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Fuzzy analysis for stability of steel frame with fixity factor modeled as triangular fuzzy number

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Abstract. This study presents algorithms for determining the fuzzy critical loads of planar steel frame structures with fixity factors of beam - column and column - base connections are modeled as triangular fuzzy numbers. The finite element method with linear elastic semi-rigid connection and Response Surface Method (RSM) in mathematical statistic are applied for problems with symmetric triangular fuzzy numbers. The α - level optimization using the Differential Evolution (DE) involving integrated finite element modeling is proposed to apply for problems with any triangular fuzzy numbers. The advantage of the proposed methodologies is demonstrated through some example problems relating to for the twenty - story, four - bay planar steel frames.

Keywords: steel frame; critical load; fuzzy connection; response surface method; differential evolution algorithm

1. Introduction

When we analyze the stability of semi-rigid connection steel frame structures, the fixity factor of connection has a significant influence on the buckling resistance capacity of a steel frame (Biggs *et al*. 2015, Piyawat *et al*. 2013). In practice, however, many parameters like worker skill, quality of welds, properties of material and type of the connecting elements affect the behavior of a connection, and this fixity factor is difficult to determine exactly. Therefore, in a practical analysis of structures, a systematic approach is needed to include the uncertainty in the joints behavior and the fixity factor of a connection modeled as a fuzzy number is reasonable (Keyhani *et al*. 2012).

In recent years, the static analysis for planar steel frame structure with the fuzzy connection has been reported (Keyhani *et al*. 2012). However, the buckling analysis for determining the fuzzy critical load by using exact approach has been limited. For the rigid frame, Tuan *et al*. (2015) presented an approach by using Response Surface Method (RSM) for fuzzy free vibration analysis

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of linear elastic structure in which response surfaces (surrogate functions) in terms of complete quadratic polynomials are presented for model quantities and all fuzzy variables are standardized. The usage of the RSM shows that this approach has effectiveness for the complex structural problems with a large number of fuzzy variables. However, the RSM is only suitable for problems which all fuzzy variables are modeled as symmetric triangular fuzzy numbers. For the problems with non-symmetric triangular fuzzy numbers, the fuzzy structural analysis must use another approach. Anh *et al*. (2014) presented an optimization algorithm for fuzzy analysis by combining the Differential Evolution (DE) with the α - level optimization. DE is a global optimization technique, which combines the evolution strategy and the Monte Carlo simulation, and is simple and easy to use.

In this paper, the fuzzy critical load of planar steel frame structure with fuzzy fixity factor is determined by using two approaches for solutions. The first approach is based on the classical finite element method in combination with the response surface method for fuzzy fixity factor input and obtained fuzzy critical load output. This is implemented similarly to the approach which can be found by Tuan (2015), however, the finite element is extended with the linear elastic semi-rigid connection which can be found by Anh (2002). The second approach is based on finite element model by combining the α - level optimization with the Differential Evolution algorithm which is a population-based optimizer. The DE is similar to the genetic algorithm (GA), but it is simple, easy for application and its global convergence and robustness are better than most other GAs (Storn *et al*. 1995, Mezura-Montes 2013). Two solution approaches are different and applied to problems with various fuzzy inputs. In the first approach, the fuzzy fixity factor modeled as the non-symmetric triangular fuzzy number has not considered yet. This is implemented in the second approach and that is the advantage of DE. A comparison of the fuzzy critical loads between the RSM and the DE is presented by considering the twenty floor, four bay planar steel frame structure subjected to concentrated loads at nodes, in which the fixity factors are modeled as symmetric triangular fuzzy numbers. The obtained results are not significantly different. Hence, the α - level optimization in combination with the Differential Evolution algorithm is applied to this analysis, in which considering the fuzzy fixity factors at the boundary constrain are modeled as non-symmetric triangular fuzzy numbers. In addition, the determinant results of the proposed algorithms are also compared with ones of the SAP2000 software. Moreover, the computational efficiency and applicability of the DE optimization in the context of fuzzy critical load analysis is demonstrated through on the example of that frame subjected to uniform loads uniformly distributed on the beams.

2. Finite element with linear semi-rigid connection

The critical load is determined by solving the Eigenvalue equation

$$Det\left(\left[K\right]-\lambda\left[K_G\right]\right)=0 \tag{1}$$

where $[K]$ is the assembled stiffness matrix of the frame and $[K_G]$ is the assembled geometric stiffness matrix of the frame.

The frame element with linear semi - rigid connection as shown in Fig. 1, in which E - the elastic modulus, A - the section area, I - the inertia moment, and k_i - rotation resistance stiffnesses at connections (i=1,2).

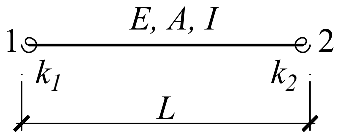


Fig. 1 Frame element with linear semi-rigid connection

The element stiffness matrix - [K^{el}] and the element geometric stiffness matrix - [K_G^{el}] of the frame are given as following (Anh 2002)

$$\left[K^{el}\right]=\begin{bmatrix} \frac{EA}{L} & & & & & \\ 0 & k_{22} & & & symmetric & \\ 0 & k_{32} & k_{33} & & & \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & & \\ 0 & k_{52} & k_{53} & 0 & k_{55} & \\ 0 & k_{62} & k_{63} & 0 & k_{65} & k_{66} \end{bmatrix} \tag{2}$$

where

$$k_{22}=k_{55}=-k_{52}=\frac{12EI}{L^3}\frac{\left(s_1+s_2+s_1s_2\right)}{\left(4-s_1s_2\right)} \tag{2a}$$

$$k_{32}=-k_{53}=\frac{6EI}{L^2}\frac{s_1\left(s_2+2\right)}{\left(4-s_1s_2\right)} \tag{2b}$$

$$k_{33}=2k_{63}=\frac{12EI}{L}\frac{s_1}{\left(4-s_1s_2\right)} \tag{2c}$$

$$k_{62}=-k_{65}=\frac{6EI}{L^2}\frac{s_2\left(s_1+2\right)}{\left(4-s_1s_2\right)} \tag{2d}$$

$$k_{66}=\frac{12EI}{L}\frac{s_2}{\left(4-s_1s_2\right)} \tag{2e}$$

and

$$\left[K_G^{el}\right] = \frac{1}{30L\left(4-s_1 s_2\right)^2}\begin{bmatrix} 0 & & & & & \\ 0 & k_{22}^g & & symmetric & & \\ 0 & k_{32}^g & k_{33}^g & & & \\ 0 & 0 & 0 & 0 & & \\ 0 & k_{52}^g & k_{53}^g & 0 & k_{55}^g & \\ 0 & k_{62}^g & k_{63}^g & 0 & k_{65}^g & k_{66}^g \end{bmatrix} \tag{3}$$

where

$$k_{22}^g = k_{55}^g = -k_{52}^g = 12\left(s_1 s_2\left(s_2 - 34\right) + 8\left(s_2^2 + 5\right) + \left(3s_2^2 + s_2 + 8\right)s_1^2\right) \tag{3a}$$

$$k_{32}^g = -k_{53}^g = 3Ls_1\left(4s_2\left(4s_2 - 7\right) + s_1\left(s_2^2 - 12s_2 + 32\right)\right) \tag{3b}$$

$$k_{33}^g = 12L^2 s_1^2\left(2s_2^2 - 7s_2 + 8\right) \tag{3c}$$

$$k_{62}^g = -k_{65}^g = 3Ls_2\left(32s_2 + \left(s_2 + 16\right)s_1^2 - 4s_1\left(3s_2 + 7\right)\right) \tag{3d}$$

$$k_{63}^g = -3L^2 s_1 s_2\left(28 - 16s_2 + s_1\left(7s_2 - 16\right)\right) \tag{3e}$$

$$k_{66}^g = 12L^2 s_2^2\left(2s_1^2 - 7s_1 + 8\right) \tag{3f}$$

in which $s_i = Lk_i/(3EI + Lk_i)$ denote the fixity factor of semi - rigid connection at the boundaries (i=1,2). In Eq. (1), when fixity factors of connections are given by fuzzy numbers, the critical load is also the fuzzy number. In steel structures, the common connections can be defined by linguistic terms as shown in Fig. 2. Eleven linguistic terms are assigned numbers from 0 to 10. These include 0-Ideal Hinged (Absolutely Hinged), 1-Very Hinged (e.g., single web angle), 2-Almost Hinged (e.g., single web plate), 3- Fairly Hinged (e.g., double web angle), 4-More and Less Hinged (e.g., header plate), 5-Half Rigid-Half Hinged (e.g., top & seat angle), 6-More and Less Rigid (e.g., top

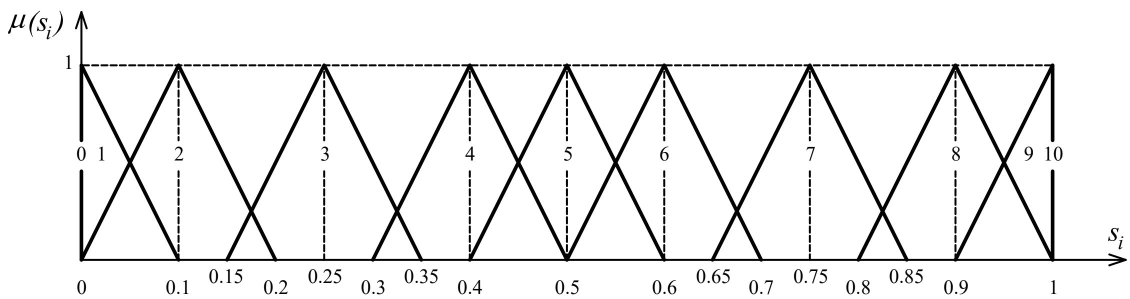


Fig. 2 Membership functions of fuzzy fixity factors

plate & seat angle), 7-Fairly Rigid (e.g., top & seat plate), 8-Almost Rigid (e.g., end plate), 9-Very Rigid (e.g., t-stub & web angle), 10-Ideal Rigid (Absolutely Rigid) (Keyhani *et al*. 2012).

3. Two algorithms for fuzzy structural analysis

3.1 Response Surface Method (RSM)

In the statistical theory, surrogate models are often used including polynomial regression model, Kringing model, radial basis function. In this paper, to determine the critical loads, a complete quadratic polynomial regression model is used as surrogate model, in which all variables are standardized and assumed to be uncorrelated (Tuan *et al*. 2015)

$$y(X) = a_0 + \sum_{i=1}^{n} a_i X_i + \sum_{i=1,i<j}^{n-1} a_{ij} X_i X_j + \sum_{i=1}^{n} a_{ii} X_i^2 \tag{4}$$

wthere X_i are the standardized fuzzy variables; $a_0=y(X=0)$, and a_i, a_{ij} are the unknown coefficients which determined by the method of least squares; $y(X)$ represents the surrogate function of critical load and uncertain structural parameters are assumed as symmetric triangular fuzzy numbers, $x_i=(a,l,l)_{LR}$. The standardized fuzzy variables X_i is defined as following

$$X_i = \frac{x_i - a}{(l/3)} \tag{5}$$

For the above definition, the original fuzzy variables $x_i=(a,l,l)_{LR}$ are transformed to standardized fuzzy variables $X_i=(0,3,3)_{LR}$.

To complete the surrogate polynomial functions of Eq. (4), all coefficients a_i, a_{ij} shall be determined by a fitting procedure, which minimizes the difference (error) between the outputs of surrogate function and the outputs of classical finite element model. Normally, some experiments with deterministic input data are carried out and the best fitting function can be obtained by minimizing the sum of the square errors from the given output data. In RSM, with the number of experiments not too large, and in fact, maximum, minimum responses usually occur on the surface of the cube, the face-centered cube design, and the Box-Behnken designs are often used (Mason *et al*. 2003). Fig. 3 shows an illustration of the Box-Behnken design with three input variables.

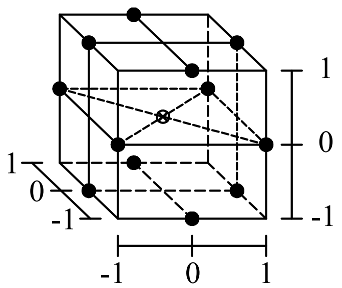


Fig. 3 The Box-Behnken design with three variables

To select the suitable design, the quality of the response surface is assessed by error estimation. The most prominent methods are split sample, cross-validation and bootstrapping, in which the split sample and the cross-validation are easy to use (Queipo *et al*. 2005). In this study, the leave-one-out cross-validation is applied, in which, each response point is tested once and trained k-2 times (since the center point has been used to determine a_0) with k is number of the Box-Behnken designs. The error estimation of j_{th} design (using $X(j)$ as the test set) is determined by the formulas

$$GSE_j = \left(y_j - \hat{y}_j^{(-j)}\right)^2 \to \min \tag{6}$$

where GSE_j - the square error of j^{th} design; y_j - output value at $\mathbf{X}^{(j)}$, determined by classical FEM; $\hat{y}_j^{(-j)}$- estimated value at $\mathbf{X}^{(j)}$ design of j^{th} design.

3.2 α - level optimization using Differential Evolution (DE)

For fuzzy structural analysis, the α-level optimization is known as a general approach in which all the fuzzy inputs are discretized by the intervals that are equal α-levels. The output intervals are then searched by the optimization algorithms. The optimization process is implemented directly by the finite element model and the goal function is evaluated many times in order to reach to an acceptable value. In this study, the solution procedure is proposed by combining the Differential Evolution (DE) with the α-level optimization. DE which is a population-based optimizer, is suggested by Storn and Price (1995). The DE algorithm has shown better than the genetic algorithm (GA) and is simple and easy to use. Basic procedure of DE is described as following.

For an objective function $f(x)$, we want to search for the global optima of $f(x)$ over a continuous space domain: $x=\{x_i\}$, $x_i \in [x_{i,\min}, x_{i,\max}]$, $i=1,2,\ldots n$.

For each generation G, a population of NP parameter vectors $x_k(G)$, $k = 1,2,\ldots NP$, is utilized. The initial population is generated as

$$x_{k,i}\left(0\right) = x_{i,\min} + rand[0,1].\left(x_{i,\max} - x_{i,\min}\right), i = 1, 2, \ldots n \tag{7}$$

where $rand[0,1]$ is the uniformly distributed random real value in the interval [0,1].

For each target vector in a population $x_k(G)$, k=1,2,…NP, a mutant vector y is generated according to

$$y = x_{r_1}\left(G\right) + F.\left(x_{r_2}\left(G\right) - x_{r_3}\left(G\right)\right) \tag{8}$$

with r_1, r_2, r_3 are randomly chosen integers and $1 \le r_1 \ne r_2 \ne r_3 \ne k \le NP$; F is a real and constant factor usually chosen in the interval [0,1] to control the amplification of the differential variation $\left(x_{r_2}\left(G\right) - x_{r_3}\left(G\right)\right)$.

In order to increase the diversity of the perturbed parameter vectors, the crossover is introduced. To this end, the trial vector z with its elements determined by

$$z_i = \begin{cases} y_j & \text{if}\left(rand[0,1] \le Cr\right) or \left(r = i\right) \\ x_{k,i} & \text{if}\left(rand[0,1] > Cr\right) and \left(r \ne i\right) \end{cases} \tag{9}$$

Here, r is randomly chosen integer in the interval [1,n]; C_r is use-defined crossover constant in the interval [0,1].

The new vector z is then compared to $x_k(G)$. If z yields better objective function value then z

becomes a member of the next generation (G+1); otherwise, the old value $x_k(G)$ is retained.

4. Numerical illustration

4.1 Twenty - story, four - bay planar steel frame subjected to loads concentrated at ends

The first example is considered by analysis a twenty - story, four - bay semi-rigid planar steel frame structural system subjected to loads P concentrated at ends as shown in Fig. 3. The elastic modulus E=2.1E+08 kN/m^2, the fixity factor at column base is s_1, the fixity factor at the ends of beams from story 1 to story 4 is s_2, from story 5 to story 8 is s_3, from story 9 to story 14 is s_4, and from story 9 to story 14 is s_5. The section properties used for analysis of the frame are shown in Table 1. Five fuzzy cases for analysis of fuzzy critical load were considered as in Table 2. The fixity factors in Case 1 are symmetric triangular fuzzy numbers, so the fuzzy critical loads are calculated with two different techniques. Since the fixity factors are non-symmetric triangular fuzzy numbers, the critical loads in other cases are solved by using differential evolution algorithm.

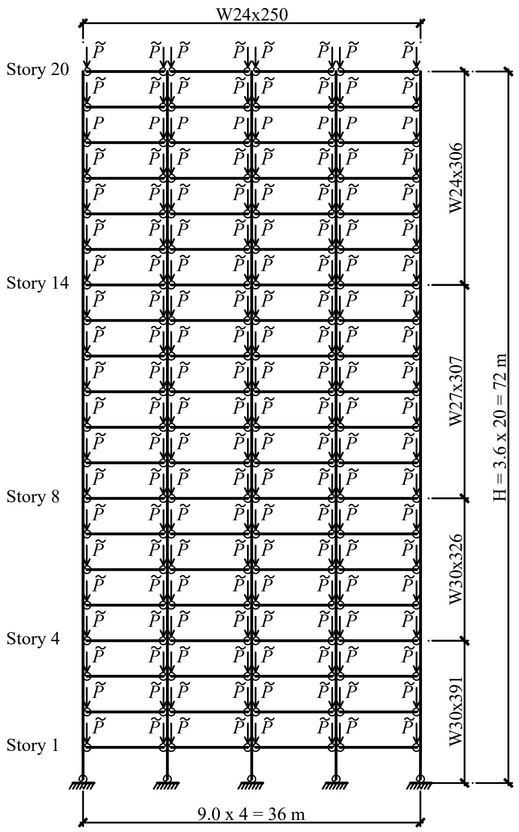


Fig. 3 Semi-rigid planar steel frame subjected to concentrated loads

Table 1 Section properties used for analysis of semi - rigid frame

Member	Section	Cross - section area, A (m^2)	Moment of inertia, I (m^4)
Column (1^{st} to 4^{th} story)	W30×391	7.35E-02	8.616E-03
Column (5^{th} to 8^{th} story)	W30×326	6.17E-02	6.993E-03
Column (9^{th} to 14^{th} story)	W27×307	5.82E-02	5.453E-03
Column (15^{th} to 20^{th} story)	W24×306	5.79E-02	4.454E-03
Beam (1^{st} to 20^{th} story)	W24×250	4.74E-02	3.534E-03

Table 2 Five fuzzy cases for analysis of the fuzzy critical loads

Case	Fuzzy fixity factors of connections				
	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	$\tilde{s}_4$	$\tilde{s}_5$
Case 1	8	7	7	7	7
Case 2	10	1	1	1	1
Case 3	9	9	9	9	9
Case 4	9	5	5	5	5
Case 5	9	8	7	6	5

4.1.1 Solving by RSM

In Case 1, the Box-Behnken designs with two input fuzzy variables ($\tilde{s}_1$ and $\tilde{s}_2 = \tilde{s}_3 = \tilde{s}_4 = \tilde{s}_5$) are presented in Table 3. In this table, the critical loads P_{cr} are calculated by using the classical FEM programmed by MATLAB (Khennane 2013). Table 3 also shows a comparison of the critical loads obtained by the present study and those from the SAP2000 software. The results of the coefficients of the surrogate function for the critical load are shown in Table 4. The intervals of the critical loads are shown in Table 5.

Table 3 The Box - Behnken designs and the comparison of the critical loads

No.	x_1=s_1	X_1	x_2=s_2	X_2	P_{cr} (kN), MATLAB	P_{cr} (kN), SAP2000	Difference (%)
0	0.80	0	0.75	0	5019.870	5016.255	+0.0720
1	0.80	0	0.85	3	5924.240	5918.627	+0.0947
2	0.80	0	0.65	-3	4206.770	4204.470	+0.0547
3	0.90	3	0.75	0	5034.120	5033.360	+0.0151
4	0.90	3	0.85	3	5941.020	5939.338	+0.0283
5	0.90	3	0.65	-3	4218.770	4218.499	+0.0064
6	0.70	-3	0.75	0	5000.760	4993.027	+0.1546
7	0.70	-3	0.85	3	5901.470	5890.079	+0.1930
8	0.70	-3	0.65	-3	4190.850	4185.652	+0.1240

Table 4 Coefficients of surrogate function for the critical load

Coefficients	P_{cr} (kN)
a_0	5019.87000000
a_1	5.61270833
a_2	286.13000000
a_{12}	0.32305556
a_{11}	-0.26548611
a_{22}	5.06402778

Table 5 The intervals of the critical loads by RSM - Case 1

α - *cut*	P_{cr} (kN)
α=1	[5019.8700; 5019.8700]
α=0.8	[4846.7845; 5196.7594]
α=0.6	[4677.6191; 5377.3364]
α=0.4	[4512.3738; 5561.6009]
α=0.2	[4351.0487; 5749.5529]
α=0	[4193.6438; 5941.1925]

4.1.2 Solving by DE

The output intervals of critical loads are calculated by using DE programmed by MATLAB for all Cases. The parameters for DE are: the NP=50, F=0.5, Cr=0.9. The optimization process is stopped after 30 iterations. The results of the critical load intervals are shown in Tables 6-7. It is found from the analysis results that the critical loads in Case 2 (Very Hinged) are smaller than the critical loads in Case 3 (Very Rigid). Fig. 1 shows the comparison of the fuzzy critical loads of Case 1 by two different techniques. Figs. 5 and 6 show the membership functions of the fuzzy critical loads for different Cases of analysis.

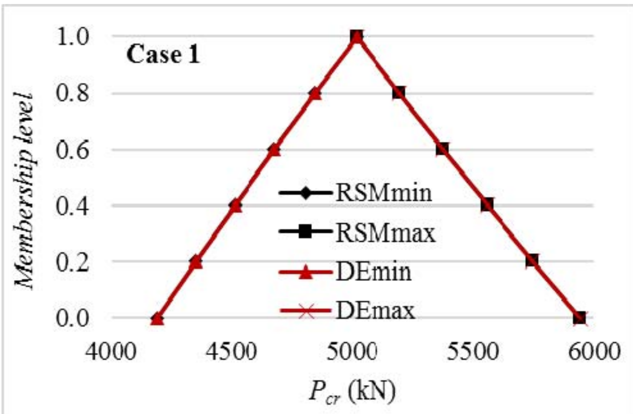
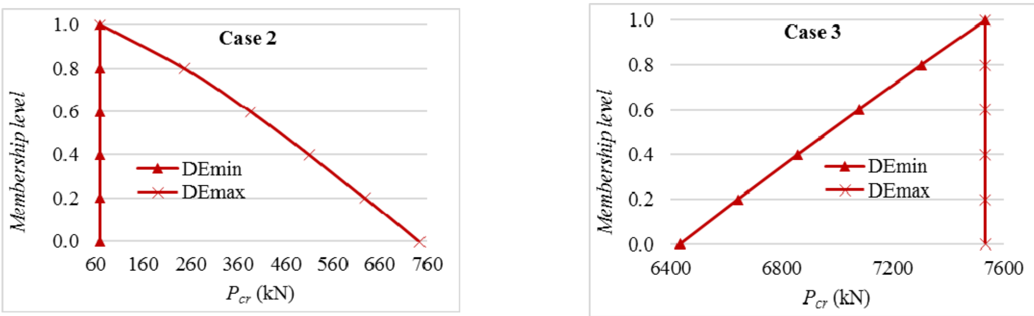
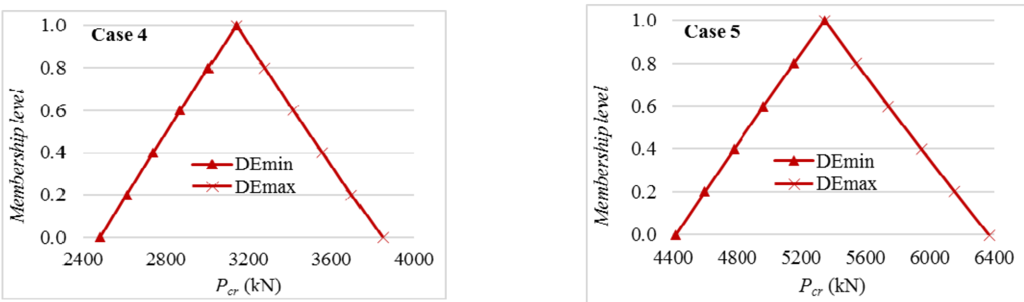


Fig. 4 Fuzzy critical load P_{cr} by RSM and DE - Case 1



(a) Case 2 (b) Case 2

Fig. 5 Fuzzy critical load P_{cr} by DE - Cases 2 and 3



(a) Case 4 (b) Case 5

Fig. 6 Fuzzy critical load P_{cr} by DE - Cases 4 and 5

Table 6 The intervals of the critical loads by DE -Cases 1, 2 and 3

α - *cut*	P_{cr} (kN)		
	Case 1	Case 2	Case 3
α=1	[5019.8700; 5019.8700]	[68.2094; 68.2094]	[7532.8500; 7532.8500]
α=0.8	[4847.2500; 5196.1700]	[68.2094; 247.7710]	[7302.1600; 7532.8500]
α=0.6	[4678.1700; 5376.2800]	[68.2094; 387.2150]	[7077.2300; 7532.8500]
α=0.4	[4512.4900; 5560.2000]	[68.2094; 511.8260]	[6857.8200; 7532.8500]
α=0.2	[4350.0900; 5748.5500]	[68.2094; 629.9270]	[6643.7200; 7532.8500]
α=0	[4190.8500; 5940.5400]	[68.2094; 744.7640]	[6434.7100; 7532.8500]

Table 7 The intervals of the critical loads by DE -Cases 4 and 5

α - *cut*	P_{cr} (kN)	
	Case 4	Case 5
α=1	[3140.5100; 3140.5100]	[5349.5900; 5349.5900]
α=0.8	[3004.7300; 3277.3000]	[5157.1100; 5545.2200]
α=0.6	[2871.1500; 3416.3500]	[4967.9300; 5744.6000]
α=0.4	[2740.0600; 3558.0300]	[4781.8700; 5947.6200]
α=0.2	[2610.6400; 3699.2700]	[4598.8600; 6154.2200]
α=0	[2482.7800;3848.9300]	[4418.6400; 6365.4100]

4.2 Twenty - story, four - bay planar steel frame subjected to uniform loads on beams

As a second example of analysis, a twenty - story, four - bay planar steel frame structural system subjected to uniform loads q on beams as shown in Fig. 7 is considered. The properties and Cases used for analysis of the frame are the same as in the first example. The output intervals of critical loads are calculated by using DE programmed by MATLAB for all Cases. The parameters for DE are: the NP=50, F=0.5, C_r=0.9. The optimization process is stopped after 30 iterations. The results of the critical load intervals are shown in Tables 8 and 9. Figs. 8, 9 and 10 show the membership functions of the fuzzy critical loads for all analysis Cases.

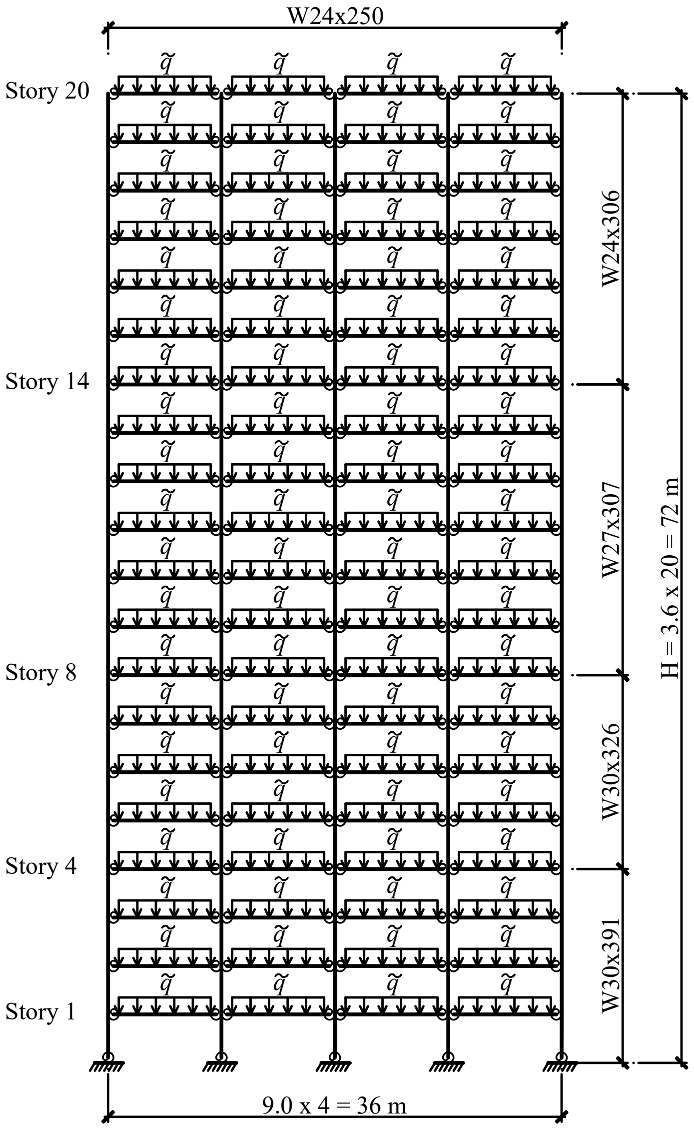


Fig. 7 Semi-rigid planar steel frame subjected to uniform loads on beams

Table 7 The intervals of the critical loads by DE - Cases 1, 2 and 3

α - *cut*	q_{cr} (kN/m)		
	Case 1	Case 2	Case 3
α=1	[1115.5400; 1115.5400]	[15.1577; 15.1577]	[1674.0400; 1674.0400]
α=0.8	[1077.1800; 1154.7200]	[15.1577; 55.0603]	[1622.7700; 1674.0400]
α=0.6	[1039.6100; 1194.7400]	[15.1577; 86.0479]	[1572.7800; 1674.0400]
α=0.4	[1002.7900; 1235.6300]	[15.1577; 113.7390]	[1524.0200; 1674.0400]
α=0.2	[966.6960; 1277.4700]	[15.1577; 139.9840]	[1476.4300; 1674.0400]
α=0	[931.3080; 1320.0500]	[15.1577; 165.5030]	[1429.9800; 1674.0400]

Table 8 The intervals of the critical loads by DE - Cases 4 and 5

α - *cut*	q_{cr} (kN/m)	
	Case 4	Case 5
α=1	[697.8920; 697.8920]	[1188.7200; 1188.7200]
α=0.8	[667.7170; 728.2880]	[1145.9500; 1232.1900]
α=0.6	[638.0340; 759.1900]	[1103.9200; 1276.4800]
α=0.4	[608.8210; 790.6730]	[1062.5800; 1321.5900]
α=0.2	[580.1350; 822.6110]	[1021.9200; 1367.4900]
α=0	[551.7280; 855.3110]	[981.8750; 1414.4100]

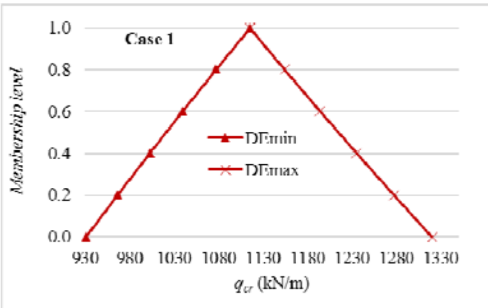
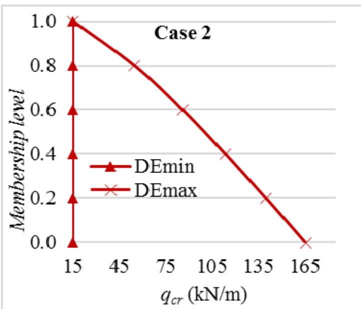
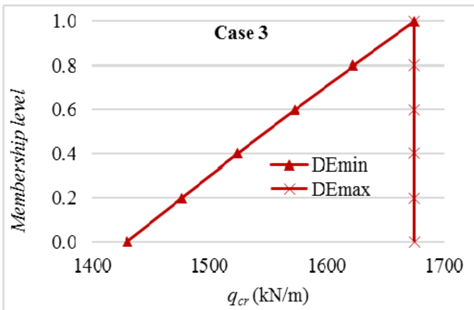


Fig. 8 Fuzzy critical load q_{cr} by RSM and DE - Case 1

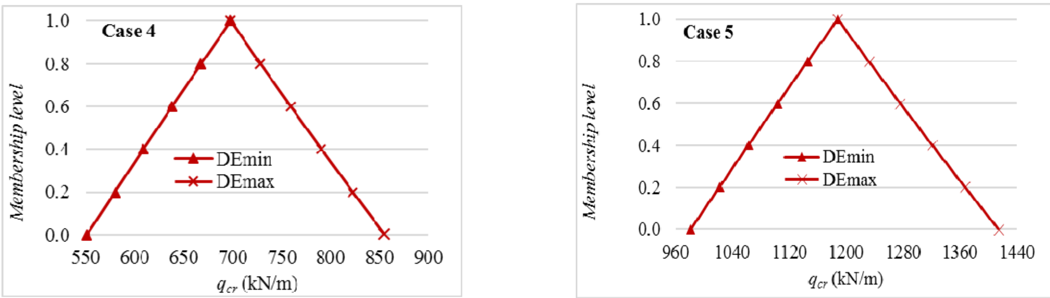


(a) Case 2



(b) Case 2

Fig. 9 Fuzzy critical load q_{cr} by DE - Cases 2 and 3



(a) Case 4 (b) Case 5

Fig. 10 Fuzzy critical load q_{cr} by DE - Cases 4 and 5

5. Conclusions

This study presents two solution techniques solutions which are applicable for fuzzy structural analysis to determine the fuzzy critical load of planar steel frame structure in which fixity factors of beam - column and column - base connections are modeled as triangular fuzzy numbers. From the results of the numerical examples the following comments are obtained:

• The fuzzy finite element analysis based on the response surface method, the result is obtained the fuzzy critical loads by using the response surface with the surrogate function is the complete quadratic polynomial. This approach is suitable for the fuzzy input variables modeled as symmetric triangles. The benefit in the application of this methodology is demonstrated through an analysis of the twenty - story, four - bay planar steel frames with a lot of elements and fuzzy variables. This Case is also carried out by other approach using the Differential Evolution (DE) in combination with the α - level optimization, and the comparison of the fuzzy critical loads between two solution approaches solutions give a good agreement.

• From the accuracy of the result implemented by using DE in the Case 1, this paper is extended for the other Cases in which the fuzzy input variables modeled as any triangles. The results are obtained by using this approach show that the α - level optimization algorithm in combination with DE is more advantageous than the RSM in combination with GA in which the finite element method is applied to linear elastic semi-rigid connection with multi-degree-of-freedom systems and non-symmetric triangular fuzzy variables. The computational benefits and applicability of the DE optimization are demonstrated on determining the fuzzy critical loads of that frame subjected to uniform loads on the beams.

• Using simple linear elastic semi-rigid connection model is suitable for the structural system assumed that its displacement is small. As the displacement is large, the relationship of moment-rotation is nonlinear, this may be subject of studies in the context of fuzzy analysis in the future.

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