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Deformation in a homogeneous isotropic thermoelastic solid with multi-dual-phase-lag heat & two temperature using modified couple stress theory

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Abstract. The objective of this paper is to study the deformation in a homogeneous isotropic thermoelastic solid using modified couple stress theory subjected to inclined load with two temperatures with multi-dual-phase-lag heat transfer. Uniformly distributed and linearly distributed forces have been applied to find the functionality of the problem. Laplace and Fourier transform technique is applied to obtain the solutions of the governing equations. The displacement components, conductive temperature, stress components and couple stress are obtained in the transformed domain. A numerical inversion technique has been used to obtain the solutions in the physical domain. The effect of two temperature and inclined load is depicted graphically on the resulted quantities.

Keywords: modified couple stress theory; two temperature; isotropic solid; inclined load; Laplace and Fourier transform; couple stress moment tensor

1. Introduction

Classical continuum mechanics predicts the behavior of structures under loads at macro scale, but careful experiments have shown that it deviates in capturing behavior of materials at micro/nano scale.

In classical elasticity theory, forces are transmitted at an infinitesimal element surface as tractions or more specifically force tractions. On the other hand, in size dependent theories, moments are transmitted on an infinitesimal element surface as moment or couple tractions in addition to force tractions. These force and moment tractions can then be represented by tensorial (force) stresses and couple stresses on infinitesimal element. Correspondingly new measures of deformation, such as curvatures, are presented in addition to strains. First mathematical model to examine the material with couple stresses was presented by Cosserat and Cosserat (1909). Displacements and independent rotations, known as microrotations, were used as the kinematical quantities. Their work was further revived by Mindlin (1964), Eringen (1999), Nowacki (1986) and Chen and Wang (2001). Another branch of theories, known as second gradient or strain gradient theories were developed by Mindlin and Eshel (1968), Yang *et al.* (2002) and Lazar *et al.* (2005). These involve gradients of strains,

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rotations and their various combinations all originating from the displacement field to avoid the microrotations. One other branch of theories based on Voigt (1887) was developed by Toupin (1962), Mindlin and Tiersten (1962) and Koiter (1964) in which displacements and macrorotations were taken as the kinematical quantities. These macrorotations are the continuum mechanical rotations, which are defined as one half the curl of displacements. Finally, the curvatures are defined as gradient of these macrorotations. But these theories had some indeterminacy/ inconsistencies in the couple stress and force stress tensors due to the limited number of relations. So, Hadjesfandiari and Dargush (2011) offered consistent couple stress theory (C-CST) with the skew-symmetric couple-stresses to resolve the inconsistencies in previous models. This theory was not applicable to anisotropic materials. Therefore, Chen and Li (2013) introduced the new modified couple stress theory (NM-CST) for anisotropic materials containing three length scale parameters. A finite element method is developed based on the skew symmetric couple stress theory Hadjesfandiari and Dargush (2011). Tsiatas and Yiotis (2010) and Guo *et al.* (2016) applied the modified couple stress theory to model anisotropic elasticity at the microscale. Chen *et al*. (2012), Chen and Si (2013), Chen and Li (2014), studied laminated composite beams based on new modified couple stress theory. Yang and He (2017) studied the vibration and buckling of orthotropic functionally graded micro-plates on the basis of a re-modified couple stress theory. Ke and Wang (2011) presented a size-dependent beam model for dynamic stability as well as nonlinear free vibration analysis using the Timoshenko beam theory. Reddy (2011) presented a microstructure-dependent nonlinear model for the static bending, vibration and buckling analysis of FG micro-beams. Nateghi *et al.* (2012) performed the buckling analysis of FG micro-beams within the framework of several higher-order beam models and discussed the effect of boundary conditions. Zihao and He (2019) studied orthotropic functionally graded micro-plates based on re-modified couple stress theory. Asghari *et al.* (2010, 2011) examined the static and dynamic behaviors of FG Euler–Bernoulli and Timoshenko micro-beams. Despite of that several researchers worked on the theory that uses the similar technique such as Arif *et al.* (2018), Othman *et al.* (2013), Abbas and Youssef (2013), Abbas and Zenkour (2014), Kumar *et al.* (2016), Lata (2018a, 2018b), Lata and Kaur (2019a, 2019b), Bhatti *et al.* (2020a, 2020b, 2020c), Riaz *et al.* (2019), Marin (1996, 1997, 2010a, 2010b), Vlase *et al.* (2019).

The present investigation deals with the deformation in isotropic thermoelastic solid using modified couple stress theory subjected to inclined load with two temperatures with multi-dual-phase-lag heat transfer in context with modified couple stress theory proposed by Yang (2002). This theory contains one couple stress parameter to determine the size effects. Distributed and concentrated forces have been applied to find the utility of the problem. Laplace and Fourier transform technique is applied to obtain the solutions of the governing equations. The displacement components, conductive temperature, stress components and couple stress are obtained in the transformed domain. A numerical inversion technique has been used to obtain the solutions in the physical domain. The effect of two temperature and inclined load is depicted graphically on the resulted quantities.

2. Basic equations

Following Kumar *et al.* (2015) and Zenkour (2020), the field equations for isotropic modified couple stress thermoelastic medium without mass diffusion, without energy dissipation in the absence of body forces, body couples are given by

(a) Constitutive relationships are

$$t_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \frac{1}{2}e_{kij}m_{lk,l} - \beta_1 T\delta_{ij}, \tag{1}$$

$$m_{ij} = 2\alpha\chi_{ij}, \tag{2}$$

$$\chi_{ij} = \frac{1}{2}\left(\omega_{i,j} + \omega_{j,i}\right), \tag{3}$$

$$\omega_i = \frac{1}{2}e_{ijk}u_{k,j}. \tag{4}$$

(b) Equation of motion is

$$\left(\lambda + \mu + \frac{\alpha}{4}\Delta\right)\nabla(\nabla.\vec{u}) + \left(\mu - \frac{\alpha}{4}\Delta\right)\nabla^2\vec{u} - \beta_1\nabla T = \rho\ddot{\vec{u}}, \tag{5}$$

(c) Equation of heat conduction is

$$K\mathcal{L}_\theta\nabla^2\varphi = \mathcal{L}_q\frac{\partial}{\partial t}(\rho C_e T + \beta_1 T_0 u_{i,j}), \tag{6}$$

where

$$T = (1 - a\nabla^2)\varphi. \tag{7}$$

Here $u = (u, v, w)$ is the components of displacement vector, σ_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, e_{ijk} is alternate tensor, m_{ij} are the components of couple-stress, a is the two temperature parameter, T is the thermodynamical temperature, $\varphi$is the conductive temperature, χ_{ij} is curvature, ω_i is the rotational vector,$\rho$is the density, K is the thermal conductivity, C_e is the specific heat at constant strain, T_0 is the reference temperature assumed to be such that$T/_{T_0} \ll 1$ and $\beta_1 = (3\lambda + 2\mu)\alpha_t$. Here α_t is the coefficients of linear thermal expansion and diffusion expansion respectively, α is the couple stress parameter, Δ is the Laplacian operator, ∇ is del operator, δ_{ij} is Kronecker's delta, $\mathcal{L}_\theta = 1 + \sum_{r=1}^{R_1}\frac{\tau_\theta^r\partial^r}{r!\partial t^r}$, $\mathcal{L}_q = \varrho + \tau_0\frac{\partial}{\partial t} + \sum_{r=2}^{R_2}\frac{\tau_q^r\partial^r}{r!\partial t^r}$. where ϱ is non-dimension parameter(=0 or 1 according to the thermoelasticity theory),and value of R_1 and R_2 may vary according to multi-dual-phase-lag theory required. The thermal relaxation time parameters τ_0, τ_θ and τ_q are the thermal memories in which τ_q is the phase-lag (PL) of the heat flux, $(0 \le \tau_\theta < \tau q)$, while τ_θ is the PL of the temperature gradient.

3. Formulation and solution of the problem

We consider a two dimensional homogeneous isotropic modified couple stress thermoelastic medium initially at uniform temperature T_0 occupying the region of a half space $z \ge 0$. A rectangular coordinate system (x, y, z) having origin on the surface $z = 0$ has been taken. All the field quantities depend on (x, z, t). The half surface is subjected to isolated and insulated boundary conditions.

The initial and regularity conditions are given by

$$\begin{gathered} u(x,z,0) = 0 = \dot{u}\,(x,z,0), \\ v(x,z,0) = 0 = \dot{v}\,(x,z,0), \\ \varphi(x,z,0) = 0 = \dot{\varphi}(x,z,0) \quad for \;\; z \geq 0, -\infty < x < \infty, \\ u(x,z,t) = v(x,z,t) = \varphi(x,z,t) = 0 \, for \;\; t > 0 \; when \, z \to \infty. \end{gathered} \tag{8}$$

Using Eq. (8) in the Eqs. (1)-(7) yields

$$(\lambda+\mu)\frac{\partial e}{\partial x} + \mu\nabla^2 u + \frac{\alpha}{4}\nabla^2\left(\frac{\partial e}{\partial x} - \nabla^2 u\right) - \beta_1\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{9}$$

$$(\lambda+\mu)\frac{\partial e}{\partial z} + \mu\nabla^2 w + \frac{\alpha}{4}\nabla^2\left(\frac{\partial e}{\partial z} - \nabla^2 w\right) - \beta_1\frac{\partial T}{\partial z} = \rho\frac{\partial^2 w}{\partial t^2}, \tag{10}$$

$$K\mathcal{L}_\theta\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2}\right) = \mathcal{L}_q\frac{\partial}{\partial t}(\rho C_e T + \beta_1 T_0 e), \tag{11}$$

$$t_{33} = \left(\frac{\partial w}{\partial z} + \frac{\partial u}{\partial x}\right) + 2\mu\frac{\partial w}{\partial x} - \beta_1(1 - a\nabla^2)\varphi, \tag{12}$$

$$t_{31} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) - \frac{\alpha}{4}\,\nabla^2\left(-\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \tag{13}$$

$$m_{32} = \frac{\alpha}{2}\left(\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 w}{\partial x\partial z}\right), \tag{14}$$

where $e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$. $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$

To facilitate the solution, the dimensionless quantities are defined as

$$x' = \frac{\omega^*}{c_1}x,\;\; z' = \frac{\omega^*}{c_1}z,\;\; u' = \frac{\omega^*}{c_1}u,\;\; w' = \frac{\omega^*}{c_1}w,\;\; t' = \omega^* t,\;\; t_{ij}' = \frac{t_{ij}}{\beta_{1T_0}}\,,\;\; m_{ij}' = \frac{m_{ij}}{c_1\beta_{1T_0}}, T' = \frac{\beta_1 T}{\rho c_1^2}\,,\; c_1^2 = \frac{\lambda+2\mu}{\rho}\,,\;\; \varphi' = \frac{\beta_1\varphi}{\rho c_1^2},\; \omega^{*2} = \frac{\lambda}{\mu t^2+\rho\alpha}\,,\;\; a' = \left(\frac{\omega^*}{c_1}\right)^2 a\;. \tag{15}$$

where $\omega*$ and c_1 are the characteristic frequency and longitudinal wave velocity in the media.

Using the dimensionless quantities defined by (15) in the Eqs. (9)-(14), and after suppressing the primes and further applying scalar potentials $\Phi(x,z,t)$ and $\Psi(x,z,t)$ in dimensionless form defined by

$$u = \frac{\partial\Phi}{\partial x} - \frac{\partial\Psi}{\partial z}, \qquad w = \frac{\partial\Phi}{\partial z} + \frac{\partial\Psi}{\partial x} \tag{16}$$

On the resulting equations, we obtain

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\Phi - (1 - a\nabla^2)\varphi = 0, \tag{17}$$

$$(a_1\nabla^2 + \frac{\alpha}{4}\,a_2\nabla^2(\nabla^2) - \frac{\partial^2}{\partial t^2})\Psi = 0, \tag{18}$$

$$\mathcal{L}_\theta\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2}\right) - \mathcal{L}_q\frac{\partial}{\partial t}(\varepsilon_1(1 - a\nabla^2)\varphi + \varepsilon_2\nabla^2\Phi) = 0, \tag{19}$$

$$t_{33} = \frac{\lambda}{\beta_1 T_0}\left(\frac{\partial^2\Phi}{\partial z^2} + \frac{\partial^2\Phi}{\partial x^2}\right) + \frac{2\mu}{\beta_1 T_0}(\frac{\partial^2\Phi}{\partial z^2} + \frac{\partial^2\Psi}{\partial x\partial z}) - \frac{\rho c_1^2}{\beta_1 T_0}(1 - a\nabla^2)\varphi, \tag{20}$$

$$t_{31} = \frac{\mu}{\beta_1 T_0}\left(\frac{\partial^2\Psi}{\partial x^2} + \frac{2\partial^2\Phi}{\partial x\partial z} - \frac{\partial^2\Psi}{\partial z^2}\right) + \frac{\alpha}{4\beta_1 T_0}\left(\frac{\omega^*}{c_1}\right)^2\nabla^4\Psi, \tag{21}$$

$$m_{32} = -\frac{\alpha\omega^*}{2\beta_1 T_0 c_1^2}\nabla^2\frac{\partial\Psi}{\partial z}, \tag{22}$$

where $a_1 = \frac{\mu}{\rho c_1^2}$, $a_2 = \frac{\omega^{*2}}{\rho c_1^4}$, $a_3 = \frac{(\lambda+\mu)}{\rho c_1^2}$, $\varepsilon_1 = \frac{\rho c_1^2 C_e}{\omega^* K}$, $\varepsilon_2 = \frac{\beta_1^2 T_0}{\rho\omega^* K}$, $\nabla^2\Phi = e$, $\Psi = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$.

Applying Laplace and Fourier transformation defined by

$$\overline{f}(x_1, x_{3,} s) = \int_0^\infty f\,(x_1, x_{3,} t)e^{-st}dt, \tag{23}$$

$$\hat{f}(\xi, z, s) = \int_{-\infty}^{\infty}\overline{f}(x, z, s)e^{i\xi x}d\,x. \tag{24}$$

On the set of Eqs. (17)-(19), we obtain system of three homogeneous equations. These resulting equations have non trivial solution if the determinant of the coefficients of $(\widehat{\Phi}, \widehat{\Psi}, \hat{\varphi},)$ vanishes, which yields the following characteristic equation

$$(PD^8 + QD^6 + RD^4 + SD^2 + T) = 0, \tag{25}$$

where

$$P = a_4\gamma_4 + a a_6 a_4,$$

$$Q = -(\xi^2 + s^2)a_4\gamma_4 + \gamma_2\gamma_4 + a_4(-\mathcal{L}_\theta\xi^2 - \gamma_3) - a_6((1 + a\xi^2)a_4 - a(\gamma_2 - a_4\xi^2)).$$

$$R = \gamma_2(-\mathcal{L}_\theta\xi^2 - \gamma_3) - (\xi^2 + s^2)(\gamma_2\gamma_4 + a_4(-\mathcal{L}_\theta\xi^2 - \gamma_3))$$
$$-a_6((1 + a\xi^2)(\gamma_2 - a_4\xi^2) - a(\gamma_1 - \gamma_2\xi^2)),$$

$$S = -(\xi^2 + s^2)(\gamma_2(-\mathcal{L}_\theta\xi^2 - \gamma_3) + \gamma_1\gamma_4) + \gamma_1(-\mathcal{L}_\theta\xi^2 - \gamma_3)$$
$$-a_6((1 + a\xi^2)(\gamma_1 - \gamma_2\xi^2) + a\gamma_1\xi^2),$$

$$T = -\gamma_1(\xi^2 + s^2)(-\mathcal{L}_\theta\xi^2 - \gamma_3) + a_6\gamma_1\xi^2(1 + a\xi^2).$$

The roots of the Eq. (28) are $\pm\lambda_i\,(i = 1, 2, 3, 4, 5)$, Using the radiation condition that $\widehat{\Phi}, \widehat{\Psi}, \hat{\varphi} \to 0$ as $z \to \infty$, the solution of Eq. (30) may be written as

$$(\widetilde{\Phi}, \widetilde{\Psi}, \tilde{\varphi}) = \sum_{i=1}^{4}(1, R_i, S_i)A_i e^{-\lambda_i z}, \tag{26}$$

where

$$R_i = \frac{(\xi^2+s^2)(\mathcal{L}_\theta\xi^2+\gamma_3)-\xi^2(1+a\xi^2)+\left((\xi^2+s^2)\gamma_4-\mathcal{L}_\theta\xi^2-\gamma_3+a_6(1+2a\xi^2)\right)\lambda_i{}^2+(\gamma_4-aa_6)\lambda_i{}^4}{\gamma_1(-\mathcal{L}_\theta\xi^2-\gamma_3)+(\gamma_2(-\mathcal{L}_\theta\xi^2-\gamma_3)+\gamma_2\gamma_4)\lambda_i{}^2+(\gamma_2\gamma_4+a_4(-\mathcal{L}_\theta\xi^2-\gamma_3))\lambda_i{}^4+a_4\gamma_4\lambda_i{}^6},$$

$$S_i = \frac{-(\xi^2+s^2)\gamma_1+(-(\xi^2+s^2)\gamma_2+\gamma_1)\lambda_i{}^2+(-(\xi^2+s^2)a_4+\gamma_2)\lambda_i{}^4+\ a_4\lambda_i{}^6}{\gamma_1(-\mathcal{L}_\theta\xi^2-\gamma_3)+(\gamma_2(-\mathcal{L}_\theta\xi^2-\gamma_3)+\gamma_2\gamma_4)\lambda_i{}^2+(\gamma_2\gamma_4+a_4(-\mathcal{L}_\theta\xi^2-\gamma_3))\lambda_i{}^4+a_4\gamma_4\lambda_i{}^6},$$

$$a_4 = \frac{\alpha}{4}a_2, \qquad a_5 = \mathcal{L}_q s, \qquad a_6 = a_5 \in_2, \qquad a_7 = a_5 \in_1, \qquad a_8 = aa_7,$$

$$\gamma_1 = -a_1\xi^2 + a_4\xi^4 - s^2, \quad \gamma_2 = a_1 - 2a_4\xi^2, \quad \gamma_3 = a_7 + a_8\xi^2, \quad \gamma_4 = \mathcal{L}_\theta + a_8.$$

4. Boundary conditions

We consider a normal line load F_1 per unit length acting in the positive z axis on the plane boundary $z = 0$ along the y axis and a tangential load F_2 per unit length, acting at the origin in the positive x axis. The boundary conditions are

$$t_{33}(x,z,t) = -F_1\psi_1(x)H(t), \tag{27}$$

$$t_{31}(x,z,t) = -F_2\psi_2(x)H(t), \tag{28}$$

$$m_{32} = 0 \tag{29}$$

$$\frac{\partial\varphi(x,z,t)}{\partial z} = 0, \tag{30}$$

where F_1 and F_2 are the magnitude of forces applied, $\psi_1(x)$ and $\psi_2(x)$ specify the vertical and horizontal load distribution functions respectively along x-axis, $H(t)$ is the Heaviside unit step function. (Fig. 2)

Substituting the values of $\widehat{u_1}$, $\widehat{u_3}$, $\hat{T}$ from Eq. (26) in the boundary conditions Eqs. (27)-(30) and with the aid of Eqs. (1), (5)-(8), (15), (23)-(24) we obtain the components of displacement, normal stress, tangential stress, tangential couple stress and temperature change as

$$\tilde{u} = -\frac{F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{4}(\mathrm{i}\xi+\lambda_i R_i)B_{1i}\,e^{-\lambda_i z} - \frac{F_2\widehat{\psi_2}(\xi)}{\Delta}\sum_{i=1}^{4}(\mathrm{i}\xi+\lambda_i R_i)B_{3i}\,e^{-\lambda_i z}, \tag{31}$$

$$\widetilde{w} = -\frac{F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{4}(-\lambda_i+\mathrm{i}\xi R_i)B_{1i}\,e^{-\lambda_i z} - \frac{F_2\widehat{\psi_2}(\xi)}{\Delta}\sum_{i=1}^{4}(-\lambda_i+\mathrm{i}\xi R_i)B_{3i}\,e^{-\lambda_i z}, \tag{32}$$

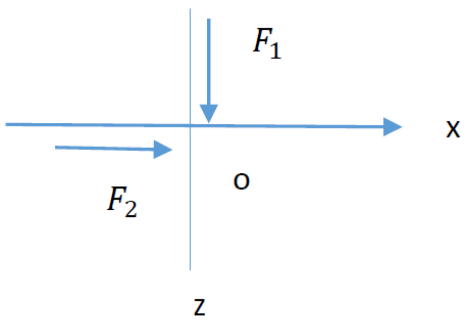

Fig. 1 Normal and tangential loadings

$$\tilde{\varphi} = \frac{F_1\widehat{\psi_1}(\xi)\rho c_1^2}{\beta_1\Delta}\sum_{i=1}^{4}\lambda_i S_i B_{1i}\, e^{-\lambda_i z} + \frac{F_2\widehat{\psi_2}(\xi)\rho c_1^2}{\beta_1\Delta}\sum_{i=1}^{4}\lambda_i S_i B_{3i}\, e^{-\lambda_i z}, \tag{33}$$

$$\widetilde{t_{33}} = -\frac{F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{4} A_{1i}B_{1i}\, e^{-\lambda_i z} - \frac{F_2\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{4} A_{1i}B_{2i}\, e^{-\lambda_i z}, \tag{34}$$

$$\widetilde{t_{31}} = \frac{-F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{4} A_{2i}B_{1i}e^{-\lambda_i z} + \frac{-F_2\widehat{\psi_2}(\xi)}{\Delta}\sum_{i=1}^{4} A_{2i}B_{2i}\, e^{-\lambda_i z}, \tag{35}$$

$$\widetilde{m_{32}} = \frac{-F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{4} A_{3i}\, B_{1i}e^{-\lambda_i z} + \frac{-F_2\widehat{\psi_2}(\xi)}{\Delta}\sum_{i=1}^{4} A_{3i}\, B_{2i}e^{-\lambda_i z}. \tag{36}$$

where

$A_{1i} = \dfrac{\lambda\left(-\xi^2+\lambda_i^2\right)}{\beta_1 T_0} + \dfrac{2\mu\lambda_i(\lambda_i - \mathrm{i}\xi R_i)}{\beta_1 T_0} - \dfrac{\rho c_1^2}{\beta_1 T_0}\left(\left(1 - a\left(-\xi^2+\lambda_i^2\right)\right)S_i\right)$,

$A_{2i} = \dfrac{1}{\beta_1 T_0}\left(\mu(-2\mathrm{i}\xi\lambda_i - \left(-\xi^2+\lambda_i^2\right)) - \dfrac{\alpha}{4}\left(\dfrac{\omega^*}{c_1}\right)^2\left(-\xi^4 - \lambda_i^4 + 2\xi^2\lambda_i^2\right)R_i\right)$,

$A_{3i} = \dfrac{\alpha\omega^*}{2\beta_1 T_0 c_1^2}\left(\lambda_i^3 - \xi^2\lambda_i\right)R_i$,

$A_{4i} = \dfrac{-\rho c_1^2}{\beta_1}\lambda_i S_i$,

$\Delta = \Delta_1 - \Delta_2 + \Delta_3 - \Delta_4$,

$\Delta_1 = A_{11}A_{22}(A_{33}A_{44} - A_{43}A_{34}) - A_{11}A_{23}(A_{32}A_{44} - A_{42}A_{34}) + A_{11}A_{24}(A_{32}A_{43} - A_{42}A_{33})$,

$\Delta_2 = A_{12}A_{21}(A_{33}A_{44} - A_{43}A_{34}) - A_{12}A_{23}(A_{31}A_{44} - A_{41}A_{34}) + A_{24}A_{12}(A_{31}A_{43} - A_{41}A_{33})$,

$\Delta_3 = A_{13}A_{21}(A_{32}A_{44} - A_{42}A_{34}) - A_{22}A_{13}(A_{31}A_{44} - A_{41}A_{34}) + A_{13}A_{24}(A_{31}A_{42} - A_{41}A_{32})$,

$\Delta_4 = A_{14}A_{21}(A_{32}A_{43} - A_{42}A_{33}) - A_{22}A_{14}(A_{31}A_{43} - A_{41}A_{33}) + A_{14}A_{23}(A_{31}A_{42} - A_{41}A_{32})$,

$B_{1i} = \dfrac{(-1)^{1+i}\Delta_i}{A_{1i}}$,

$B_{21} = -A_{21}(A_{33}A_{44} - A_{43}A_{34}) + A_{23}(A_{31}A_{44} - A_{41}A_{34}) - A_{24}(A_{31}A_{43} - A_{41}A_{33})$,

$B_{22} = A_{11}(A_{33}A_{44} - A_{43}A_{34}) - A_{13}(A_{31}A_{44} - A_{41}A_{34}) + A_{14}(A_{31}A_{43} - A_{41}A_{33})$,

$B_{23} = -A_{11}(A_{23}A_{44} - A_{43}A_{24}) + A_{13}(A_{21}A_{44} - A_{41}A_{24}) - A_{14}(A_{21}A_{43} - A_{41}A_{23})$,

$B_{34} = A_{11}(A_{23}A_{34} - A_{33}A_{24}) - A_{13}(A_{21}A_{34} - A_{31}A_{24}) + A_{14}(A_{21}A_{33} - A_{31}A_{23})$,

$A_i = -\dfrac{1}{\Delta}\left(B_{1i}F_1\widehat{\psi_1}(\xi) + (B_{2i}F_2\widehat{\psi_2}(\xi)\right)$.

4.1 Influence function

The method to obtain the half-space influence function, i.e., the solution due to distributed force/source applied on the half space is obtained by setting

$$\{\psi_1(x), \quad \psi_2(x)\} = \begin{cases} 1 & if\ |\,x\,|\ \leq m \\ 0 & if\ \ |\,x\,|\ > m \end{cases} . \tag{37}$$

The Laplace and Fourier transforms of $\psi_1(x)$ with respect to the pair (x, ξ) for the case of a uniform strip load of non-dimensional width 2 m applied at origin of co-ordinate system $x = z = 0$ in the dimensionless form after suppressing the primes becomes

$$\{\widehat{\psi_1}(\xi), \widehat{\psi_2}(\xi)\} \ = \ [\,2\,sin\,(\,\xi m\,)\,/\xi\,]\,, \xi \neq 0. \tag{38}$$

The expressions for displacement components, stress components, conductive temperature and couple stress can be obtained for uniformly distributed normal force and thermal source by replacing $\widehat{\psi_1}\,(\xi)$ and $\widehat{\psi_2}(\xi)$ from Eq. (38) respectively in Eqs. (31)-(36).

4.2 Linearly distributed Force

The solution due to linearly distributed force applied on the half space is obtained by setting

$$\{\psi_1(x), \psi_2(x)\} = \begin{cases} 1 - \dfrac{|\,x\,|}{m} & if\ |\,x\,| \leq m \\ 0 & if\ \ |\,x\,| > m \end{cases} . \tag{39}$$

Here 2 m is the width of the strip load, using Eq. (15) and applying the transform defined by Eq. (27) on Eq. (44), we obtain

$$\{\widehat{\psi_1}(\xi), \widehat{\psi_2}(\xi)\} = \left[\frac{2\{1 - \cos(\xi m))}{\xi^2 m}\right], \xi \neq 0. \tag{40}$$

Using Eq. (40) in the Eqs. (31)-(36), we obtain the components of displacement, stress, conductive temperature and couple stress.

4.3 Applications

Inclined line load. Suppose an inclined load F_0, per unit length is acting on the y axis and its inclination with z axis is δ, we have (see Fig. 2)

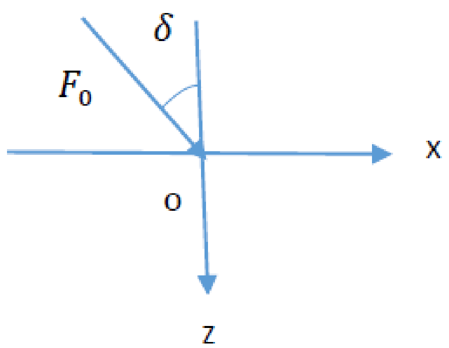

Fig. 2 Inclined load over a thermoelastic solid

$$F_1 = F_0 \cos \delta, \qquad F_2 = F_0 \sin \delta\,. \tag{41}$$

Using Eq. (41) in Eqs. (31)-(36) and with aid of Eqs. (37)-(40) we obtain the expressions for displacements, conductive temperature, stresses and couple stress for concentrated force, uniformly distributed force and linearly distributed force on the surface of isotropic thermoelastic without energy dissipation.

5. Particular cases

In the foregoing sections, the analytical solution is previously given for the multi-dual-phase-lag theory. The transfer heat conduction equation given in Eq. (6) with their two parameters $\mathcal{L}_\theta$ and $\mathcal{L}_q$ covers at least five models of the generalized thermoelasticity.

The easier thermoelasticity theory is the coupled one (CTE) in which $\tau_\theta = \tau_q = \tau_0 = 0$ and $\varrho = 1$ (1965) That is

$$\mathcal{L}_\theta = \mathcal{L}_q = 1 \tag{42}$$

In the L–S thermoelasticity theory (1967) we set $\tau_\theta, \tau_q \to 0, \varrho = 1$ and $\tau_0 > 0$. In this case

$$\mathcal{L}_\theta = 1\,,\ \mathcal{L}_q = 1 + \tau_0 \frac{\partial}{\partial t}. \tag{43}$$

Also, the heat conduction equation based on the G–N thermoelasticity theory of type II (1993) is given by setting $\tau_\theta, \tau_q \to 0, \varrho = 0$ and $\tau_0 = 1$. So, their parameters are reduced to

$$\mathcal{L}_\theta = 1\,,\ \mathcal{L}_q = \frac{\partial}{\partial t}. \tag{44}$$

Now, the simplest form of the heat equation with dual-phase-lag (SdPL) is applied by setting (1965)

$$\mathcal{L}_\theta = 1 + \tau_\theta \frac{\partial}{\partial t}\,,\ \mathcal{L}_q = 1 + \tau_q \frac{\partial}{\partial t}. \tag{45}$$

Additional refined dual-phase-lag (RdPL) theory appeared in the literature (Tzou 1995) is defined by including the effect of the term containing ${\tau_q}^2$ in the above equation as

$$\mathcal{L}_\theta = 1 + \tau_\theta \frac{\partial}{\partial t}\,,\ \mathcal{L}_q = 1 + \tau_q \frac{\partial}{\partial t} + \frac{{\tau_q}^2}{2} \frac{\partial^2}{\partial t^2}. \tag{46}$$

In fact, Eq. (46) represents the first type of the present refined multi-dual-phase-lag (RPL) theory with $\varrho = 1,\ \tau_0 \to \tau_q$ and $R_1 = 1,\ R_2 = 2$. Additional types are presented here for $R1 = R2 = R \geq 2$

If $a = 0$, from Eqs. (34)-(39), we obtain the corresponding expressions for displacements, conductive temperature, stresses and couple stress for isotropic thermoelastic solid without two temperature. If $a \neq 0$, from Eqs. (34)-(39), we obtain the corresponding expressions for displacements, conductive temperature, stresses and couple stress for isotropic thermoelastic solid with two temperature.

6. Inversion of the transformations

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs.

(34)-(39). Here the displacement components, normal and tangential stresses and temperature change, couple stress are functions of z, the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f\ (\xi\ ,z,s)$. To obtain the function $f(x,z,t)$ in the physical domain, we first invert the Fourier transform using

$$\bar{f}(x,z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\mathrm{i}\xi x}\hat{f}(\xi\ ,z\ ,s)\,d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} |cos\ (\xi\ x)f_e - \ isin(\xi\ x)f_0|d\xi\ . \tag{47}$$

where f_e and f_0 are respectively the odd and even parts of $\hat{f}(\xi\ ,x_3\ ,s)$. Thus the expression (47) gives the Laplace transform $\bar{f}(\xi\ ,x_3,\mathrm{s})$ of the function $f(x,x_3,t)$. Following Honig and Hirdes (1984), the Laplace transform function $\bar{f}(\xi\ ,x_3,\mathrm{s})$ can be inverted to $f(x,x_3,t)$.

The last step is to calculate the integral in Eq. (47). The method for evaluating this integral is described in Press *et al.* (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Results and discussions

For numerical computations following Sherief and Saleh (2005) and Zenkour (2020), we take the copper material which is isotropic as

$\lambda\ = 7.76\times 10^{10}Kgm^{-1}s^{-1}$, $\mu = 3.86\times 10^{10}Kgm^{-1}s^{-1}$, $T_0 = 293\ K$,
$C^* = 3831\times 10^3 JKg^{-1}K^{-1}$, $\alpha_t = 1.78\times 10^{-5}K^{-1}$, $\rho = 8.954\times 10^3 Kgm^{-3}$,
$K_{11} = K_{33} = .383\times 10^3 Wm^{-1}K^{-1}$, $\alpha = .05Kgms^{-2}$, $R_1 = 1, R_2 = 2$,
$\tau_\theta = .05$, $\tau_q = .1$, $\tau_0 = .1$, $\varrho = 1$

Software GNU octave has been used to determine the components of displacements, conductive temperature, normal stress, tangential stress and couple stress for homogeneous isotropic thermoelastic medium with distance x for two different values of two temperature parameter for $\delta = 0^0$(initial angle), $\delta = 45^0$(intermediate angle) and $\delta = 90^0$(extreme angle).

The solid lines in black, red and blue respectively corresponds to the $a = 0$ with angle of inclination $\delta = 0^0$, $\delta = 45^0$ and $\delta = 90^0$ (without two temperature).

The solid lines in green, purple and yellow respectively corresponds to the a=.03 with angle of inclination $\delta = 0^0$, $\delta = 45^0$ and $\delta = 90^0$ (with two temperature).

Uniformly distributed Force: In Fig. 3 displacement component u has oscillatory behaviour for both the cases $a = 0$ and $a = .03$ for all angle of inclinations. for $\delta = 90^0$ value of u is lower than $\delta = 0^0$, $\delta = 45^0$ in the range $0 \le x \le 1$, $3 \le x \le 5.5$ and $7 \le x \le 10$. In Fig. 4 w has oscillatory effect with the distance x.For $\delta = 0^0$curves are descending oscillatory. For $\delta = 45^0$and $\delta = 90^0$ curves are inverse oscillatory in the range $0 \le x \le 6$ irrespective of the twotemperature parameter. In Fig. 5 φ has oscillatory trend for all angles of inclination (initial angle, intermediate angle and extreme angle) for both $a = 0$ and $a = .03$. value of φ is highest at the pole. Near the loading surface values are higher for $\delta = 45^0$ than $\delta = 0^0$and $\delta = 90^0$. In Fig. 6 value of t_{33} decreases for $0 \le x \le 6$ and $8.5 \le x \le 10$ and increases in the rest irrespective of angle of inclination and two temperature parameter. In Fig. 7 value of t_{31} decreases for $0 \le x \le 3$, $4 \le x \le 6$ and $9 \le x \le 10$,and increases in the remaining range of x except for $\delta = 90^0$, $a = 0$. For $\delta = 90^0$, $a = 0$ curve increases monotonically $0 \le x \le 2$ and

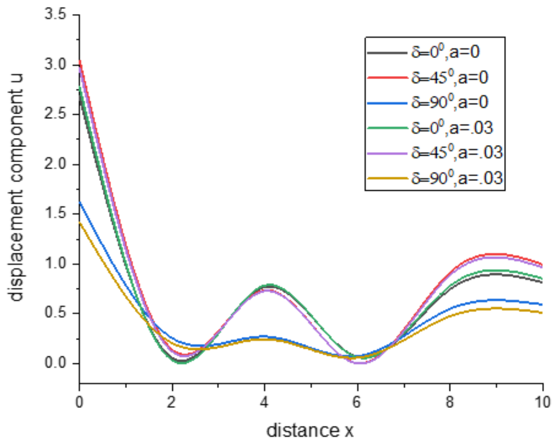

Fig. 3 variation of displacement component u with the distance x (uniformly distributed force)

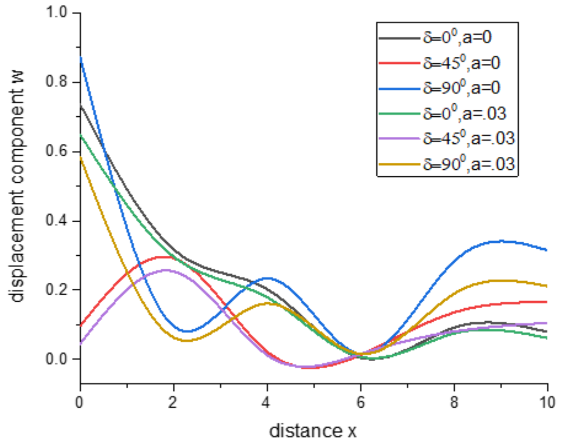

Fig. 4 variation of displacement component w with the distance x (uniformly distributed force)

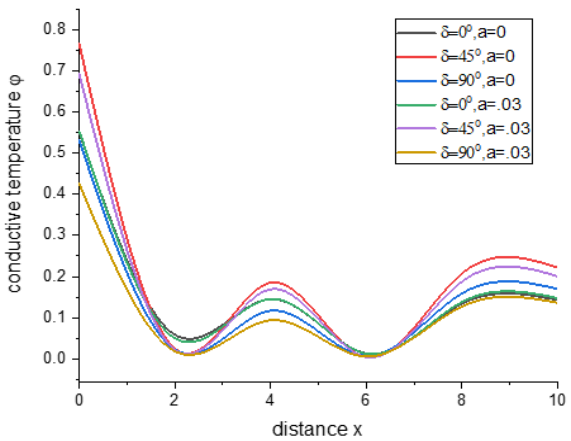

Fig. 5 variation of conductive temperature φ with the distance x (uniformly distributed force)

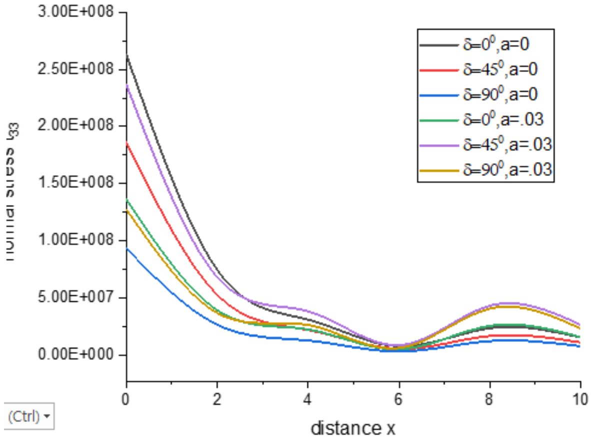

Fig. 6 variation of normal stress t_{33} with the distance x (uniformly distributed force)

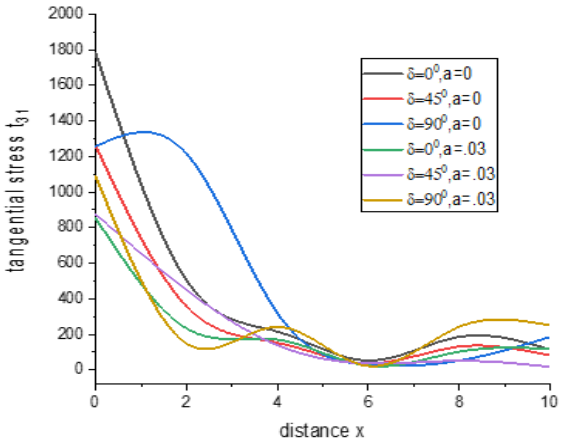

Fig. 7 variation of tangential stress t_{31} with the distance x (uniformly distributed force)

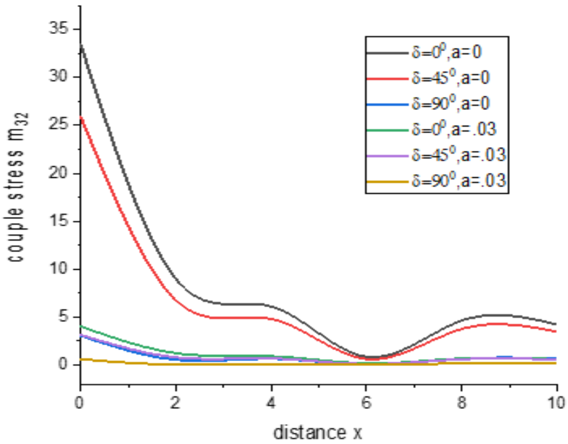

Fig. 8 variation of couple stress m_{32} with the distance x (uniformly distributed force)

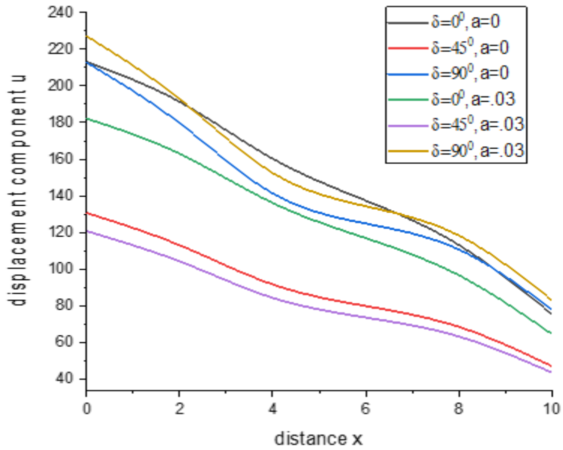

Fig. 9 Variation of displacement component u with the distance x (linearly distributed force)

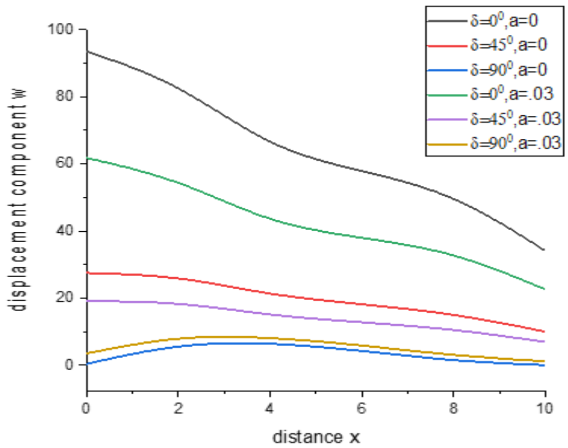

Fig. 10 variation of displacement component w with the distance x (linearly distributed force)

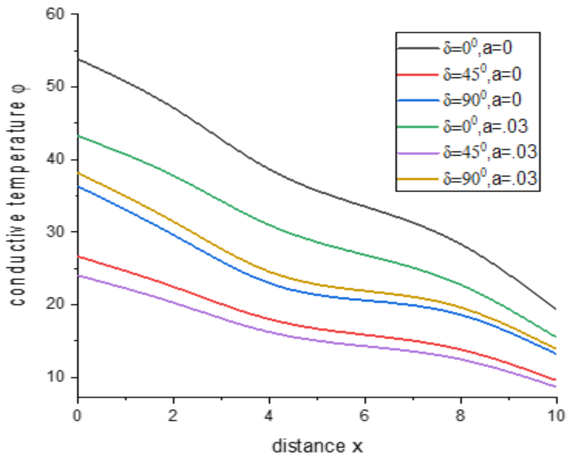

Fig. 11 Variation of conductive temperature φ with the distance x (linearly distributed force)

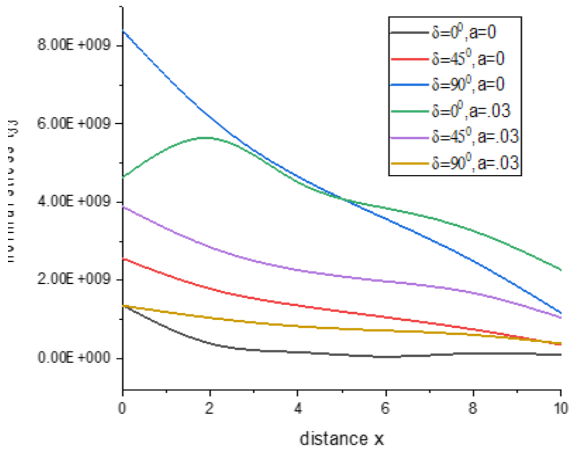

Fig. 12 variation of normal stress t_{33} with the distance x (linearly distributed force)

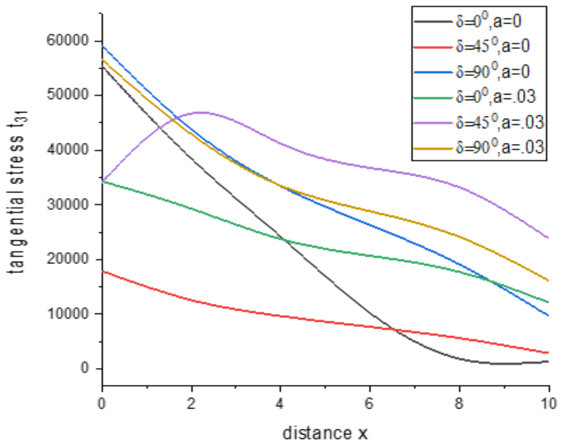

Fig. 13 variation of tangential stress t_{31} with the distance x (linearly distributed force)

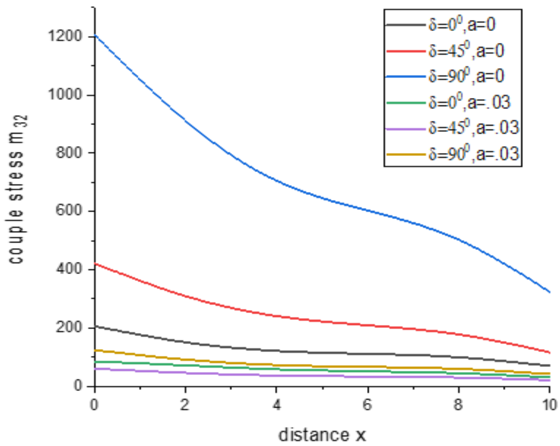

Fig. 14 variation of couple stress m_{32} with the distance x (linearly distributed force)

$0 \le x \le 2$ and $6 \le x \le 10$, increases in the rest. In Fig.6 couple stress m_{32} assumes oscillatory trend for $\delta = 0^0,\ a = 0$and $\delta = 90^0,\ a = 0$. Value of m_{32} is higher for$\delta = 0^0,\ a = 0$ than. $\delta = 90^0,\ a = 0$. In the remaining cases irrespective if the angle of inclination and two temperature parameter value decreases for $0 \le x \le 5$ and assumes constant value in the rest of the range.

Linearly distributed Force: In Fig. 9 u follows descending oscillatory trend with small amplitude of oscillation in all the six cases. Value of u is lesser in case of $\delta = 45^0$ than initial inclination and extreme inclination. For $\delta = 45^0$ value of u is greater in case of $a = 0$ than $a = .03$ in the whole range. In Fig. 10 for the cases $\delta = 0^0,\ a = 0,\ \delta = 45^0,\ a = 0,\ \delta = 0^0,\ a = .03$ and $\delta = 45^0,\ a = .03$ curves coresponding to w follow descending oscillatory trend with very small amplitude of oscillation. For $\delta = 90^0,\ a = 0$ and $\delta = 90^0,\ a = .03$ curves increase in the range $0 \le x \le 4$ and decreases in the rest. It is observed from the graph that value of w is higher for $a = 0$ than $a = .03$ for same angle of inclination. In Fig. 11 φ follows oscillatory behaviour. Value is least for intermediate angle than the initial angle and extreme angle. In Fig. 12 t_{33} in case of $\delta = 45^0,\ a = 0,\ \delta = 90^0,\ a = 0,\ \delta = 90^0,\ a = .03,\ \delta = 45^0,\ a = .03$ decreases with the increase of x. For $\delta = 0^0,\ a = .03$ t_{33} increases in the range $0 \le x \le 2$ and decreases in the remaining range. For $\delta = 0^0,\ a = 0$ curve decreases in first half and increases in the next half of range. In Fig. 13 t_{31} in case of $\delta = 0^0,\ a = 0,\ \delta = 45^0,\ a = 0,\ \delta = 90^0,\ a = 0,\ \delta = 0^0,\ a = .03,\ \delta = 90^0,\ a = .03$ decreases with the increase of x. For $\delta = 45^0,\ a = 0$ curve increases for $0 \le x \le 2$ and follows oscillatory trend with small amplitude in the remaining range. In Fig. 14 m_{32} follows descending oscillatory trend for $a = 0$. The variations for $a = .03$ monotonically decrease.

8. Conclusions

Effect of two temperature and inclined load have significant impact on components of normal displacement, tangential displacement, conductive temperature, normal stress, tangential stress and couple stress. As disturbance travels through the constituents of the medium, it suffers sudden changes resulting in an inconsistent / non uniform pattern of graphs. Examining the graphs, we reach at certain conclusions:

1. Due to two temperature and inclination, there is change in magnitude of deformation.
2. In case of uniformly distributed force pattern of variation of displacement component w changes effectively with the increase of angle of inclination. Pattern of variation is same for fix angle of inclination for both the without two temperature and with two temperature, except the magnitude.
3. In case of uniformly distributed force conductive temperature φ has oscillatory trend irrespective of the angle of inclination and two temperature parameter.
4. In case of uniformly distributed force all curves for the variation of normal stress t_{33} assume same value at $x = 6$.
5. In case of linearly distributed force it is observed from the graph that value of displacement component w increases in the presence of two temperature.

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