Composite Materials and Engineering, *Vol. 3, No. 3 (2021) 241-262*
https://doi.org/10.12989/cme.2021.3.3.241

Effect of two-temperature in an orthotropic thermoelastic media with fractional order heat transfer

Parveen Lata[a] and Himanshi Himanshi*

Department of Basic and Applied Sciences Punjabi University Patiala, Punjab India

(Received July 31, 2021, Revised December 8, 2021, Accepted December 10, 2021)

Abstract. In this article we studied the effect of two-temperature in a two-dimensional orthotropic thermoelastic media with fractional order heat transfer in generalized thermoelasticity with three-phase-lags due to thermomechanical sources. The boundary of the surface is subjected to linearly distributed and concentrated loads (mechanical and thermal source). The solution of the problem is obtained with the help of Laplace and Fourier transform techniques. The expressions for displacement components, stress components and conductive temperature are derived in transformed domain. Numerical inversion technique is used to obtain the results in physical domain. The effect of two-temperature on all the physical quantities has been depicted with the help graphs. Some special cases are also discussed in the present investigation.

Keywords: concentrated and linearly distributed loads; Fourier transform; fractional calculus; laplace transform; orthotropic; two-temperature, three-phase-lags

1. Introduction

The elastic solid body deforms due to the action of an external loads and heat sources. The study of the dynamical system in which both thermal and mechanical effects are introduced give rise to the subject called thermoelasticity. When a material body is subjected to an external force and heat source it changes its shape due to the change in its temperature. Thermoelasticity deals with the flow of heat, stresses and strain produced in the material body. From many years, an intense amount of interest has been given to the non-classical theories which are referred to as generalized thermoelastic theories. These are developed to overcome the shortcomings of classical old theories. According to these theories the heat signals propagate with finite speed and also applicable in those cases where the heat flux rate is very high. The thermoelastic theories with two-temperature are also included in the field of non-classical theories. Many researchers worked on the two-temperature thermoelastic problems. Chen and Gurtin (1968) and Chen *et al*. (1968, 1969) formulated two-temperature theory of thermoelasticity for deformable bodies.

Heat conduction equation of this theory depends on two different types of temperatures thermodynamical temperature (T) and the conductive temperature(ϕ). The difference between these two temperatures is proportional to the heat supply. The two temperatures are equal in the absence

*Corresponding author, Ph.D., E-mail: heena.zakhmi@gmail.com
[a] Associate Professor, E- mail: parveenlata@pbi.ac.in

http://www.techno-press.org/?journal=cme&subpage=7 ISSN: 2671-4930 (Print), 2671-5120 (Online)

of heat supply for time independent problems. For time dependent problems, the two temperatures are different regardless of the presence of heat supply. Moreover, fractional order theory came into existence during the second half of the 19th century and has been used to model polymer materials. The applications of fractional order theory are widely spread over the areas like in dynamics, fluid mechanics, biology, physics, mechanics of solids etc. Fractional order differential equations have been focus of many studies due to their non-localization property. Most of the problems of physical processes are solved with the help of fractional order heat conduction equations. The application of fractional calculus was first introduced by Abel to solve the tautochrone problem. Marin (1999) obtained the existence and uniqueness solutions for the mixed initial-boundary value problems in thermoelasticity of dipolar bodies. Youssef (2006) formulated a new theory of generalized thermoelasticity by considering two-temperature generalized thermoelasticity theory for a homogeneous isotropic body without energy dissipation. Palani and Abbas (2009) investigated the combined effects of magnetohydrodynamics and radiation on free convection flow past an impulsively started isothermal vertical plate with Rossel and diffusion approximation. Abbas et. al. (2009) studied the effect of thermal dispersion on free convection in a fluid saturated porous medium. Sur and Kanoria (2012) constructed the theory of two-temperature thermoelasticity by considering fractional order heat equation. Bassiouny and Sabry (2013) constructed the two-temperature fractional order thermoelastic model for piezoelectric materials. Marin et al. (2013) modelled a micro stretch thermoelastic body with two-temperatures and eliminated divergences among the classical elasticity and research. Naggar *et al*. (2013) studied the effect of initial stress, magnetic field, rotation, voids and thermal field on plane waves in generalized thermoelasticity. Abbas and Kumar (2014) studied the interactions due to thermal source in micropolar generalized thermoelastic half-space by finite element method Ezzat *et al*. (2014) constructed the two temperature magneto-thermoelastic theory by using fractional order heat conduction equation. Bhattacharya and Kanoria (2014) studied the interactions in a two-dimensional isotropic generalized thermoelastic diffusive spherical shell with fractional order heat transfer and two-temperature. Mondal *et al*. (2014) investigated the thermomechanical interactions in isotropic homogenous semi-infinite generalized thermoelastic solids with two-temperature and fractional order. Zenkour and Abouelregal (2015) studied the fractional effect in a generalized semi-infinite solid induced by pulse laser heating. Marin *et al*. (2015a) studied the double porosity structure for micropolar bodies. Marin *et al*. (2015b) extended the domain of influence theorem for generalized thermoelasticity of anisotropic material with voids. Abbas and Marin (2017a) examined the analytical solution of thermoelastic interaction in a half space due to pulsed laser heating. Abbas and Marin (2017b) also studied the two-dimensional generalized thermoelastic diffusion problem due to laser pulse. Kumar *et al*. (2017) studied the effect of hall current and rotation in a modified couple stress generalized thermoelastic half space due to ramp type heating. Abbas (2018) studied the effect of fractional parameter in a two dimensional problem in the context of thermal shock with three types of conductivity weak, normal and strong conductivity. Molla *et al*. (2019) studied the fractional and two-temperature effect on the stress components for an unbounded generalized thermoelastic media with spherical cavity. Lata and Kaur (2019) studied the thermomechanical interactions in transversely isotropic magneto- thermoelastic solids with rotation due to time harmonic sources. Saeed *et al*. (2020) studied the thermo-elastic interactions in a poroelastic material using finite-element method. Zhang *et al*. (2020) studied the Entropy Analysis on the Blood Flow through Anisotropically Tapered Arteries Filled with Magnetic Zinc-Oxide (ZnO) Nanoparticles. Khamis *et al*. (2020) examined the thermoelastic interactions in a homogeneous isotropic fractional ordered thermoelastic infinite medium with a cylindrical cavity. Bekkaye (2020) studied the mechanical behaviour of FG plate using refined trigonometric shear deformation theory. Othmane *et al*. (2020) studied the bending the and free vibration analysis of laminated composite and sandwich plates. Alzahrani and Abbas (2020) studied the photo-thermal interactions in a semiconducting medium with spherical cavity and two-temperature. Biswas and Abo-Dahab (2020) studied the magneto-thermoelastic interactions in a three-dimensional orthotropic medium. Zenkour (2020) studied the magnetic thermal shock problem for a fibre-reinforced anisotropic half space with refined multi-dual-phase-lag model of heat transfer. Guellil *et al*. (2021) studied the Influences of porosity distributions and boundary conditions on mechanical bending response of functionally graded plates resting on Pasternak foundation. Bendenia *et al*. (2021) studied the Deflections, stresses and free vibration studies of FG-CNT reinforced sandwich plates resting on Pasternak elastic foundation. Zerrouki *et al*. (2021) studied the effect of non-linear FG-CNT distribution on mechanical properties of functionally graded nano-composite beam. Tahir *et al*. (2021) investigated the wave propagation of a ceramic-metal FGM plate with various porosity distributions in a hygrothermal environment and resting on a viscoelastic foundation. Mudhaffar *et al*. (2021) studied the Hygro-thermo-mechanical bending behaviour of advanced functionally graded ceramic metal plate resting on a viscoelastic foundation. Merazka *et al*. (2021) analyzed the Hygro-thermo-mechanical response of FG plates resting on elastic foundations. Houari *et al*. (2021) Bending analysis of functionally graded plates using a new refined quasi-3D shear deformation theory and the concept of the neutral surface position. Bakoura *et al*. (2021) studied the buckling analysis of functionally graded plates using HSDT in conjunction with the stress function method. Lata and Himanshi (2021a, b, c) studied the various orthotropic thermoelastic problems of generalized thermoelasticity with fractional order heat transfer. Bassiouny (2021) studied the behaviour of homogeneous isotropic thermoelastic semi-infinite material with fractional order heat transfer subject to thermal loading.

Including all the above work, we observed that a two dimensional problem in an orthotropic media with and without energy dissipation in the context of fractional order heat transfer with three phase-lags and two temperatures has not been considered yet. So, in this attempt we have examined the effect of two temperature in an orthotropic thermoelastic solid by using fractional order heat conduction equation of type GN-III with three phase-lags due to thermomechanical sources. Laplace and Fourier transforms are used to obtain the components of stress, displacement and temperature change subjected to concentrated load and linearly distributed load. The effect of two-temperature on various components with and without fractional effect has been examined with the help of graphs.

2. Basic equations

Following Lata and Zakhmi (2021b), the constitutive relations, equation of motion and heat conduction for anisotropic thermoelastic solid with three-phase-lags in the absence of body forces and heat sources are given as

$$\sigma_{ij} = c_{ijkl}e_{kl} - \beta_{ij}\, T, \tag{1}$$

$$\sigma_{ij,j} = \rho\, \ddot{u}_i, \tag{2}$$

$$K_{ij}\left(1+ \frac{\tau_t^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\dot{\phi}_{,ji} + K_{ij}^{*}\left(1+\frac{\tau_v^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\phi_{,ji} = \left(1 + \frac{\tau_q^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}} + \frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right)\left[\rho\, C_E\ddot{T} + \beta_{ij}T_0\ddot{e}_{ij}\right], \tag{3}$$

where $\beta_{ij} = C_{ijkl}e_{kl},\ \beta_{ij} = \beta_i\delta_{ij},\ K_{ij} = K_i\,\delta_{ij},\ K_{ij}^{*} = K_i^{*}\delta_{ij}$ $(i\,, j = 1, 2, 3)$ i is not summed and

δ_{ij} is Kronecker delta.

The strain displacement relations are taken as

$$e_{ij} = \frac{1}{2}\,(u_{i,j} + u_{j,i}), \quad i,j = 1,2,3 \tag{4}$$

Following Youssef (2006), the two temperature relation is given by $T = (\phi - a_{ij}\phi_{,ij})$, where $i,j = 1,2,3$

Here, dot (.) represents the partial derivative w.r.t time and (,) denote the partial derivative w.r.t spatial coordinate, c_{ijkl} (= c_{klij} = c_{jikl} = c_{ijlk}) is the tensor of elastic constant, ρ is the density, T_0 is the reference temperature, u_i are the components of displacement vector $\boldsymbol{u}$, C_E is the specific heat at constant strain, σ_{ij} = (σ_{ji}), e_{ij} are the components of stress and strain tensors, T is the absolute temperature, ϕ is the conductive temperature. Also τ_q , τ_t and τ_v are respectively the phase-lag of the heat flux, the phase lag of the temperature gradient and the phase lag of the thermal displacement respectively. α_{ij} is the coefficient of linear thermal expansion and β_{ij} is the tensor of thermal moduli, a_{ij} are the two-temperature parameters, K_{ij} and K_{ij}^* are the constants of thermal conductivity and material characteristic constant respectively.

Here, the symmetries of elastic parameters C_{ijkl} is due to

i. The stress tensor is symmetric, which is only possible if $(C_{ijkl} = C_{jikl})$
ii. If a strain energy density exists for the material, the elastic stiffness tensor must satisfy $C_{ijkl} = C_{klij}$
iii. From stress tensor and elastic stiffness tensor symmetries infer ($C_{ijkm} = C_{ijmk}$) and $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$

3. Formulation of the problem

We consider a homogeneous orthotropic thermoelastic body initially at uniform temperature T_0 with and without energy dissipation in generalized thermoelasticity with fractional order heat transfer and two-temperature. We take a rectangular coordinate system (x, y, z) having origin on the surface $z = 0$ with z-axis pointing vertically downwards into the medium is introduced. The surface of the half space is subjected to the thermomechanical sources i.e., a normal force F_1 and a thermal source F_2 is acting at $z = 0$. For two-dimensional problem in xz-plane, we take

$$\vec{u} = (u, 0, w) \tag{5}$$

Following Kumar and Chawla (2014), constitutive relations for orthotropic thermoelastic media using Eq. (1) can be written as

$$\sigma_{xx} = C_{11}\, e_{xx} + C_{13}\, e_{zz} - \beta_1 \mathrm{T}, \tag{6}$$

$$\sigma_{zz} = C_{13}\, e_{xx} + C_{33}\, e_{zz} - \beta_3 \mathrm{T}, \tag{7}$$

$$\sigma_{xz} = 2\, C_{55}\, e_{xz}, \tag{8}$$

where

$$e_{xx} = \frac{\partial u}{\partial x}, \; e_{zz} = \frac{\partial w}{\partial z}, \; e_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \; T = \phi - \left(a_1 \frac{\partial^2 \phi}{\partial x^2} + a_3 \frac{\partial^2 \phi}{\partial z^2}\right), \tag{9}$$

Eqs. (2) and (3) with the help of Eqs. (6)-(9) takes the form

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{55}\frac{\partial^2 u}{\partial z^2} + (C_{13} + C_{55})\frac{\partial^2 w}{\partial x \partial z} - \beta_1 \frac{\partial}{\partial x}\{\phi - (a_1\frac{\partial^2 \phi}{\partial x^2} + a_3\frac{\partial^2 \phi}{\partial z^2})\} = \rho \frac{\partial^2 u}{\partial t^2}, \quad (9)$$

$$(C_{13} + C_{55})\frac{\partial^2 u}{\partial x \partial z} + C_{55}\frac{\partial^2 w}{\partial x^2} + C_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3 \frac{\partial}{\partial z}\left\{\phi - \left(a_1\frac{\partial^2 \phi}{\partial x^2} + a_3\frac{\partial^2 \phi}{\partial z^2}\right)\right\} = \rho \frac{\partial^2 w}{\partial t^2}, \quad (10)$$

$$K_1(1+\frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha})\,\dot{\phi}_{,11} + K_3(1+\frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha})\,\dot{\phi}_{33} + K_1^*(1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\,\phi_{,11} + K_3^*(1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\phi_{,33} =$$
$$\left[1 + \frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha} + \frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right]\left[\rho C_E \frac{\partial^2}{\partial t^2}\{\phi - \left(a_1\frac{\partial^2 \phi}{\partial x^2} + a_3\frac{\partial^2 \phi}{\partial z^2}\right)\} + T_0\{\beta_1 \ddot{u}_{1,1} + \beta_3 \ddot{u}_{3,3}\}\right]. \quad (11)$$

In the above equations we use the contracting subscript notations ($11 \to 1,\ 22 \to 2,\ 33 \to 3, 23 \to 4,\ 13 \to 5,\ 12 \to 6$) to relate C_{ijkl} to C_{mn} , where $i,j,k,l = 1,2,3$ and $m,n = 1,2,3,4,5,6$.

We assume that the medium is initially at rest, then the initial and regularity conditions are given by

$$u_1(x,z,0) = 0 = \dot{u}_1(x,z,0),$$
$$u_3(x,z,0) = 0 = \dot{u}_3(x,z,0),$$
$$\phi(x,z,0) = 0 = \dot{\phi}(x,z,0), \text{ For } z \geq 0, -\infty < x < \infty;$$
$$u_1(x,z,t) = u_3(x,z,t) = \phi(x,z,t) = 0, \text{ For } t > 0 \ when\ z \to \infty$$

We define the following dimensionless quantities to facilitate the solution

$$x' = \frac{x}{L},\ z' = \frac{z}{L},\ u' = \frac{\rho c_1^2}{L T_0 \beta_1}u,\ w' = \frac{\rho c_1^2}{L T_0 \beta_1}w,\ t' = \frac{C_1}{L}t,$$
$$\sigma'_{33} = \frac{\sigma_{33}}{T_0\beta_1},\ \sigma'_{31} = \frac{\sigma_{31}}{T_0\beta_1},\ \phi' = \frac{\phi}{T_0},\ a_1' = \frac{a_1}{L},\ a_3' = \frac{a_3}{L},\ T' = \frac{T}{T_0}. \quad (13)$$

where $c_1^2 = \frac{c_{11}}{\rho}$.

By the use of dimensionless quantities given by Eq. (13) in Eqs. (10)-(12) and suppressing the primes for convenience yield

$$\frac{\partial^2 u}{\partial x^2} + \delta_1\frac{\partial^2 u}{\partial z^2} + \delta_2\frac{\partial^2 w}{\partial x \partial z} - \frac{\partial}{\partial x}\{\phi - \left(\frac{a_1}{L}\frac{\partial^2 \phi}{\partial x^2} + \frac{a_3}{L}\frac{\partial^2 \phi}{\partial z^2}\right)\} = \frac{\partial^2 u}{\partial t^2}, \quad (14)$$

$$\delta_3\frac{\partial^2 w}{\partial z^2} + \delta_1\frac{\partial^2 w}{\partial x^2} + \delta_2\frac{\partial^2 u}{\partial x \partial z} - \varepsilon\frac{\partial}{\partial z}\left\{\phi - \left(\frac{a_1}{L}\frac{\partial^2 \phi}{\partial x^2} + \frac{a_3}{L}\frac{\partial^2 \phi}{\partial z^2}\right)\right\} = \frac{\partial^2 w}{\partial t^2}, \quad (15)$$

$$\epsilon_1\tau_t\frac{\partial}{\partial t}\left(\frac{\partial^2 \phi}{\partial x^2}\right) + \epsilon_2\tau_t\frac{\partial}{\partial t}\left(\frac{\partial^2 \phi}{\partial z^2}\right) + \epsilon_3\tau_v\left(\frac{\partial^2 \phi}{\partial x^2}\right) + \epsilon_4\tau_v\left(\frac{\partial^2 \phi}{\partial z^2}\right)$$
$$= \tau_q\left[\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left(\frac{a_1}{L}\frac{\partial^2 \phi}{\partial x^2} + \frac{a_3}{L}\frac{\partial^2 \phi}{\partial z^2}\right) + \epsilon_5\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x} + \varepsilon\frac{\partial w}{\partial z}\right)\right], \quad (16)$$

where

$\delta_1 = \frac{c_{55}}{c_{11}},\ \delta_2 = \frac{c_{13}+c_{15}}{c_{11}},\ \delta_3 = \frac{c_{33}}{c_{11}},\ \tau_t = (1+\frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha}),\ \tau_v = (1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}),\ \tau_q = \left(1 + \frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha} + \frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right),\ \epsilon_1 = \frac{K_1}{\rho L C_1 C_E},\ \epsilon_2 = \frac{K_3}{\rho L C_1 C_E},\ \epsilon_3 = \frac{K_1^*}{\rho c_1^2 C_E},\ \epsilon_4 = \frac{K_3^*}{\rho c_1^2 C_E},\ \epsilon_5 = \frac{\beta_1^2 T_0}{\rho^2 c_1^2 C_E}.$

Apply Laplace and Fourier transforms defined by

$$\bar{f}(x,z,s) = \int_0^\infty f(x,z,t)e^{-st}\,dt, \quad (17)$$

$$\hat{f}(\xi,z,s)=\int_{-\infty}^{\infty}\overline{f}\ (x,z,s)e^{i\xi x}\,dx, \tag{18}$$

On Eqs, (14)-(16), we obtain a system of three homogeneous equations in $(\hat{u},\hat{w},\hat{\phi})$

$$[\delta_1 D^2-\ p_1]\hat{u}\ +[p_7\ D]\hat{w}+[-i\xi p_8+i\xi p_{10}D^2]\ \hat{\phi}=0, \tag{19}$$

$$[\delta_1 D^2-\ p_1]\hat{u}\ +[p_7\ D]\hat{w}+[-i\xi p_8+i\xi p_{10}D^2]\ \hat{\phi}=0, \tag{20}$$

$$\begin{aligned}&\left[-i\xi s^2\varepsilon_5\,\tau_q'\right]\hat{u}\ +\left[-\varepsilon\varepsilon_5 s^2\tau_q' D\right]\hat{w}\\&+\left[\ p_3\tau_t'+s\varepsilon_2\tau_t' D^2+p_4\tau_v'+\varepsilon_4\tau_v' D^2+\tau_q' p_5+p_6 D^2\tau_q'\right]\hat{\phi}=0,\end{aligned} \tag{21}$$

where

$$\begin{gathered}\mathrm{D}\ =\frac{d}{dz},\ p_1=(s^2+\xi^2),\ \ p_2=(\delta_1\xi^2+s^2),\ p_3=-s\,\xi^2\varepsilon_1,\\ p_4\ =\ -\xi^2\varepsilon_3,\ p_5=-s^2\left(1+\tfrac{a_1}{L^2}\xi^2\right),\ \ p_6=s^2\left(\tfrac{a_3}{L^2}\right),\ p_7=i\xi\delta_2,\\ p_8=\left(1+\tfrac{a_1}{L^2}\xi^2\right),\ \ p_9=-\varepsilon\varepsilon_5 i\xi\delta_2 s^2,\ p_{10}=\tfrac{a_3}{L^2},p_{11}=\tfrac{a_1}{L^2},\\ \tau_t'\ =1+\tfrac{\tau_t^\alpha}{\alpha!}s^\alpha,\quad \tau_v'=1+\tfrac{\tau_v^\alpha}{\alpha!}s^\alpha\ ,\ \ \tau_q'=1+\tfrac{\tau_q^\alpha}{\alpha!}s^\alpha+\tfrac{\tau_q^{2\alpha}}{2\alpha!}s^{2\alpha}.\end{gathered}$$

These resulting equations have non- trivial solution if the determinant of the coefficient $(\hat{u},\hat{w},\hat{\phi})$ vanishes, which yield to the following characteristic equation.

$$(\mathrm{P}D^6+QD^4+RD^2+S\,)\left(\hat{u},\hat{w},\hat{\phi}\right)=0, \tag{22}$$

where

$$\begin{gathered}\mathrm{D}=\tfrac{d}{dz}\ ,\ P=\{\ \tau_t'(\delta_3\delta_1\epsilon_2 s)+(\delta_3\delta_1\epsilon_4)\,\tau_v'+(\ \delta_3\delta_1 p_6+\delta_1\varepsilon^2\varepsilon_5\,p_6)\ \tau_q'\},\\ Q=\{\ \tau_t'(\delta_3\delta_1 p_3-\delta_1 s\epsilon_2 p_2-\delta_3 s\epsilon_2 p_1-s\epsilon_2 p_7^2)+\ \tau_v'(\delta_3\delta_1 p_4-\ \delta_1\epsilon_4 p_2-\ \delta_3\epsilon_4 p_1-p_7^2\epsilon_4)+\\ \tau_q'(\delta_3\delta_1 p_5-\ \delta_1 p_2\,p_6\ -\delta_3 p_1\,p_6\ -p_6 p_7^2+\delta_1 p_5\varepsilon^2\varepsilon_5-\ p_1 p_6\varepsilon^2\varepsilon_5+\ \xi^2\varepsilon_5\delta_3\,p_6)\},\\ R=\ \{\tau_t'(-\delta_1 p_2 p_3-\ p_1 p_3\delta_3+p_1 p_2 s\epsilon_2-\ p_3 p_7^2)+\ \tau_v'\,(-\delta_1 p_2 p_4-\delta_3 p_1 p_4+p_1 p_2\epsilon_4\ -\ p_4 p_7^2)+\\ \tau_q'\,(\,-\delta_1 p_2 p_5-\delta_3 p_1 p_5+\ p_1 p_2 p_6-p_1 p_5\varepsilon^2\varepsilon_5-\ p_5 p_7^2-i\xi p_7 p_5\varepsilon_5\varepsilon-i\xi p_8 p_9+\ \xi^2 s^2\varepsilon_5\delta_3 p_8-\\ p_6\xi^4\varepsilon_5\delta_1-\ \xi^2 s^2\varepsilon_5 p_6)\},\\ S=\ \tau_t'(p_1 p_2 p_3)+\ \tau_v'(\,p_1 p_2 p_4)+\ \tau_q'(\,p_1 p_2 p_5-\ \xi^4 s^2\varepsilon_5\delta_1 p_8-\ \xi^2 s^4\varepsilon_5 p_8).\end{gathered}$$

The roots of the Eq. (22) are $\pm\lambda_i$ (i = 1,2,3); the solution of the equation satisfying the radiation conditions can be written as

$$\tilde{u}=A_1 e^{-\lambda_1 z}+A_2 e^{-\lambda_2 z}+\ A_3 e^{-\lambda_3 z}, \tag{23}$$

$$\widetilde{w}=\ d_1 A_1 e^{-\lambda_1 z}+\ d_2 A_2 e^{-\lambda_2 z}+\ d_3 A_3 e^{-\lambda_3 z}, \tag{24}$$

$$\tilde{\phi}=\ l_1 A_1 e^{-\lambda_1 z}+\ l_2 A_2 e^{-\lambda_2 z}+l_3 A_3 e^{-\lambda_3 z}\ \ , \tag{25}$$

Also the coupling constants d_i and l_i are given by $d_i=\frac{\lambda_i^4\ A^*+\lambda_i^2 B^*+C^*}{\lambda_i^4 A'+\ \lambda_i^2 B'+C'}$, $i=1,2,3$, $l_i=\frac{\lambda_i^4 P'+\lambda_i^2 Q'+R'}{\lambda_i^4 A'+\lambda_i^2 B'+C'}$, $i=1,2,3$, where

$$A'=\ \{\tau_t'(\ \delta_3\ \epsilon_2\mathrm{s})\ +\tau_v'(\ \delta_3\epsilon_4)+\ \ \tau_q'(\ \delta_3 p_6+\ p_6\varepsilon^2\varepsilon_5)\},$$

$$B' = \{ \tau_t' \ (\delta_3 p_3 - \epsilon_2 s\, p_2) + \tau_v' \ (\delta_3 p_4 - p_2 \ \epsilon_4) + \tau_q' (\delta_3 p_5 - p_2 p_6 + p_5 \varepsilon^2 \varepsilon_5)\},$$
$$C' = \{ \tau_t' \ (- p_3 p_2) + \tau_v' \ (-p_4 p_2) + \tau_q' \ (- p_5 p_2) \},$$
$$A^* = \{\tau_t'(\delta_1 \ \epsilon_2 s) + \tau_v'(\delta_1 \ \epsilon_4) + \tau_q'(\delta_1 p_6)\},$$
$$B^* = \{ \tau_t' \ (\delta_1 p_3 - p_1 s \epsilon_2) + \tau_v'(\delta_1 p_4 - p_1 \ \epsilon_4) + \tau_q' \ (\delta_1 p_5 - p_1 p_6 - \xi^2 s^2 \varepsilon_5 p_{10})\},$$
$$C^* = \{\tau_t' \ (-p_1 p_3) + \tau_v'(- p_1 p_4) + \tau_q'(- p_5 p_1 + \xi^2 s^2 \varepsilon_5 p_8)\},$$
$$P' = \{ \ \delta_1 \delta_3 \}, \ Q' = \{-p_1 \delta_3 - p_2 \delta_1 - p_7^2\}, \ R' = \{ p_1 p_2 \}.$$

4. Boundary conditions

Following Lata and Zakhmi (2019), the boundary conditions are taken as

$$\sigma_{zz} = - F_1 \psi_1 (x)\, \delta\, (t), \tag{26}$$

$$\sigma_{xz} = 0, \tag{27}$$

$$\frac{\partial \phi}{\partial z} = F_2 \psi_2(x)\, \delta\, (t) \ at \ z = 0, \tag{28}$$

where F_1 is the magnitude of force applied, F_2 is the constant temperature applied on the boundary, $\psi_1 (x)$ and $\psi_2 (x)$ is the source distribution function along x -axis.

With the help of Laplace and Fourier transform defined by Eqs. (17)-(18) on the boundary conditions (26)-(28) and with the help of Eqs. (1), (9)-(13), (16) and (23)-(25), we obtain the displacement components, stress components and conductive temperature as follows

$$\begin{aligned} \tilde{u} = & -\frac{F_1 \widetilde{\psi}_1 (\xi)}{\Delta} \left(\Delta_1 e^{-\lambda_1 z} + \Delta_2 e^{-\lambda_2 z} + \Delta_3 e^{-\lambda_3 z} \right) \\ & + \frac{F_2 \widetilde{\psi}_2 (\xi)}{\Delta} \left(\Delta_1^* e^{-\lambda_1 z} + \Delta_2^* e^{-\lambda_2 z} + \Delta_3^* e^{-\lambda_3 z} \right), \end{aligned} \tag{29}$$

$$\begin{aligned} \widetilde{w} = & -\frac{F_1 \widetilde{\psi}_1 (\xi)}{\Delta} \left(\mathrm{d}_1 \Delta_1 e^{-\lambda_1 z} + \mathrm{d}_2 \Delta_2 e^{-\lambda_2 z} + \mathrm{d}_3 \Delta_3 e^{-\lambda_3 z} \right) \\ & + \frac{F_2 \widetilde{\psi}_2 (\xi)}{\Delta} \left(\mathrm{d}_1 \Delta_1^* e^{-\lambda_1 z} + \mathrm{d}_2 \Delta_2^* e^{-\lambda_2 z} + \mathrm{d}_3 \Delta_3^* e^{-\lambda_3 z} \right), \end{aligned} \tag{30}$$

$$\begin{aligned} \tilde{\phi} = & -\frac{F_1 \widetilde{\psi}_1 (\xi)}{\Delta} \left(\mathrm{l}_1 \Delta_1 e^{-\lambda_1 z} + \mathrm{l}_2 \Delta_2 e^{-\lambda_2 z} + \mathrm{l}_3 \Delta_3 e^{-\lambda_3 z} \right) \\ & + \frac{F_2 \widetilde{\psi}_2 (\xi)}{\Delta} \left(\mathrm{l}_1 \Delta_1^* e^{-\lambda_1 z} + \mathrm{l}_2 \Delta_2^* e^{-\lambda_2 z} + \mathrm{l}_3 \Delta_3^* e^{-\lambda_3 z} \right), \end{aligned} \tag{31}$$

$$\begin{aligned} \widetilde{\sigma_{33}} = & -\frac{F_1 \widetilde{\psi}_1 (\xi)}{\Delta} \left(\Delta_{11} \Delta_1 e^{-\lambda_1 z} + \Delta_{12} \Delta_2 e^{-\lambda_2 z} + \Delta_{13} \Delta_3 e^{-\lambda_3 z} \right) \\ & + \frac{F_2 \widetilde{\psi}_2 (\xi)}{\Delta} \left(\Delta_{11} \Delta_1^* e^{-\lambda_1 z} + \Delta_{12} \Delta_2^* e^{-\lambda_2 z} + \Delta_{13} \Delta_3^* e^{-\lambda_3 z} \right), \end{aligned} \tag{32}$$

$$\begin{aligned} \widetilde{\sigma_{13}} = & -\frac{F_1 \widetilde{\psi}_1 (\xi)}{\Delta} \left(\Delta_{21} \Delta_1 e^{-\lambda_1 z} + \Delta_{22} \Delta_2 e^{-\lambda_2 z} + \Delta_{23} \Delta_3 e^{-\lambda_3 z} \right) \\ & + \frac{F_2 \widetilde{\psi}_2 (\xi)}{\Delta} \left(\Delta_{21} \Delta_1^* e^{-\lambda_1 z} + \Delta_{22} \Delta_2^* e^{-\lambda_2 z} + \Delta_{23} \Delta_3^* e^{-\lambda_3 z} \right), \end{aligned} \tag{33}$$

where

$$\Delta = \Delta_{11}(\Delta_{22}\Delta_{33} - \Delta_{32}\Delta_{23}) - \Delta_{12}(\Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{31}) + \Delta_{13}(\Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31}),$$

$$\Delta_1 = (\Delta_{33}\Delta_{22} - \Delta_{32}\Delta_{23}),\ \Delta_2 = -(\Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{31}),\ \Delta_3 = (\Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31}),$$

$$\Delta_1^* = (\Delta_{12}\Delta_{23} - \Delta_{13}\Delta_{22}),\ \Delta_2^* = (\Delta_{13}\Delta_{21} - \Delta_{11}\Delta_{23}),\ \Delta_3^* = (\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}),$$

$$\Delta_{1j} = \frac{C_{13}\ i\xi}{\rho c_1^2} - \frac{C_{33} d_j \lambda_j}{\rho c_1^2} - \varepsilon l_j\left(1 + \frac{a_1}{L}\xi^2 - \frac{a_3}{L}\lambda_j^2\right), j = 1,2,3$$

$$\Delta_{2j} = \frac{C_{55}}{\rho c_1^2}(-\lambda_j + i\xi d_j),\quad j = 1,2,3$$

$$\Delta_{3j} = \frac{-T_0}{L}(l_j\lambda_j).\qquad j = 1,2,3$$

4.1 Mechanical force on the surface of half –space

Taking $F_2 = 0$ in Eqs. (29)- (33), we obtain the components of tangential stress, normal stress, displacement and conductive temperature due to mechanical force.

4.2 Thermal source on the surface of half-space

Taking $F_1 = 0$ in Eqs. (29)- (33), we obtain the components of tangential stress, normal stress, displacement and conductive temperature due to thermal source.

5. Applications

5.1 Concentrated force

The solution due to concentrated normal force is obtained by setting

$$\psi_1(x) = \delta(x),\ \psi_2(x) = \delta(x), \tag{34}$$

In Eqs. (26) and (28), where $\delta(x)$ is the Dirac delta function. By applying Laplace and Fourier transformations defined in Eqs. (17)-(18) on (34), we get

$$\widehat{\psi_1}(\xi) = 1,\ \widehat{\psi_2}(\xi) = 1, \tag{35}$$

Using Eq. (35) in Eqs. (29)-(33), we obtain the components of stress, displacements and conductive temperature.

5.2 Uniformly distributed force

The solution due to uniformly distributed force is obtained by setting

$$\{\psi_1(x),\ \psi_2(x)\} = \begin{Bmatrix} 1\ if\ |x| \le m \\ 0\ if\ |x| > m \end{Bmatrix}, \tag{36}$$

In Eqs. (26) and (28). The Laplace and Fourier transforms of $\psi_1(x)$and $\psi_2(x)$with respect to the pair (x,ξ) in case of uniformly distributed load of non-dimensional width 2 m applied at origin of co-ordinate system $x = z = 0$ is given by

$$\{ \widehat{\psi_1}(\xi),\ \widehat{\psi_2}(\xi) \} = [2\,sin(\xi m)/\,\xi]\ ,\xi\ \neq 0. \tag{37}$$

Using Eq. (37) in Eqs. (29)-(33), we get the stress components, displacement components and conductive temperature.

6. Inversion of transformation

To obtain the solution of the problem in physical domain, we must invert the transformations in Eqs (29)-(33). Here, the displacement components, tangential and normal stresses and conductive temperature are functions of z, the parameters of Laplace and Fourier transforms s and ξ respectively and are of the form f (ξ, z, s). To obtain the function f (x, z, t) in the physical domain, we first invert the Fourier transform using

$$\bar{f}(x, z, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\xi x_1}\hat{f}\ (\xi, z, s)\,d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty}|\cos(\xi x)\,f_e - isin(\xi x)f_0|\ d\xi, \tag{38}$$

where f_0 and f_e are respectively the odd and even parts of $\hat{f}(\xi$, z, s). Thus the expression Eq. (38) gives the Laplace transform $\bar{f}(x, z, s)$of the function f(x, z, t) Following Honig and Hirdes (1984), the Laplace transform function $\bar{f}(x, z, s)$ can be inverted to f (x, z, t).The last step is to calculate the integral in Eq. (38). The method of evaluating this integral is described in Press *et al.* (1986), It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Numerical results and discussion

For numerical computations, we take the following values of the relevant parameter for an orthotropic thermoelastic material (Biswas *et al.* 2017). (Table 1)

By using above values of parameters, the numerical simulated results are presented graphically by using octave software to show the effect of two-temperature on stress components, displacement components and conductive temperature 'ϕ' with distance x for an orthotropic thermoelastic body by using different values of fractional order parameter α = 0,0.8 and two temperature parameter a_1= 0.05, $a_3 = 0.08$ respectively.

7.1 Effect of two-temperature

(1) The red solid line with centre symbol triangle (Δ) holds for an orthotropic material with two-temperature and without fractional parameter effect i.e., $a_1 = 0.05,\ a_3 = 0.08,\ \alpha = 0$

(2) The green solid line with centre symbol plus (□) holds for an orthotropic material with two-temperature and fractional parameter effect i.e., $a_1 = 0.05, a_3 = 0.08, \alpha = 0.8$

(3) The blue solid line with centre symbol circle (◊) holds for an orthotropic material without two-temperature and without fractional parameter effect i.e., $a_1 = a_3 = 0,\ \alpha = 0$

(4) The black solid line with centre symbol circle (o) holds for an orthotropic material without two-temperature and with fractional parameter effect i.e., $a_1 = a_3 = 0,\ \alpha = 0.8$

Table 1

Quantity	Value	Unit
c_{11}	18.78×10^{10}	$\text{Kg}m^{-1}s^{-2}$
c_{13}	8.0×10^{10}	$\text{Kg}m^{-1}s^{-2}$
c_{33}	10.2×10^{10}	$\text{Kg}m^{-1}s^{-2}$
c_{55}	10.06×10^{10}	$\text{Kg}m^{-1}s^{-2}$
c_E	4.27×10^{2}	$\text{JK}g^{-1}K^{-1}$
β_1	1.96×10^{-5}	$\text{N}m^{2}K^{-1}$
β_3	1.4×10^{-5}	$\text{N}m^{2}K^{-1}$
T_0	293	K
K_1	0.12×10^{3}	$\text{W}m^{-1}K^{-1}$
K_3	0.33×10^{3}	$\text{W}m^{-1}K^{-1}$
K_1^*	1.313×10^{2}	$\text{W}s^{-1}$
K_3^*	1.54×10^{2}	$\text{W}s^{-1}$
ρ	8.836×10^{3}	Kgm^{-3}
τ_t	1×10^{-7}	S
τ_v	1.5×10^{-8}	S
τ_q	2.0×10^{-7}	S

8. Particular cases

(1) If we put $K_1 = K_3 = 0$ in Eqs. (29)-(33), we get the resulting expressions for an orthotropic thermoelastic solid without energy dissipation and with two-temperature with TPL (three-phase-lag) fractional order model of heat transfer.

(2) If $a_1 = a_3 = 0$, in Eqs. (29)-(33), we get the resulting expressions for an orthotropic thermoelastic solid with and without energy dissipation with TPL fractional order theory of generalized thermoelasticity without two-temperature.

(3) The problem reduces for the case GN-I type fractional order dual-phase-lag model with two-temperature if we put $K_1^* = K_3^* = 0$ in Eqs. (29)-(33).

(4) If $C_{11} = C_{33,}\ 2C_{55} = C_{11} - C_{33}$, we get the expressions for displacement components, stress components and conductive temperature in transversely isotropic thermoelastic medium with three-phase-lag fractional order heat conduction model and two-temperature in generalized thermoelasticity.

(5) If $C_{11} = C_{33} = \lambda + 2\mu, C_{13} = \lambda, C_{55} = \mu,\ \beta_1 = \beta_3 = \beta, K_1 = K_3 = K, K_1^* = K_3^* = K^*$, we get the expressions for isotropic thermoelastic solid with TPL fractional order theory in generalized thermoelasticity with two-temperature.

(6) The present problem reduces for the case GN-III type model of thermoelasticity with two temperature, If we put $\tau_t = \tau_v = \tau_q = 0$ in Eqs. (29)-(33).

(7) If we take α =1 and $\tau_t \neq 0$, $\tau_v \neq 0$, $\tau_q \neq 0$, then the problem reduces for an orthotropic thermoelastic solid with TPL model of heat conduction in generalized thermoelasticity with two-temperature.

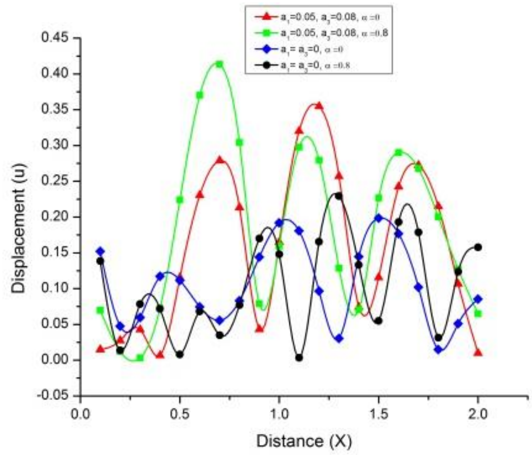

Fig. 1 variation of tangential displacement u with distance x (concentrated mechanical force)

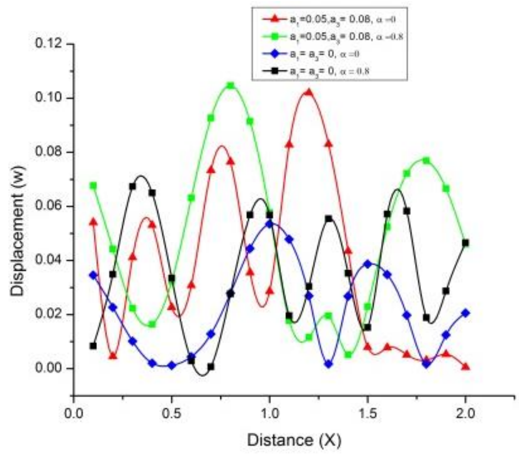

Fig. 2 variation of normal displacement w with distance x (concentrated mechanical force)

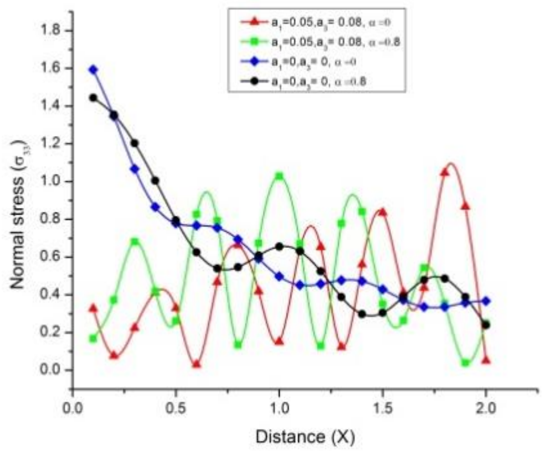

Fig. 3 variation of normal stress σ_{33} with distance x (concentrated mechanical force)

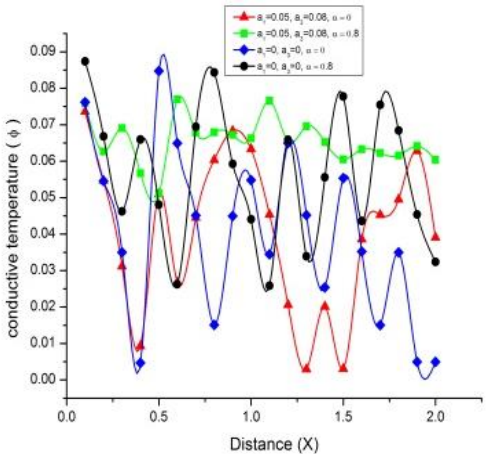

Fig. 4 variation of conductive temperature ϕ with distance x (concentrated mechanical force)

9. Mechanical forces on the surface of half space

9.1 Concentrated mechanical force

Figs. 1 and 2 shows the variation of tangential and normal displacements with distance x with and without two temperature corresponding to two different values of fractional parameter α= 0, 0.8 respectively. We noticed that the pair of curves with two-temperature (and the pair of curves without two-temperature) follows an oscillatory pattern with difference in magnitude of oscillations in the whole range of distance. With two-temperature amplitude of oscillations are high as compared to without two-temperature for both with and without fractional effect. Fig. 2 depicts the behavior of normal displacement with distance x. Here, in all the four cases trends are similar (i.e., oscillatory) with little difference in their magnitude of oscillations. Figs. 3 and 4 describe the behaviour of normal stress σ_{33} and conductive temperature ϕ with distance x. It can be noticed that for the case with two-temperature near the boundary surface the value of normal stress decreases for α= 0 whereas it increases for α = 0.8 after that it exhibits oscillatory behaviour in the remaining range with increasing value of distance. However, in the case without two-temperature it reduces from maximum to minimum value with small oscillations corresponding to α = 0, 0.8 respectively. The nature of conductive temperature ϕ with distance x shows an oscillatory behaviour in all the four cases with and without two-temperature corresponding to α = 0 0.8 respectively with increasing value of distance x.

9.2 Linearly distributed mechanical force

In linearly distributed mechanical force, Figs. 6-10 as in case of concentrated force shows the variation of tangential displacement, normal displacement, normal stress, conductive temperature and tangential stress with distance x (with and without two-temperature) corresponding to α = 0, 0.8 respectively. In Figs. 6 and 7, we noticed that the variation of all the four curves with distance x (in the both cases with and without two-temperature) for both values of fractional parameter exhibits the same trends (i.e., oscillatory) with change in their amplitude of oscillations. Fig. 8 depicts the

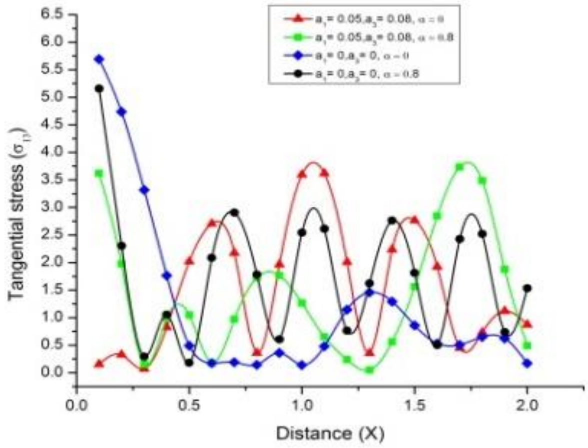

Fig. 5 variation of tangential stress σ_{31} with distance x (concentrated mechanical force)

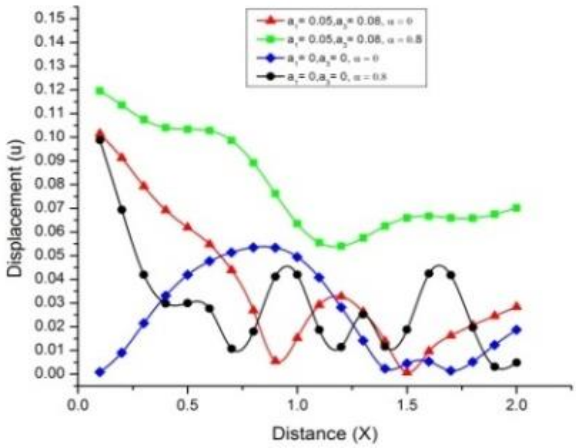

Fig. 6 Variation of tangential displacement u with distance x (linearly distributed mechanical force**)**

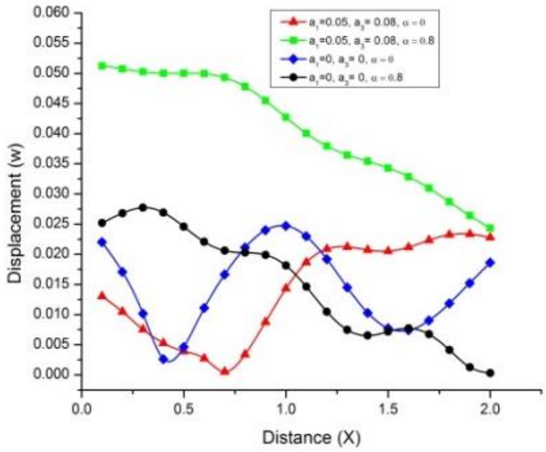

Fig. 7 Variation of normal displacement w with distance x (linearly distributed mechanical force)

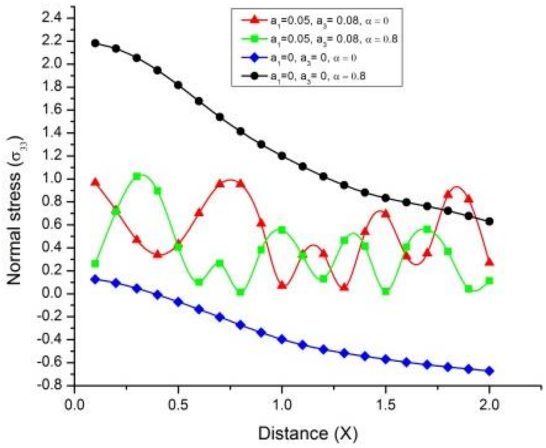

Fig. 8 Variation of normal stress σ_{33} withdistance x (linearly distributed mechanical force)

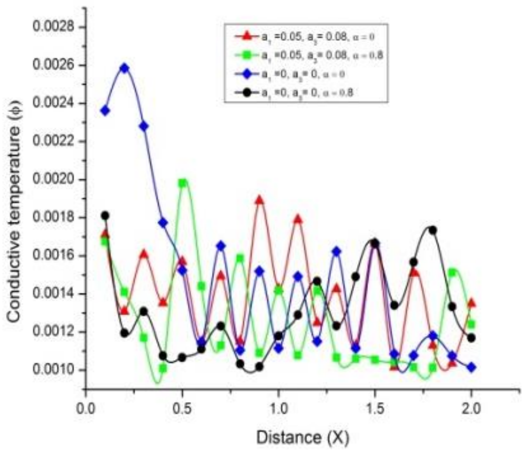

Fig. 9 Variation of conductive temperature ϕ with distance x (linearly distributed mechanical force)

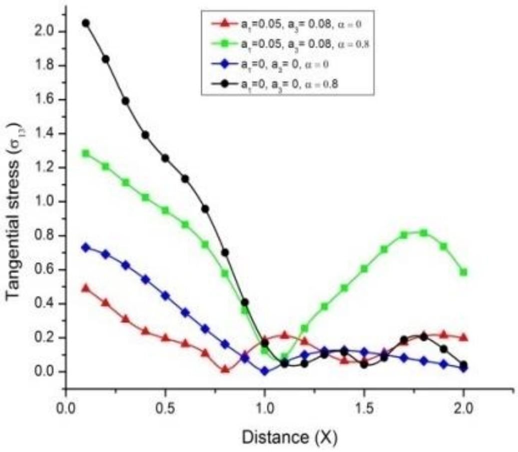

Fig. 10 Variation of tangential stress σ_{31} with distance x (linearly distributed mechanical force)

variation of normal stress. We see that for the case with two-temperature near the boundary surface the value of normal stress decreases for $\alpha = 0$ whereas it increases for $\alpha = 0.8$ and then follows an oscillatory trends in the rest of the range with increasing value of distance. While in the case without two-temperature there is a slow decrease from high to low value in the overall range of distance corresponding to $\alpha = 0$, 0.8 respectively. Fig. 9 illustrates the variation of conductive temperature. It can be seen that all the four curves shows same behaviour i.e., oscillatory with increasing value of distance x. Also in fig 10 variation of tangential stress σ_{31} with distance $x$has shown. We see that in the initial range $0 < x < 1$, the value of tangential stress decreases sharply near the loading surface with and without two-temperature for both values of α and in the remaining range its value increases sharply for the case with $\alpha = 0.8$ and with two-temperature. While for the remaining three cases with $\alpha = 0$ (for both with and without two-temperature) and for the case $\alpha = 0.8$ (without two-temperature) its value oscillates slowly and all the curves intersect each other when the distance approaches to its maximum value.

10. Deformation due to thermal source

10.1 Concentrated thermal source

Figs. (11)-(15) give the characteristics for concentrated thermal source. It is depicted from the graphs that the distribution curves for both the displacement components (u, w) normal stress (σ_{33}), conductive temperature(ϕ) and tangential stress (σ_{13}) for concentrated thermal source follow the same pattern as in case of concentrated mechanical force i.e., all the distributions curves are oscillatory in nature with difference in their amplitude of oscillations for all the four cases with and without two-temperature and with α = 0, 0.8 respectively.

10.2 Linearly distributed thermal source

As in all the above cases, here in concentrated thermal source Figs. 16 and 17 depict the variation of tangential and normal displacements with distance x with and without two-temperature and with

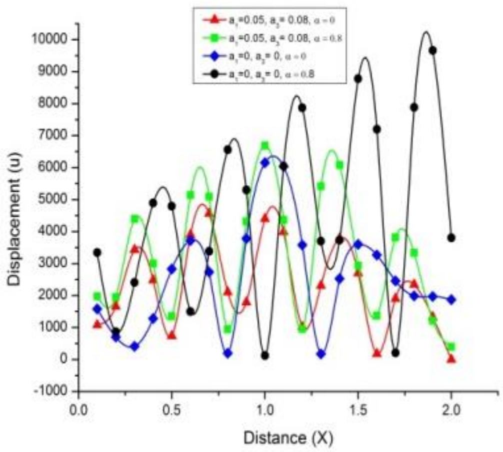

Fig. 11 Variation of tangential displacement u with distance x (concentrated thermal source)

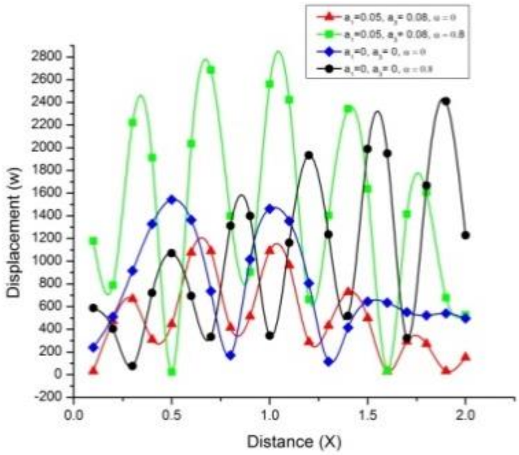

Fig. 12 Variation of normal displacement w with distance x (concentrated thermal source)

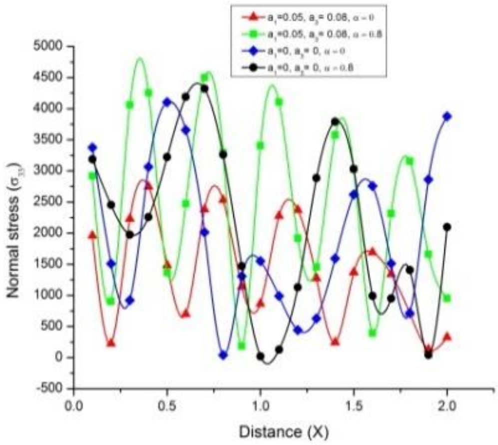

Fig. 13 Variation of normal stress σ_{33} with distance x (concentrated thermal Source)

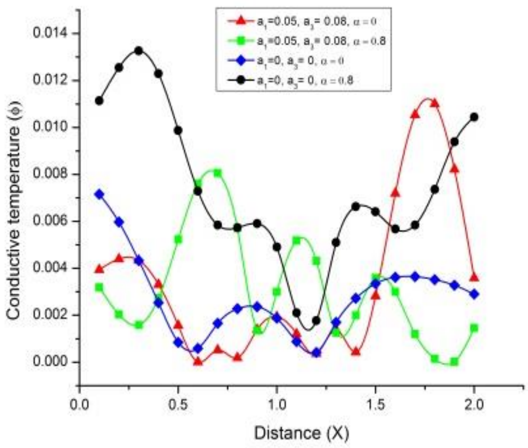

Fig. 14 Variation of conductive temperature ϕ with distance x (concentrated thermal source)

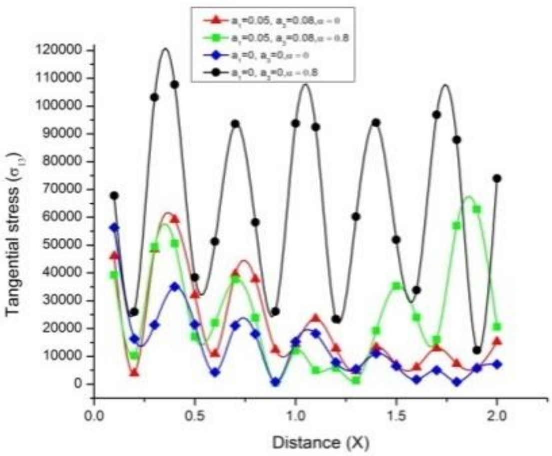

Fig. 15 Variation of tangential stress σ_{31} with distance x (concentrated thermal source)

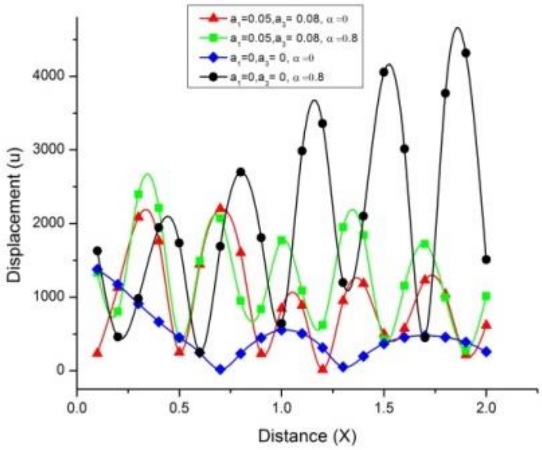

Fig. 16 Variation of tangential displacement u with distance x (uniformly distributed thermal force)

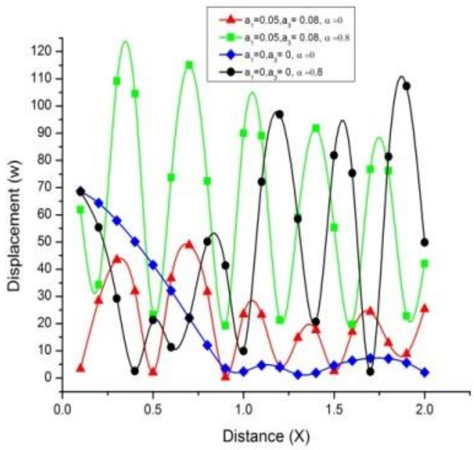

Fig. 17 Variation of normal displacement w with distance x (uniformly distributed thermal force)

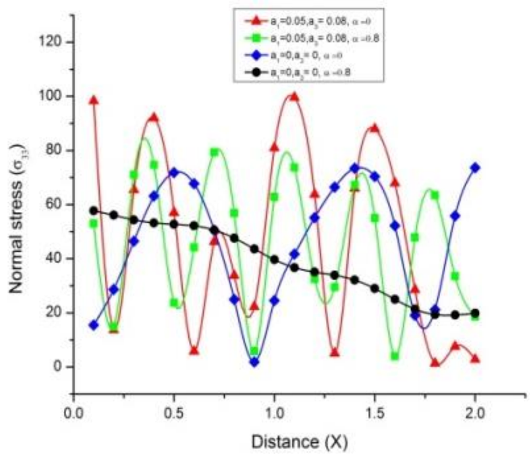

Fig. 18 Variation of normal stress σ_{33} with distance x (uniformly distributed thermal source)

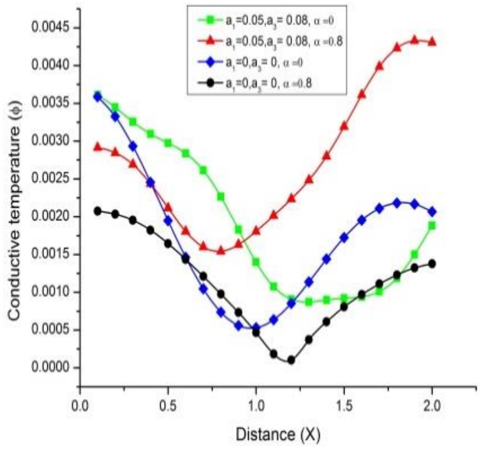

Fig. 19 Variation of conductive temperature ϕ with distance x (uniformly distributed thermal source)

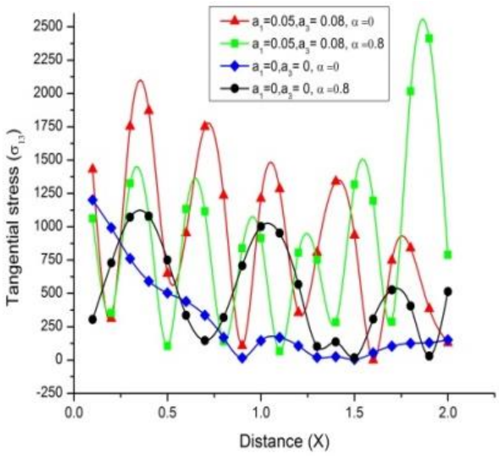

Fig. 20 Variation of tangential stress σ_{33} with distance x (uniformly distributed thermal source)

$\alpha = 0$, 0.8 respectively. We observed that the value of displacement u for the case $\alpha = 0$ and with two-temperature increases first near the boundary surface then decreases and shows an oscillatory behaviour in the whole range of distance x. However for the case $\alpha = 0.8$ and with two-temperature its variation is just opposite i.e., decreases near the boundary surface then increases and follow an oscillatory pattern in the rest of the range after that there comes a point when both the curves meet each other. Without two-temperature and for $\alpha = 0$, 0.8 its value oscillates with different magnitudes in the whole range of distance x. From Fig. 17, we observed that the behaviour of normal displacement is same (i.e., oscillatory) for all the four cases. Figs. 18-20 show the variation of normal stress, conductive temperature and tangential stress with distance x. It can be seen that for both the cases with and without two-temperature corresponding to $\alpha = 0$, 0.8 the distribution curves are oscillatory in nature with different magnitudes.

11. Conclusions

From the above discussion, we concluded that the change in the value of two-temperature with different values of fractional parameter has great impact on all the field components when the both thermal sources and mechanical forces are applied to an orthotropic thermoelastic body. It can be noticed that the distribution curves for all the physical quantities follow an oscillatory pattern with different magnitude and amplitude of oscillations. In other words, we can say that two-temperature tends to move in oscillatory manner. The problem is theoretical but it is useful as an improvement in the field of generalized with fractional order heat conduction equation. According to this theory, fractional order thermoelasticity has significant importance in dynamical problems of solid mechanics and structural mechanics as in most of the practical problems of physical processes differential equations with fractional order are used to obtain the solution.

References

Abbas, I.A., EL-Amin, M.F. and Salama, A. (2009), “Effect of thermal dispersion on free convection in a fluid saturated porous medium”, *Int. J. Heat Fluid Fl.*, **30**(2), 229-236. http://doi.org/10.1016/j.ijheatfluidflow.2009.01.004.

Abbas, I.A. and Kumar, R. (2014), “Deformation due to thermal source in micropolar generalized thermoelastic half-space by finite element method”, *J. Computat. Theor. Nanosci.*, **11**(1), 185-190. http://doi.org/10.1166/jctn.2014.3335.

Abbas, I.A. and Marin, M. (2017a), “Analytical solution of thermoelastic interaction in a half-space by pulsed laser heating”, *Physica E*, **87**, 254-260. http://doi.org/10.2016/j.physe.2016.10.048

Abbas, I.A. and Marin, M. (2017b), “Analytical solution of a two-dimensional generalized thermoelastic diffusion problem due to laser pulse”, *Iran. J. Sci. Technol.*, **42**(3), 57-71. http://doi.org/10.1007/s40097-017-0077-1.

Abbas, I.A. (2018), “A study on fractional order theory in thermoelastic half-space under thermal loading” *Phys. Mesomech.*, **21**(2), 150-156. https://doi.org /10.1134/S102995991802008X.

Abouelregal, A.E. (2018), “The effect of temperature-dependent physical properties and fractional thermoelasticity on non-local nano beams, *J. Math. Theor. Phys.*, **1**(2), 49-58. http://doi.org/10.15406/oajmtp.2018.01.00009.

Alzahrani, F.S. and Abbas, I.A. (2020), “Photo-thermal interactions in a semiconducting media with a spherical cavity under hyperbolic two-temperature model”, *Mathematics*, **8**(585), 1-11. http://doi.org/10.3390/math8040585.

Allam, O., Draiche, K., Bousahla, A.A., Bourada, F., Tounsi A., Benrahou, K.H., Mahmoud S.R., Adda Bedia E.A. and Tounsi A., (2020), "A generalized 4-unknown refined theory for bending and free vibration analysis of laminated composite and sandwich plates and shells", *Comput. Concrete*, **26**(2), 185-201. http://doi.org/10.12989/cac.2020.26.2.185.

Bakoura, A., Bourada, F., Bousahla, A., Tounsi A., Benrahou, K.H., Tounsi, A., Al-Zahrani, M.M. and Mahmoud, S.R., (2021), "Buckling analysis of functionally graded plates using HSDT in conjunction with the stress function method", *Comput. Concrete*, **27**(1), 73-83. http://doi.org/10/12989/cac.2021.27.1.073.

Bassiouny, E. (2021), "Mathematical model for hyperbolic two temperature fractional-order thermoelastic materials subjected to thermal loading", *Appl. Math. Inform. Sci.*, **15**(1), 23-29. http://doi.org/10.18576.amis/150104.

Bassiouny, E. and Sabry, R. (2013), "Fractional order two temperature thermoelastic behaviour of piezoelectric materials", *J. Appl. Math. Phys.*, **1**, 110-120. http://doi.org/10.4236/jamp.2013.15017.

Bekkaye, T.H.L., Fahsi, B., Bousahla, A.A., Bourada, F., Tounsi A., Benrahou, K.H., Tounsi, A. and Al-Zahrani, M.M., (2021), "Porosity-dependent mechanical behaviors of FG plate using refined trigonometric shear deformation theory", *Comput. Concrete*, **26**(5), 439-450. http://doi.org/10/12989/cac.2021.26.5.439.

Bendenia, N., Zidour, M., Bousahla, A.A., Bourada, F., Tounsi A., Benrahou, K.H., Adda Bedia, E.A., Mahmoud, S.R. and Tounsi, A. (2020), "Deflections, stresses and free vibration studies of FG-CNT reinforced sandwich plates resting on Pasternak elastic foundation", *Comput. Concrete*, **26**(3), 213-226. http://doi.org/10.12989/cac.2020.26.3.213.

Bhattacharya, D. and Kanoria, M. (2014), "The influence of two-temperature fractional order generalized thermoelastic diffusion inside a spherical shell", *Int. J. Appl. Innov. Eng. Manag.*, **3**(8), 96-108.

Chen, P.J. and Gurtin, E.M. (1968), "On a theory of heat conduction involving two temperatures", *Zeitschrift für Angewandte Mathematik und Physik*, **19**(4), 614-627. https://doi.org/10.1007/BF01594969.

Chen, P.J., Gurtin, M.E. and Williams, W.O. (1968), "A note on non-simple heat conduction", *Zeitschrift für Angewandte Mathematik und Physik ZAMP*, **19**(4), 969-970.

Chen, P.J., Gurtin, E.M. and Williams, O.W. (1969), "On the thermodynamics of non-simple elastic materials with two temperatures", *Zeitschrift für angewandte Mathematik und Physik*, **20**(1), 107-112.

E-L Naggar, A.M., Kishka, Z., Abd-Alla, A.M., Abbas, I.A., Abo-Dahab, S.M. and Elsagheer, M. (2013), "On the initial stress, magnetic field, voids and rotation effects on plane waves in generalized thermoelasticity", *J. Computat. Theor. Nanosci.*, **10**(6), 1408-1417. http://doi.org/10.1166/jctn.2013.2862.

Ezzat, M.A., EL- Karamany, A.S. and EL-Bary, A.A. (2014), "Magneto-thermoelasticity with two fractional order heat transfer", *J. Assoc. Arab Univ. Basic Appl. Sci.*, **19**, 70-79. http://doi.org/10.1016/j.jau.bas.2014.06.009.

Guellil, M., Saidi, H., Bourada, F., Bousahla, A.A., Tounsi, A., Al-Zahrani, M.M., Hussain, M. and Mahmoud, S.R. (2021), "Influences of porosity distributions and boundary conditions on mechanical bending response of functionally graded plates resting on Pasternak foundation", *Steel Compos. Struct.*, **38**(1), 1-15. http://doi.org/10.12989/scs.2021.38.1.001.

Hachemi, H., Bousahla, A.A., Kaci, A., Bourada, F., Tounsi, A., Benrahou, K.H., Tounsi, A., Al-Zahrani, M.M. and Mahmoud, S.R., (2021), "Bending analysis of functionally graded plates using a new refined quasi-3D shear deformation theory and the concept of the neutral surface position", *Steel Compos. Struct.*, **39**(1), 51-64. http://doi.org/10.12989/scs.2021.39.1.051.

Khamis, A.K., El-Bary, A.A., Youssef, H.M. and Bakali, A. (2020), "Generalized thermoelasticity with fractional order strain of infinite medium with a cylindrical cavity", *Int. J. Adv. Appl. Sci.*, **7**(7), 102-108. http://doi.org/10.21833/ijaas.2020.07.013.

Kumar, R. and Chawla, V. (2014), "General solution and fundamental solution for two-dimensional problem in orthotropic thermoelastic media with voids", *Theor. Appl. Mech.*, **41**(4), 247-265. http://doi.org/10.2298/TAM1404247.

Kumar, R., Devi, S. and Sharma, V. (2017), "Effects of hall current and rotation in modified couple stress generalized thermoelastic half space due to ramp-type heating", *J. Solid Mech.*, **9**(3), 527-542.

Lata, P. and Himanshi, H. (2021a), "Orthotropic magneto-thermoelastic solid with higher order dual-phase-lag model in frequency domain", *Struct. Eng. Mech.*, **77**(3), 315-327.

http://doi.org/10.12989/sem.2021.77.3.315.
Lata, P. and Himanshi, H. (2021b), "Stoneley wave propagation in an orthotropic thermoelastic media with fractional order theory", *Compos. Mater. Eng.*, **3**(1), 57-70. http://doi.org/10.12989/cme.2021.3.1.057.
Lata, P. and Himanshi, H. (2021c), "Orthotropic magneto-thermoelastic solid with multi-dual-phase-lag model and hall current", *Coupled Syst. Mech.*, **10**(2), 103-121. http://doi.org/10.12989/csm.2021.10.2.103.
Lata, P. and Kaur, I. (2019), "Thermomechanical interactions due to time harmonic sources in a transversely isotropic magneto thermoelastic solids with rotation", *Int. J. Microstruct. Mater. Properties*, **14**(6), 549-577. http://doi.org/10.1504/IJMMP.2019.103190.
Marin, M. (1999), "An evolutionary equation in thermoelasticity of dipolar bodies", *J. Math. Phys.*, **40**(3), 1391-1399. http://doi.org/10.1063/1.532809.
Marin, M., Agarwal, R.P. and Mahmoud, S.R. (2013), "Modeling a microstretch thermoelastic body with two-temperatures", *Abstract Appl. Anal.*, 583464. http://doi.org/10.1155/2013/583464.
Marin, M., Vlase, S. and Paun, M. (2015a), "Considerations on double porosity structure for micropolar bodies", *AIP Adv.*, **5**(3), 037113. http://doi.org/10.1063/1.4914912.
Marin, M., Othman, M.I.A. and Abbas, I.A. (2015b), "An extension of the domain of influence theorem for generalized thermoelasticity of anisotropic material with voids", *J. Computat. Theor. Nanosci.*, **12**(8), 1594-1598. http://doi.org/10/1166/jctn.2015.3934.
Merazka, B., Bouhadra, A., Menasria, A., Selim, M. M., Bousahla, A.A., Bourada, F., Tounsi, A., Benrahou, K. H., Tounsi, A. and Al-Zahrani, M. M., (2021), "Hygro-thermo-mechanical bending response of FG plates resting on elastic foundations", *Steel Compos. Struct.*, **39**(5), 631-643. http://doi.org/10.12989/scs.2021.39.5.631.
Molla, M.A.K., Mondal, N. and Mallik, S.H. (2019), "Effects of fractional and two-temperature parameters on stress distributions for an unbounded generalized thermoelastic medium with spherical cavity", *Arab. J. Basic Appl. Sci.*, **26**(1), 302-310. http://doi.org/10.1080/25765299.2019.1621511.
Mondal, S., Maliik, S.H. and Kanoria, M. (2014), "Fractional order two-temperature dual-phase-lag thermoelasticity with variable thermal conductivity", *Int. Scholar. Res. Notice.*, 1-13. http://doi.org/10.1155/2014/646049.
Mudhaffar, M.I., Tounsi, A., Chikh, A., AL-Osta, M.A., Alzahrani, M.M. and Al-Dulaijan, S.U. (2021), "Hygro-thermo-mechanical bending behavior of advanced functionally graded ceramic metal plate resting on a viscoelastic foundation", *Structures*, **33**, 2177-2189. https://doi.org/10.1016/j.istruc.2021.05.090.
Oldham, K.B. and Spainer, J. (1974), *The Fractional Calculus*, Academic Press, New York, U.S.A.
Palani, G. and Abbas, I. A. (2009), "Free convection MHD flow with thermal radiation from an impulsively-started vertical plate", *Nonlinear Anal. Modell. Control*, **14**(1), 73-84. http://doi.org/10.15388/NA.2009.14.1.14531.
Saeed, T., Abbas, I.A. and Marin, M. (2020), "A gl model on thermo-elastic interactions in a poroelastic material using finite element method", *Symmetry*, **12**(3), 488. http://doi.org/10.3390/sym12030488.
Sur, A. and Kanoria, M. (2012), "Fractional order two-temperature thermoelasticity with finite wave speed", *Acta Mechanica*, **223**, 2685-2701. http://doi.org/10.1007/s00707-012-0736-7.
Tahir, S.I., Chikh, A., Tounsi, A., AL-Osta, M.A., Al-Dulaijan, S.U. and Alzahrani, M.M., (2021), "Wave propagation analysis of a ceramic-metal functionally graded sandwich plate with different porosity distributions in a hygro-thermal environment", *Compos. Struct.*, **269**, 114030. https://doi.org/10.1016/j.compstruct.2021.114030.
Press, W.H., Teukolsky, S.A., Vetterling, W.T. and Flannery, B.P. (1986), *Numerical Recipes in Fortran*, Cambridge University Press, New York, U.S.A.
Youssef, H.M. (2006), "Theory of two temperature generalized thermoelasticity", *IMA J. Appl. Math.*, **71**(3), 383-390. https://doi.org/10.1093/imamat/hxh101.
Yadav, R., Kumar, K. and Deswal, S. (2015), "Two temperature thermal viscoelasticity with fractional order strain subjected to moving heat source", *J. Math.*, 487513. http://doi.org/10.1155/2015/487513.
Zenkour, A.M. and Abouelregal, A.E. (2015), "The fractional effects of a two-temperature generalized thermoelastic semi-infinite solid induced by pulsed laser heating", *Arch. Mech.*, **67**(1), 53-73.
Zenkour, A.M. (2020), "Magneto-thermal shock for a fiber-reinforced anisotropic half-space studied with a

refined multi-dual-phase-lag model", *J. Phys. Chem. Solids*, **137**, 109213. http://doi.org/10.1016/j.jpcs.2019.109213.

Zerrouki, R., Karas, A., Zidour, M., Bousahla, A.A., Tounsi, A., Bourada, F., Tounsi, A., Benrahou, K.H. and Mahmoud, S.R., (2021), "Effect of nonlinear FG-CNT distribution on mechanical properties of functionally graded nano-composite beam", *Struct. Eng. Mech.*, **78**(2), 117-124. http://doi.org/ 10.12989/sem.2021.78.2.117.

Zhang, L., Bhatti, M.M., Marin, M. and Mekheimer, M. (2020), "Entropy analysis on the blood flow through anisotropically tapered arteries filled with magnetic zinc-oxide (ZnO) nanoparticles", *Entropy*, **22**(10), 1070. http://doi.org/10.3390/e22101070.

GY