Advances in Materials Research, *Vol. 11, No. 4* (*2022*) *279-298*
https://doi.org/10.12989/amr.2022.11.4.279

Buckling and bending analyses of a sandwich beam based on nonlocal stress-strain elasticity theory with porous core and functionally graded facesheets

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(*Received September 12, 2021, Revised January 8, 2022, Accepted January 20, 2022*)

Abstract. In this paper, the important novelty and the defining a physical phenomenon of the resent research is the development of nonlocal stress and strain parameters on the porous sandwich beam with functionally graded materials in the top and bottom face sheets. Also, various beam models including Euler-Bernoulli, Reddy and the generalized formulation of two-variable beam theories are obtained in this research. According to a nonlocal strain elasticity theory, the strain at a reference point in the body is dependent not only on the stress state at that point, but also on the stress state at all of the points throughout the body. Thus, the nonlocal stress-strain elasticity theory is defined that can be actual at micro/nano scales. It can be seen that the critical buckling load and transverse deflection of sandwich beam by considering both nonlocal stress-strain parameters is higher than the nonlocal stress parameter. On the other hands, it is noted that by considering the nonlocal stress-strain parameters simultaneously becomes the actual case.

Keywords: a nonlocal stress-strain elasticity theory; bending and buckling analysis; functionally graded facesheets; porous core; sandwich beam

1. Introduction

Sandwich structures are made of three layers including flexible core layer and two facesheets layers. These structures are used due to high strength to weight ratio in various industries including aerospace, automobile, mechanics and civil structures.

The classical theory (CT) discovered in 1660. This theory stated that the stress at a reference point in the body is dependent on the strain state at that point.

The nonlocal elasticity theory (the nonlocal stress elasticity theory) was first presented by Eringen's (1972) & (1983). According to nonlocal stress elasticity theory, the stress at a reference point in the body is dependent not only on the strain state at that point, but also on the strain state at all of the points throughout the body. Polizzotto (2001) presented nonlocal elasticity and related variational principles. Based on nonlocal elasticity theory (NET), Reddy (2007) considered bending, buckling and vibration of various beam theories including the Euler–Bernoulli, Timoshenko, Reddy, Levinson-Reddy. Yang *et al*. (2008) illustrated pull-in instability of nano-switches subjected to an electrostatic force due to an applied voltage using NET. Ghorbanpour *et al*. (2009) considered

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http://www.techno-press.org/?journal=amr&subpage=5 ISSN: 2234-0912 (Print), 2234-179X (Online)

buckling analysis of a double-walled carbon nanotube embedded in an elastic medium using the energy method. Pradhan (2009) studied buckling of single layer graphene sheet based on nonlocal elasticity and higher order shear deformation theory. Murmu and Adhikari (2010) employed transverse vibration of double-nanobeam-systems based on NET. Mohammadimehr *et al*. (2010) investigated torsional buckling of a double-walled carbon nanotubes (DWCNTs) embedded on Pasternak foundations using NET. Narendar (2011) presented buckling analysis of micro-/nano-scale plates based on two-variable refined plate theory incorporating nonlocal scale effects. Wang and Wang (2011) studied vibration of nanoscale plates with surface energy via nonlocal elasticity. Zenkour and Sobhy (2013) illustrated thermal buckling of nano-plates lying on Pasternak elastic substrate medium using NET. Eltaher *et al*. (2016) considered a review on nonlocal elastic models for bending, buckling, vibrations, and wave propagation of nanoscale beams.

There are various theories to consider size dependent effect at micro/nano scales. One of important size dependent effect is a modified couple stress theory (MCST) that proposed by Yang *et al*. (2002) for first time. Jomehzadeh *et al*. (2011) considered the size-dependent vibration analysis of micro-plates using MCST. They investigated the effect of material length scale parameter on the natural frequency. Al-Basyouni *et al*. (2015) presented the size dependent effect on the bending and vibration analysis of functionally graded micro beams using MCST and neutral surface position.

Babaei and Eslami (2019) depicted thermally induced large deflection of FGM shallow micro-arches with integrated surface piezoelectric layers based on MCST. Gao (2015) studied a new Timoshenko beam model incorporating microstructure and surface energy effects.

The other size dependent effect such as modified strain gradient theory (MSGT) (proposed by Lam *et al*. (2003) for first time; Ansari *et al*. (2013), Thai *et al*. (2018), Li *et al*. (2020)), most general strain gradient theory (MGSGT) (Ansari *et al*. 2013, Shooshtari and Razavi 2015) and nonlocal strain gradient theory (NSGT) (Ebrahimi *et al*. 2016, Karami and Shahsavari 2020, Ghayesh *et al*. 2020). Yazdani *et al*. (2019) considered free vibration of Cooper-Naghdi micro saturated porous sandwich cylindrical shells with reinforced CNT face sheets under combined loadings including magneto-hydro-thermo-mechanical loadings. Alavi and Eipakchi (2019) investigated geometry and load effects on transient response of a VFGM annular plate using an analytical approach. Ebrahimi *et al*. (2019) presented wave dispersion characteristics of porous graphene platelet-reinforced composite shells. Mohammadimehr *et al*. (2018) illustrated buckling and vibration analyses of MGSGT double-bonded micro composite sandwich SSDT plates reinforced by CNTs and BNNTs with isotropic foam & flexible transversely orthotropic cores. In the other work, Mohammadimehr and Alimirzaei (2016) presented nonlinear static and vibration analysis of Euler-Bernoulli composite beam model reinforced by FG-SWCNT with initial geometrical imperfection using FEM. Yazdani and Mohammadimehr (2019) showed double bonded Cooper-Naghdi micro sandwich cylindrical shells with porous core and CNTRC face sheets: Wave propagation solution. Bendenia *et al*. (2020) presented deflections, stresses and free vibration studies of FG-CNT reinforced sandwich plates resting on Pasternak elastic foundation. Daikh *et al*. (2020) considered static bending of multilayered carbon nanotube-reinforced composite plates. Altekin (2020) investigated combined effects of material properties and boundary conditions on the large deflection bending analysis of circular plates on a nonlinear elastic foundation. Nejadi and Mohammadimehr (2020) illustrated analysis of a functionally graded nanocomposite sandwich beam considering porosity distribution on variable elastic foundation using DQM: Buckling and vibration behaviors.

Rahi *et al*. (2021) presented a simplified numerical method for nonlocal static and dynamic analysis of a graphene nanoplate. Canbay *et al*. (2021) studied thermostructural shape memory effect observations of ductile Cu-Al-Mn smart alloy. Rabia *et al*. (2020) considered predictions of the maximum plate end stresses of imperfect FRP strengthened RC beams: study and analysis. Namayandeh *et al*. (2020) studied temperature and thermal stress distributions in a hollow circular cylinder composed of anisotropic and isotropic materials. Hadji and Bernard (2020) investigated bending and free vibration analysis of functionally graded beams on elastic foundations with analytical validation.

Some researchers worked about nonlinear buckling and dynamic of porous FGM plates and shells as following references (Duc 2013, 2014, 2016a, b, Duc *et al*. 2017a, b, 2018a, b, Cong *et al*. 2018). Also, Duc and Quan (2015) considered the nonlinear dynamic analysis of imperfect FGM double curved thin shallow shells with temperature-dependent properties on elastic foundation. Moreover, Duc *et al*. (2016~~e~~) illustrated the nonlinear thermal dynamic response of shear deformable FGM plates on elastic foundations. In the other work, they (Duc *et al*. 2017c) studied the nonlinear dynamic response and vibration of imperfect shear deformable functionally graded plates subjected to blast and thermal loads. Also, Anh *et al*. (2015) presented the nonlinear buckling analysis of thin FGM annular spherical shells on elastic foundations under external pressure and thermal loads.

Some researchers worked about numerical methods to solve Partial Differential Equations (PDEs) as follows

Anitescu *et al*. (2019) and Guo *et al*. (2019) worked about artificial and deep neural networks that are a topic of great interest in the machine and deep learning community due to their ability to solve very difficult problems, respectively. Thus, Samaniego *et al*. (2020) presented an energy approach to the solution of partial differential equations in computational mechanics via machine learning including concepts, implementation and applications. Also, Nguyen-Thanh (2020) presented a deep energy method for finite deformation hyper-elasticity. Ren *et al*. (2020a, b) presented a higher order nonlocal operator method for solving partial differential equations.

The uncertainties method consists of three ingredients: (1) sampling method, (2) surrogate models, (3) sensitivity analysis (SA) method are usually used for data collection, for example, to measure temperature and pressure, Vu-Bac *et al*. (2016) investigated this effect on the efficiency of thousands of data devices for temperature and pressure. It has been said that using the available methods to check the uncertainty of data accuracy, the degree of uncertainty and the effect of each of the parameters on the output of the system is also examined. However, in the present work, the input data for finding system frequencies has not been measured by the authors and has been considered as a nominal value in the analysis (as well as other references from the same nominal values for their analysis and its effect on behavior of the system is used). The present work also provides a theoretical method for examining the behavior of the system, and the construction process is not performed to address the uncertainty issues regarding the dimensional measurements of the structure. Therefore, in the end, it can be said that due to the theoretical nature of the analysis and the lack of measured data, it is not possible to check the uncertainty.

Nguyen *et al*. (2021) presented a size-dependent isogeometric approach for vibration analysis of FG piezoelectric porous microplates using modified strain gradient theory. Also, they (Duc *et al*. 2017a) considered a novel three-variable shear deformation plate formulation based on Theory and Isogeometric implementation. In the other work, they (Duc *et al*. 2017b) illustrated NURBS-based postbuckling analysis of functionally graded carbon nanotube-reinforced composite shells.

In this research, bending and buckling analyses of a sandwich beam theory with various distributions of porous core and functionally graded facesheets is investigated based on a nonlocal stress-strain elasticity theory. In this paper, the important novelty and the defining a physical phenomenon of the resent research is the development of nonlocal stress and strain parameters on the porous sandwich beam with functionally graded materials in the top and bottom face sheets.

Also, various beam models including Euler-Bernoulli, Reddy and the generalized formulation of two-variable beam theory are obtained in this research. It is noted that by considering only nonlocal stress parameter ($(e_0a)_{stress}$), the stiffness of structures decreases, also if you consider only nonlocal strain parameter ($(e_0a)_{strain}$) the stiffness of structures increases, but when both $(e_0a)_{stress}$ and $(e_0a)_{strain}$ consider the stiffness of structures increases while the value of stiffness in this case ($(e_0a)_{stress}$ and $(e_0a)_{strain}$ simultaneously) is lower than nonlocal strain parameter ($(e_0a)_{strain}$) only that can be more actual at micro/nano scales.

2. A nonlocal stress-strain elasticity theory

The previous nonlocal elasticity theory (the nonlocal stress elasticity theory) was first presented by Eringen (1983). According to nonlocal stress elasticity theory, the stress at a reference point in the body is dependent not only on the strain state at that point, but also on the strain state at all of the points throughout the body. Thus, the constitutive equation of the nonlocal stress elasticity theory can be written as follows Eringen (1983)

$$(1-(e_0a)_{stress}^2\nabla^2)\sigma_{xx}=(\lambda+2\mu)\varepsilon_{xx} \tag{1}$$

where σ_{ij} and ε_{ij} are second order stress and strain tensors, respectively. λ and μ are Lame's constants. Moreover, $(e_0a)_{stress}$ denotes the small scale effect based on the nonlocal stress elasticity theory. δ_{ij} is Kronecker delta.

Based on a nonlocal strain elasticity theory, the constitutive equation of a nonlocal strain elasticity theory can be stated as follows

$$\sigma_{xx}=(1-(e_0a)_{strain}^2\nabla^2)(\lambda+2\mu)\varepsilon_{xx} \tag{2}$$

Based on superposition principle for linear elastic theory, we can combine Eqs. (1) and (2), thus the nonlocal stress-strain elasticity theory can be considered as follows

$$(1-(e_0a)_{stress}^2\nabla^2)\sigma_{xx}=(1-(e_0a)_{strain}^2\nabla^2)(\lambda+2\mu)\varepsilon_{xx} \tag{3}$$

where the Lame's constant are defined as follows

$$\lambda=\frac{E\upsilon}{(1+\upsilon)(1-2\upsilon)},\mu=\frac{E}{2(1+\upsilon)} \tag{4}$$

It is shown in Eq. (3) that the Euler-Bernoulli beam model based on the simplified higher-order nonlocal strain gradient theory could be reduced to either the common strain gradient model of Aifantis (1992) by taking $(e_0a)_{stress}$ to approach zero, or the nonlocal stress model of Eringen (1983) by taking $(e_0a)_{strain}$ to approach zero. It could also be reduced to the classical model when both $(e_0a)_{stress}$ and $(e_0a)_{strain}$ vanish. Hence, this model (higher-order nonlocal strain gradient theory) bridges the nonlocal stress theory of Eringen (1972, 1983, 2002) and the strain gradient theory of Mindlin (1965), Aifantis (1992), etc. Thus, with the above noticeable, the advantage of this model becomes the combinations of the two size dependent at nano/micro scales. It is noted that by considering only nonlocal stress parameter ($(e_0a)_{stress}$), the stiffness of structures decreases, also if you consider only nonlocal strain parameter ($(e_0a)_{strain}$) the stiffness of structures increases, but when both $(e_0a)_{stress}$ and $(e_0a)_{strain}$ consider the stiffness of structures increases while the value of stiffness in this case ($(e_0a)_{stress}$ and $(e_0a)_{strain}$ simultaneously) is lower than nonlocal strain parameter ($(e_0a)_{strain}$) only that can be more actual at micro/nano scales. The disadvantage

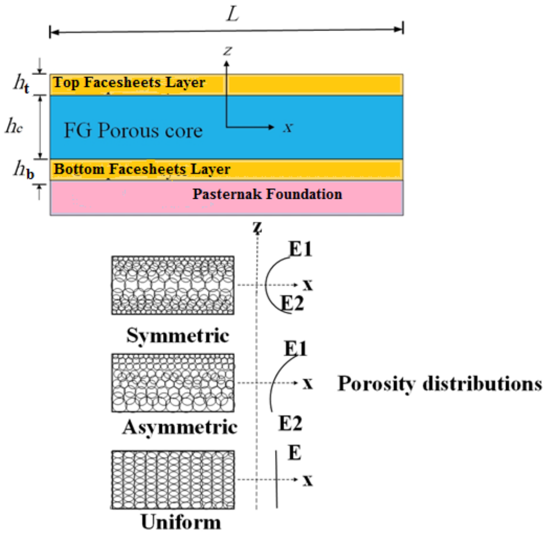


Fig. 1 The schematic view of a sandwich beam with various distributions of porous core and functionally graded top and bottom facesheet layers

of this model is that the volume of the equations becomes more and more complex, while the results of this model become closer to actual.

In Eq. (3), with combinations of two concepts (Eqs. (1) and (2)), the nonlocal stress-strain elasticity theory is defined that can be actual in micro/nano scales.

The limitations and assumptions of the present model have been stated as follows

(1) The nonlocal stress and strain parameters has been used to consider the size-dependent effect on the governing equation of equilibrium of the beam.

(2) The constitutive equations of this work are assumed linear in the elastic region.

(3) It assumed that the equations of the equilibrium become linear.

(4) The sandwich cylindrical shell is assumed to be as a continuous body that no delamination occurs between the layers of the sandwich shell during movement.

(5) There are no debonding between functionally graded materials.

Fig. 1 shows a sandwich beam with various distributions of porous core and functionally graded top and bottom facesheet layers. h_t and h_b, illustrate the thickness of top and bottom facesheet layers. Also, h_c show the thickness of porous core.

2.1 Euler-Bernoulli beam model

The displacement fields for Euler-Bernoulli beam theory are considered as follows

$$\begin{aligned} u(x,y,z) &= u_0(x) - z\frac{\partial w(x)}{\partial x} \\ v(x,y,z) &= 0 \\ w(x,y,z) &= w(x) \end{aligned} \tag{5}$$

where u, v, w denote the displacements in x, y, and z directions, respectively.

Using Eq. (5), the kinematic equations for Euler-Bernoulli beam are written as

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \tag{6}$$

By employing Eq. (6), the variation of strain energy can be written as follows

$$\delta U = \int_A \left(-\frac{\partial N_x}{\partial x}\delta u_0(x) - \frac{\partial^2 M_x}{\partial x^2}\delta w(x)\right)dA \tag{7a}$$

where

$$\begin{aligned} N_x &= \int_{-0.5H}^{0.5H}\sigma_{xx}\,dz = \int_{-0.5h_c-h_b}^{-0.5h_c}\sigma_{xx)b}\,dz + \int_{-0.5h_c}^{0.5h_c}\sigma_{xx)c}\,dz + \int_{0.5h_c}^{0.5h_c+h_t}\sigma_{xx)t}\,dz \\ M_x &= \int_{-0.5H}^{0.5H}\sigma_{xx}\,zdz = \int_{-0.5h_c-h_b}^{-0.5h_c}\sigma_{xx)b}\,zdz + \int_{-0.5h_c}^{0.5h_c}\sigma_{xx)c}z\,dz + \int_{0.5h_c}^{0.5h_c+h_t}\sigma_{xx)t}z\,dz \end{aligned} \tag{7b}$$

Then the variational method for the external work due to elastic foundation, transverse loadings, and axial force can be considered as follows

$$\delta W_{ext} = -\int_A \{(+N_{x0}\frac{\partial^2 w(x)}{\partial x^2} - F_{elastic} + q(x))\delta w(x) + F_{axial}(x)\delta u_0(x)\}dA \tag{8}$$

where N_{x0}, $q(x)$, $F_{axial}(x)$ and $F_{elastic}$ are axial buckling load, axial loadings, transverse loadings, and elastic foundation, respectively.

Based on the total potential energy, we have

$$\delta\prod = 0 \Rightarrow \delta U + \delta W_{ext} = 0 \tag{9}$$

Substituting Eqs. (7a), (8) into Eq. (9) yields the following equations of equilibrium for sandwich beam theory

$$\delta u_0(x): -\frac{\partial N_x}{\partial x} = F_{axial} \tag{10a}$$

$$\delta w(x): \frac{\partial^2 M_x}{\partial x^2} + N_{x0}\frac{\partial^2 w(x)}{\partial x^2} - F_{elastic} + q(x) = 0 \tag{10b}$$

By substituting Eq. (3) into Eq. (7b) and based on a nonlocal stress-strain elasticity theory, we have

$$\begin{aligned} (1-(e_0a)^2_{stress}\nabla^2)N_x &= \left(1-\left(e_0a\right)^2_{strain}\nabla^2\right)\int_{-0.5H}^{0.5H}(\lambda+2\mu)\left(\frac{\partial u_0(x)}{\partial x} - z\frac{\partial^2 w(x)}{\partial x^2}\right)dz \\ (1-(e_0a)^2_{stress}\nabla^2)M_x &= \left(1-\left(e_0a\right)^2_{strain}\nabla^2\right)\int_{-0.5H}^{0.5H}(\lambda+2\mu)\left(\frac{\partial u_0(x)}{\partial x} - z\frac{\partial^2 w(x)}{\partial x^2}\right)zdz \end{aligned} \tag{11a}$$

where

$$(\overset{(0)}{Q_{11}}, \overset{(1)}{Q_{11}}, \overset{(2)}{Q_{11}}) = \int_{-0.5H}^{0.5H}(\lambda+2\mu)(1,z,z^2)dz \tag{11b}$$

Based on a stress-strain elasticity theory, and substituting Eq. (11b) into Eq. (11a) yields the following equation

$$\begin{aligned} (1-(e_0a)^2_{stress}\nabla^2)N_x &= (1-\left(e_0a\right)^2_{strain}\nabla^2)\left(\overset{(0)}{Q_{11}}\frac{\partial u_0(x)}{\partial x} - \overset{(1)}{Q_{11}}\frac{\partial^2 w(x)}{\partial x^2}\right) \\ (1-(e_0a)^2_{stress}\nabla^2)M_x &= (1-\left(e_0a\right)^2_{strain}\nabla^2)\left(\overset{(1)}{Q_{11}}\frac{\partial u_0(x)}{\partial x} - \overset{(2)}{Q_{11}}\frac{\partial^2 w(x)}{\partial x^2}\right) \end{aligned} \tag{11c}$$

Based on a nonlocal stress-strain elasticity theory, the governing equations of equilibrium for sandwich beam theory are derived as follows

$$\delta u_0(x): -(1-(e_0a)^2_{strain}\nabla^2)\left(\overset{(0)}{Q}_{11}\frac{\partial^2 u_0(x)}{\partial x^2}-\overset{(1)}{Q}_{11}\frac{\partial^3 w(x)}{\partial x^3}\right) = (1-(e_0a)^2_{stress}\nabla^2)F_{axial} \tag{12a}$$

$$\delta w(x): -(1-(e_0a)^2_{strain}\nabla^2)\left(\overset{(1)}{Q}_{11}\frac{\partial^3 u_0(x)}{\partial x^3}-\overset{(2)}{Q}_{11}\frac{\partial^4 w(x)}{\partial x^4}\right) = (1-(e_0a)^2_{stress}\nabla^2)\left(N_{x0}\frac{\partial^2 w(x)}{\partial x^2}-F_{elastic}+q(x)\right) \tag{12b}$$

where

$$F_{axial}(x)=\sum_{m=1}^{\infty}F_m\cos\left(\frac{m\pi x}{L}\right),\ q(x)=\sum_{m=1}^{\infty}q_m\sin\left(\frac{m\pi x}{L}\right)$$
$$N_{x0}=-P_{cr} \tag{12c}$$
$$F_{elastic}=K_w w(x)-K_G\nabla^2 w(x)$$

The displacements for Euler-Bernoulli sandwich beam theory can define as follows

$$u_0(x)=\sum_{m=1}^{\infty}U_m\cos\left(\frac{m\pi x}{L}\right)$$
$$w(x)=\sum_{m=1}^{\infty}W_m\sin\left(\frac{m\pi x}{L}\right) \tag{13}$$

By substituting Eq. (13) into Eqs. (12a) and (12b), the matrix form for sandwich beam theory is obtained as

$$([K]+[B])\{X\}=\{F\} \tag{14}$$

where the stiffness and buckling matrices and force vector are obtained as follows

$$[K]=(1+(e_0a)^2_{strain}\alpha^2)\begin{bmatrix}\overset{(0)}{Q}_{11}\alpha^2 & -\overset{(1)}{Q}_{11}\alpha^3\\ -\overset{(1)}{Q}_{11}\alpha^3 & \overset{(2)}{Q}_{11}\alpha^4\end{bmatrix} \tag{14}$$

$$[B]=(1+(e_0a)^2_{stress}\alpha^2)\begin{bmatrix}0 & 0\\ 0 & K_w+K_G\alpha^2-P_{cr}\alpha^2\end{bmatrix}$$
$$\{F\}=(1+(e_0a)^2_{stress}\alpha^2)\begin{Bmatrix}F_m\\ q_m\end{Bmatrix}$$
$$\alpha=\frac{m\pi}{L} \tag{15}$$
$$\{X\}=\begin{Bmatrix}U_m\\ W_m\end{Bmatrix}$$

2.2 Reddy beam model

The displacement fields for Reddy beam model are considered as follows

$$u(x,y,z) = u_0(x) + z\psi(x) - \frac{4z^3}{3H^2}\left(\psi(x) + \frac{\partial w(x)}{\partial x}\right)$$
$$v(x,y,z) = 0 \qquad (16)$$
$$w(x,y,z) = w(x)$$

Using Eq. (16), the kinematic equations for Reddy beam are written as

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + z\frac{\partial \psi(x)}{\partial x} - \frac{4z^3}{3h^2}\left(\frac{\partial \psi(x)}{\partial x} + \frac{\partial^2 w(x)}{\partial x^2}\right)$$
$$\gamma_{xz} = +\psi - \frac{4z^2}{h^2}\left(\psi + \frac{\partial w}{\partial x}\right) + \frac{\partial w}{\partial x} \qquad (17)$$

Using Eq. (17), the variation of strain energy can be written as follows

$$\delta U = \int_A \left(-\frac{\partial N_x}{\partial x}\delta u_0(x) - \frac{\partial M_x^{(1)}}{\partial x}\delta\psi(x) + \frac{\partial M_x^{(3)}}{\partial x}\delta\psi(x) - \frac{\partial^2 M_x^{(3)}}{\partial x^2}\delta w(x) + Q_x^{(0)}\delta\psi(x) - Q_x^{(2)}\delta\psi(x) + \frac{\partial Q_x^{(2)}}{\partial x}\delta w(x) - \frac{\partial Q_x^{(0)}}{\partial x}\delta w(x)\right) dA \qquad (18a)$$

where

$$N_x = \int_{-0.5H}^{0.5H} \sigma_{xx}\, dz = \int_{-0.5h_c-h_b}^{-0.5h_c} \sigma_{xx)b}\, dz + \int_{-0.5h_c}^{0.5h_c} \sigma_{xx)c}\, dz + \int_{0.5h_c}^{0.5h_c+h_t} \sigma_{xx)t}\, dz$$
$$M_x^{(1)} = \int_{-0.5H}^{0.5H} \sigma_{xx}\, zdz = \int_{-0.5h_c-h_b}^{-0.5h_c} \sigma_{xx)b}\, zdz + \int_{-0.5h_c}^{0.5h_c} \sigma_{xx)c} z\, dz + \int_{0.5h_c}^{0.5h_c+h_t} \sigma_{xx)t} z\, dz$$

$$M_x^{(3)} = \frac{4}{3H^2}\int_{-0.5H}^{0.5H} \sigma_{xx}\, z^3 dz = \frac{4}{3H^2}\left(\int_{-0.5h_c-h_b}^{-0.5h_c} \sigma_{xx)b}\, z^3 dz + \int_{-0.5h_c}^{0.5h_c} \sigma_{xx)c} z^3\, dz + \int_{0.5h_c}^{0.5h_c+h_t} \sigma_{xx)t} z^3\, dz\right)$$
$$Q_x^{(0)} = \int_{-0.5H}^{0.5H} \tau_{xz}\, dz = \int_{-0.5h_c-h_b}^{-0.5h_c} \tau_{xz)b}\, dz + \int_{-0.5h_c}^{0.5h_c} \tau_{xz)c}\, dz + \int_{0.5h_c}^{0.5h_c+h_t} \tau_{xz)t}\, dz \qquad (18b)$$
$$Q_x^{(2)} = \frac{4}{H^2}\int_{-0.5H}^{0.5H} \tau_{xz} z^2\, dz = \frac{4}{H^2}\left(\int_{-0.5h_c-h_b}^{-0.5h_c} \tau_{xz)b} z^2\, dz + \int_{-0.5h_c}^{0.5h_c} \tau_{xz)c}\, z^2 dz + \int_{0.5h_c}^{0.5h_c+h_t} \tau_{xz)t}\, z^2 dz\right)$$

Substituting Eqs. (18a), (8) into Eq. (9) yields the following equations of equilibrium for sandwich beam theory

$$\delta u_0(x): -\frac{\partial N_x}{\partial x} = F_{axial} \qquad (19a)$$

$$\delta w(x): -\frac{\partial^2 M_x^{(3)}}{\partial x^2} + \frac{\partial Q_x^{(2)}}{\partial x} - \frac{\partial Q_x^{(0)}}{\partial x} - N_{x0}\frac{\partial^2 w(x)}{\partial x^2} + F_{elastic} - q(x) = 0 \qquad (19b)$$

$$\delta\psi(x): -\frac{\partial M_x^{(1)}}{\partial x} + \frac{\partial M_x^{(3)}}{\partial x} + Q_x^{(0)} - Q_x^{(2)} = 0 \qquad (19c)$$

Based on Navier's type solution, the displacements fields can be considered as follows

$$u_0(x) = \sum_{m=1}^{\infty} U_m \cos\frac{m\pi x}{L}$$

$$\psi(x) = \sum_{m=1}^{\infty} \psi_m \cos\frac{m\pi x}{L}$$
$$w(x) = \sum_{m=1}^{\infty} W_m \sin\frac{m\pi x}{L} \tag{20}$$

By substituting Eq. (20) into Eqs. (19), the matrix form for sandwich Reddy beam theory is obtained as

$$([K] + [B])\{X\} = \{F\} \tag{21}$$

where K, B and F are the stiffness and buckling matrices and force vector. Also, X vector is defined as follows

$$\{X\} = \begin{Bmatrix} U_m \\ \psi_m \\ W_m \end{Bmatrix} \tag{22}$$

2.3 The generalized formulation of two-variable beam theory

The displacement fields for the generalized formulation of two-variable beam theory are considered as follows (Nguyen *et al*. 2017a or b)

$$u(x,y,z) = u_0(x) - z\frac{\partial w_b}{\partial x} - \alpha\frac{4z^3}{3H^2}\frac{\partial}{\partial x}\left(\frac{\partial^2 w_b}{\partial x^2}\right)$$
$$v(x,y,z) = 0$$
$$w(x,y,z) = w_b(x) + \alpha\frac{\partial^2 w_b(x)}{\partial x^2} \tag{23a}$$
$$\alpha = \frac{z_0\int_{-H/2}^{H/2} z\frac{E(z)}{1-\upsilon^2}dz - \int_{-H/2}^{H/2} z^2\frac{E(z)}{1-\upsilon^2}dz}{\int_{-H/2}^{H/2}(1-4\frac{z^2}{H^2})\frac{E(z)}{(2(1+\upsilon))}dz}$$

where z_0 indicates the distance of neutral plane from the mid-plane of FG plate as follows

$$z_0 = \frac{\int_{-H/2}^{H/2} z\frac{E(z)}{1-\upsilon^2}dz}{\int_{-H/2}^{H/2}\frac{E(z)}{1-\upsilon^2}dz} \tag{23b}$$

Using Eq. (23a), the kinematic equations for the generalized formulation of two-variable beam theory are written as

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} - \frac{4z^3}{3h^2}\alpha\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w_b}{\partial x^2}\right)$$
$$\gamma_{xz} = -\frac{4z^2}{h^2}\alpha\frac{\partial}{\partial x}\left(\frac{\partial^2 w_b}{\partial x^2}\right) + \alpha\frac{\partial^3 w_b}{\partial x^3} \tag{24}$$

Using Eq. (24), the variation of strain energy can be written as follows

$$\delta U = \int_A \begin{pmatrix} -\frac{\partial N_x}{\partial x}\delta u_0(x) - \frac{\partial^2 M_x^{(1)}}{\partial x^2}\delta w_b(x) - \alpha\frac{\partial^4 M_x^{(3)}}{\partial x^4}\delta w_b(x) \\ +\alpha\frac{\partial^3 Q_x^{(2)}}{\partial x^3}\delta w_b(x) - \alpha\frac{\partial^3 Q_x^{(0)}}{\partial x^3}\delta w_b(x) \end{pmatrix} dA \tag{25a}$$

where

$$N_x = \int_{-0.5H}^{0.5H} \sigma_{xx}\, dz = \int_{-0.5h_c-h_b}^{-0.5h_c} \sigma_{xx)b}\, dz + \int_{-0.5h_c}^{0.5h_c} \sigma_{xx)c}\, dz + \int_{0.5h_c}^{0.5h_c+h_t} \sigma_{xx)t}\, dz$$

$$M_x^{(1)} = \int_{-0.5H}^{0.5H} \sigma_{xx}\, zdz = \int_{-0.5h_c-h_b}^{-0.5h_c} \sigma_{xx)b}\, zdz + \int_{-0.5h_c}^{0.5h_c} \sigma_{xx)c} z\, dz + \int_{0.5h_c}^{0.5h_c+h_t} \sigma_{xx)t} z\, dz$$

$$M_x^{(3)} = \frac{4}{3H^2}\int_{-0.5H}^{0.5H} \sigma_{xx}\, z^3 dz = \frac{4}{3H^2}\Big(\int_{-0.5h_c-h_b}^{-0.5h_c} \sigma_{xx)b}\, z^3 dz + \int_{-0.5h_c}^{0.5h_c} \sigma_{xx)c} z^3\, dz + \int_{0.5h_c}^{0.5h_c+h_t} \sigma_{xx)t} z^3\, dz\Big) \tag{25b}$$

$$Q_x^{(0)} = \int_{-0.5H}^{0.5H} \tau_{xz}\, dz = \int_{-0.5h_c-h_b}^{-0.5h_c} \tau_{xz)b}\, dz + \int_{-0.5h_c}^{0.5h_c} \tau_{xz)c}\, dz + \int_{0.5h_c}^{0.5h_c+h_t} \tau_{xz)t}\, dz$$

$$Q_x^{(2)} = \frac{4}{H^2}\int_{-0.5H}^{0.5H} \tau_{xz} z^2\, dz = \frac{4}{H^2}\Big(\int_{-0.5h_c-h_b}^{-0.5h_c} \tau_{xz)b} z^2\, dz + \int_{-0.5h_c}^{0.5h_c} \tau_{xz)c}\, z^2 dz + \int_{0.5h_c}^{0.5h_c+h_t} \tau_{xz)t}\, z^2 dz\Big)$$

Substituting Eqs. (25a), (8) into Eq. (9) yields the following equations of equilibrium for sandwich beam theory

$$\delta u_0(x): -\frac{\partial N_x}{\partial x} = F_{axial} \tag{26a}$$

$$\begin{aligned}\delta w_b(x): &-\frac{\partial^2 M_x^{(1)}}{\partial x^2} - \alpha\frac{\partial^4 M_x^{(3)}}{\partial x^4} + \alpha\frac{\partial^3 Q_x^{(2)}}{\partial x^3} - \alpha\frac{\partial^3 Q_x^{(0)}}{\partial x^3} \\ &-N_{x0}\frac{\partial^2 w(x)}{\partial x^2} + F_{elastic} - q(x) = 0\end{aligned} \tag{26b}$$

Based on Navier's type solution, the displacements fields can be considered as follows

$$\begin{aligned} u_0(x) &= \textstyle\sum_{m=1}^{\infty} U_m \cos\frac{m\pi x}{L} \\ w_b(x) &= \textstyle\sum_{m=1}^{\infty} W_m \sin\frac{m\pi x}{L}\end{aligned} \tag{27}$$

By substituting Eq. (27) into Eqs. (26), the matrix form for sandwich Reddy beam theory is obtained as

$$([K] + [B])\{X\} = \{F\} \tag{28}$$

where K, B and F are the stiffness and buckling matrices and force vector, and X vector is defined as follows

$$\{X\} = \begin{Bmatrix} U_m \\ W_m \end{Bmatrix} \tag{29}$$

2.4 Young modulus for core and face sheets layers

The Young's modulus and density of porous core for two types of different distributions is defined as follows (Chen *et al*. (2017))

$$\begin{aligned} E_c &= E_{0c}\left(1 - e_0\cos\left(\frac{\pi z}{2\hbar_c} + \frac{\pi}{4}\right)\right) \\ \rho_c &= \rho_{0c}\left(1 - e_m\cos\left(\frac{\pi z}{2\hbar_c} + \frac{\pi}{4}\right)\right)\end{aligned} \qquad \text{Asymmetric type} \tag{30a}$$

$$E_c = E_{0c}\left(1 - e_0 \cos\left(\frac{\pi z}{\hbar_c}\right)\right)$$
$$\rho_c = \rho_{0c}\left(1 - e_m \cos\left(\frac{\pi z}{\hbar_c}\right)\right) \qquad \text{Symmetric type} \qquad (30b)$$
$$e_m = 1 - \sqrt{1 - e_0}$$

Based on functionally graded material properties, the Young's modulus for top and bottom facesheets of sandwich beam are written as

$$E_{11}{}^{t} = E_{Al} + (E_{st} - E_{Al}) \times \left(\frac{1}{2} + \frac{z_t}{\hbar_t}\right)^n$$
$$E_{11}{}^{b} = E_{Al} + (E_{st} - E_{Al}) \times \left(\frac{1}{2} + \frac{z_b}{\hbar_b}\right)^n \qquad (31)$$

where n is the power law index.

3. Numerical results and discussions

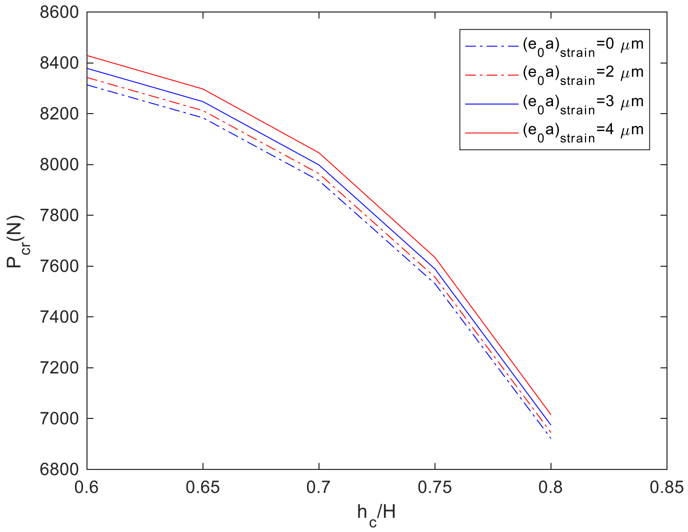
In this article, a nonlocal stress-strain elasticity theory on the bending and buckling analysis of sandwich Euler-Bernoulli beam theory with porous core and functionally graded facesheets is considered.

The mechanical and geometrical parameters for sandwich beam with porous cores and functionally graded facesheets are considered in Table 1.

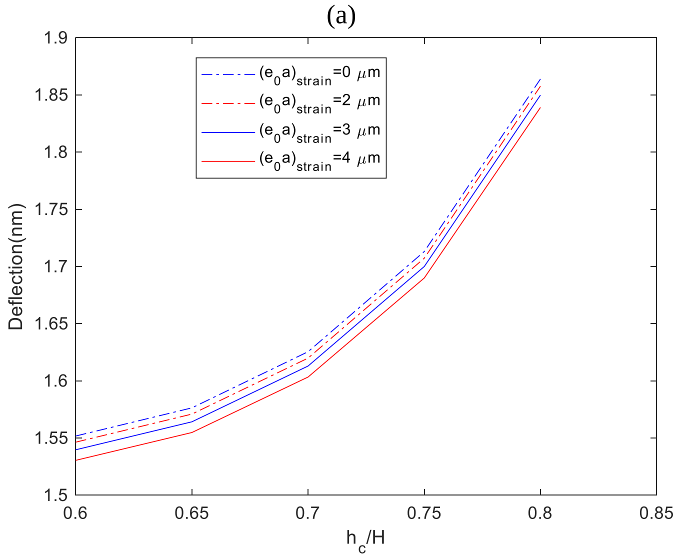
But because short time in revised (two week) and suggest the reviewer's to add Reddy and the generalized formulation of two-variable beam theories, thus I obtain only the formulation, I could not plot and compared Euler-Bernoulli, Reddy and the generalized formulation of two-variable beam theories. Thus, all figures plot only for Euler-Bernoulli beam theory. Figs. 2 and 3 illustrate the critical buckling load versus core thickness to total thickness ratio (h_c/H) for different values of nonlocal strain parameter ($(e_0a)_{strain}$) and nonlocal stress parameter ($(e_0a)_{stress}$) based on a nonlocal strain elasticity theory, nonlocal stress elasticity theory (Eringen's theory), respectively. It is seen that with increasing of the thickness ratio, the critical buckling load decreases; while the transverse deflection increases. It can be seen that with increasing of $(e_0a)_{strain}$, the critical buckling load increases because with increasing this parameter, the stiffness of sandwich beam increases; while, the transverse deflection of sandwich beam decreases. Also, it can be seen from Fig. 3 that it is vice

Table 1 The mechanical and geometrical parameters for sandwich beam

The geometric and loadings of sandwich beam	$H = 10\ \mu m,\ h_c = 0.8\ H,\ m = 1,$ $q = F_{axial} = -10\ KN/m$
The properties of facesheets layers	$E_{Al} = 70\ GPa,\ E_{st} = 200\ GPa,$ $n = 2;\ \upsilon_{Al} = 0.33,\ \upsilon_{st} = 0.3,$
The properties of core	$E_{0c} = 120\ GPa,\ \upsilon_c = 0.34,\ e_0 = 0.3,$
Nonlocal stress-strain parameters	$(e_0a)_{strain} = 1\ \mu m,\ (e_0a)_{stress} = 1\ \mu m$
Elastic foundation parameters	$K_w = 1 \times 10^6\ N/m^3,\ K_G = 1000\ N/m$



(a)

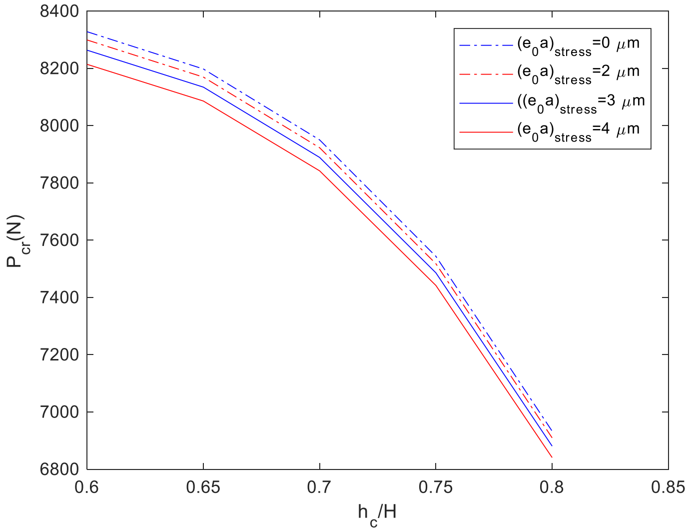


(b)

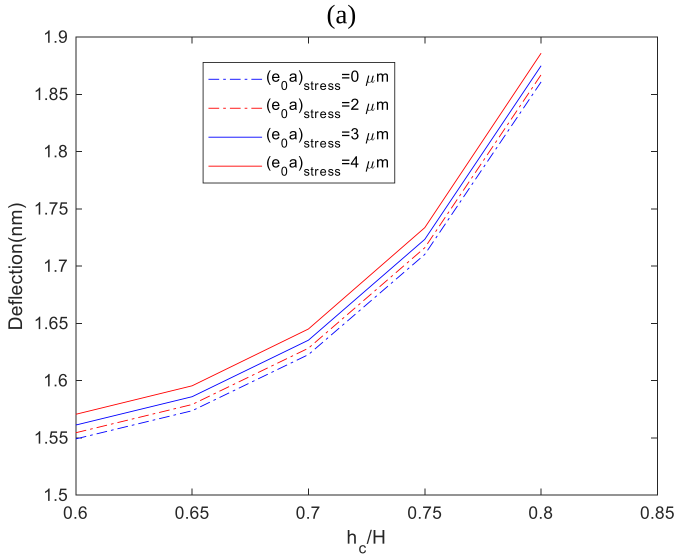
Fig. 2 The effect of nonlocal strain parameter on (a) Critical buckling load and (b) Transverse deflection of sandwich beam

versa for $(e_0a)_{stress}$. It is noted that the nonlocal strain parameter leads to increase the stiffness of structures; while, the nonlocal stress parameter leads to enhance the softness of structures.

The effect of nonlocal both stress and strain parameters on (a) critical buckling load (b) transverse deflection of sandwich Euler-Bernoulli beam theory is shown in Fig. 4. Based on a nonlocal stress-strain elasticity theory. It is concluded that with an increase in the nonlocal strain parameter, the critical buckling load increases; while it is vice versa for nonlocal stress elasticity, because the stiffness of sandwich beam enhances with an increase in the nonlocal strain parameter; in which, the nonlocal stress parameter leads to reduce the stiffness of structures at micro/nano scale. It is



(a)

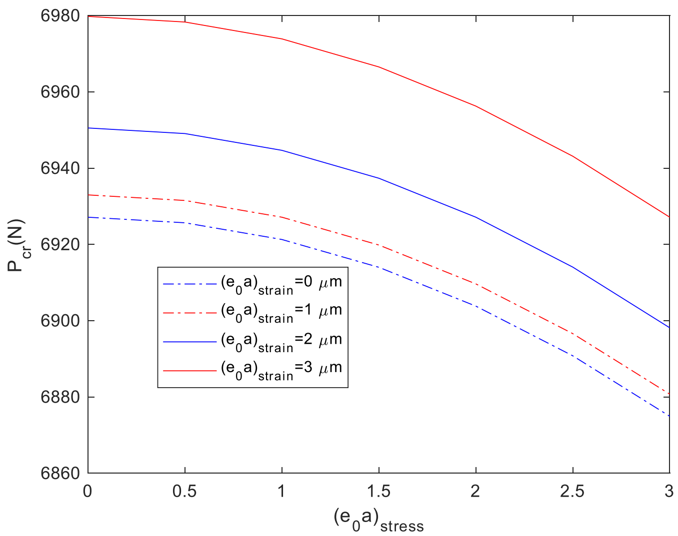


(b)

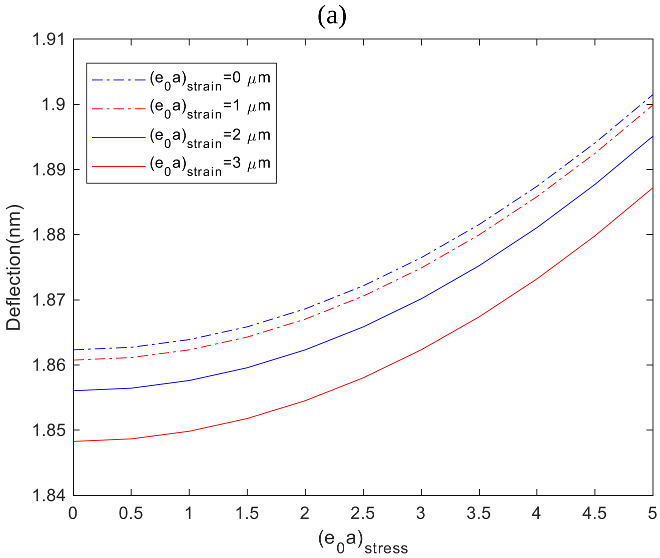
Fig. 3 The effect of nonlocal stress parameter on (a) Critical buckling load and (b) Transverse deflection of sandwich beam

seen from Fig. 4 that the critical buckling load and transverse deflection of sandwich beam by considering both nonlocal stress and nonlocal strain parameters is higher than the nonlocal stress parameter only. On the other hands, it is noted that by considering the nonlocal stress-strain parameters simultaneously becomes the actual case.

Fig. 5 illustrates the effect of power law index (n) on (a) critical buckling load (b) transverse deflection of sandwich beam based on a nonlocal stress-strain elasticity theory. It is seen that with increasing of power law index, the stiffness of micro sandwich beam decreases, then it leads to decrease the critical buckling load and increase the transverse deflection of sandwich beam. On the



(a)



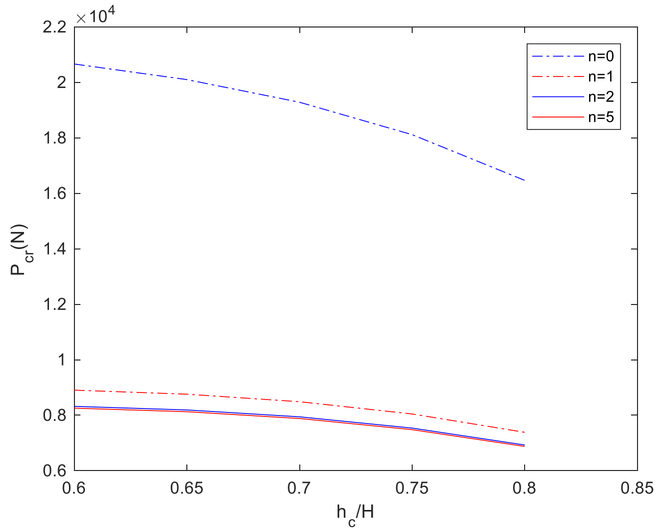
(b)

Fig. 4 The effect of nonlocal both stress and strain parameters on (a) Critical buckling load and (b) Transverse deflection of sandwich beamy

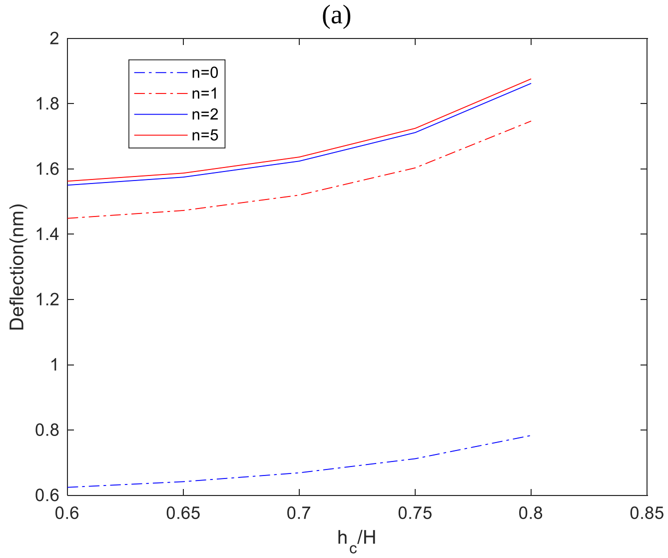
other hands, with increasing of power law index, the flexibility of sandwich structures increases.

4. Conclusions

In this paper, the important novelty and the defining a physical phenomenon of the resent research is the development of nonlocal stress and strain parameters on the porous sandwich beam with functionally graded materials in the top and bottom face sheets. Also, various beam models including



(a)



(b)

Fig. 5 The effect of power law index on (a) Critical buckling load and (b) Transverse deflection of sandwich beam

Euler-Bernoulli, Reddy and the generalized formulation of two-variable beam theories are investigated in this research. It is seen that with an increase in the nonlocal strain parameter, the critical buckling load increases; while it is vice versa for nonlocal stress elasticity, because the stiffness of sandwich beam enhances with an increase in the nonlocal strain parameter; in which, the nonlocal stress parameter leads to reduce the stiffness of structures at micro/nano scale. It can be seen that the critical buckling load and transverse deflection of sandwich beam by considering both nonlocal stress - strain parameters is higher than the nonlocal stress parameter. On the other hands, it is noted that by considering the nonlocal stress-strain parameters simultaneously becomes the actual case. It can be shown that with an enhance in the power law index, the stiffness of micro sandwich beam decreases, then it leads to decrease the critical buckling load and increase the transverse deflection of sandwich beam. On the other hands, with increasing of power law index, the flexibility of sandwich structures increases.

Acknowledgments

The authors would like to thank the referees for their valuable comments. Also, they are thankful to thank the Iranian Nanotechnology Development Committee for their financial support and the University of Kashan for supporting this work by Grant No. 891238/29.

References

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), “Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position”, *Compos. Struct.*, **125**, 621-630. https://doi.org/10.1016/j.compstruct.2014.12.070.

Alavi, S.H. and Eipakchi, H. (2019), “Geometry and load effects on transient response of a VFGM annular plate: An analytical approach”, *Struct. Eng. Mech., Int. J.*, **70**(2), 179-197. https://doi.org/10.12989/sem.2019.70.2.179.

Altekin, M. (2020), “Combined effects of material properties and boundary conditions on the large deflection bending analysis of circular plates on a nonlinear elastic foundation”, *Comput. Concrete, Int. J.*, **25**(6), 537-549. https://doi.org/10.12989/cac.2020.25.6.537.

Anh, V.T.T., Bich, D.H. and Duc, N.D. (2015), “Nonlinear buckling analysis of thin FGM annular spherical shells on elastic foundations under external pressure and thermal loads”, *Eur. J. Mech. – A/Solids*, **50**, 28-38. https://doi.org/10.1016/j.euromechsol.2014.10.004.

Anitescu, C., Atroshchenko, E., Alajlan, N. and Rabczuk, T. (2019), “Artificial neural network methods for the solution of second order boundary value problems”, *Comput. Mater. Continua*, **59**(1), 345-359. https://doi.org/10.32604/cmc.2019.06641.

Ansari, R., Gholami, R., Faghih Shojaei, M., Mohammadi, V. and Sahmani, S. (2013), “Size-dependent bending, buckling and free vibration of functionally graded Timoshenko micro beams based on the most general strain gradient theory”, *Compos. Struct.*, **100**, 385-397. https://doi.org/10.1016/j.compstruct.2012.12.048.

Babaei, H. and Eslami, M.R. (2019), “Thermally induced large deflection of FGM shallow micro-arches with integrated surface piezoelectric layers based on modified couple stress theory”, *Acta Mechanica*, **230**, 2363-2384. https://doi.org/10.1007/s00707-019-02384-0.

Bahaadini, R. and Saidi, A.R. (2018), “Aeroelastic analysis of functionally graded rotating blades reinforced with graphene nanoplatelets in supersonic flow”, *Aerosp. Sci. Technol.*, **80**, 381-391. https://doi.org/10.1016/j.ast.2018.06.035.

Bendenia, N., Zidour, M., Bousahla, A.N., Bourada, F., Tounsi, A., Benrahou, K.H., Bedia, E.A.A., Mahmoud, S.R. and Tounsi, A. (2020), “Deflections, stresses and free vibration studies of FG-CNT reinforced sandwich plates resting on Pasternak elastic foundation”, *Comput. Concrete, Int. J.*, **26**(3), 213-226. https://doi.org/10.12989/cac.2020.26.3.213.

Canbay, C.A., Karaduman, O., Ibrahim, P.A. and Őzkul, I. (2021), “Thermostructural shape memory effect observations of ductile Cu-Al-Mn smart alloy”, *Adv. Mater. Res., Int. J.*, **10**(1), 45-56. https://doi.org/10.12989/amr.2021.10.1.045.

Chen, D., Kitipornchai, S. and Yang, J. (2017), “Free vibration and elastic buckling of functionally graded porous beams reinforced by graphene platelets”, *Mater. Des.*, **116**, 656-665.

https://doi.org/10.1016/j.matdes.2016.12.061.

Cong, P.H., Chien, T.M., Khoa, N.D. and Duc, N.D. (2018), "Nonlinear thermo-mechanical buckling and post-buckling response of porous FGM plates using Reddy's HSDT", *J. Aerosp. Sci. Technol.*, **77**, 419-428. https://doi.org/10.1016/j.ast.2018.03.020.

Daikh, A.A., Bensaid, I., Bachiri, A., Houari, M.S.A., Tounsi, A. and Merzouki, T. (2020), "On static bending of multilayered carbon nanotube-reinforced composite plates", *Comput. Concrete, Int. J.*, **26**(2), 137-150. https://doi.org/10.12989/cac.2020.26.2.137.

Duc, N.D. (2013), "Nonlinear dynamic response of imperfect eccentrically stiffened FGM double curved shallow shells on elastic foundation", *Compos. Struct.*, **102**, 306-314. https://doi.org/10.1016/j.compstruct.2012.11.017.

Duc, N.D. (2014), *Nonlinear Static and Dynamic Stability of Functionally Graded Plates and Shells*, Vietnam National University Press, Hanoi, Vietnam.

Duc, N.D. (2016a), "Nonlinear thermal dynamic analysis of eccentrically stiffened S-FGM circular cylindrical shells surrounded on elastic foundations using the Reddy's third-order shear deformation shell theory", *Eur. J. Mech. – A/Solids*, **58**, 10-30. https://doi.org/10.1016/j.euromechsol.2016.01.004.

Duc, N.D. (2016b), "Nonlinear thermo-electro-mechanical dynamic response of shear deformable piezoelectric Sigmoid functionally graded sandwich circular cylindrical shells on elastic foundations", *J. Sandw. Struct. Mater.*, **20**(3), 351-378. https://doi.org/10.1177/1099636216653266.

Duc, N.D. and Quan, T.Q. (2015), "Nonlinear dynamic analysis of imperfect FGM double curved thin shallow shells with temperature-dependent properties on elastic foundation", *J. Vib. Control*, **21**(7), 1340-1362. https://doi.org/10.1177/1077546313494114.

Duc, N.D., Cong, P.H., Anh, V.M., Quang, V.D., Tran, P., Tuan, N.D. and Thinh, N.H. (2015), "Mechanical and thermal stability of eccentrically stiffened functionally graded conical shell panels resting on elastic foundations and in thermal environment", *Compos. Struct.*, **132**, 597-609. https://doi.org/10.1016/j.compstruct.2015.05.072.

Duc, N.D., Bich, D.H. and Cong, P.H. (2016c), "Nonlinear thermal dynamic response of shear deformable FGM plates on elastic foundations", *J. Thermal Stresses*, **39**(3), 278-297. https://doi.org/10.1080/01495739.2015.1125194.

Duc, N.D., Nguyen, P.D. and Khoa, N.D. (2017a), "Nonlinear dynamic analysis and vibration of eccentrically stiffened S-FGM elliptical cylindrical shells surrounded on elastic foundations in thermal environments", *Thin-Wall. Struct.*, **117**, 178-189. https://doi.org/10.1016/j.tws.2017.04.013.

Duc, N.D., Tuan, N.D., Tran, P. and Quan, T.Q. (2017b), "Nonlinear dynamic response and vibration of imperfect shear deformable functionally graded plates subjected to blast and thermal loads", *Mech. Adv. Mater. Struct.*, **24**(4), 318-329. https://doi.org/10.1080/15376494.2016.1142024.

Duc, N.D., Tuan, N.D., Tran, P. and Quan, T.Q. (2017c), "Nonlinear dynamic response and vibration of imperfect shear deformable functionally graded plates subjected to blast and thermal loads", *Mech. Adv. Mater. Struct.*, **24**(4), 318-329. https://doi.org/10.1080/15376494.2016.1142024.

Duc, N.D., Quang, V.D., Nguyen, P.D. and Chien, T.M. (2018a), "Nonlinear dynamic response of FGM porous plates on elastic foundation subjected to thermal and mechanical loads using the first order shear deformation theory", *J. Appl. Computat. Mech.*, **4**(4), 245-259. https://doi.org/10.22055/jacm.2018.23219.1151.

Duc, N.D., Seung-Eock, K. and Chan, D.Q. (2018b), "Thermal buckling analysis of FGM sandwich truncated conical shells reinforced by FGM stiffeners resting on elastic foundations using FSDT", *J. Thermal Stresses*, **41**(3), 331-365. https://doi.org/10.1080/01495739.2017.1398623.

Ebrahimi, F., Barati, M.R. and Dabbagh, A. (2016), "A nonlocal strain gradient theory for wave propagation analysis in temperature-dependent inhomogeneous nanoplates", *Int. J. Eng. Sci.*, **107**, 169-182. https://doi.org/10.1016/j.ijengsci.2016.07.008.

Ebrahimi, F., Seyfi, A. and Tornabene, F. (2019), "Wave dispersion characteristics of porous graphene platelet-reinforced composite shells", *Struct. Eng. Mech., Int. J.*, **71**(1), 99-107. https://doi.org/10.12989/sem.2019.71.1.099.

Eltaher, M.A., Khater, M.E. and Emam, S.A. (2016), "A review on nonlocal elastic models for bending, buckling, vibrations, and wave propagation of nanoscale beams”, *Appl. Mathe. Modell.*, **40**(5-6), 4109-4128. https://doi.org/10.1016/j.apm.2015.11.026.

Eringen, A.C. (1972), “Nonlocal polar elastic continua”, *Int. J. Eng. Sci.*, **10**(1), 1-16. https://doi.org/10.1016/0020-7225(72)90070-5.

Eringen, A.C. (1983), “On direrential equations of nonlocal elasticity and solutions of screw dislocation and surface waves”, *J. Appl. Phys.*, **54**, 4703-4710. https://doi.org/10.1063/1.332803.

Gao, X.-L. (2015), “A new Timoshenko beam model incorporating microstructure and surface energy effects”, *Acta Mechanica*, **226**, 457-474. https://doi.org/10.1007/s00707-014-1189-y.

Ghayesh, M.H., Farajpour, A. and Farokhi, H. (2020), “Effect of flow pulsations on chaos in nanotubes using nonlocal strain gradient theory”, *Commun. Nonlinear Sci. Numer. Simul.*, **83**, 105090. https://doi.org/10.1016/j.cnsns.2019.105090.

Ghorbanpour, A.A., Shokravi, M., Mohammadimehr, M. (2009), “Buckling analysis of a double-walled carbon nanotube embedded in an elastic medium using the energy method”, *J. Solid Mech.*, **1**(4), 289-299.

Guo, H., Zhuang, X. and Rabczuk, T. (2019), “A deep collocation method for the bending analysis of Kirchhoff plate”, *Comput. Mater. Continua*, **59**(2), 433-456. https://doi.org/ 10.32604/cmc.2019.06660.

Hadji, L. and Bernard, F. (2020), “Bending and free vibration analysis of functionally graded beams on elastic foundations with analytical validation”, *Adv. Mater. Res., Int. J.*, **9**(1), 63-98. https://doi.org/10.12989/amr.2020.9.1.063.

Jomehzadeh, E., Noori, H.R. and Saidi, A.R. (2011), “The size-dependent vibration analysis of micro-plates based on a modified couple stress theory”, *Physica E: Low-dimens. Syst. Nanostruct.*, **43**(4), 877-883. https://doi.org/10.1016/j.physe.2010.11.005.

Karami, B. and Shahsavari, D. (2020), “On the forced resonant vibration analysis of functionally graded polymer composite doubly-curved nanoshells reinforced with graphene-nanoplatelets”, *Comput. Methods Appl. Mech. Eng.*, **359**, 112767. https://doi.org/10.1016/j.cma.2019.112767.

Lam, D., Yang, F., Chong, A., Wang, J. and Tong, P. (2003), “Experiments and theory in strain gradient elasticity”, *J. Mech. Phys. Solids*, **51**, 1477-1508. https://doi.org/10.1016/S0022-5096(03)00053-X.

Li, Q., Wu, D., Gao, W. Tin-Loi, F. (2020), “Size-dependent instability of organic solar cell resting on Winkler–Pasternak elastic foundation based on the modified strain gradient theory”, *Int. J. Mech. Sci.*, **177**, 105306. https://doi.org/10.1016/j.ijmecsci.2019.105306.

Ma, H.M., Gao, X.-L. and Reddy, J.N. (2008), “A microstructure-dependent Timoshenko beam model based on a modified couple stress theory”, *J. Mech. Phys. Solids*, **56**(12), 3379-3391. https://doi.org/10.1016/j.jmps.2008.09.007.

Merzouki, T., Ahmed, H.M.S., Bessaim, A., Haboussi, M., Dimitri, R. and Tornabene, F. (2022), “Bending analysis of functionally graded porous nanocomposite beams based on a non-local strain gradient theory”, *Mathe. Mech. Solids*, **27**(1), 66-92. https://doi.org/10.1177/10812865211011759.

Mohammadimehr, M. and Alimirzaei, S. (2016), “Nonlinear static and vibration analysis of Euler-Bernoulli composite beam model reinforced by FG-SWCNT with initial geometrical imperfection using FEM”, *Struct. Eng. Mech., Int. J.*, **59**(3), 431-454. https://doi.org/10.12989/sem.2016.59.3.431.

Mohammadimehr, M., Saidi, A.R., Ghorbanpour Arani, A., Arefmanesh, A. and Han, Q. (2010), “Torsional buckling of a DWCNT embedded on winkler and pasternak foundations using nonlocal theory”, *J. Mech. Sci. Technol.*, **24**(6), 1289-1299. https://doi.org/10.1007/s12206-010-0331-6.

Mohammadimehr, M., Nejad, E.S. and Mehrabi, M. (2018), “Buckling and vibration analyses of MGSGT double-bonded micro composite sandwich SSDT plates reinforced by CNTs and BNNTs with isotropic foam & flexible transversely orthotropic cores”, *Struct. Eng. Mech., Int. J.*, **65**(4), 491-504. https://doi.org/10.12989/sem.2018.65.4.491.

Murmu, T. and Adhikari, S. (2010), “Nonlocal transverse vibration of double-nanobeam-systems”, *J. Appl. Phys.*, **08**, 083514. https://doi.org/10.1063/1.3496627.

Namayandeh, M.J., Mohammadimehr, M., Mehrabi, M. and Sadeghzadeh-Attar, A. (2020), “Temperature and thermal stress distributions in a hollow circular cylinder composed of anisotropic and isotropic materials”, *Adv. Mater. Res., Int. J.*, **9**(1), 15-32. https://doi.org/10.12989/amr.2020.9.1.015.

Narendar, S. (2011), “Buckling analysis of micro-/nano-scale plates based on two-variable refined plate theory incorporating nonlocal scale effects", *Compos. Struct.*, **93**(12), 3093-3103. https://doi.org/10.1016/j.compstruct.2011.06.028.

Nejadi, M.M. and Mohammadimehr, M. (2020), "Analysis of a functionally graded nanocomposite sandwich beam considering porosity distribution on variable elastic foundation using DQM: Buckling and vibration behaviors", *Comput. Concrete, Int. J.*, **25**(3), 215-224. https://doi.org/10.12989/cac.2020.25.3.215.

Nguyen, T.N., Ngo, T.D. and Nguyen-Xuan, H. (2017a), "A novel three-variable shear deformation plate formulation: Theory and Isogeometric implementation", *Comput. Methods Appl. Mech. Eng.*, **326**, 376-401. https://doi.org/10.1016/j.cma.2017.07.024.

Nguyen, T.N., Thai, C.H., Luu, A.T., Nguyen-Xuan, H. and Lee, J. (2017b), "NURBS-based postbuckling analysis of functionally graded carbon nanotube-reinforced composite shells", *Comput. Methods Appl. Mech. Eng.*, **326**, 376-401. https://doi.org/10.1016/j.cma.2019.01.011.

Nguyen, L.B., Thai, C.H., Duong-Nguyen, N. and Nguyen-Xuan, H. (2021), "A size-dependent isogeometric approach for vibration analysis of FG piezoelectric porous microplates using modified strain gradient theory", *Eng. Comput.* https://link.springer.com/article/10.1007/s00366-021-01468-7.

Nguyen-Thanh, V.M., Zhuang, X. and Rabczuk, T. (2020), "A deep energy method for finite deformation hyperelasticity", *Eur. J. Mech. - A/Solids*, **80**, 103874. https://doi.org/10.1016/j.euromechsol.2019.103874

Polizzotto, C. (2001), "Nonlocal elasticity and related variational principles", *Int. J. Solids Struct.*, **38**(42-43), 7359-7380. https://doi.org/10.1016/S0020-7683(01)00039-7.

Pradhan, S.C. (2009), "Buckling of single layer graphene sheet based on nonlocal elasticity and higher order shear deformation theory", *Phys. Lett. A*, **373**(45), 4182-4188. https://doi.org/10.1016/j.physleta.2009.09.021.

Rabia, B., Daouadji, T.H. and Abderezak, R. (2020), "Predictions of the maximum plate end stresses of imperfect FRP strengthened RC beams: study and analysis", *Adv. Mater. Res., Int. J.*, **9**(4), 265-287. https://doi.org/10.12989/amr.2020.9.4.265.

Rahi, M.J., Firoozjaee, A.R. and Dehestani, M. (2021), "Simplified numerical method for nonlocal static and dynamic analysis of a graphene nanoplate", *Adv. Mater. Res., Int. J.*, **10**(1), 1-22. https://doi.org/10.12989/amr.2021.10.1.001.

Reddy, J.N. (2007), "Nonlocal theories for bending, buckling and vibration of beams", *Int. J. Eng. Sci.*, **45**, 288-307. https://doi.org/10.1016/j.ijengsci.2007.04.004

Ren, H., Zhuang, X. and Rabczuk, T. (2020a), "A higher order nonlocal operator method for solving partial differential equations", *Comput. Methods Appl. Mech. Eng.*, **367**, 113132. https://doi.org/10.1016/j.cma.2020.113132.

Ren, H., Zhuang, X. and Rabczuk, T. (2020b), "A nonlocal operator method for solving partial differential equations", *Comput. Methods Appl. Mech. Eng.*, **358**, 112621. https://doi.org/10.1016/j.cma.2020.112621.

Samaniego, E., Anitescu, C., Goswami, S., Nguyen-Thanh, V.M., Guo, H., Hamdia, K., Zhuang, X. and Rabczuk, T. (2020), "An energy approach to the solution of partial differential equations in computational mechanics via machine learning: Concepts, implementation and applications", *Comput. Methods Appl. Mech. Eng.*, **362**, 112790. https://doi.org/10.1016/j.cma.2019.112790.

Shooshtari, A. and Razavi, S. (2015), "Nonlinear vibration analysis of rectangular magneto-electro-elastic thin plates", *J. Eng.*, **28**, 139-147. https://doi.org/10.5829/idosi.ije.2015.28.01a.18.

Thai, C.H., Ferreira, A.J.M. and Nguyen, H. (2018), "Isogeometric analysis of size-dependent isotropic and sandwich functionally graded microplates based on modified strain gradient elasticity theory", *Compos. Struct.*, **192**, 274-288. https://doi.org/10.1016/j.euromechsol.2018.07.012.

Vu-Bac, N., Lahmer, T., Zhuang, X., Nguyen-Thoi, T. and Rabczuk, T. (2016), "A software framework for probabilistic sensitivity analysis for computationally expensive models", *Adv. Eng. Software*, **100**, 19-31. https://doi.org/10.1016/j.advengsoft.2016.06.005.

Wang, K.F. and Wang, B.L. (2011), "Vibration of nanoscale plates with surface energy via nonlocal elasticity", *Phys. E: Low-dimens. Syst. Nanostruct.*, **44**(2), 448-453. https://doi.org/10.1016/j.physe.2011.09.019.

Yang, F., Chong, A.C.M., Lam, D.C.C. and Tong, P. (2002), "Couple stress based strain gradient theory for elasticity", *Int. J. Solids Struct.*, **39**(10), 2731-2743. https://doi.org/10.1016/S0020-7683(02)00152-X.

Yang, J., Jia, X.L. and Kitipornchai, S. (2008), "Pull-in instability of nano-switches using nonlocal elasticity theory", *J. Phys. D: Appl. Phys.*, **41**, 035103. https://doi.org/10.1088/0022-3727/41/3/035103.

Yazdani, R., Mohammadimehr, M. and Navi, B.R. (2019), "Free vibration of Cooper-Naghdi micro saturated porous sandwich cylindrical shells with reinforced CNT face sheets under magneto-hydro-thermo-mechanical loadings", *Struct. Eng. Mech., Int. J.*, **70**(3), 351-365. https://doi.org/10.12989/sem.2019.70.3.351.

Yazdani, R. and Mohammadimehr, M. (2019), "Double bonded Cooper-Naghdi micro sandwich cylindrical shells with porous core and CNTRC face sheets: Wave propagation solution", *Comput. Concrete, Int. J.*, **24**(6), 499-511. https://doi.org/10.12989/cac.2019.24.6.499.

Zenkour, A.M. and Sobhy, M. (2013), "Nonlocal elasticity theory for thermal buckling of nanoplates lying on Winkler–Pasternak elastic substate medium", *Phys. E: Low-dimens. Syst. Nanostruct.*, **53**, 251-259. https://doi.org/10.1016/j.physe.2013.04.022.

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