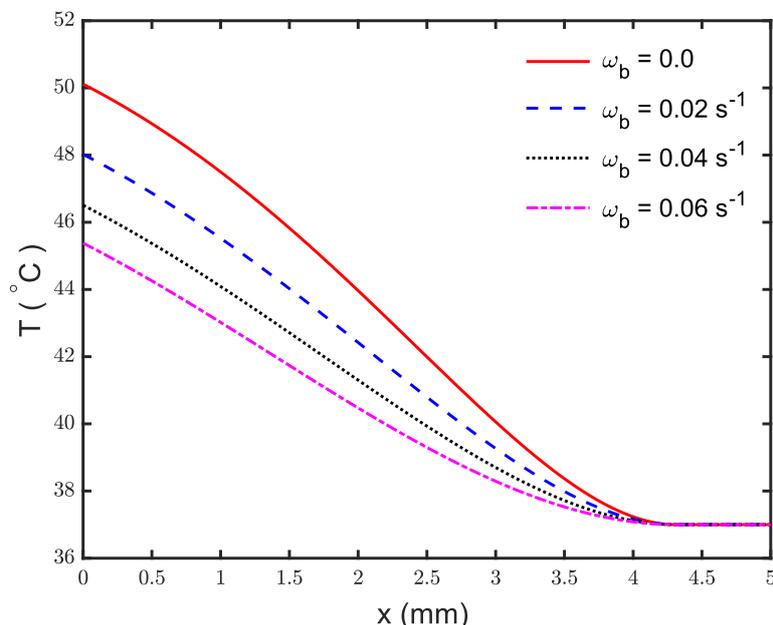


Fig. 1 The effect of blood perfusion rate  $\omega_b$  in the temperature variations  $T$  when  $t_p = 32$  s

of  $B_1, B_2, B_3$  and  $B_4$  can be determined by utilizing the boundary conditions of the problem. To obtain the final solutions for temperature, displacement and strain distribution, the Stehfest method (Stehfest 1970) is employed as a numerical inversion strategy.

#### 4. Numerical results and discussions

In order to illustrate the theoretical outcomes discussed in the preceding parts, we present the calculated numerical values of the physical constants. The material constants for skin tissue at the reference temperature are computed as demonstrated below (Li *et al.* 2018)

$$\begin{aligned} \rho_b &= 1060 \text{ (kg)(m}^{-3}\text{)}, c_b = 3770 \text{ (J)(kg}^{-1}\text{)(k}^{-1}\text{)}, \mu = 3.446 \times 10^7 \text{ (N)(m}^{-2}\text{)}, \\ K &= 0.235 \text{ (W)(m}^{-1}\text{)(k}^{-1}\text{)}, \rho = 1190 \text{ (kg)(m}^{-3}\text{)}, c_e = 3600 \text{ (J)(kg}^{-1}\text{)(k}^{-1}\text{)}, \\ \lambda &= 8.27 \times 10^8 \text{ (N)(m}^{-2}\text{)}, Q_m = 1.19 \times 10^3 \text{ (W)(m}^{-3}\text{)}, \alpha_t = 1 \times 10^{-4} \text{ (k}^{-1}\text{)}, T_o = 310 \text{ (k)}. \end{aligned}$$

Using the same set of parameters as before, the computed numerical values for physical quantities under the generalized biothermoelastic model without energy dissipation are presented below. These results are illustrated in Fig. 1-6. At time  $t = 40$  s, numerical computations were conducted to determine the variations in displacement, temperature and strain along the distance  $x$ . These quantities were evaluated for several values of the studied parameters, as illustrated in Figs. 1-6. The temperature variation in relation to the distance  $x$  are depicted in Figs. 1 and 4. The results demonstrate that the temperature initially peaks at the skin surface ( $x = 0$ ) as a result of the exponentially diminishing pulse boundary heating flux. As the distance  $x$  continues to increase, the temperature gradually diminishes until it reaches nearly zero. Figs. 2 and 5 show the displacement variations in relation to the distance  $x$ . The data clearly shows that the displacement magnitudes begin at their maximum values and progressively diminish as the distance  $x$  increases, eventually

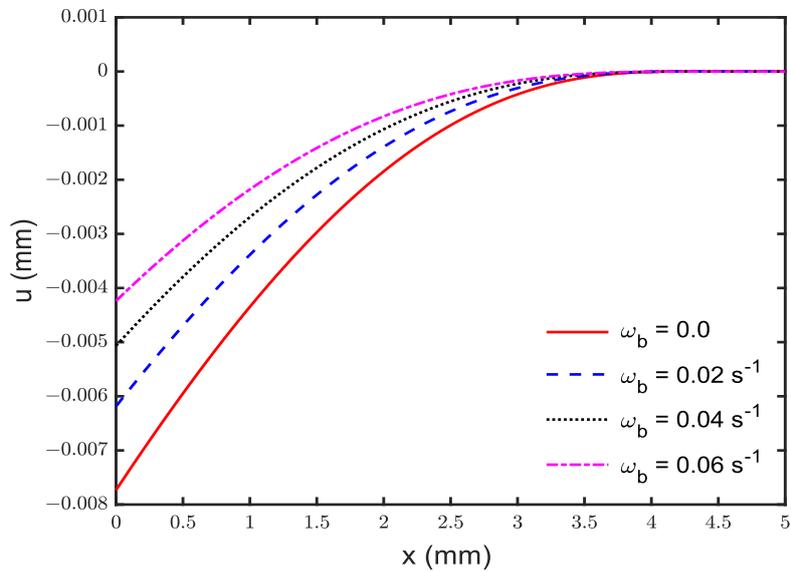


Fig. 2 The effect of blood perfusion rate  $\omega_b$  in the displacement variation  $u$  when  $t_p = 32 \text{ s}$

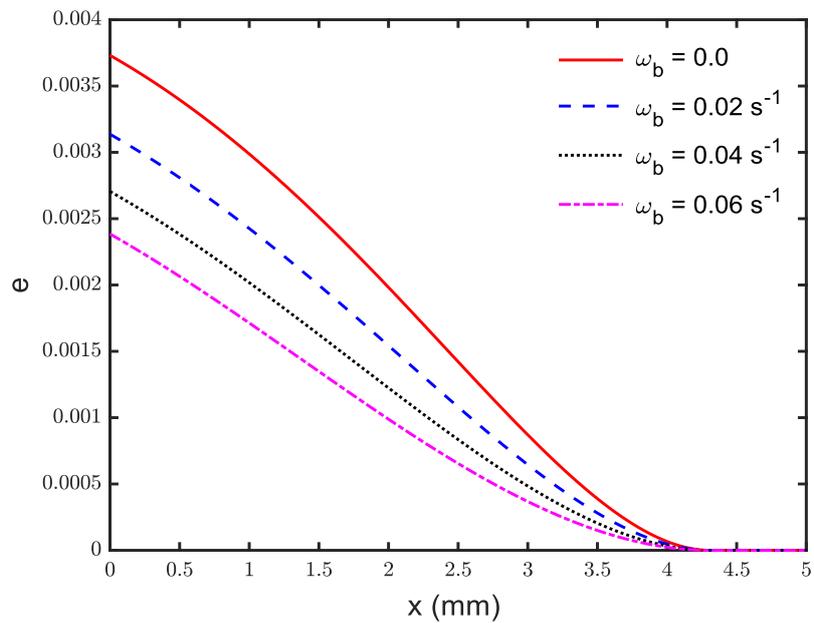


Fig. 3 The effect of blood perfusion rate  $\omega_b$  in the strain variations  $e$  when  $t_p = 32 \text{ s}$

approaching zero. The variations of strain with respect to the distance  $x$  are illustrated in Figs. 3 and 6. It is clear from the data that the strain initially reaches its maximum values and then gradually decreases until it approaches zero. The first group of Figures, namely Figs. 1, 2, and 3, portray the variations of temperature, displacement, and strain, respectively, under different blood perfusion rates. The data clearly demonstrates that the blood perfusion rate has a notable impact on

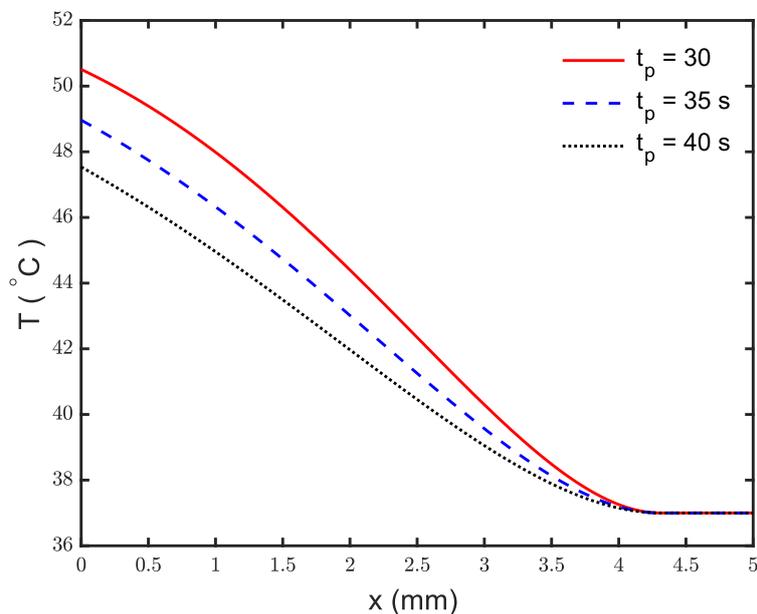


Fig. 4 The temperature variations  $T$  for different values of characteristic time of pulsing heat flux  $t_p$

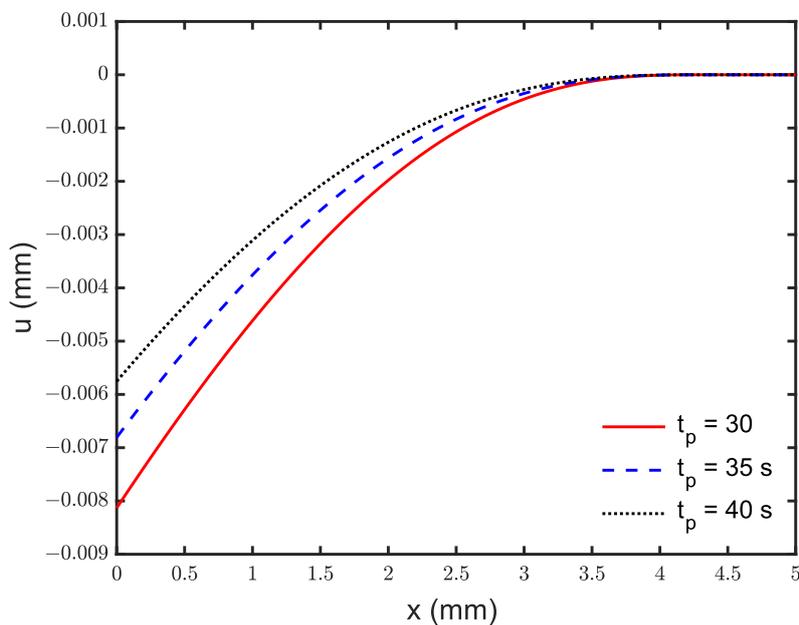


Fig. 5 The displacement variation  $u$  for various values of characteristic time of pulsing heat flux  $t_p$

the studied variables. With an increase in the blood perfusion rate, the maximum amplitudes of temperature, displacement, and strain decrease. This suggests that higher blood perfusion rates facilitate greater convective heat loss due to faster blood flow, leading to reduced magnitudes of temperature, displacement, and strain. The second group of Figures, namely Figs. 4, 5, and 6,

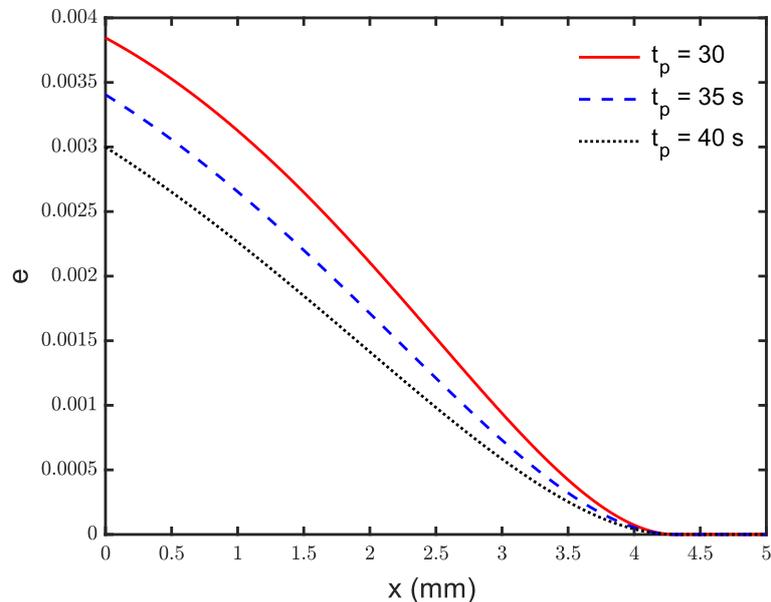


Fig. 6 The strain variation  $e$  for various values of characteristic time of pulsing heat flux  $t_p$

depict the variations of temperature, displacement, and strain, respectively, under different characteristic times of pulsing heat flux. The data clearly shows that the characteristic time of the pulsing heat flux has a significant influence on the studied variables. As the characteristic time increases, the maximum amplitudes of temperature, displacement, and strain decrease. This suggests that the characteristic time of the pulsing heat flux tends to attenuate the effects of thermomechanical propagation.

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