

## Tuned vibration control in aeroelasticity of slender wood bridges

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**Abstract.** Tuned vibration control in aeroelasticity of slender wood bridges is treated in present paper. The approach suggested takes into account multiple functions in aeroelastic analysis and flutter of slender wood bridges subjected to laminar and turbulent wind flow. Tuned vibration control approach is presented with application on actual bridge. Some results obtained are discussed.

**Keywords:** Bernoulli equation; damping; flutter response; mechanics of turbulent wind motion; slender wood bridge; tuned vibration control; wind model

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### 1. Introduction

Advanced slender wood bridges (Fig. 1) are designed in such a way that no traffic or environmental load can decrease their ultimate reliability. However, the experiments sampled up indicate that ultimate behaviour of such structures occurring due to the wind can initiate unpredictable ultimate response influencing their safety. Ultimate flutter behaviour of such bridges occurs by laminar and turbulent air-flow along the surface of the main girder. Linear theory specifies a critical pressure at which the bridge motion becomes unstable. The linear theory specifies the flutter boundary but cannot give information about ultimate flutter response. For large amplitude oscillations the nonlinear effects restrain the motion to a bounded value with growing amplitudes as dynamic pressure increases.

One measure to control such response is the application of tuned vibration control (TVC) in special joints adopted on the bridge. The monitoring and identification of actual parameters, the selection of target reliability and optimal tuning by evaluation of amplification curves are made in tuning joints either automatically for each forcing situation occurring or are set up stationary for the assumed range of forcing. TVC controls the length of time interval in which the flutter response remains stable with limited amplitudes.

Slender main girders of bridges studied are made of laminated wood. Carbon fiber composites are adopted for cables. Such bridges, when subjected to laminar or turbulent air flow, can be forced into ultimate flutter response with large amplitudes and unstable aeroelastic behaviour. The monitoring submits all data for TVC. The forces in wind cables are automatically varied in order to control the

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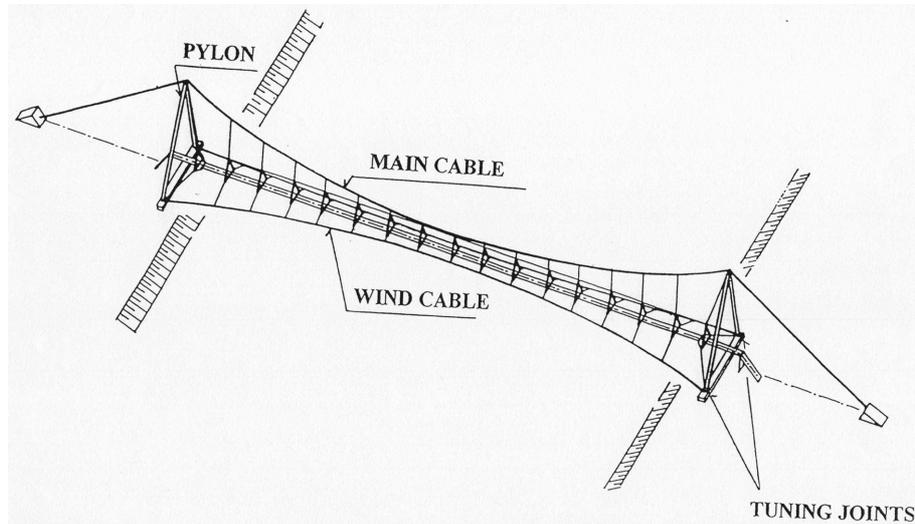


Fig. 1 Slender wood bridge with TVC-equipment

response. The TVC-software in tuning joints allows updated identification of all structural and forcing data, their evaluation, monitoring, optimization and consequently the control of structural response.

The treatment and modeling of turbulent air flow in artificial boundary layer around the bridge is a research domain based on advanced scientific technologies. They are imposed by necessity of studying the turbulent air movement in the proximity of slender structures. The models of turbulent air flow are used in the assessments being validated by tunnel testing of parameters integrated in calculation. Aeroelastic response depends on wind speed, wind direction, wind flow (laminar or turbulent), wind temperature and humidity, snow and ice loads, geometry and configuration of the wood bridges studied and dynamic properties of all structural elements adopted.

There occur turbulent air flows on edges of the main girder which increase the wind speeds and pressures. Regarding the variability of configurations of the bridges with artificial boundary layer there appear the combined laminar and turbulent wind flows. The measurements in aerodynamic tunnel submit the data required for the analysis of the problem.

## 2. Analysis

Slender wood bridges are prone to wind-induced vibrations for various reasons. Some issues considered in their wind resistant design are mentioned by:

(1) Wind turbulences force the bridge with a considerable power and the movements owing to turbulences and associated mechanisms are stochastic in nature.

(2) There is produced a strong vortex wake associated with aerodynamic drag force experienced by the bridge. Depending on the wind speed and cross-section's shape, the shedding of vortices is more or less regular with shedding periods inversely proportional to the wind speed. In resonance conditions the structure's oscillation can control the rhythm of the vortex shedding.

(3) Aside the known vortex trail type excitation the more general types of forcing appear in the bridge. The vortices generated by the local geometry and movement of the bridge contribute to such

periodic forcing.

(4) Aeroelastic forces proportional to the movement of the bridge produce the self-induced divergent vibrations at some wind speeds.

(5) In the design of the bridge is to be avoided that absolute value of negative aerodynamic damping exceeds the positive mechanical damping producing across-wind flexural mode instability. Associated critical wind speed is the flutter velocity while corresponding circular frequency is the flutter frequency.

(6) At the onset of the divergence the aerodynamic instability of the bridge is initiated.

In this paper the wind induced structural phenomena are treated by transient dynamics. Laminar and turbulent wind forcing is studied adopting the wave propagation approach. The goal is to develop the approach based on transient dynamics combined with wave propagation forcing and adopted for the analysis of aeroelastic response of slender wood bridges.

### 3. Basic principles

The wind flow field is described by the velocity field  $\hat{w}$  which is the function of location vector  $r$  and of time  $t$  and is given by

$$\hat{w} = f(r, t) \quad (1)$$

The location vector in Cartesian system is

$$r = i.x + j.y + k.z \quad (2)$$

and the velocity vector is

$$\hat{w} = i.v + j.u + k.w \quad (3)$$

with parameters

$$v = f_1(x, y, z, t) \quad (4)$$

$$u = f_2(x, y, z, t) \quad (5)$$

$$w = f_3(x, y, z, t) \quad (6)$$

As velocity potential is introduced, the scalar function  $\Phi(x, y, z)$  is given by

$$\hat{w} = \text{grad } \Phi \quad (7)$$

The scalar terms of Eq. (3) are specified by velocity components

$$v = \partial \Phi / \partial x \quad (8)$$

$$u = \partial \Phi / \partial y \quad (9)$$

$$w = \partial \Phi / \partial z \quad (10)$$

The Bernoulli equation for non-stationary wind flow is then given by

$$\partial \Phi / \partial t + w \cdot w / 2 + \int 1 / \rho dp = F(t) \quad (11)$$

In order to obtain energetic relations for volume unit of the wind flow, each term of Eq. (11) is to be multiplied by the density of the wind flow  $\rho$ . The term  $\rho(\partial \Phi / \partial t + w \cdot w / 2)$  represents the kinetic energy available,  $dp$  is the energy of the dynamic flow occurring and  $\rho \cdot F(t)$  is the energetic balance of the wind flow studied. The lift force on the bridge is given by

$$F = \int (p_d - p_u) dp \quad (12)$$

with  $p_d$  and  $p_u$  as values of the pressures on lower and upper surface of the main girder studied. The pressure is modeled by the translation flow with constant speed and by the flow with circulation  $\Gamma \neq 0$  for each closed profile of the main girder of the bridge. There pays

$$F = \int \rho b \cdot \hat{w} \cdot \Gamma \quad (13)$$

with  $b$  as width of the bridge girder studied. The circulation  $\Gamma$  depends on the air velocity, air flow angle, geometry and environment of the bridge.

#### 4. Wind model

Turbulent air flow and wind gusts are given by intensity, spectral distribution and coherence. In order to generate the wind effects as occurring actually, the wind models are used giving the data and parameters for 10-minute constant wind velocity steps, as well as the frequency spectrum and the coherence properties of turbulences appearing. The basic parameter of such wind model (König 1972) is a 10-minute average step  $v_G$  of a standard 50-year wind velocity in atmospheric boundary layer studied. The profile of 10-minute wind velocity  $v_w$  is established in accordance with exponential law by

$$v_w = v_G (z/z_G)^\alpha \quad (14)$$

with height  $z$ , with  $z_G$  as corresponding gradient height and with  $\alpha$  as the exponent for the wind profile studied (Ruscheweyh 1982, König 1970). The variability of the wind velocity  $\sigma$  is defined as a standard aberation of the gusts in the wind direction. In accordance with the wind model used  $\sigma$  is constant along  $z_G$  and is given by

$$\sigma = z_o \cdot v_G \cdot \sqrt{(6 \cdot \beta) / z_G} \quad (15)$$

where  $z_o = 10$  is the comparative height and  $\beta$  is the roughness parameter due to reference (König and Zilch (1970).

As further parameter of the wind model appears the atmospheric coherence. It describes the

similarity of the speed variability in various points  $n$  along the span and height of the bridge and is given by

$$C_H(A,B,n) = \sqrt{\{[G_{AB}(n).G_{AB}(n) + Q_{AB}(n).Q_{AB}(n)]/[S_{AA}(n) + S_{BB}(n)]\}} \quad (16)$$

with  $A$  and  $B$  as two nodes of the bridge model used,  $S_{AA}(n)$  and  $S_{BB}(n)$  as turbulence spectra measured in  $A$  and  $B$ ,  $G_{AB}(n)$  and  $Q_{AB}(n)$  as covariance and quadrature spectra of  $v_w(A,t)$  and  $v_w(B,t)$ , respectively. Eq. (16) specifies time vs propagation of the wind gusts appearing.

### 5. Aerodynamic forces

Studied is the plane panel of the main girder of the wood bridge subjected to a wind flow initiating aerodynamic forces as shown in Fig. 2.

In case of simultaneous action of critical velocity of the air flow and of the resonance frequency of bridge vibration there appears the flutter combination of flexural and torsional oscillations. For linear analysis of the problem the cross-section studied is an ideal smooth panel and the bridge is forced by laminar air flow along the whole length studied. The aerodynamic forces in accordance with the theory of Theodorsen (1935) are given by

$$L_1 = -2. \pi. \rho. b. v_w. C(k). [v_w. v_T + i. \omega u_i + i. b / (2. \omega v_T)] \quad (17)$$

$$L_2 = 2. \pi. \rho. \omega b^2. \omega u_i \quad (18)$$

$$L_3 = -\pi. \rho. b^2. v_w \quad (19)$$

$$L_4 = \pi. \omega^2. \rho. v_T. b^4 / 8 \quad (20)$$

where  $u_i$  and  $v_T$  are flexural and torsional deformations,  $i$  is the complex unit and  $C(k)$  is the complex Theodorsen function given by

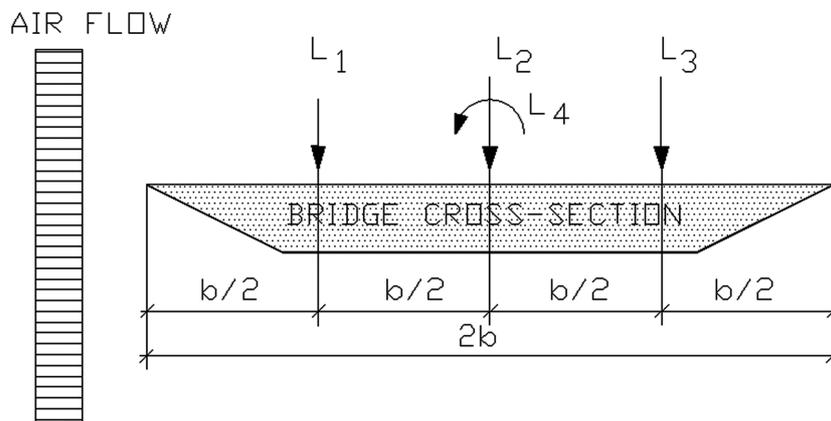


Fig. 2 Aeroelastic forces on the bridge

$$C(k) = H_1^{(2)}(k)/[H_1^{(2)} + i.H_0^{(2)}(k)] \quad (21)$$

with  $H_i^{(2)}$  as Hankel cylindrical functions of second order and with the frequency

$$k = 2.\pi.b/\lambda \quad (22)$$

where  $\lambda$  is the period of natural vibration of the bridge.

## 6. Mechanism of damping

The wind response of slender wood bridges is influenced by material and structural damping taking into account the viscoelastic properties of wood, dissipative capacity of environment as well as aerodynamic damping due to interactions bridge vs environment. The equivalent damping represents the total energy dissipation and is given by several mechanisms which specify relations between damping forces and strain velocities. The energy transformations due to material damping are given by nonstationary thermic variations caused by friction and motion of atomic groups in the wood material used. The dissipative capacity depends on frequency, stress and thermal effects. The interactions of all these mechanisms are significant in the assessment of the problem.

For the analysis of material and structural damping the theory of hysteretic damping is adopted (Sorokin 1957, Lazan 1968). The stress-strain relation in complex form is given by

$$\sigma = E_o (\eta_1 + i \eta_2) . \varepsilon \quad (23)$$

with stress and strain  $\sigma$  and  $\varepsilon$ , respectively, and with  $E_o$  as complex modulus of elasticity when the real part of strain converges to zero. The parameters  $\eta_1$  and  $\eta_2$  are the real functions of the damping factor  $\eta$  and are given by

$$\eta_1 = (1 - \eta^2/4)/(1 + \eta^2/4) \quad (24)$$

$$\eta_2 = \eta/(1 + \eta^2/4) \quad (25)$$

The damping factor  $\eta$  and logarithmic decrement of damping  $\delta_f$  are given by

$$\eta = \delta_f / \pi \quad (26)$$

If the stress  $\sigma$  varies sinusoidally, the strain changes with frequency  $\omega$  and with phase shift  $\alpha$ . If the stress is

$$\sigma = \sigma_o . e^{i\omega t} \quad (27)$$

with the stress amplitude  $\sigma_o$ , then corresponding strain is given by

$$\varepsilon = \varepsilon_o . e^{i(\omega t - \alpha)} \quad (28)$$

For the complex modulus of elasticity there holds

$$E = E_1 + i.E_2 = \sigma/\varepsilon \quad (29)$$

with real and imaginary components given by

$$E_1 = \sigma_0 \cdot \cos \alpha / \varepsilon_0 \quad (30)$$

$$E_2 = \sigma_0 \cdot \sin \alpha / \varepsilon_0 \quad (31)$$

The damping factor  $\eta$  is given by

$$\eta = \operatorname{tg} \alpha = E_2/E_1 \quad (32)$$

and is included into complex moduli of elasticity

$$E = E_0 (1 + i.\eta) \quad (33)$$

$$G = G_0 (1 + i.\eta_e) \quad (34)$$

where  $\eta_e$  is the shear damping factor. The structural damping is approximated by complex spring characteristics

$$k_B = k_0 (1 + i.\eta_B) \quad (35)$$

specified by elastic supports or joints. The parameters  $k_0$  and  $\eta_B$  are defined as the spring constant and the factor of structural damping. The structural damping appears in the interaction with the material damping.

In the hysteretic model the damping forces generated by material friction are proportional to deformations. The loss factor is equivalent to energy dissipated. There can appear theoretical non-causalities because hysteretic damping approach holds only for steady state harmonic oscillations. However, the numerical and laboratory experiments have experienced negligible significance of such theoretical malfunctioning.

The TVC of wood bridges is made by computer operated variation of hysteretic, viscous and viscoelastic parameters in the damping facilities of tuning joints as well as in the energy absorbing members adopted. In the hysteretic members in tuning joints the damping forces generated by material friction are proportional to the deformations occurring. The tuning thus depends on stress vs displacement dependence of the stiffness appearing. The specific work of the total damping is given by

$$D = J \sigma^n \quad (36)$$

where  $J$  and  $n$  are experimentally found parameters (Lazan 1968). The total work of damping is obtained by integrating specific works of material and structural members adopted. The variability of stress causes that each material particle has its own hysteresis curve contributing to total damping.

Within the element volume  $V_g$  the maximum stress  $\sigma_{max}$  corresponds to the maximum work of damping  $D_{max}$  and the work of tuning is given by

$$D_g = D_{max} V_g \beta_1 \quad (37)$$

with nondimensional parameter  $\beta_1$  (Lazan 1968). The energy cumulated in analyzed volume is

$$U_g = 0.5 V_g \sigma_{max}^2 \beta_2 / E \quad (38)$$

with nondimensional parameter  $\beta_2$  (Lazan 1968). The factor of damping for analyzed volume is given by

$$\eta_s = D_g / (2\pi U_g) \quad (39)$$

and is implemented into complex modulus of elasticity in accordance with the stress available. For linear damping there holds  $\beta_1/\beta_2 = 1$ . The ultimate analysis with nonlinear damping is based on the assessment of  $\beta_1$  and  $\beta_2$  for given geometry and stress. An iterative scheme is used for specification of damping factors in each element of the bridge. In the first iteration the parameters  $\beta_1^{(1)}$ ,  $\beta_2^{(1)}$  and  $\eta_s^{(1)}$  are specified for the stress level available. Such parameters are the basis for following iteration steps specifying actual incremental stress and corresponding parameters  $\beta_1^{(i)}$ ,  $\beta_2^{(i)}$  and  $\eta_s^{(i)}$ . The analysis is continued until satisfaction of convergence criterion

$$|\eta_s^{(i+1)} / \eta_s^{(i)} - 1| \leq 0.01 \quad (40)$$

When analyzing the viscous damping in TVC-joints adopted, the substitution of complex stiffness by an equivalent viscous damping is to be made. Such a shift from one model of damping into another one has to be made in order to ensure the same amount of energy dissipation per cycle of vibration in the hysteretic model

$$\Delta W_{hysteretic} = 2 \pi \eta K u_o^2 \quad (41)$$

as well as in the viscous damping model

$$\Delta W_{viscous} = 2 \pi \omega_o C u_o^2 \quad (42)$$

The terms  $K$  and  $C$  are stiffness and damping matrices, respectively, and  $u_o$  is the vector of deformations of the bridge oscillating with frequency  $\omega_o$ . The damping capacity of the viscous member is defined as the ratio of the energy  $\Delta W$  dissipated per cycle of vibration vs maximal stored energy per cycle

$$\psi = \Delta W / W = 2 \pi \xi. \quad (43)$$

The damping capacity is in such a way related to hysteretic loss factor  $\eta$  based on complex modulus of elasticity and on viscous damping ratio  $\xi$ . The damping capacity of the viscoelastic damping member

$$\psi^0 = \psi_{vol} + \psi_{dev} \quad (44)$$

is splitted up into volumetric ( $\psi_{vol}$ ) and deviatoric ( $\psi_{dev}$ ) parts. The volumetric part is considered to be simply elastic and viscoelastic behaviour is principally related to the deviatoric part. The stress is splitted up into elastic and dissipative parts. In constitutive equation there holds

$$\sigma = \sigma_{elastic} + \sigma_{dissipative} = E \varepsilon + \lambda \dot{\varepsilon} \quad (45)$$

with Young modulus  $E$ , with strain  $\varepsilon$ , strain rate  $\dot{\varepsilon}$  and with  $\lambda$  as viscosity constant of damping member studied. There pays

$$\lambda = \nu E \quad (46)$$

where  $\nu$  is the relaxation time. In strain  $\varepsilon = B.u$  and strain rate  $\dot{\varepsilon} = B.\dot{u}$  the parameters  $u$  and  $\dot{u}$  are nodal vectors of deformations and velocities, respectively. The matrix  $B$  consists of the derivatives of shape functions applied.

Environmental air causes additional interactive damping in the aeroelastic response of wood bridges. For the assessment of total damping is to be dealt with residual value of dissipative energy given by

$$W = \Delta W + \Delta L \quad (47)$$

with dissipative components of structural damping  $\Delta W$  as mentioned above and dissipative part of environmental damping  $\Delta L$ . When taking into account the air incompressibility which holds for the wind velocities  $v < 50$  m/sec (density variability of air for such velocities is less than 1%), for constant air pressure  $q$  there holds

$$q = \rho v^2 / 2 \quad (48)$$

where  $\rho$  is the air density. Such pressure appears on the motionless bridge forced by constant air flow with velocity  $v$ . The additional damping force is given by

$$C_L = c A \rho v^2 / 2 \quad (49)$$

with area  $A$  of the bridge and with the coefficient  $c$  as explained below. The damping force  $C_L$  is valid for stationary process, e.g., for the constant velocity of the air flow. However, when dealing with ultimate response, the non-stationary circumstances are to be considered. The resulting force acting on the bridge is then given by

$$C_R = c_d A \rho v^2 + c_m A_D \rho a \quad (50)$$

with wind acceleration  $a$ . In Eq. (50) besides constant pressure term there appears one additional term corresponding to the gyration mass. The coefficients  $c$ ,  $c_d$  and  $c_m$  are given in (Davenport 1961). The gyration forces depend on additional virtual mass of the air pressed in front of the bridge. For small amplitudes the air damping is proportional to velocity of the bridge. For large amplitudes the

air damping depends on the resistance force given by constant pressure of wind flow on the bridge. Aeroelastic interaction bridge vs air flow causes further additional damping of the dynamic motion. There appear aerodynamic dissipative forces and damping is given by phase shifts of motions and forces appearing. The amplitudes of aerodynamic forces increase linearly with the wind velocity.

## 7. Turbulences

All turbulences in the wind forcing are considered as a special family of motions from one space region into another one (Tesar 1988). Their updated configuration is specified by location of the air displacements in space and time. The variations of configurations are continuous and during deformation there appear no new boundary conditions. Each new configuration is related to a reference position stated.

When taking into account the Cartesian coordinates  $x, y, z$  and corresponding displacements  $u, v, w$ , the Green strain tensor is given by

$$E_{xx} = \partial u_x / \partial x + [(\partial u_x / \partial x)^2 + (\partial u_y / \partial x)^2 + (\partial u_z / \partial x)^2] / 2 \quad (51)$$

$$E_{xy} = [(\partial u_y / \partial x) + (\partial u_x / \partial y) + (\partial u_x / \partial x)(\partial u_x / \partial y) + (\partial u_y / \partial x)(\partial u_y / \partial y) + (\partial u_z / \partial x)(\partial u_z / \partial y)] / 2, \dots, \text{etc.} \quad (52)$$

In order to set up the constitutive equations, the stress tensor with the same reference is needed. The second Piola-Kirchhoff stress tensor  $S_{ij}$  has the properties required and the generalized equation of the air flow is then given by

$$S_{ij} = g(E_{ij}) \quad (53)$$

with  $g$  as function of the Green strain tensor  $E_{ij}$ .

When analysing the air flow with volume, surface area and density,  $B, S$  and  $\rho_0$ , respectively, the volume forces of the mass unit are given by  $F_{o,i}$  and the strains by  $T_i$ . The system in equilibrium is submitted to a virtual displacement  $\delta u_i$  being kinematically consistent with initial conditions assumed. The equilibrium of the virtual work is given by

$$\int S_{ij} \delta E_{ij} dB - \int T_i \delta u_i dS - \int P_i \delta u_i dB = 0 \quad (54)$$

with substitution

$$P_i = \rho_0 F_{o,i} \quad (55)$$

Eq. (54) specifies the stationary value of the potential energy in all deformations  $u_i$ . The incremental equivalent of corresponding variation principle is given by

$$\int S_{ij}^{(1)} \delta E_{ij}^{(1)} dB - \int T_i^{(1)} \delta u_i^{(1)} dS - \int P_i^{(1)} \delta u_i^{(1)} dB = 0 \quad (56)$$

$$\int S_{ij}^{(2)} \delta E_{ij}^{(2)} dB - \int T_i^{(2)} \delta u_i^{(2)} dS - \int P_i^{(2)} \delta u_i^{(2)} dB = 0, \quad (57)$$

with superscripts (1) and (2) for neighbouring configurations studied. The strains and volume forces have the same reference configuration and there holds

$$\Delta T_i = T_i^{(2)} - T_i^{(1)} \quad (58)$$

$$\Delta P_i = P_i^{(2)} - P_i^{(1)} \quad (59)$$

The variations of both deformation fields are the same

$$\delta u_i = \delta u_i^{(1)} = \delta u_i^{(2)} \quad (60)$$

The incremental virtual work equation is given by Eqs. (56) and (57)

$$\int (S_{ij}^{(2)} \delta E_{ij}^{(2)} - S_{ij}^{(1)} \delta E_{ij}^{(1)}) dB - \int \Delta T_i \delta u_i dS - \int \Delta P_i \delta u_i dB = 0 \quad (61)$$

when taking into account the virtual variations of both configurations studied. Eq. (61) specifies the configuration (2) from the known configuration (1) and known load increments. When the work made by mass and damping forces on virtual displacements  $\delta u_i$  is added, the principle of virtual work for the problem studied is given by

$$\int S_{ij} \delta E_{ij} dB + \int \rho u_i \delta u_i dB + \int C_i u_i \delta u_i dB - \int T_i \delta u_i dS - \int P_i \delta u_i dB = 0 \quad (62)$$

where  $\rho$  and  $C$  are mass and damping terms.

The turbulence in the air flow is described by instantaneous wind speed as a function of space and time with mean and fluctuation components given by

$$u(x, y, z, t) = U(x, y, z) + u'(x, y, z) \quad (63)$$

$$v(x, y, z, t) = V(x, y, z) + v'(x, y, z) \quad (64)$$

$$w(x, y, z, t) = W(x, y, z) + w'(x, y, z) \quad (65)$$

The mean values of  $U$ ,  $V$ ,  $W$  are the result of averaging in a certain interval of time the wind speed and the fluctuating components.

The turbulence scales of the instantaneous wind speed are the measure of representative dimensions of the vortices induced by the turbulences inside the air flow. Their importance lies in the fact that they describe the turbulences which „wrap“ the bridge periphery in a certain time.

The assessment of a turbulence motion starts with the specification of the correlation functions of fluctuating components which may run in longitudinal, transversal and vertical directions. In general, the characteristics of the air flow are well defined if the correlation functions are specified for the mean streamwise components longitudinally and transversally. The correlation in time is specified by formulae

$$\rho_{u(i)u(j)}(\tau) = R_{u(i)u(j)}(\tau) / [(\sqrt{(u')^2(t)}) \cdot (\sqrt{(u')^2(t + \tau)})] \quad (66)$$

$$R_{u(i)u(j)}(\tau) = u_i(t).u_j(t + \tau) = \lim_{T \rightarrow \infty} 1/T \int [u_i(t).u_j(t + \tau)]dt \quad (67)$$

Eq. (67) represents the covariance function of the process  $u(t)$  being determined by measuring in two different points in space at the difference of time  $\tau$  (Teleman 2008).

According to Taylor's hypothesis (Hautoy 1990) the inter-correlation between any of the fluctuating parts, discarding the wind instantaneous speed measured in two points being separated by distance  $\Delta x$  in the direction of the wind flow, is equal with the auto-covariance determined for the period studied. The inter-correlation functions give information concerning the dimensions of turbulences in direction of the wind action. The existence of the mean values of the wind speed inside of turbulent flow is given by fact that in a certain point  $i$  the turbulence has a certain periodicity in time. After a certain period the phenomenon repeats itself in space. These two idioms specify the turbulence scales in time and space. The turbulence scales define the frequency of the gusts in the wind action. The integral length scales correspond to spatial nature of the wind action specifying the longitudinal, lateral and vertical scales given by

$$L_x = \int \rho_{u'(i)u'(j)}(\Delta x, 0, 0) d(\Delta x) \quad (68)$$

$$L_y = \int \rho_{u'(i)u'(j)}(0, \Delta y, 0) d(\Delta y) \quad (69)$$

$$L_z = \int \rho_{u'(i)u'(j)}(0, 0, \Delta z) d(\Delta z) \quad (70)$$

with integration from 0 until  $\infty$ . The most important of these three is the longitudinal scale, the other two being practically its derivatives. The integral time scale of the turbulence is defined by

$$A_T = \int \rho_{u'(i)u'(j)}(\tau) d\tau \quad (71)$$

According to above Taylor's hypothesis, the longitudinal scale of a turbulence may be specified by integral time scale and by mean wind speed  $V$  in the streamwise direction given by

$$L_x = V.A_T \quad (72)$$

The studies for determination of the turbulence scale, both at natural scale and in laboratory, have produced the empirical Davenport's formula

$$A_T = 0.084 L/V \quad (73)$$

given in sec, where  $L$  is the longitudinal scale of the wind speed and  $V$  is the mean wind speed.

The incorporation of above forcing into behaviour of the bridge is specified by the wave propagation with corresponding interactions and reflexions of laminar and turbulent air flows. The waves initiated are specified by the spectral evolution describing the occurrence of wind turbulences.

The spectral evolution is based on following definitions

1. Each stationary function  $x(t)$  is given in integral form

$$x(t) = \int e^{i\omega t} dA(\omega) \quad (74)$$

with symbol  $A(\omega)$  for orthogonal complex process studied.

2. Linear transformation  $y(t)$  of the function  $x(t)$  in Eq. (74) is given by

$$y(t) = \int H(i\omega) e^{i\omega t} dA(\omega) \quad (75)$$

with  $H(\omega)$  as corresponding admittance function.

3. Spectral densities of functions  $x(t)$  and  $y(t)$  are connected by

$$S_y(\omega)/S_x(\omega) = |H(i\omega)|^2 \quad (76)$$

Turbulent air flow is defined by a wave number  $r_i(\omega)$ , with longitudinal ( $d$ ) and shear ( $s$ ) waves. Stationary waves are emitted with complex amplitude  $F(\omega, z_0)$ , e.g.,  $z = z_0$ . The wave superposition is given by

$$w_i(t, z) = \int e^{-i\omega t} e^{ir(\omega z)} dF(\omega, z_0) \quad (77)$$

For wave interactions in turbulences the forcing spectrum is given by

$$S(\omega, 0) = S(\omega, z_0) |H(\omega, 0)|^2 e^{-2 \text{Im}[r(\omega)]} \quad (78)$$

with response  $H(\omega, 0)$ .

Recent approaches are given in Refs. (Lu 2012, Belver 2012, Cheggaga 2012, McWilliam 2011).

## 8. Tuned vibration control

The system identification for each forcing situation appearing, together with structural response, optimization and monitoring, are principal operations made in the TVC-joints. The system identification is a part of modeling with data basis available from updated structural and forcing situations measured. The analysis of structural response considers all linear and nonlinear interaction effects appearing. The optimization and monitoring take into account the target functions adopted in order to control the bridge response. The tuning joints contain the facilities for variability of forces in wind cables in TVC, taking account of:

- updated frequency spectrum of the bridge studied; is initiated by variability of forces in the wind cables,
- updated damping parameters of the bridge studied; are influenced by damping facilities and energy absorbers in structural system and in the TVC-joints adopted,
- updated monitoring of time response of the bridge.

Above items change the frequency spectrum as well as structural damping parameters of the bridge for each forcing situation occurring.

Other examples of TVC-systems are based on utilization of cables located interior of the bridge girder studied (see Fig. 3 - alternatives 2 and 3). The principal scheme of such TVC facility is in Fig. 4. The TVC is performed either by the variability of forces in the wind cables or by the variability of distances of structural elastic supports in contact points girder vs wind cables interior of the girder. The TVC is activated by adoption of simple locking mechanisms in contact points.

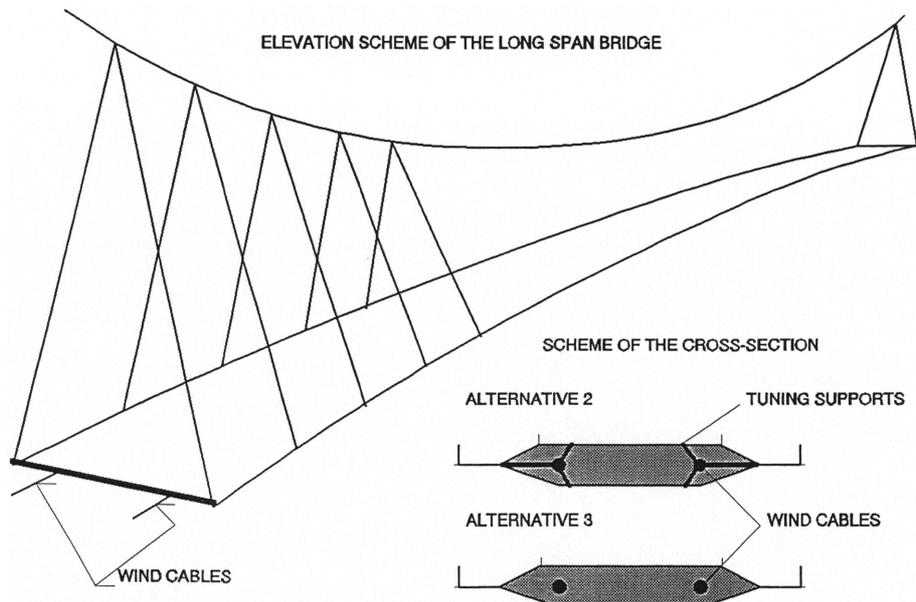


Fig. 3 TVC-alternatives

### TUNED VIBRATION CONTROL JOINT

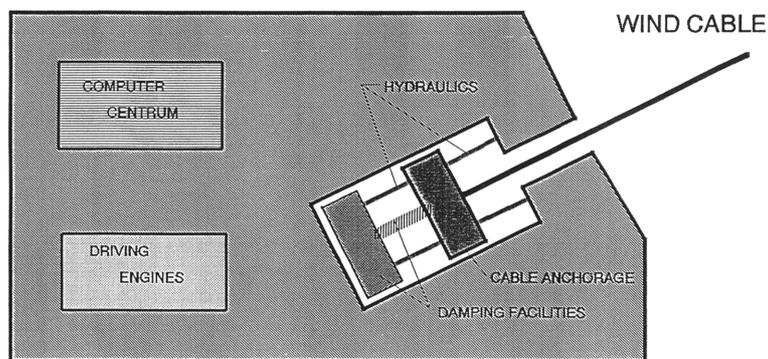


Fig. 4 TVC-joint

The TVC in scope of the alternative 3 consists of the wind cables placed interior of the bridge girder. Axial forces in the wind cables are varied in order to control the torsional/flexural response of the bridge.

## 9. Application

Studied is the ultimate flutter response of the slender wood bridge as shown in Fig. 5. The span of the bridge is 100 m. The main girder of the bridge is made of laminated wood. The carbon fiber composites are adopted for the cables. The structural parameters of the bridge are: girder width

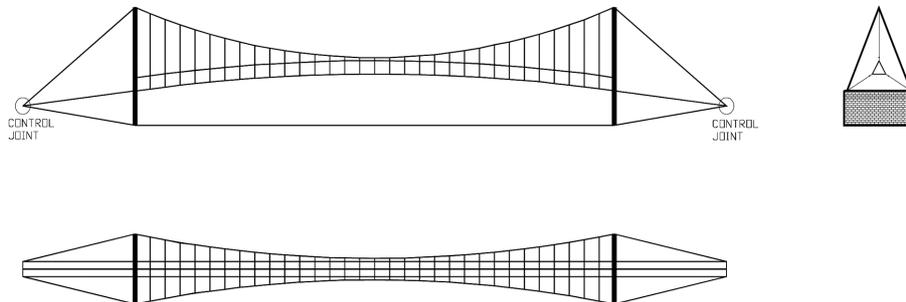


Fig. 5 Geometry of the wood bridge studied

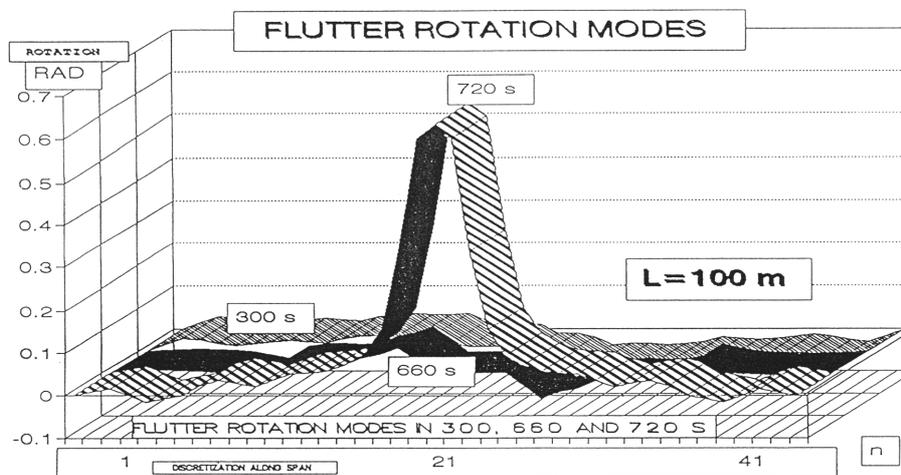


Fig. 6 Ultimate bridge response

7.9 m, girder height 4.1 m and its mass per  $\text{m}^2$  is 1830 kg. The bridge was forced by standard laminar and turbulent air flows measured in southern territory of Slovakia (Tesar 2011). The ultimate flutter time response during simultaneous action of flutter eigenvalues given by resonance frequency of the bridge 0.66 Hz and by critical wind velocity 23.6 m/sec was studied.

The assessment has shown the dominant influence of flutter rotation modes on resulting ultimate response of the bridge. Starting with simultaneous occurrence of both eigenvalues and assuming the discretization of the bridge span into  $n$ -nodes, the structural time response until the bridge collapse was studied. In scope of the TVC the wind cables of the bridge were submitted to variable axial tensile forces. The bridge response in time points 300 sec (for axial tension force 0.1 MN), 660 sec (for axial tension force 0.25 MN) and 720 sec (for axial tension 0.5 MN) after initiation of simultaneous action of both eigenvalues, is plotted in Fig. 6.

## 10. Conclusions

The efficiency of the present system for tuned vibration control of slender wood bridges, adopting the variability of forces in the wind cables, is illustrated. The TVC appears as efficient tool for the

aided reliability of slender wood bridges subjected to laminar and turbulent wind forcing.

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