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On buckling behavior of thick advanced composite sandwich plates

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Abstract. In this work, a novel higher-order shear deformation theory (HSDT) is presented for buckling analysis of functionally graded plates. The present theory accounts for both shear deformation and thickness stretching effects by a parabolic variation of all displacements across the thickness, and satisfies the stress-free boundary conditions on the upper and lower surfaces of the plate without requiring any shear correction factor. The number of independent unknowns of this theory is four, as against five in other shear deformation theories. Unlike the conventional HSDT, the present one has a new displacement field which introduces undetermined integral variables. The material properties of the faces of sandwich plate are assumed to vary according to a power law distribution in terms of the volume fractions of the constituents. The core layer is made of an isotropic ceramic material. The governing equations are obtained by the principle of virtual work. Analytical solutions for the buckling analyses are solved for simply supported sandwich plate. Numerical examples are given to show the effects of varying gradients, thickness stretching, and thickness to length ratios on the critical buckling loads of functionally graded sandwich plate.

Keywords: functionally graded plates; refined plate theory; buckling analysis; stretching effects

1. Introduction and literature review

Functionally graduated plates (FGM) are widely used in various topics of engineering such as mechanics, aerospace, chemistry, electricity, etc (Avcar 2019, Shahsavari *et al.* 2018, Faleh *et al.*

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2018, Abualnour et al. 2018, Avcar and Mohammed 2018, Mouli *et al.* 2018, Kar *et al.* 2017, Kar and Panda 2016 and 2017, Kar *et al.* 2016, Akbas 2015, Kar and Panda 2015, Behravan Rad 2015, Avcar 2015 and 2016, Ahmed 2014). The advantages of FGM structures have a high thermal resistance and a gradual change in material characteristics along the chosen direction, like thickness, for example. For the design of FGM plates for application in a high temperature environment, thermomechanical stresses and deflections are important parameters to consider.

The study of plates designed with this type of material requires more precise theories to predict their response. Shear deformation has a significant effect on functional gradient plate responses; shear deformation theories are used to account for the effects of warping for thick plates. These theories include the first-order shear deformation theory (FSDT), which accounts for the shear deformation effect represented by a linear variation in plane displacements across the thickness, and verifies the null conditions of transverse shear stresses on the plane. The upper and lower surfaces of the plate, a shear correction factor that depends on many parameters is required to correct the error due to a constant shear stress assumption across the thickness (Belifa *et al.* 2016, Semmah *et al.* 2014, Benzair *et al.* 2008).

The higher order shear deformation theories (HSDT) that account for shear deformation effects, and satisfy the nullity of transverse shear stresses on the upper and lower surfaces of the plate; which leads to the no need for a correction factor (Berrabah *et al.* 2013, Panda and Katariya 2015, Katariya and Panda 2016); these theories are proposed assuming higher order variations of in-plane displacements (Reddy 2000, Xiang et al. 2011; Kolahchi *et al.* 2015, Aldousari 2016) or both in plane and transversal displacements across the thickness (Matsunaga 2008, Chen *et al.* 2009, Reddy 2011, Akavci 2016).

The use of sandwich structures offers great potential for large civil infrastructure projects, such as industrial buildings and vehicular bridges. In recent years, the functionally graded materials (FGMs) are taken into account in the sandwich structure industries (Mehar *et al.* 2017, Katariya *et al.* 2017a, Abdelaziz *et al.* 2017, Mehar and Panda 2018, Belabed *et al.* 2018, Katariya *et al.* 2018, Sharma *et al.* 2018, Dash *et al.* 2018 and 2019, Mehar *et al.* 2019).

Two categories of FG sandwich structures commonly exist: homogeneous face sheet with FG-core and FG face sheet with homogeneous core. For the case of homogeneous core, the soft core is commonly employed because of the light weight and high bending stiffness in the structural design. The homogeneous hardcore is also used in other fields such as control or in the thermal environments. These two categories of FG sandwich structures are widely used in various problems such as vibration, bending or buckling. For functional materials, the theory of plane elasticity has made great progress for both plates and beams. However, for FG sandwich structures, related studies are so limited.

Zenkour and Sobhy (2010) investigated the thermal buckling of various types of FG sandwich plate using sinusoidal shear deformation plate theory. An investigation of bending response of a simply supported FGM viscoelastic sandwich beam with elastic core resting on Pasternak’s elastic foundations was presented by Zenkour *et al.* (2010). Bhangale and Ganesan (2006) studied vibration and buckling analysis of a FG sandwich beam having constrained viscoelastic layer in thermal environment by using finite element formulation.

Bui *et al.* (2013) investigated transient responses and natural frequencies of sandwich beams with inhomogeneous FG core using a truly mesh free radial point interpolation method. Sobhy (2013) studied the vibration and buckling behavior of exponentially graded material sandwich plate resting on elastic foundations under various boundary conditions. Swaminathan and Naveenkumar (2014) presented some higher order refined computational models for the stability analysis of FGM plates. Ait Yahia *et al.* (2015) studied wave propagation in order to compare different shear theories and porosities solution in FG plates. Ait Atmane *et al.* (2015) studied a computational shear displacement model for vibrational analysis of functionally graded beams with porosities. Beldjelili *et al.* (2016) analyzed the hygro-thermo-mechanical bending response of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory. Bouderba *et al.* (2016) studied the thermal stability of FG sandwich plates using a simple shear deformation theory. Becheri *et al.* (2016) analyzed the buckling of symmetrically laminated plates using nth-order shear deformation theory with curvature effects. Houari *et al.* (2016) presented a new simple three-unknown sinusoidal shear deformation theory for FG plates. Benbakhti *et al.* (2016) proposed an analytical formulation of the static thermomechanical problem of functionally graduated sandwich plates by employing a new type of quasi-3D plate shear deformation theory, with the addition of the integral term in the displacement field which leads to a reduction in the number of variables and governing equations. Benadoudada *et al.* (2017) studied wave propagation in order to understand the dispersive law and porosities solution in FG beams.

A large application of the thickness stretching effect in FG plates has been proved in the study of Carrera *et al.* (2011). Bennai *et al.* (2015) used a new higher-order shear and normal deformation theory for buckling, bending and vibration of functionally graded sandwich beams; Ait Atmane *et al.* (2017) studied the effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations. This effect has an important role in thick FG plates and beams; it was taken into account by (Hebali *et al.* 2014, Belabed *et al.* 2014, Hamidi *et al.* 2015, Larbi Chaht *et al.* 2015, Bourada *et al.* 2015, Draiche *et al.* 2016, Bennoun *et al.* 2016, Benahmed *et al.* 2017, Sekkal *et al.* 2017, Bouafia *et al.* 2017, Katariya *et al.* 2017b, Benchohra *et al.* 2018, Karami *et al.* 2018, Khiloun *et al.* 2019, Zaoui *et al.* 2019).

In this work, a new four-variable plate theory is developed to investigate the buckling of sandwich plates by introducing the thickness stretching effect of FGM plates. In the present theory, the displacement field introduces undetermined integral variables. As the plate is supposed to be isotropic at any point in its volume, with a Young's modulus varying through the thickness according to a power law as a function of the volume fraction of the constituents of the plate, whereas, the Poisson's ratio is assumed to be constant.

Equilibrium equations for FGM sandwich plates are obtained using the virtual works principle. The analytical relationships of the plate are obtained using Navier's solutions.

Numerical results for uni-axial and bi-axial critical buckling loads have been studied and presented to illustrate the accuracy and efficiency of the quasi-3D theory model by comparing the results obtained with those determined by the TSDPT and SSDPT theories of Zenkour (2005) , Neves *et al.* (2012) , and Mahmoud and Tounsi (2017).

2. Theory and formulation for functionally graded sandwich plate

2.1 Problem formulation

In this work, a rectangular sandwich plate of length a, width b and thickness h is considered. The coordinate system is chosen such that the x-y plane coincides with the mid-plane of the plate ($z \in [-h/2, +h/2]$). The core of the sandwich plate is made of a ceramic material and skins are consisting of FGM within the thickness direction. In the lower skin, a mixture of

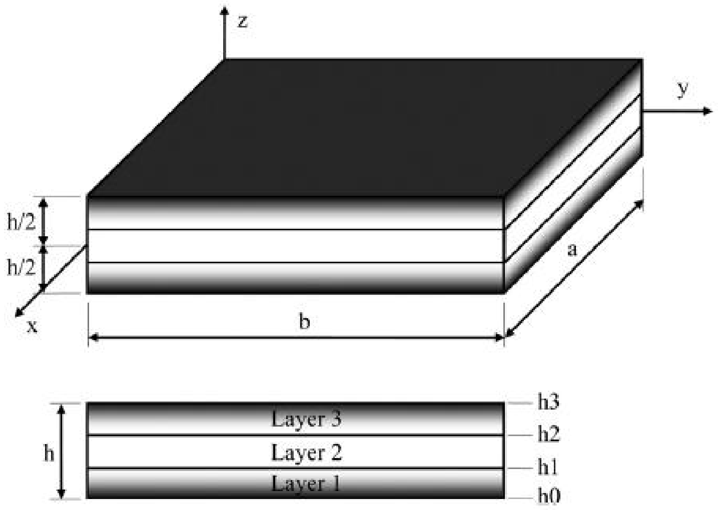


Fig. 1 Geometry and coordinates of rectangular FGM sandwich plate

ceramics and metals is changing from pure metal ($z = h_0 = -h/2$) to pure ceramic while the top skin face changes continuously from pure ceramic surface to pure metal surface ($z = h_3 = +h/2$) as shown in Fig. 1. A simple power law in terms of the volume fraction of the ceramic phase is considered:

$$V^{(1)} = \left(\frac{z - h_0}{h_1 - h_0}\right)^k, \quad z \in [h_0, h_1] \tag{1a}$$

$$V^{(2)} = 1, \qquad z \in [h_1, h_2] \tag{1b}$$

$$V^{(3)} = \left(\frac{z - h_3}{h_2 - h_3}\right)^k, \quad z \in [h_2, h_3] \tag{1c}$$

where $V^{(n)}$, ($n = 1,2,3$) denotes the volume fraction function of layer n; k is the volume fraction index ($0 \le k \le +\infty$), which dictates the material variation profile through the height of plate. The effective material properties, like Young's modulus E, Poisson's ratio ν, and mass density ρ, then can be expressed by the rule of mixture Zenkour (2010), Bourada *et al.* (2012) as follows:

$$P^{(n)}(z) = P_2 + (P_1 - P_2)V^{(n)} \tag{2}$$

where $P^{(n)}$ is the effective material property of FGM of layer n. Where, P_1 and P_2 are the properties of the top and bottom faces of layer 1, respectively, and vice versa for layer 3 depending on the volume fraction $V^{(n)}$, ($n = 1,2,3$).For simplicity, Poisson's ratio of plate is assumed to be constant in this study for that the effect of Poisson's ratio on the deformation is much less than that of Young's modulus Dellal and Erdogan (1983).

2.2 Kinematics and Elastic stress-strain relations

The displacement field of the present theory is formulated based on the following hypotheses: (1) The transverse deflection is superposed into three parts namely: bending, shear and stretching components; (2) the in-plane displacements are superposed also into three parts namely: extension, bending and shear components; (3) the bending components of the in-plane displacements are identical to those used in the classical plate theory (CPT); and (4) the shear parts of the in-plane displacements lead to the hyperbolic variations of shear strains as well as the shear stresses across the thickness of the plate in such a way that the shear stresses becomes zero on the top and bottom surfaces of the plate. Based on these assumptions, the following displacement field relations can be obtained

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+k_1\, f(z)\int\theta(x,y,t)\,dx \tag{3a}$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+k_2\, f(z)\int\theta(x,y,t)\,dy \tag{3b}$$

$$w(x,y,z,t)=w_0(x,y,t)+g(z)\theta(x,y) \tag{3c}$$

The coefficients k_1 and k_2 depends on the geometry. It can be seen that the kinematic in Eq. (3) introduces only four unknowns (u_0, v_0, w_0 and θ).

In this study, the present HSDT is obtained by setting:

$$f(z)=z-\frac{1}{\left[\cosh(\pi/2)-1\right]}\left(\frac{h}{\pi}\sinh\left(\frac{\pi}{h}z\right)-z\right) \qquad g(z)=\frac{df(z)}{dz} \tag{4}$$

The strain-displacement expressions, based on this formulation, are given as follows:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} \tag{5a}$$

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=f'(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}+g(z)\begin{Bmatrix}\gamma_{yz}^1\\ \gamma_{xz}^1\end{Bmatrix}, \tag{5b}$$

$$\varepsilon_z=g'(z)\,\varepsilon_z^0 \tag{5c}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \theta \\ k_2 \theta \\ k_1 \dfrac{\partial}{\partial y}\int \theta \, dx + k_2 \dfrac{\partial}{\partial x}\int \theta \, dy \end{Bmatrix}, \tag{6a}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \int \theta \, dy \\ k_1 \int \theta \, dx \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^1 \\ \gamma_{xz}^1 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \theta}{\partial y} \\ \dfrac{\partial \theta}{\partial x} \end{Bmatrix}, \quad \varepsilon_z^0 = \theta$$

and

$$g'(z) = \frac{dg(z)}{dz} \tag{6b}$$

The integrals presented in the above equations shall be resolved by a Navier type method and can be expressed as follows:

$$\frac{\partial}{\partial y}\int \theta \, dx = A' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x}\int \theta \, dy = B' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \int \theta \, dx = A' \frac{\partial \theta}{\partial x}, \quad \int \theta \, dy = B' \frac{\partial \theta}{\partial y} \tag{7}$$

where the coefficients A' and B' are considered according to the type of solution employed, in this case via Navier method. Therefore, A', B', k_1 and k_2 are expressed as follows:

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = -\alpha^2, \quad k_2 = -\beta^2 \tag{8}$$

where α and β are defined in expression (23).

The linear constitutive relations are given below:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{9}$$

where C_{ij} are the three-dimensional elastic constants defined by

$$C_{11} = C_{22} = C_{33} = \frac{(1-\nu)E(z)}{(1-2\nu)(1+\nu)}, \tag{10a}$$

$$C_{12} = C_{13} = C_{23} = \frac{\nu\, E(z)}{(1-2\nu)(1+\nu)}, \tag{10b}$$

$$C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{10c}$$

2.3 Governing Equations

Hamilton's principle is herein employed to deduce the equations of motion (Bouazza *et al.* 2016 and 2018, Bellifa *et al.* 2017, Attia *et al.* 2018, Bakhadda *et al.* 2018, Bourada *et al.* 2018 and 2019):

$$0 = \int_0^t (\delta\, U + \delta\, V)\, dt \tag{11}$$

Where δU is the variation of strain energy; δV is the variation of the external work done by external load applied to the plate.

The variation of strain energy of the plate is expressed by

$$\begin{aligned}\delta\, U &= \int_V \left[\sigma_x \delta\, \varepsilon_x + \sigma_y \delta\, \varepsilon_y + \sigma_z \delta\, \varepsilon_z + \tau_{xy} \delta\, \gamma_{xy} + \tau_{yz} \delta\, \gamma_{yz} + \tau_{xz} \delta\, \gamma_{xz} \right] dV \\ &= \int_\Omega \Big[N_x\, \delta\, \varepsilon_x^0 + N_y\, \delta\, \varepsilon_y^0 + N_z\, \delta\, \varepsilon_z^0 + N_{xy}\, \delta\, \gamma_{xy}^0 + M_x^b\, \delta\, k_x^b + M_y^b\, \delta\, k_y^b + M_{xy}^b\, \delta\, k_{xy}^b \\ &\quad + M_x^s\, \delta\, k_x^s + M_y^s\, \delta\, k_y^s + M_{xy}^s\, \delta\, k_{xy}^s + Q_{yz}^s\, \delta\, \gamma_{yz}^0 + S_{yz}^s\, \delta\, \gamma_{yz}^1 + Q_{xz}^s\, \delta\, \gamma_{xz}^0 + S_{xz}^s\, \delta\, \gamma_{xz}^1 \Big]\, dA \end{aligned} \tag{12}$$

where Ω is the top surface and the stress resultants N , M , S and Q are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i dz \quad , \quad (i = x, y, xy) \quad ; \quad N_z = \int_{-h/2}^{h/2} g'(z)\sigma_z dz \tag{13a}$$

and

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \quad , \quad \left(Q_{xz}^s, Q_{yz}^s\right) = \int_{-h/2}^{h/2} f'\left(\tau_{xz}, \tau_{yz}\right) dz \tag{13b}$$

The variation of the external work can be written as

$$\delta\, V = -\int_A \overline{N} \delta w dA \tag{14}$$

with

$$\overline{N} = \left[N_x^0 \frac{\partial^2 w}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 w}{\partial x \partial y} + N_y^0 \frac{\partial^2 w}{\partial y^2} \right] \tag{15}$$

Where (N_x^0, N_y^0, N_{xy}^0) are in-plane applied loads.

By substituting Eqs. (12) and (14) into Eq. (11), the following can be derived:

$$\delta\, u_0: \quad \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0$$

$$\delta\, v_0: \quad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0$$

$$\delta\, w_0: \quad \frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x \partial y}+\frac{\partial^2 M_y}{\partial y^2}+N_x^0\frac{\partial^2 w}{\partial x^2}+2N_{xy}^0\frac{\partial^2 w}{\partial x \partial y}+N_y^0\frac{\partial^2 w}{\partial y^2}=0 \tag{16}$$

$$\delta\, \theta: \quad -\mathrm{k}_1 A'\frac{\partial^2 S_x}{\partial x^2}--\mathrm{k}_2 B'\frac{\partial^2 S_y}{\partial y^2}\; -\left(k_1 A'+k_2 B'\right)\frac{\partial^2 S_{xy}}{\partial x \partial y}+k_1 A'\frac{\partial R_{xz}}{\partial x}+k_2 B'\frac{\partial R_{yz}}{\partial y}$$

$$+g(0)\left(N_x^0\frac{\partial^2 w}{\partial x^2}+2N_{xy}^0\frac{\partial^2 w}{\partial x \partial y}+N_y^0\frac{\partial^2 w}{\partial y^2}\right)=\; 0$$

Substituting Eq. (5) into Eq. (9) and the subsequent results into Eqs. (13), the stress resultants are obtained in terms of strains as following compact form:

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix}=\begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix}\begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}+\varepsilon^0\begin{Bmatrix} L \\ L^a \\ R \end{Bmatrix}, \quad \begin{Bmatrix} Q \\ S \end{Bmatrix}=\begin{bmatrix} F^s & X^s \\ X^s & A^s \end{bmatrix}\begin{Bmatrix} \gamma^0 \\ \gamma^1 \end{Bmatrix} \tag{17a}$$

$$N_z=L(\varepsilon_x^0+\varepsilon_y^0)+L^a(k_x^b+k_y^b)+R(k_x^s+k_y^s)+R^a\varepsilon_z^0 \tag{17b}$$

in which

$$N=\left\{N_x, N_y, N_{xy}\right\}^t, \quad M^b=\left\{M_x^b, M_y^b, M_{xy}^b\right\}^t, \quad M^s=\left\{M_x^s, M_y^s, M_{xy}^s\right\}^t \tag{18a}$$

$$\varepsilon=\left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t, \quad k^b=\left\{k_x^b, k_y^b, k_{xy}^b\right\}^t, \quad k^s=\left\{k_x^s, k_y^s, k_{xy}^s\right\}^t \tag{18b}$$

$$A=\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B=\begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D=\begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{18c}$$

$$B^s=\begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s=\begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s=\begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix} \tag{18d}$$

$$Q=\left\{Q_{xz}^{s},Q_{yz}^{s}\right\}^{t},\; S=\left\{S_{xz}^{s},S_{yz}^{s}\right\}^{t},\; \gamma^{0}=\left\{\gamma_{xz}^{0},\gamma_{yz}^{0}\right\}^{t},\; \gamma^{1}=\left\{\gamma_{xz}^{1},\gamma_{yz}^{1}\right\}^{t} \tag{18e}$$

$$F^{s}=\begin{bmatrix}F_{55}^{s} & 0\\ 0 & F_{44}^{s}\end{bmatrix},\; X^{s}=\begin{bmatrix}X_{55}^{s} & 0\\ 0 & X_{44}^{s}\end{bmatrix},\; A^{s}=\begin{bmatrix}A_{55}^{s} & 0\\ 0 & A_{44}^{s}\end{bmatrix} \tag{18f}$$

$$\begin{Bmatrix}L\\ L^{a}\\ R\\ R^{a}\end{Bmatrix}=\int_{z}\lambda(z)\begin{Bmatrix}1\\ z\\ f(z)\\ g'(z)\dfrac{1-\nu}{\nu}\end{Bmatrix}g'(z)dz \tag{18g}$$

and stiffness components are given as:

$$\begin{Bmatrix}A_{11} & B_{11} & D_{11} & B_{11}^{s} & D_{11}^{s} & H_{11}^{s}\\ A_{12} & B_{12} & D_{12} & B_{12}^{s} & D_{12}^{s} & H_{12}^{s}\\ A_{66} & B_{66} & D_{66} & B_{66}^{s} & D_{66}^{s} & H_{66}^{s}\end{Bmatrix}=\int_{z}\lambda(z)\left(1,z,z^{2},f(z),z\,f(z),f^{2}(z)\right)\begin{Bmatrix}\dfrac{1-\nu}{\nu}\\ 1\\ \dfrac{1-2\nu}{2\nu}\end{Bmatrix}dz \tag{19a}$$

$$\left(A_{22},B_{22},D_{22},B_{22}^{s},D_{22}^{s},H_{22}^{s}\right)=\left(A_{11},B_{11},D_{11},B_{11}^{s},D_{11}^{s},H_{11}^{s}\right) \tag{19b}$$

$$\left(F_{44}^{s},X_{44}^{s},A_{44}^{s}\right)=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+\nu)}\left([f'(z)]^{2},f'(z)g(z),g^{2}(z)\right)dz \tag{19c}$$

$$\left(F_{55}^{s},X_{55}^{s},A_{55}^{s}\right)=\left(F_{44}^{s},X_{44}^{s},A_{44}^{s}\right) \tag{19d}$$

By substituting Eq. (18) into Eq. (16), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0, θ) and the appropriate equations take the form:

$$\begin{aligned}&A_{11}d_{11}u_{0}+A_{66}\,d_{22}u_{0}+\left(A_{12}+A_{66}\right)d_{12}v_{0}-B_{11}d_{111}w_{0}-\left(B_{12}+2B_{66}\right)d_{122}w_{0}\\&+\left(B_{66}^{s}\left(k_{1}A'+k_{2}\,B'\right)+B_{12}^{s}k_{2}\,B'\right)\,d_{122}\theta+B_{11}^{s}k_{1}A'\,d_{111}\theta+Ld_{1}\theta=0,\end{aligned} \tag{20a}$$

$$\begin{aligned}&A_{22}\,d_{22}v_{0}+A_{66}\,d_{11}v_{0}+\left(A_{12}+A_{66}\right)\,d_{12}u_{0}-B_{22}\,d_{222}w_{0}-\left(B_{12}+2B_{66}\right)\,d_{112}w_{0}\\&+\left(B_{66}^{s}\left(k_{1}A'+k_{2}\,B'\right)+B_{12}^{s}k_{1}\,A'\right)\,d_{112}\theta+B_{22}^{s}k_{2}\,B'd_{222}\theta+Ld_{2}\theta=0,\end{aligned} \tag{20b}$$

$$
\begin{aligned}
&B_{11}\, d_{111}u_0 + \left(B_{12} + 2B_{66}\right) d_{122}u_0 + \left(B_{12} + 2B_{66}\right) d_{112}v_0 + B_{22}\, d_{222}v_0 - D_{11}d_{1111}w_0 \\
&-2\left(D_{12} + 2D_{66}\right) d_{1122}w_0 - D_{22}\, d_{2222}w_0 + D_{11}^s k_1\, A' d_{1111}\theta \; + \left((D_{12}^s + 2D_{66}^s)\left(k_1A' + k_2B'\right)\right) d_{1122}\theta \\
&+ D_{22}^s k_2 B' d_{2222}\theta + L^a (d_{11}\theta + d_{22}\theta) + N_x^0 d_{11}w + 2\, N_{xy}^0 d_{12}w + N_y^0 d_{22}w = 0,
\end{aligned} \tag{20c}
$$

$$
\begin{aligned}
&-k_1A'B_{11}^s d_{111}u_0 - (B_{12}^s k_2 B' + B_{66}^s (k_1A' + k_2B'))d_{122}u_0 - (B_{12}^s k_1 A' + B_{66}^s (k_1A' + k_2B'))d_{112}v_0 - B_{22}^s k_2 B' d_{222}v_0 \\
&+ D_{11}^s k_1 A' d_{1111}w_0 + ((D_{12}^s + 2\, D_{66}^s)(k_1A' + k_2B'))d_{1122}w_0 + D_{22}^s k_2 B' d_{2222}w_0 - H_{11}^s (k_1A')^2 d_{1111}\theta \\
&- H_{22}^s (k_2B')^2 d_{2222}\theta - (2H_{12}^s k_1k_2A'B' + (k_1A' + k_2B')^2 H_{66}^s) d_{1122}\theta + ((k_1A')^2 F_{55}^s + 2k_1A'X_{55}^s + A_{55}^s) d_{11}\theta \\
&+ ((k_2B')^2 F_{44}^s + 2k_2B'X_{44}^s + A_{44}^s) d_{22}\theta - 2R(k_1A'd_{11}\theta + k_2B'd_{22}\theta) - L(d_1u_0 + d_2v_0) \\
&+ L^a (d_{11}w_0 + d_{22}w_0) - R^a\theta + g(0)(N_x^0 d_{11}w + 2N_{xy}^0 d_{12}w + N_y^0 d_{22}w) = 0
\end{aligned} \tag{20d}
$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators:

$$
d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \quad (i, j, l, m = 1{,}2). \tag{21}
$$

3. Analytical solutions

The Navier solution procedure is employed to determine the analytical solutions for which the displacement variables are expressed as product of arbitrary parameters and known trigonometric functions to respect the equations of motion and boundary conditions.

$$
\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}e^{i\omega t}\cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn}e^{i\omega t}\sin(\alpha\, x)\cos(\beta\, y) \\ W_{mn}e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \\ X_{mn}e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{22}
$$

where ω is the frequency of free vibration of the plate, $\sqrt{i} = -1$ the imaginary unit.
with

$$
\alpha = m\pi / a \text{ and } \beta = n\pi / b \tag{23}
$$

Considering that the plate is subjected to in-plane compressive forces of form: $N_x^0 = -N_0$, $N_y^0 = -\gamma\, N_0$, $N_{xy}^0 = 0$, $\gamma = N_y^0 \big/ N_x^0$ (here γ are non-dimensional load parameter). Substituting Eq. (22) into Eq. (20), the following problem is obtained:

$$
\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33}+k & S_{34}+g(0)k \\ S_{14} & S_{24} & S_{34}+g(0)k & S_{44}+\left[g(0)\right]^2 k \end{bmatrix} \right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{24}
$$

Where

$$S_{11}=A_{11}\alpha^2+A_{66}\beta^2,\quad S_{12}=\alpha\beta\left(A_{12}+A_{66}\right),$$
$$S_{13}=-\alpha\left(B_{11}\alpha^2+(B_{12}+2B_{66})\beta^2\right),$$
$$S_{14}=\alpha\left((k_2B'B^s_{12}+\left(k_1A'+k_2B'\right)B^s_{66})\beta^2+k_1A'B^s_{11}\alpha^2-L\right),$$
$$S_{22}=A_{66}\alpha^2+A_{22}\beta^2$$
$$S_{23}=-\beta\left(B_{22}\beta^2+(B_{12}+2B_{66})\alpha^2\right),$$
$$S_{24}=\beta\left((k_1A'B^s_{12}+\left(k_1A'+k_2B'\right)B^s_{66})\alpha^2+k_2B'B^s_{22}\beta^2-L\right),$$
$$S_{33}=D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4, \tag{25}$$
$$k=-N_0\left(\alpha^2+\gamma\beta^2\right)$$
$$S_{34}=-D^s_{11}k_1A'\alpha^4-\left[(D^s_{12}+2D^s_{66})(k_1A'+k_2B')\right]\alpha^2\beta^2-k_2B'D^s_{22}\beta^4+L^a(\alpha^2+\beta^2)$$
$$\begin{aligned}S_{44}&=H^s_{11}(k_1A')^2\alpha^4+H^s_{22}(k_2B')^2\beta^4\\&+\left[(2k_1k_2A'B')H^s_{12}+(k_1A'+k_2B')^2H^s_{66}\right]\alpha^2\beta^2\\&+\left[(k_1A')^2F^s_{55}+2k_1A'X^s_{55}+A^s_{55}\right]\alpha^2\\&+\left[(k_2B')^2F^s_{44}+2k_2B'X^s_{44}+A^s_{44}\right]\beta^2-2R(k_1A'\alpha^2+k_2B'\beta^2)+R^a\end{aligned}$$

The critical buckling loads (N_{cr}) can be obtained from the stability problem in Eq. (24).

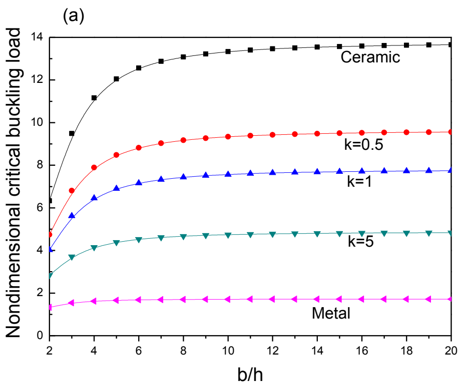
4. Numerical results and discussions

In this section, various numerical examples solved are described and discussed for establishing the efficiency and the accuracy of the present theory for the buckling analysis of FGM sandwich plates. For all the problems a simply supported (diaphragm supported) plate is considered for the analysis. The core material of the present sandwich plate is fully ceramic. The bottom skin varies from a metal-rich surface to a ceramic-rich surface while the top skin face varies from a ceramic-rich surface to a metal-rich surface.

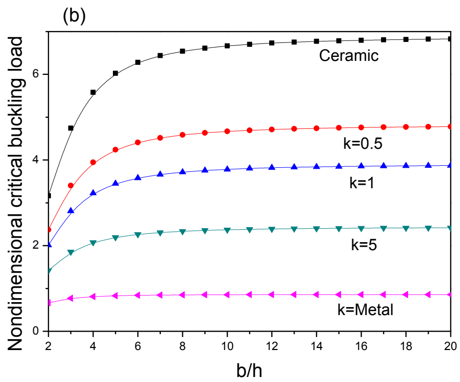
The material properties are $E_m=70E_0$ (Aluminium, Al);and $E_c=380E_0$ (Alumina, Al2O3) being $E_0=1GPa$. Poisson's ratio is $\nu_m=\nu_c=\nu=0.3$.

for both aluminum and alumina. The non-dimensional parameter used is

$$\overline{N}_{cr}=\frac{N_{cr}a^2}{100h^2E_0} \tag{26}$$

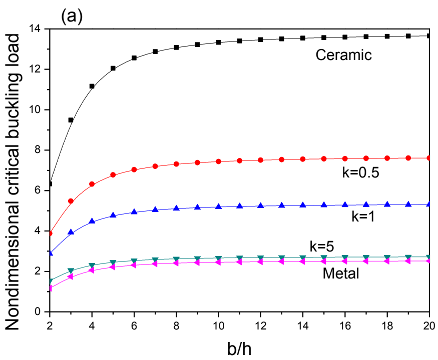


(a) Plate subjected to uniaxial compressive load $(\gamma = 0)$

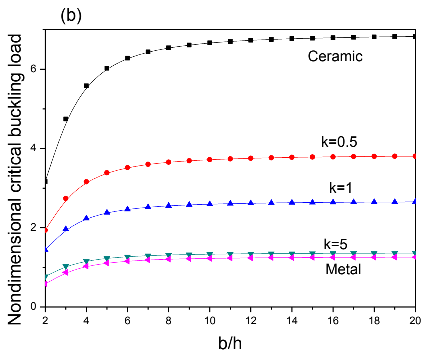


(b) Plate subjected to biaxial compressive load $(\gamma = 1)$

Fig. 2 Nondimensional critical buckling load $(\overline{N_{cr}})$ as a function of side-to-thickness ratio (b/h) of (1-2-1) FGM sandwich plates for various values of k



(a) Plate subjected to uniaxial compressive load $(\gamma = 0)$



(b) Plate subjected to biaxial compressive load $(\gamma = 1)$

Fig. 3 Nondimensional critical buckling load $(\overline{N_{cr}})$ as a function of side-to-thickness ratio (b/h) of (1-0-1) FGM sandwich plates for various values of k

Tables 1 and 2 respectively list the nondimensionalized values of uniaxial and biaxial critical buckling loads in an FGM sandwich plate for various values of power law parameter and thickness of the core with respect to face sheets. They are carried out for four types of plates with different values of the power-law index. The present theory with only four unknowns provides similar results to those predicted by the hyperbolic plate theory (HPT) proposed by Mahmoud and Tounsi (2017) with five unknowns also the results are compared with the quasi-3D hyperbolic sine shear deformation theory of Neves *et al.* (2012). In addition, the results of a third-order shear deformation plate theory (TSDPT) (Zenkour, 2005) and a sinusoidal shear deformation plate theory (SSDPT) (Zenkour 2005) are also provided to show the importance of including the thickness-stretching effect according to these tables the critical buckling loads of functionally

Table 1 Comparison of nondimensional critical buckling load of square FG sandwich plates subjected to uniaxial compressive load (a/h=10).

k	Theory	$\overline{N}_{cr}$			
		1-0-1	2-1-2	1-1-1	1-2-1
	TSDPT (*)	13.00495	13.00495	13.00495	13.00495
	SSDPT (*)	13.00606	13.00606	13.00606	13.00606
0	Neves *et al.* (2012)	12.95304	12.95304	12.95304	12.95304
	Mahmoud and Tounsi (2017)	12.98429	12.98429	12.98429	12.98429
	Present	13.32807	13.32807	13.32807	13.32807
	TSDPT (*)	7.36437	7.94084	8.43645	9.21681
	SSDPT (*)	7.36568	7.94195	8.43712	9.21670
0.5	Neves *et al.* (2012)	7.16191	7.71617	8.19283	8.94221
	Mahmoud and Tounsi (2017)	7.35541	7.93147	8.42681	9.20640
	Present	7.43811	8.02231	8.53000	9.33621
	TSDPT (*)	5.16713	5.84006	6.46474	7.50656
	SSDPT (*)	5.16846	5.84119	6.46539	7.50629
1	Neves *et al.* (2012)	5.06123	5.71125	6.31501	7.32025
	Mahmoud and Tounsi (2017)	5.16191	5.83465	6.45911	7.49996
	Present	5.19181	5.86778	6.50114	7.56568
	TSDPT (*)	2.65821	3.04257	3.57956	4.73469
	SSDPT (*)	2.66006	3.04406	3.58063	4.73488
5	Neves *et al.* (2012)	2.63658	3.00819	3.53014	4.64707
	Mahmoud and Tounsi (2017)	2.65398	3.04023	3.57873	4.73404
	Present	2.66430	3.04222	3.58076	4.74215
	TSDPT (*)	2.48727	2.74632	3.19471	4.27991
	SSDPT (*)	2.48928	2.74844	3.19456	4.38175
10	Neves *et al.* (2012)	2.47199	2.72089	3.15785	4.20550
	Mahmoud and Tounsi (2017)	2.48217	2.74301	3.19359	4.28002
	Present	2.50219	2.74495	3.19447	4.28329

(*) Zenkour (2005)

graded sandwich plate decrease with the increase of the power-law index.

Figures 2 and 3 show the variation of the critical buckling loads of the (1–2–1) and (1–0–1) types of square FG sandwich plates with homogeneous hardcore versus side-to-thickness ratio using the present new simple quasi-3D hyperbolic shear deformation theory. It can be seen that the critical buckling loads become maximum for the ceramic plates and minimum for the metal plates. It is seen that the results increase smoothly as the amount of ceramic in the sandwich plate increases. Also, the buckling load of plate under uniaxial compression is almost the twice of that of the case of the plate under biaxial compression.

For sandwich plate with homogeneous hardcore, It can be also seen, that as the material parameter increases, the critical buckling loads decrease.

Table 2 Comparison of nondimensional critical buckling load of square FG sandwich plates subjected to biaxial compressive load ($\gamma = 1$,h/b=0.1)

k	Theory	$\overline{N}_{cr}$			
		1-0-1	2-1-2	1-1-1	1-2-1
	TSDPT (*)	6.50248	6.50248	6.50248	6.50248
	SSDPT (*)	6.50303	6.50303	6.50303	6.50303
0	Neves *et al.* (2012)	6.47652	6.47652	6.47652	6.47652
	Mahmoud and Tounsi (2017)	6.49215	6.49215	6.49215	6.49215
	Present	6.66404	6.66404	6.66404	6.66404
	TSDPT (*)	3.68219	3.97042	4.21823	4.60841
	SSDPT (*)	3.68284	3.97097	4.21856	4.60835
0.5	Neves *et al.* (2012)	3.58096	3.85809	4.09641	4.47110
	Mahmoud and Tounsi (2017)	3.67770	3.96573	4.21340	4.60320
	Present	3.71905	4.01115	4.26500	4.66810
	TSDPT (*)	2.58357	2.92003	3.23237	3.75328
	SSDPT (*)	2.58423	2.92060	3.23270	3.75314
1	Neves *et al.* (2012)	2.53062	2.85563	3.15750	3.66013
	Mahmoud and Tounsi (2017)	2.58096	2.91732	3.22956	3.74998
	Present	2.59590	2.93389	3.25057	3.78284
	TSDPT (*)	1.32910	1.52129	1.78978	2.36734
	SSDPT (*)	1.33003	1.52203	1.79032	2.36744
5	Neves *et al.* (2012)	1.31829	1.50409	1.76507	2.32354
	Mahmoud and Tounsi (2017)	1.32699	1.52012	1.78936	2.36702
	Present	1.33215	1.52111	1.79038	2.37107
	TSDPT (*)	1.24363	1.37316	1.59736	2.13995
	SSDPT (*)	1.24475	1.37422	1.59728	2.19087
10	Neves *et al.* (2012)	1.23599	1.36044	1.57893	2.10275
	Mahmoud and Tounsi (2017)	1.24109	1.37150	1.5968	2.14001
	Present	1.25110	1.37247	1.59724	2.14173

(*) Zenkour (2005)

5. Summary and conclusions

A new quasi-3D hyperbolic plate theory with stretching effect for the buckling analysis of functionally graded sandwich plates is presented in this paper. The main advantage of this approach is that, in addition to incorporating the thickness stretching effect. The equations of motion are obtained by utilizing the principle of virtual work . It is based on the assumption that the transverse displacements consist of bending and shear components. Results indicate that the present approach is able to introduce the thickness stretching effect and providing very accurate results compared with the other existing higher-order plate theories.

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