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Size-dependent bending and stability analysis of FG nanobeams via a novel simplified first-order shear deformation beam theory

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Abstract. This paper is concerned to investigate the static bending and buckling response of Functionally Graded (FG) nanobeams by employing a new refined first order shear deformation beam theory. The elegancy of this novel theory is that, not only has one variable in terms of equations of motion as in classical beam theory (EBT) but also accounts for the effect of transverse shear deformation without any requirement of Shear Correction Factors (SCFs). The material properties of FG nanobeam are supposed to change gradually across the thickness direction and are evaluated via the power-law model. Nonlocal elasticity theory of Eringen is incorporated in order to capture the small scale effect into current investigation. The nonlocal governing equations of motion and boundary conditions are obtained through Hamilton's principle and they are solved using analytical solution. The obtained results are compared with some cases existing in the literature. Effect of various parameters such as length to thickness ratio, nonlocal parameter and material index on the static and stability behaviors of the FG nanobeam are perused and discussed in detail.

Keywords: FG nanobeam; nonlocal elasticity theory; bending; buckling; novel refined beam model; one variable shear deformation

1. Introduction

Recent rapid advances in the field of nanotechnology especially in the design of miniaturized devices strongly motivated the industrialists to develop and integrate structural elements such as beams and plates at nano or micro length scale. These nanoscale engineering structures show exceptional mechanical, thermal, magnetic and electrical properties, which led to stimulation in modeling of micro/nano scale structures (Heireche *et al*. 2008a, Alizada and Sofiyev 2011, Ebrahimi *et al*. 2016). It is seen that the size effect has a key role on the static and stability behaviors of material in these applications. Nanosize engineering materials have attracted wide interest in modern science and technology since the invention of Carbon Nanotubes (CNTs) by Iijima (1991). These types of nanostructures have important mechanical, thermal and electrical features that are greater

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to the usual structural materials. Lately, nanobeams and CNTs find a lot of use (Pradhan and Mandhal 2013, Wang 2005, Wang and Varadan 2006) such as actuators, sensors, transistors, probes and resonators in several NEMSs. It is well known that the classical continuum theory is very adequate in the mechanical investigation of structures at the macroscopic scales, but its applicability to capture the size dependency on the mechanical behaviors at nano and micro scale structures is very limited, say unachievable. Among techniques that are employed to capture the size effects is the well know molecular dynamic simulations which is regarded as a robust and precise implement to examine structural elements at nanoscale. But even the molecular dynamic simulation at nano scale is computationally demanding for simulation the nanostructures with huge numbers of atoms. So, there is a need for some advanced model that can capture efficiently the length scale effect. One of a new theory the most known and the most used (Eringen 1972, 1983) nonlocal elasticity theory which is the most commonly used in continuum mechanics theory and considers small scale effects with good capacity to simulate nano/micro scale devices and systems. This theory supposes that strain-stress state at every location point is a function of corresponding states of all neighbor points of the continuum body. Therefore, in order to take into account the small scales effects this theory must be considered for best design of structures at nanoscale. To achieve this objective, a number of contributions have been published based on Eringen's nonlocal elasticity theory to examine the size-dependent mechanical response of structural systems (Reddy 2007, Peddieson *et al*. 2003, Lu *et al*. 2009). Besides, some refined shear and normal deformation theories which have been developed recently by several authors were also exploited to study of bending, buckling and vibration of nanobeams and nanoplates, one can cite the following works (Tounsi *et al*. 2013a, Kheroubi *et al*. 2016, Zenkour and Sobhy 2015, Larbi Chat *et al*. 2015, Houari *et al*. 2018, Sekkal *et al*. 2017).

Functionally Graded Materials (FGMs), which are originally created by group of Japanese aerospace researchers (Koizumi 1997), are types of composite materials formed of two or more constituent phases in which material properties diverge smoothly from one surface to the other. Consequently, FGMs are able to avoid high interlaminar shear stresses, stress concentration and delamination cases which are known sometimes as defects of laminated composite materials. A FGM consisting of ceramic and metal possesses superior thermal resistance and enhanced ductility which are inspired from the ceramic and metal phases, respectively. Owning these remarkable features, FGMs are applicable to various fields of engineering including aerospace, nuclear power, chemistry and mechanical engineering. Hence, presenting these novel mechanical features, studying the mechanical responses of structural components made of non-homogenous materials has received a considerable attention by several investigators (Sallai *et al*. 2009, Barati and Shahverdi 2016, Thai and Vo 2012, Tounsi *et al*. 2013a, Zidi *et al*. 2014, 2017, Bachir Bouiadjra *et al*. 2012, 2013, 2018, Bouremana *et al*. 2013, Bourada *et al*. 2012, El Meiche *et al*. 2011, Akavci 2016, Hebali *et al*. 2016, Kar *et al*. 2016, Bensaid *et al*. 2017, El-Haina *et al.* 2017, Ebrahimi and Jafari 2016, Abualnour *et al*. 2018). Recently, FGMs find rising applications in micro- and nano-scale structures such as thin films in thin films in the form of shape memory alloys, Atomic Force Microscopes (AFMs), nano-implants in medical engineering, nanotubes in aircraft wings, nanobeams for spacecraft chassis structures etc. So, studying mechanical properties, deflection, vibration and stability behaviors of them are of significant importance to the design and manufacture of FG MEMS/NEMS, and many papers lately have been published in this context. Janghorban and Zare (2011) explored nonlocal free vibration functionally graded carbon nantubes FG-CNT nanobeams by employing the differential quadrature method. Eltaher *et al*. (2012) investigated the free vibration of FGM nanobeam based on the nonlocal Euler-Bernoulli beam theory by developing a finite element model. Şimşek and Yurtçu (2013) examined the static bending and buckling of FGM nanobeams based on analytical approach and both nonlocal Timoshenko and Euler Bernoulli beam theories. In another work, Şimşek (2014) utilized Eringen's nonlocal elasticity theory and non-classical beam model to investigate nonlinear vibration of nanobeams considering various boundary conditions. Besides a semi-analytical was implemented by Ebrahimi and his coworkers (Ebrahimi *et al*. 2015a, Ebrahimi and Salari 2015a-c, 2016) to study vibration and buckling behaviors of FG nanobeams. Sobhy (2015) researched the bending behavior, free vibration, under both mechanical and thermal buckling of FG nanoplates embedded in an elastic medium by using the four-unknown shear deformation theory in conjunction Eringen's nonlocal elasticity theory. Kolahchi *et al*. (2015) contributed to the bending behavior of FG nanoplates by using a new sinusoidal shear deformation theory. Larbi Chaht *et al*. (2015) studied the static bending and buckling of a FG nanobeam employing the nonlocal sinusoidal shear and normal beam theory. Ebrahimi and Barati (2016) formulated a nonlocal higher order shear deformation beam model for dynamic analysis of size-dependent FG nanobeams. Zemri et *al*. (2015) studied the mechanical response of FG nanosize beam via a refined nonlocal shear deformation beam theory. Ahouel et *al*. (2016) proposed a nonlocal trigonometric beam theory for bending, buckling and vibration of FG nanobeams. They also are including the neutral surface position concept in their investigation. Belkorissat *et al*. (2015) examined the vibration properties of FG nano-plate utilizing a new nonlocal refined four variable theory. Bounouara *et al*. (2016) employed for the first time a nonlocal zeroth-order shear deformation theory for free vibration analysis of FG nanoscale plates resting on elastic foundation. A unified higher order beam theory which contains various beam theories as special cases for buckling of a FG microbeam embedded in elastic Pasternak medium is proposed by Şimşek and Reddy (2013). Recently, Bouafia et *al*. (2017) employed a nonlocal quasi-3D theory to investigate the bending and free flexural vibration behaviors of functionally graded nanobeams.

Recent progress in plate and beam theories led to a new kind of refined Higher order Shear Deformation Theories (HSDTs) with a more reduced number of variables (three and single variables) for the plate and beam theories (Endo 2015, Senjanovic *et al*. 2013, Kiendl *et al*. 2015, Shimpi *et al*. 2017), meaning that they are more less expensive than the classical and refined HSDTs in term of computational cost. Based on this idea a number of investigations have been provided recently. Sayaad and Ghugal (2016) examined the bending, buckling and free vibration of homogenous beams by developing a single variable higher order refined beam theories. Thai *et al*. (2017) proposed a new simple shear deformation theory for isotropic plates, in which the number of unknowns of the presented theory was reduced from two to one as in the classical plate theory. This model has been used in another work (Thai *et al*. 2018) to investigate static bending and free vibration of isotropic nanobeams. A new three unknowns model as the case of the Classical Plate Theory (CPT) was elaborated by Tounsi *et al*. (2016) and Houari *et al*. (2016) for static, buckling and vibration analysis of both functionally graded and sandwich plates. Zidi *et al*. (2017) presented a novel simple two-unknown higher-order hyperbolic shear deformation theory for bending and dynamic behaviors of functionally graded beams. Nguyen *et al*. (2017) developed a generalized formulation of three-variable plate theory and an efficient computational approach based on IGA for analyzing functionally graded plates. Kaci *et al*. (2018) developed an exact analytical solution post-buckling analysis of shear-deformable composite beams based on a novel simple two-unknown beam theory.

The purpose of this study is to employ a new recently developed first order shear deformation beam theory for bending and buckling of FGM nanobeams. Just one unknown displacement function is used in the present refined model against three unknown displacement functions used in the existing ones. The material properties of the FG nanobeam are supposed to be graded in the thickness direction according to the power law variation. In order to capture size effect, Eringen's nonlocal

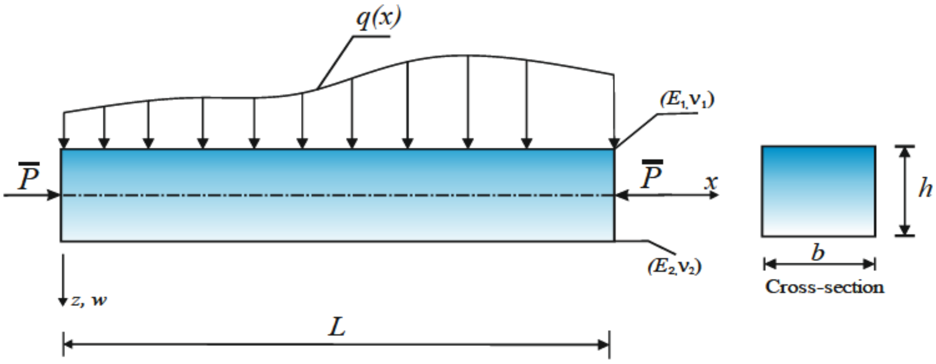

Fig. 1 Geometry and coordinates of functionally graded nano beam

elasticity theory is incorporated in the present investigation. Governing equations and associated boundary conditions for the bending and buckling of a nonlocal FG nanobeam have been derived via principal of the minimum total potential energy. These equations are solved by employing an analytical method and numerical solutions are obtained. Exactness of the results is examined using available date in the literature. The effects of small scale parameter, material graduation, and aspect ratio on the bending, buckling and vibration responses of FGM nanobeam are investigated.

2. Mathematical formulations

2.1 The material properties of FG nanobeams

One of the most constructive models for FGMs is the power-law model, in which material properties of FGMs are supposed to change according to a power law about spatial coordinates. The coordinate system for FG nano beam having a length L, width b and thickness h, is shown in Fig. 1. The FG nanobeam is assumed to be combination of ceramic and metal and material properties of the FGM nanobeam, such as Young's modulus (E), Poisson's ratio (v), and the shear modulus (G), vary continuously through the nanobeam thickness according to a power-law form (Şimşek and Yurtçu 2013, Ahouel *et al*. 2015, Bensaid *et al*. 2017), which can be described by

$$P(z) = P_m + (P_c - P_m)\left(\frac{1}{2} + \frac{z}{h}\right)^k \tag{1}$$

Here P_c and P_m are the corresponding material property at the top and bottom surfaces of the FG nanobeam. And k is the power law index which takes the value greater or equal to zero and determines the material distribution through the thickness of the beam.

2.2 Kinematics relations for present model

Based on the Novel One variable First-order Shear Deformation beam Theory (NOFSDT), the displacement field is presented as below (Malikan and Dastjerdi 2018, Malikan and Nguyen 2018, Malikan *et al*. 2019, Abdelbari *et al*. 2018).

$$u_1(x,z) = u(x) - z\frac{dw_b(x)}{dx} \tag{2a}$$

$$u_3(x,z) = w_b(x) + W' \tag{2b}$$

By assuming that the material of FG beam obeys Hooke's law, the stresses in the beam become

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{xz} \end{Bmatrix} = \begin{Bmatrix} E(z)\varepsilon_{xx} \\ 2G(z)\gamma_{xz} \end{Bmatrix} \tag{3}$$

After having Eq. (3) from S-FSDT the stresses can be found and then by substituting Eq. (3) in the S-FSDT stress resultants, Eq. (4) will be derived as

$$\begin{Bmatrix} M_x \\ Q_x \end{Bmatrix} = \int_A \begin{Bmatrix} \sigma_x z \\ \sigma_{xz} \end{Bmatrix} dA \tag{4}$$

Now applying the fourth equation of FSDT's governing equations in order to calculate w_s based on w_b.

$$\frac{M_x}{dx} - Q_x = 0 \tag{5}$$

By substituting Eq. (5) into the stress resultants of Eq. (4) we will get

$$E(z)I_c\frac{d^3w_b}{dx^3} - AG(z)\frac{dw_s}{dx} = 0 \tag{6}$$

Integrating from Eq. (6) based on x, simplifying and then passing over the integral constant terms, the shear deflection will now be obtained as follows.

$$w_s = W' = B\frac{d^2w_b}{dx^2} \tag{7}$$

The expression of the term B could be in both positive and negative sign that is explained

$$B = \frac{E(z)I_c}{AG}, \qquad G(z) = \frac{E(z)}{2(1+\nu)}$$

in which G represents the shear modulus, E is the Young's modulus, I_c ($b \times h^3/12$) denotes the moment of area of the cross-section, A is the cross-sectional area and v is the Poisson's ratio for isotropic nanobeams. Hereafter, the new beam theory will now be performed as

$$\text{at present: } w_b = w; \begin{Bmatrix} u_1(x,z) \\ u_3(x,z) \end{Bmatrix} = \begin{Bmatrix} u(x) - z\frac{dw(x)}{dx} \\ w(x) + B\frac{d^2w(x)}{dx^2} \end{Bmatrix} \tag{8}$$

Based on the total potential energy rule, the principal of the minimum total potential energy in static deflection and stability forms of the whole domain Π is employed in the variational case, and can be use as follow (Şimşek 2012, Larbi Chaht *et al*. 2015)

$$\delta\prod = \delta(Uext_{int}()) \tag{9}$$

in which Π is the total potential energy. δU_{int} is the virtual variation of the strain energy; and δW_{ext} is the variation of work induced by external forces. The first variation of the strain energy is given as

$$\delta U \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_x \delta \varepsilon_x \quad_{int} \tag{10}$$

The associated nonzero strains in Eq. (10) of the current beam model are expressed as Malikan and Dastjerdi (2018).

$$\begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{du}{dx} - z\dfrac{d^2w}{dx^3} + \dfrac{1}{2}\left(B\dfrac{d^3w}{dx^3} + \dfrac{dw}{dx}\right)^2 \\ B\dfrac{d^3w}{dx^3} \end{Bmatrix} \tag{11}$$

The variation of the work by the applied loads can be written as

$$\delta W = \int_0^L q\delta w dx + N_x \frac{dw}{dw}\frac{d\delta w}{dx} dx \tag{12}$$

where (q) and (N_x) are the transverse and axial loads, respectively.

Substituting the expressions for (δU) and (δV) from Eqs. (10) and (11) into Eq. (9) and making some mathematical manipulations, and isolating the coefficient of (δw), the following equation of motion of the novel proposed beam theory are obtained

$$\delta w = 0; \frac{d^2M_x}{dx^2} - B\frac{d^3Q_x}{dx^3} - N_x\left(B^2\frac{d^6w}{dx^6} + 2B\frac{d^4w}{dx^4} + \frac{d^2w}{dx^2}\right) = q_0 \tag{13}$$

in which (N_x), (M_x) and (Q_x) are the nonlocal stress resultants, defined as

$$\begin{Bmatrix} M_x \\ Q_x \end{Bmatrix} = \int_A \begin{Bmatrix} \sigma_x z \\ \sigma_{xz} \end{Bmatrix} dA \tag{14}$$

By substituting Eq. (11) into the Eq. (14) the stress resultants will be given as follows.

$$\begin{Bmatrix} M_x \\ Q_x \end{Bmatrix} = \begin{Bmatrix} -EI_c\dfrac{d^2w}{dx^2} \\ AGB\dfrac{d^3w}{dx^3} \end{Bmatrix} \tag{15}$$

Contrary to the classical (local) theory, in the nonlocal elasticity theory of Eringen (1972, 1983), the stress at a reference point x is considered to be a functional of the strain field at every point in the body. For example, in the non-local elasticity, the constitutive equation of strain-driven nonlocal elasticity is expressed as elasticity Eringen (1972, 1983).

$$(1 - \mu\nabla^2)\sigma_{ij} = C_{ijkl}\varepsilon_{kl}; \mu(nm^2) = (e_0a)^2, \qquad \nabla^2 = \frac{d^2}{dx^2} \tag{16}$$

$\mu = (e_0a)^2$ is a nonlocal parameter revealing the nanoscale effect on the response of nanobeams, e_0 is a constant appropriate to each material and a is an internal characteristic length. In general, a conservative estimate of the nonlocal parameter is $e_0a < 2.0$ nm for a single wall carbon nanotube (Wang 2005, Heireche *et al*. 2008a, b, Tounsi *et al*. 2013b, c, Bensaid 2017).

The material properties of the FG nano beam are supposed to be graded in the thickness direction, and are expressed according to a power law model about spatial coordinates as Bensaid and Bekhadda (2018), Zemri *et al*. (2015)

$$E(z) = E_m + (E_c - E_m)\left(\frac{1}{2} + \frac{z}{h}\right)^k \tag{17}$$

where E_c and E_m are the Young's modulus corresponding of the ceramic and metal, respectively, and k is the power-law exponent which determines the material distribution through the thickness of the beam. Due to unimportant difference of the Poisson's ratio, this variation is supposed to be constant along the thickness ($\nu(z) = \nu$). Starting of Eq. (17), when $k \to \infty$, the FG nanobeam reduces to a pure metal one and for case $k = 0$, the nanobeam looks as pure ceramic.

Now, by substituting Eqs. (15)-(16) into general Eq. (13), and also doing some mathematical manipulation, lead to the static and stability equations of the One Variable First-order Shear Deformation Theory (OVFSDT) as

$$\begin{aligned}\delta w = 0: EI_c\frac{dw^4}{dx^4} + B^2 AG\frac{d^6 w}{dx^6} - N_0\left(B^2\frac{d^6 w}{dx^6} + 2B\frac{d^4 w}{dx^4} + \frac{d^2 w}{dx^2}\right)\\ + \mu N_0\left(B^2\frac{d^8 w}{dx^8} + 2B\frac{d^6 w}{dx^6} + \frac{d^4 w}{dx^4}\right) = 0\end{aligned} \tag{18}$$

In addition from the paper of Bouremana *et al*. (2013), the S-FSDT equations could be obtained as follows.

$$\delta w_b = 0: EI_c\frac{d^4 w_b}{dx^4} - N_0\left(\frac{d^2 w_b}{dx^2} + \frac{d^2 w_s}{dx^2}\right) + \mu N_0\left(\frac{d^4 w_b}{dx^4} + \frac{d^4 w_s}{dx^4}\right) = 0 \tag{19a}$$

$$\delta w_s = 0: AG\frac{d^2 w_s}{dx^2} - N_0\left(\frac{d^2 w_b}{dx^2} + \frac{d^2 w_s}{dx^2}\right) + \mu N_0\left(\frac{d^4 w_b}{dx^4} + \frac{d^4 w_s}{dx^4}\right) = 0 \tag{19b}$$

On the other hand, the conventional FSDT equations could be obtained as follows (Simsek and Yurtçu 2013)

$$\delta w = 0: k_s AG\left(\frac{d^2 w}{dx^2} - \frac{d\varphi}{dx}\right) - N_0\frac{d^2 w}{dx^2} + \mu N_0\frac{d^4 w}{dx^4} \tag{20a}$$

$$\delta\varphi = 0: EI_c\frac{d^2\varphi}{dx^2} + k_s AG\left(\frac{dw}{dx} - \varphi\right) = 0 \tag{20b}$$

Furthermore, for CBT the bending stability equation is obtained in the following form.

$$\delta w = 0: EI_c\frac{d^4 w}{dx^4} - N_0\frac{d^2 w}{dx^2} + \mu N_0\frac{d^4 w}{dx^4} = 0 \tag{21}$$

3. Closed-form solution of simply supported FG nanobeam

This part is concerned to propose analytical solutions for solving the above nonlocal governing equations of motion, in the case of bending and buckling problems. The Navier solution technique is used to determine the analytical solutions for a simply supported FG nanobeam. The solution is assumed to be of the form

$$\begin{Bmatrix} w \\ \varphi \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} W_n \sin(\alpha x) e^{i\omega t} \\ \varphi_n \sin(\alpha x) e^{i\omega t} \end{Bmatrix} \tag{22}$$

in which U_n, W_n and φ_n are arbitrary parameters to be determined, ω is the eigenfrequency associated with nth eigenmode and $\alpha = n\pi/L$. The transverse load q is also expanded in the Fourier sine series as

$$\begin{gathered} q(x) = \sum_{n=1}^{\infty} Q_n \sin(\alpha x) \\ Q_n = \frac{2}{L}\int_0^L q(x)\sin(\alpha x)\,dx \end{gathered} \tag{23}$$

and for uniform load Q_n is expressed as

$$Q_n = \frac{4q_0}{n\pi}, \qquad n = 1,3,5\ldots. \tag{24}$$

Substituting the expansions of w, φ and q from Eqs. (27) and (28) into Eqs. (26), (25), (24), (23) the analytical solutions can be obtained from the following equations.

• OFSDT

$$W_n = \frac{\lambda q}{EI_c\alpha^4 - BAG\alpha^6} \tag{25a}$$

$$P_{cr} = \frac{EI_c\alpha^4 - \frac{(EI_c)^2}{AG}\alpha^6}{\left(\frac{EI_c}{AG}\right)^2\alpha^6 - \frac{2EI_c}{AG}\alpha^4 + \alpha^2 + \mu\left(\left(\frac{EI_c}{AG}\right)^2\alpha^8 - \frac{2EI_c}{AG}\alpha^6 + \alpha^4\right)} \tag{25b}$$

• EBT

$$W_n = \frac{\lambda q}{\alpha^4 EI_c} \tag{26a}$$

$$P_{cr} = \frac{EI_c\alpha^4}{\alpha^2 + \mu\alpha^4} \tag{26b}$$

• SFSDT

$$\begin{bmatrix} EI_c\alpha^2 + P_{cr}\alpha^2 + \mu P_{cr}\alpha^4 & P_{cr}\alpha^2 + \mu P_{cr}\alpha^4 \\ P_{cr}\alpha^2 + \mu P_{cr}\alpha^4 & -AG\alpha^2 + P_{cr}\alpha^2 + \mu P_{cr}\alpha^4 \end{bmatrix} \begin{Bmatrix} w_b \\ w_s \end{Bmatrix} = \begin{Bmatrix} \lambda q \\ \lambda q \end{Bmatrix} \tag{27}$$

• FSDT

$$\begin{bmatrix} -k_sAG\alpha^2 + P_{cr}\alpha^2 + \mu P_{cr}\alpha^4 & k_sAG\alpha \\ k_sAG\alpha & -EI_c\alpha^2 - k_sAG \end{bmatrix} \begin{Bmatrix} w \\ \varphi \end{Bmatrix} = \begin{Bmatrix} \lambda q \\ 0 \end{Bmatrix} \tag{28}$$

By making the determinant of coefficients of the above systems of Eqs. (27) and (28) equal to

Table 1 Nondimensional static deflection $\overline{w}$ of the FG nanobeam

μ	OFSDT	SFDT	TBT	RBT	OFSDT	SFDT	TBT	RBT
		$p = 0$				$p = 0.3$		
0	6.4609	6.4609	6.4867	6.4865	3.8178	3.8178	3.9090	3.9102
0.5	5.8252	5.8252	5.8487	5.8485	3.4421	3.4421	3.5245	3.5254
1	5.4438	5.4438	5.4659	5.4659	3.2167	3.2167	3.2938	3.2946
1.5	5.3166	5.3166	5.3383	5.3381	3.1416	3.1416	3.2169	3.2178
		$p = 1$				$p = 10$		
0	2.5843	2.5843	2.9401	2.9401	1.7334	1.7334	1.9190	1.9190
0.5	2.3300	2.3300	2.6508	2.6508	1.5628	1.5628	1.7310	1.7301
1	2.1775	2.1775	2.4772	2.4772	1.4605	1.4605	1.6176	1.6169
1.5	2.1216	2.1216	2.4194	2.4194	1.4264	1.4264	1.5799	1.5790

TBT: Şimşek and Yurtçu (2013); RBT: Larbi Chat *et al*. (2015)

Table 2 Nondimensional critical buckling load ($\overline{N}$) of the FG nanobeam

μ	OFSDT	SFDT	TBT	RBT	OFSDT	SFDT	TBT	RBT
		$p = 0$				$p = 0.3$		
0	2.5213	2.0631	1.9685	1.9682	3.4915	3.4915	3.2667	3.2655
0.5	2.0631	2.2948	2.1895	2.1892	3.8835	3.8835	3.6335	3.6322
1	2.2948	2.4606	2.3477	2.3473	4.1640	4.1640	3.8959	3.8945
1.5	2.4606	2.5213	2.4056	2.4052	4.2668	4.2868	3.9921	3.9906
		$p = 1$				$p = 10$		
0	5.1578	5.1578	4.3437	4.3440	7.6899	7.6899	6.6518	6.6558
0.5	5.7370	5.7370	4.8315	4.8317	8.5534	8.5534	7.3989	7.4031
1	6.1515	6.1515	5.1805	5.1808	9.1713	9.1713	7.9332	7.9379
1.5	6.3032	6.3032	5.3084	5.3086	9.3976	9.3976	8.1338	8.1397

TBT: Şimşek and Yurtçu (2013); RBT: Larbi Chat *et al*. (2015)

zero, by setting the obtained polynomial to zero, we can find the critical buckling load of S-FSDT and FSDT.

4. Results and discussion

This section is concerned to check the accuracy and reliability of this novel approach for the size-dependent static and stability responses of FG nanobeams based nonlocal on a one variable FSDT (OVFSDT) beam model. Computations have been implemented for the following material and beam properties: E_1 = 1 TPa, E_2 = 0.25 TPa, $v_1 = v_2 = 0.3$. The shear correction factor is taken as $ks = 5/6$ for Timoshenko beam theory (Şimşek and Yurtçu 2013, Larbi Chaht *et al*. 2015). For convenience, the following dimensionless amounts are used in presenting the numerical results in graphical and tabular forms.

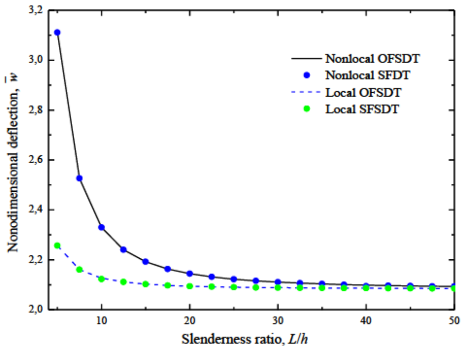

Fig. 2 Effect of the length to thickness ratio on dimensionless deflection for uniform load for k = 1, e_0a = 1 nm

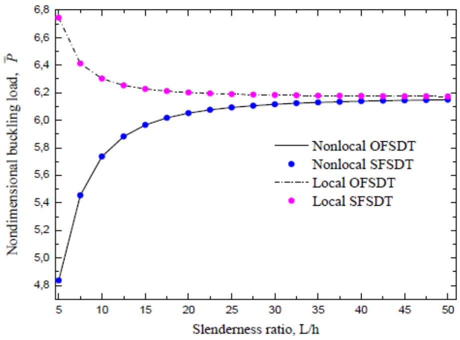

Fig. 3 Effect of the length to thickness ratio on dimensionless buckling load for k = 1, e_0a = 1 nm

$$\overline{w} = 100w\frac{E_1 I}{q_0 L^4} \qquad \text{for uniform load} \tag{29}$$

$$P_{cr} = \frac{\overline{P}L^2}{E_1 I} \tag{30}$$

For the validation purpose, the non-dimensional maximum deflection and buckling loads of the simply supported FG nanobeam with various nonlocal parameters, length to thickness ratios and power-law exponents are compared with the results reported by Şimşek and Yurtçu (2013) and Larbi Chat *et al*. (2015) without stretching effect which have been formulated by analytical models for FG Timoshenko and refined higher order nanobeams, respectively. One can see from Tables 1 and 2 that, there is a very good agreement between the obtained results and predictions from the literature confirms the high correctness of the proposed theory. The minor difference between the results obtained by the present model and the other ones is due to that, there is no shear correction factor and the present model generates just one variable governing equation of motion.

The variations of the nondimensional maximum deflections $\overline{w}$of the simply supported FG nanobeams with various values of the gradient index (k = 0, 0.3, 1, 10), nonlocal parameters (μ = 0, 0.5, 1, 1.5, 2 (nm)^2) and two different values of slenderness ratios (L/h = 10, 30) are exhibited in Table 1 based on the novel OFSDT. It is mentioned that when e_0a vanish corresponds to local beam theory. It can be concluded that the results of the present beam theory based on one variable shear

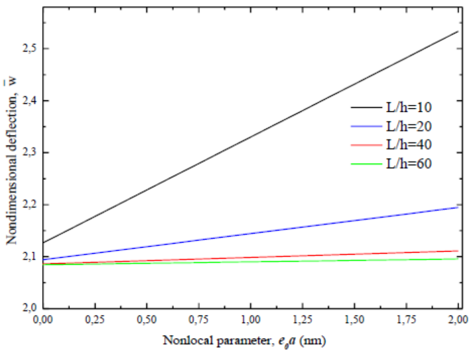

Fig. 4 Effect of nonlocal parameter on dimensionless deflection under uniform load for k = 1

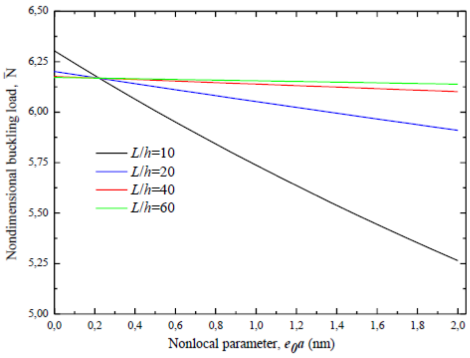

Fig. 5 Effect of nonlocal parameter on nondimensional buckling load for k = 1

deformation theory match well with those predicted by TBT (Şimşek and Yurtçu 2013) and Larbi Chat *et al*. (2015) with ($\varepsilon_z = 0$) for all values of thickness ratio L/h, power law index k and nonlocal parameter e_0a and thus confirms the proposed refined model. A variation of the material distribution parameter k leads to a significant change in the maximum deflection. On other hand, one can also say that an increase in the nonlocal parameter gives rise to an increase in the maximum deflection, which highlights the significance of the nonlocal effect.

Table 2 displays the variation of the nondimensional critical buckling loads for diverse values of thickness ratio L/h, gradient indexes k and nonlocal parameter e_0a, based on the new refined model for FG nanobeam. As can be noted also, that the obtained results are in good concordance with the results provided in the literature those of Şimşek and Yurtçu (2013) and Larbi Chat *et al*. (2015) without stretching effect again. It is seen, that the critical buckling load decreases as the nonlocal parameter rises. This emphasizes the significance of the nonlocal effect on the buckling response of beams, because the nonlocal parameter softens the nanobeam. By varying the material distribution parameter k leads to a decrement in the buckling load, because diminishing in ceramics phase constituent, and hence, stiffness of the beam.

Figs. 2 and 3 display the variation of static and buckling responses of FG nanobeam versus length to thickness ratio for two models of shear deformation namely OFSDT and SFDT. The illustrated are obtained for local and nonlocal are given by fixing the following values at e_0a = 0, e_0a = 1 nm and k = 1 for material distribution profile. We can see from these curves that the proposed OFSDT is in good in good agreement with the SFDT which highlights the effectiveness of the

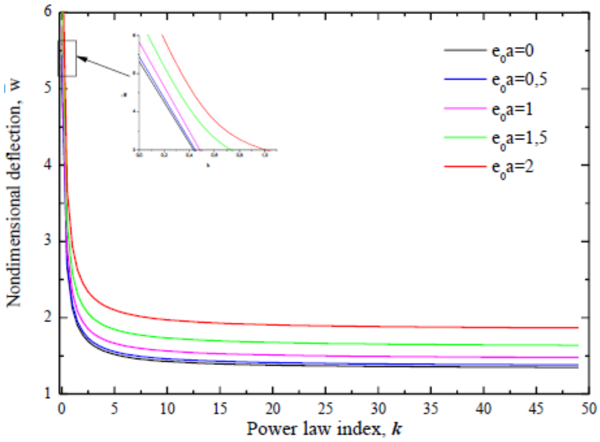

Fig. 6 Variation of the nondimensional deflection of S-S FGM nanobeam with material graduation and for different scale parameter rises (L/h = 10)

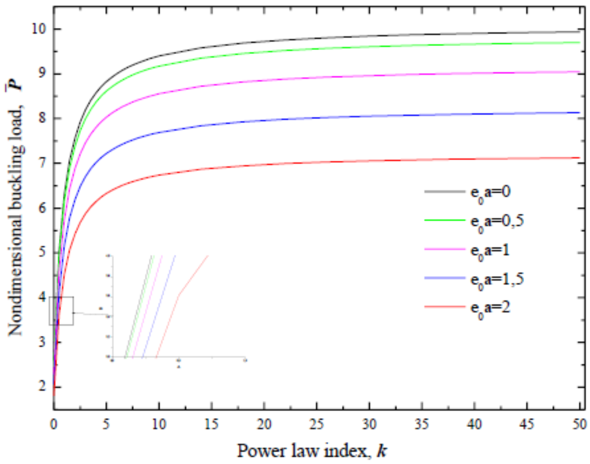

Fig. 7 Variation of the nondimensional buckling load of S-S FGM nanobeam with material graduation and for different scale parameter rises (L/h = 10)

proposed approach. Furthermore, it can be observed from the results of the figures as slenderness ratios increase, the deflection decreases and the non-dimensional buckling load increase, these influences are more important for lower values of thickness ratio (L/h), and this impact is too small for long FG nanobeams. In addition, it is noted from the obtained values of non-dimensional deflections obtained by the nonlocal model are greater to those determined by the local (classical) continuum theory, whereas for the nonlocal buckling load, the results are smaller in magnitude than the local buckling load due to stiffness softening phenomena induced by the small scale parameter.

Figs. 4 and 5 illustrate the influence of the nonlocal scale parameter on non-dimensional static deflection and buckling responses of FG nanobeam based on the new proposed nonlocal OFSD beam model for various values of geometrical report L/h. The material gradation profile is presumed to be unvarying (i.e., k = 1). These figures show that a rise in nonlocal parameter leads to an increment in transverse displacement and a decrement in the critical buckling load, which indicates the notability of the nonlocal effect. In addition, the responses vary in a linear way with the nonlocal parameter and take large values especially at relatively higher aspect ratios.

The maximum deflection and buckling load as functions of power law exponents with varying power law exponent are showed in Figs. 6-7 for fixed length to thickness ratio (L/h = 10). It is seen from these figures that the dimensionless deflection of FG nanobeam decreases and the buckling load increase with the raise of power exponent. This change takes a high rate for $0 < k < 5$ and then keeps to turn down with a low rate for $5 < k < 15$, and it can be conclude that for $k > 5$ material distribution profile has no sensible effect on the both static and stability of FG nanobeam. Also, it is observed that a variation of nonlocal parameter (μ = 0, 0.5, 1, 1.5, 2) yields to an increase and decreases into nondimensional deflection and buckling loads, respectively. Which highlight the impact of nonlocal parameter on both results, due to the softening effect.

5. Conclusions

This article has been devoted to study the static bending and stability of functionally graded nanobeams subjected to the both of transverse and axial compressive loads, based on a new first order beam approach. The highlight of this model is that, in addition to including the transverse shear strain effect, the displacement field is modeled with only one unknown, which is even less than the other shear deformation theories where we find usually three or more variables. Material distribution characteristics of FG nanobeam are supposed to change gradually along the thickness according to the power-law type. By employing the principal of the minimum total potential energy and the nonlocal differential constitutive relation of Eringen, the governing equations of motion are extracted and Navier's type solution procedure is used to solve the obtained governing equations. The obtained results based on the present OVFSDT are checked and validated with those predicted by the prior works to confirm the accuracy of the present model. At last, a comprehensive parametric study was done, and the numerical results show the significant influences of several parameters such as, nonlocal parameter, material property gradient exponent and length to thickness ratio on deflection and critical buckling loads of FG nanobeams.

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